

**Gamow-Teller strengths  
in p-, sd- and pf-shell nuclei  
as a test case of shell-model calculations**

**-Comparison of  $B(GT)$ : Exp. & SM-**

Y. Fujita, Osaka Univ.

at SM workshop, Tokyo, 2006. Jan.26-28

# Vibration Modes in Nuclei (Operators)

Microscopic classification of giant resonances

	$\Delta S = 0$ $\Delta T = 0$	$\Delta S = 0$ $\Delta T = 1$	$\Delta S = 1$ $\Delta T = 0$	$\Delta S = 1$ $\Delta T = 1$
$L = 0$		$\sum \tau_i$ IAS		$\sum \vec{\sigma}_i \tau_i$ GTR
2 <sup>nd</sup> order	$\sum r_i^2$ ISGMR	$\sum r_i^2 \tau_i$ IVGMR	$\sum r_i^2 \vec{\sigma}_i$ ISSMR	$\sum r_i^2 \vec{\sigma}_i \tau_i$ IVSMR
$L = 1$		$\sum r_i Y_m^1 \tau_i$ IVGDR	$\sum r_i Y_m^1 \vec{\sigma}_i$ ISSDR	$\sum r_i Y_m^1 \vec{\sigma}_i \tau_i$ IVSDR
2 <sup>nd</sup> order	$\sum r_i^3 Y_m^1$ ISGDR			
$L = 2$	$\sum r_i^2 Y_m^2$ ISGQR	$\sum r_i^2 Y_m^2 \tau_i$ IVGQR	$\sum r_i^2 Y_m^2 \vec{\sigma}_i$ ISSQR	$\sum r_i^2 Y_m^2 \vec{\sigma}_i \tau_i$ IVSQR
$L = 3$	$\sum r_i^3 Y_m^3$ ISGOR	$\sum r_i^3 Y_m^3 \tau_i$ IVGOR	$\sum r_i^3 Y_m^3 \vec{\sigma}_i$ ISSOR	$\sum r_i^3 Y_m^3 \vec{\sigma}_i \tau_i$ IVSOR

# B(GT) derivation

★  $\beta$  decay : fundamental, but  $E_x$  range : limited "Q-window limitation"

★  $(p, n)$  reaction at intermediate energies ( $E = 100-500$  MeV)

"proportionality" :  $B(\text{GT})$  and  $\sigma(0^\circ)$

$$\sigma(0^\circ) = KN_{\sigma\tau} |J_{\sigma\tau}(0^\circ)|^2 B(\text{GT})$$

⇒ Breakthrough against "Q-window limitation"

but resolution : rather poor ( $\Delta E = 200-400$  keV)

★  $(^3\text{He}, t)$  reaction at intermediate energies ( $E = 130-150$  MeV/u)

"high resolution" ( $\Delta E < 50$  keV)

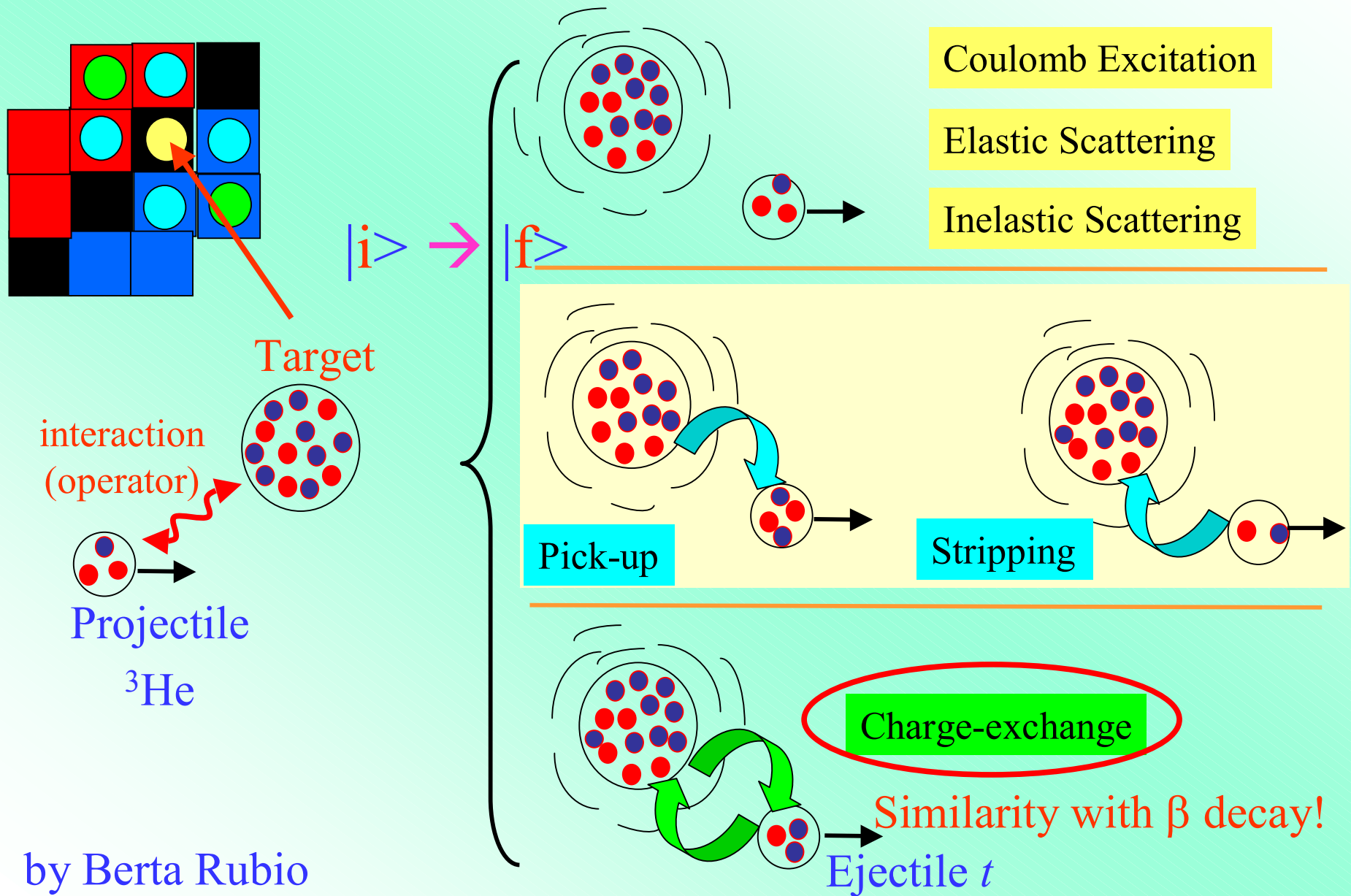
☆ magnetic spectrometer, matching techniques

"proportionality" : good ( $B(\text{GT}) > 0.03$ )

⇒ Breakthrough against "Energy resolution limitation"

⇒ Reliable  $B(\text{GT})$  values for individual transitions

# Direct Reactions with Light Projectiles





# B(GT) derivation

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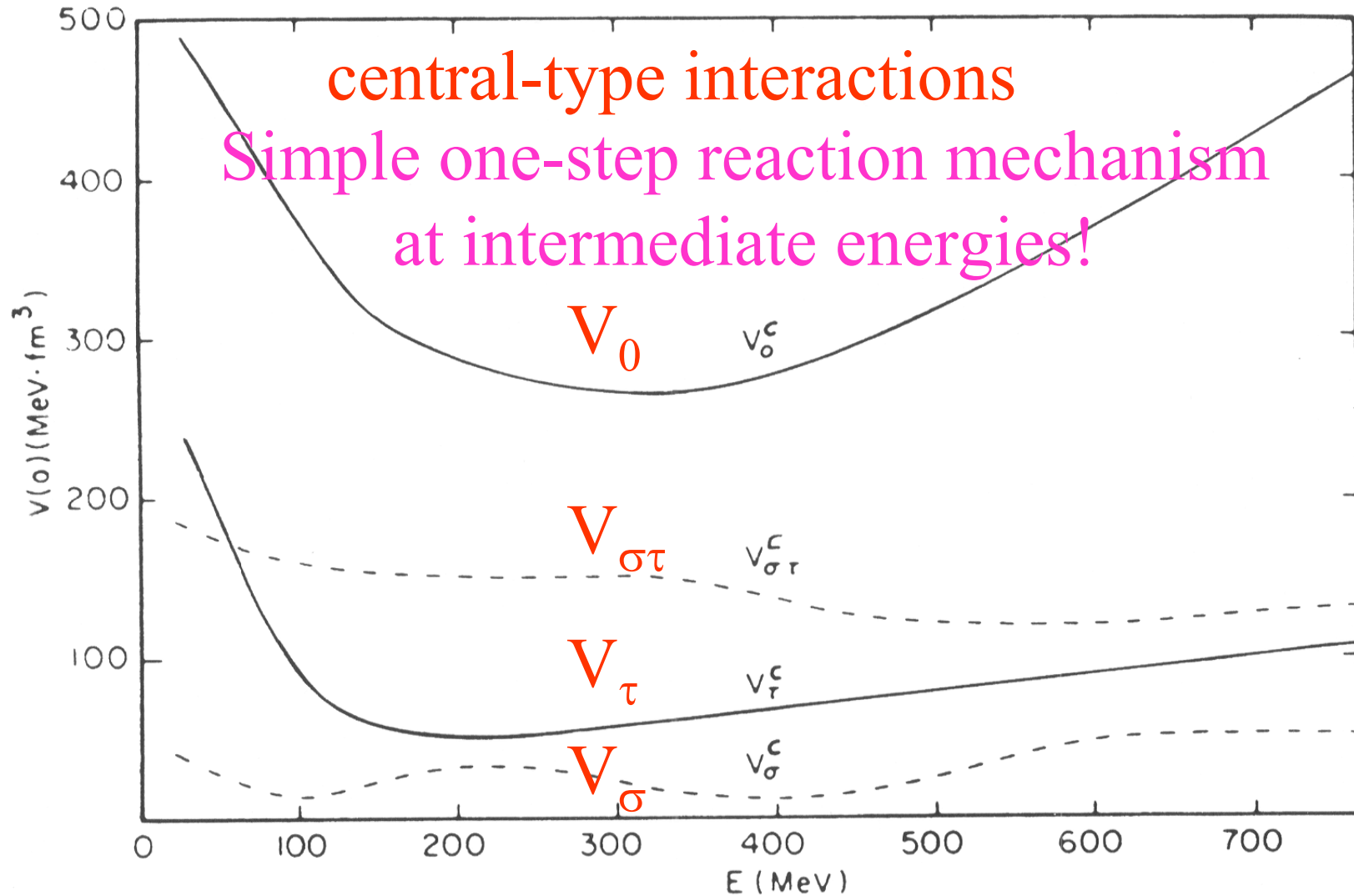
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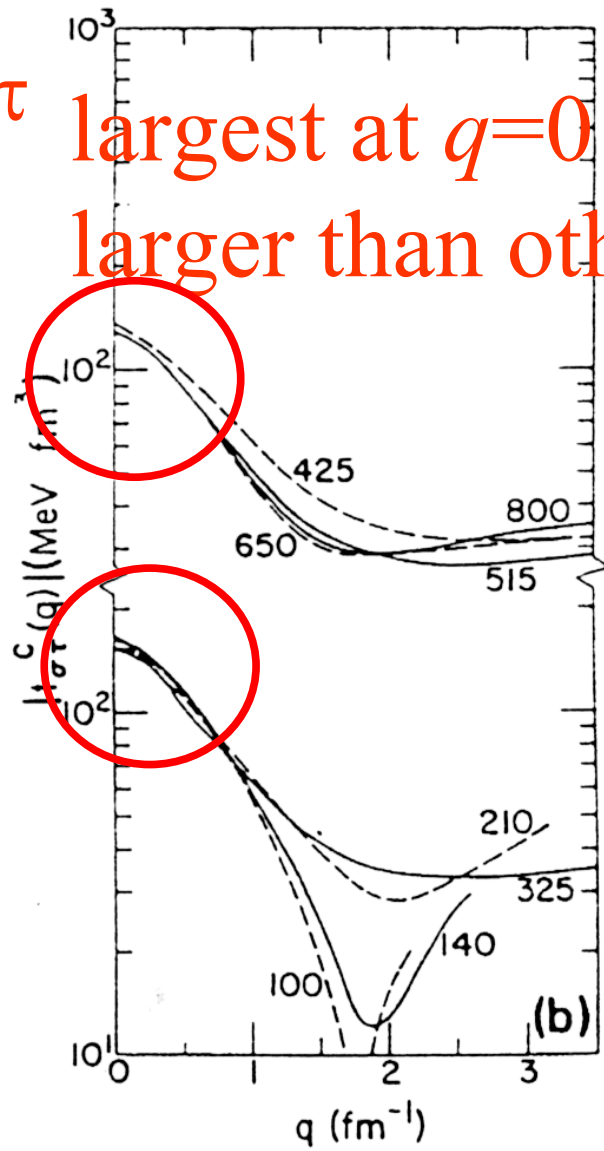
⇒ Reliable  $B(\text{GT})$  values for individual transitions

# Nucleon-Nucleon Int. : $E_{in}$ dependence at $\alpha = 0$

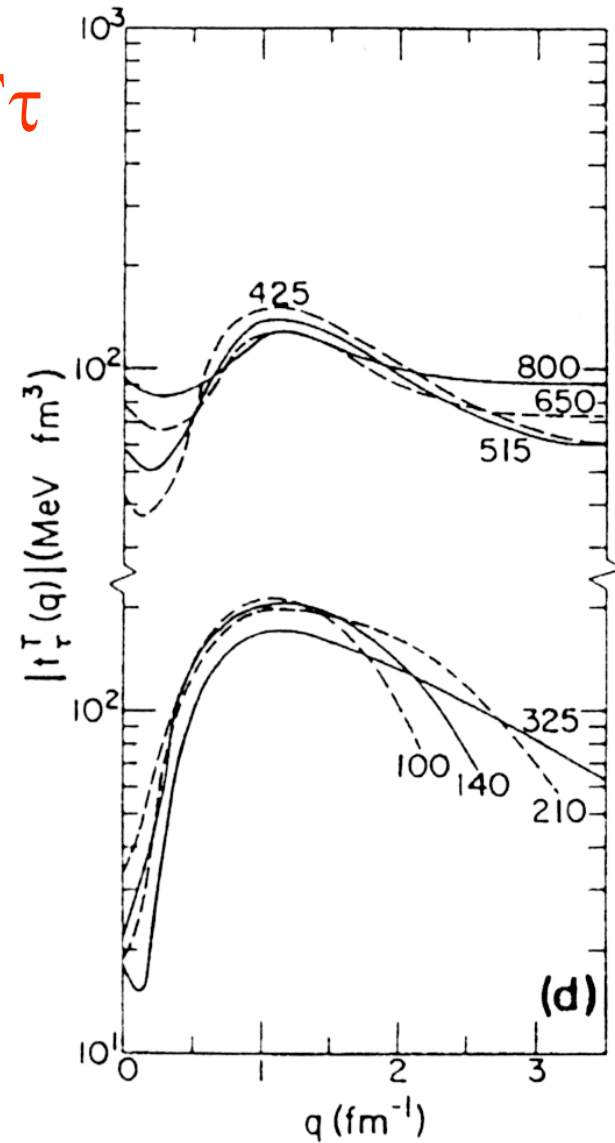


# N.-N. Int. : $\sigma\tau$ & Tensor- $\tau$ $q$ -dependence

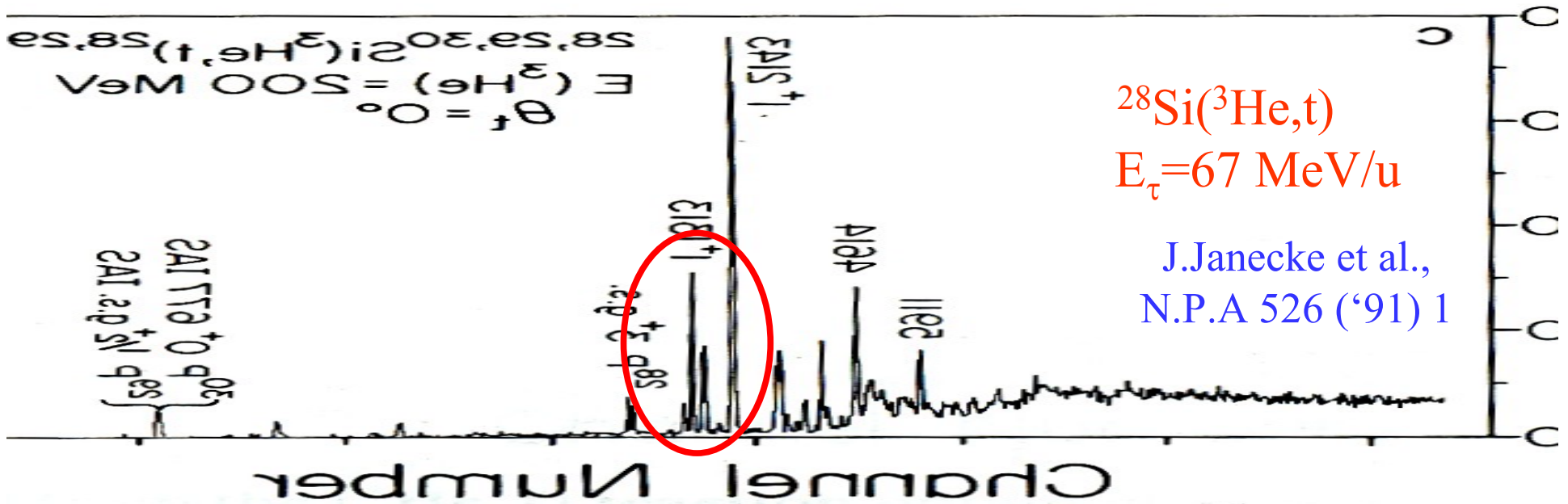
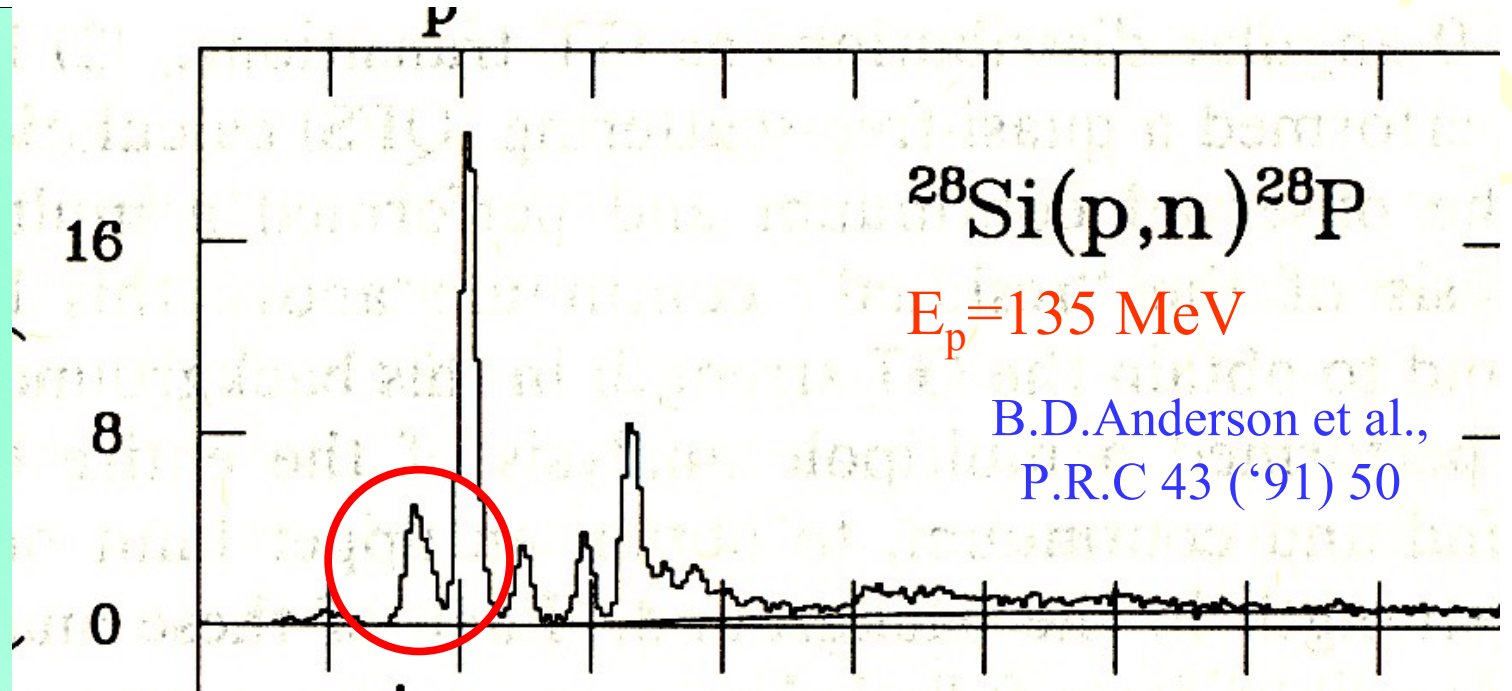
$\sigma\tau$  largest at  $q=0$  !  
larger than others !



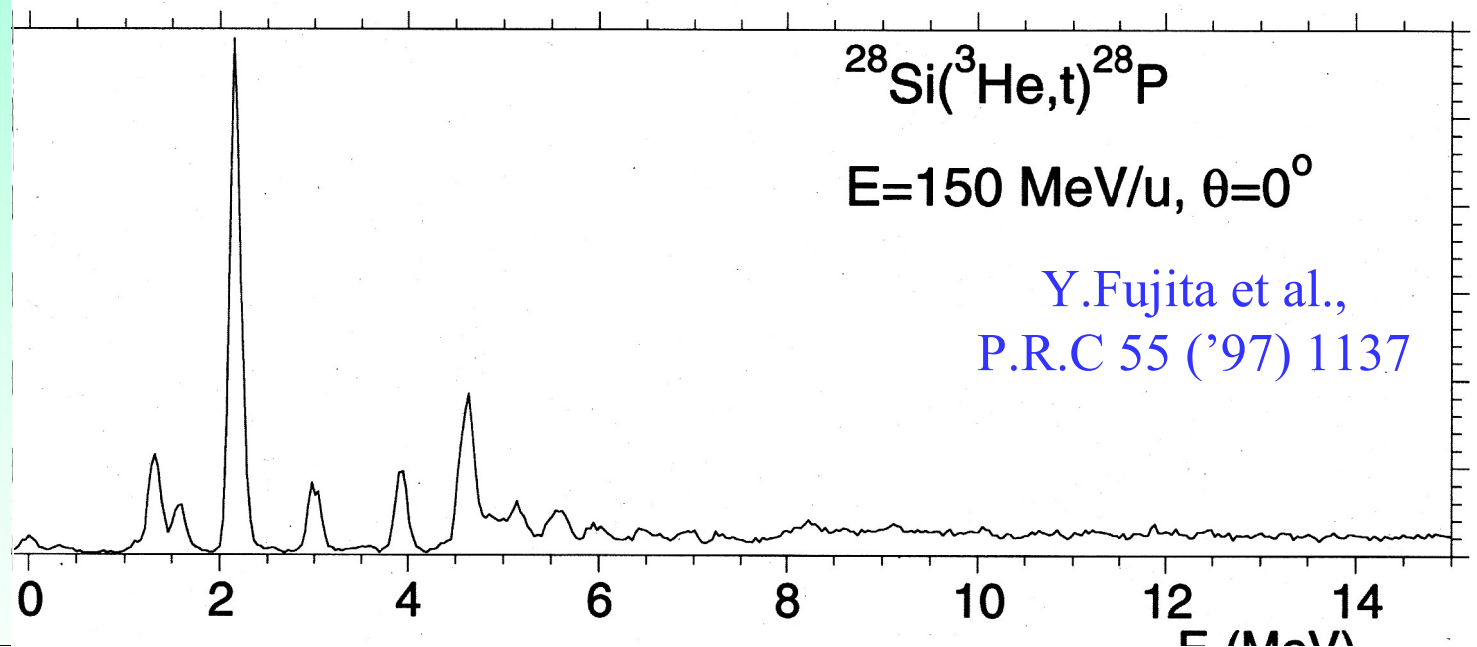
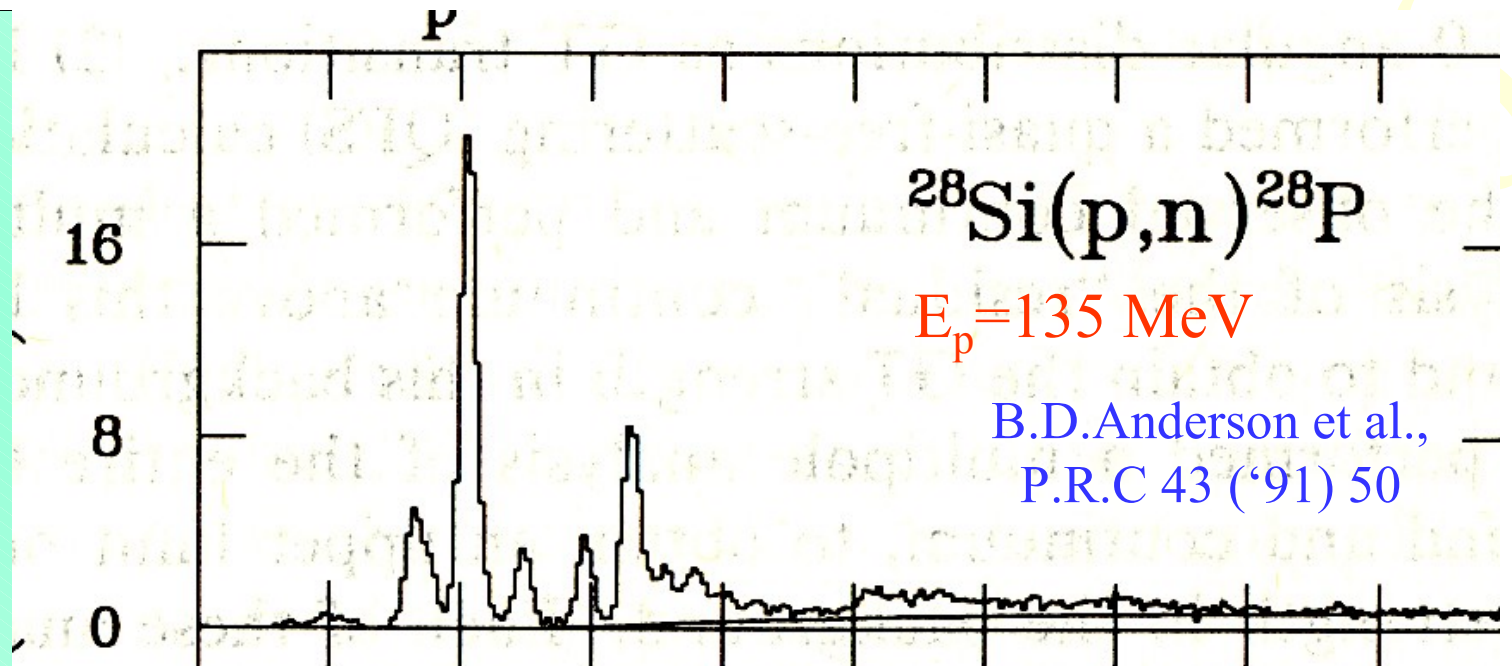
$T\tau$



$^{28}\text{Si}$ : CE  
Reactions  
(I)

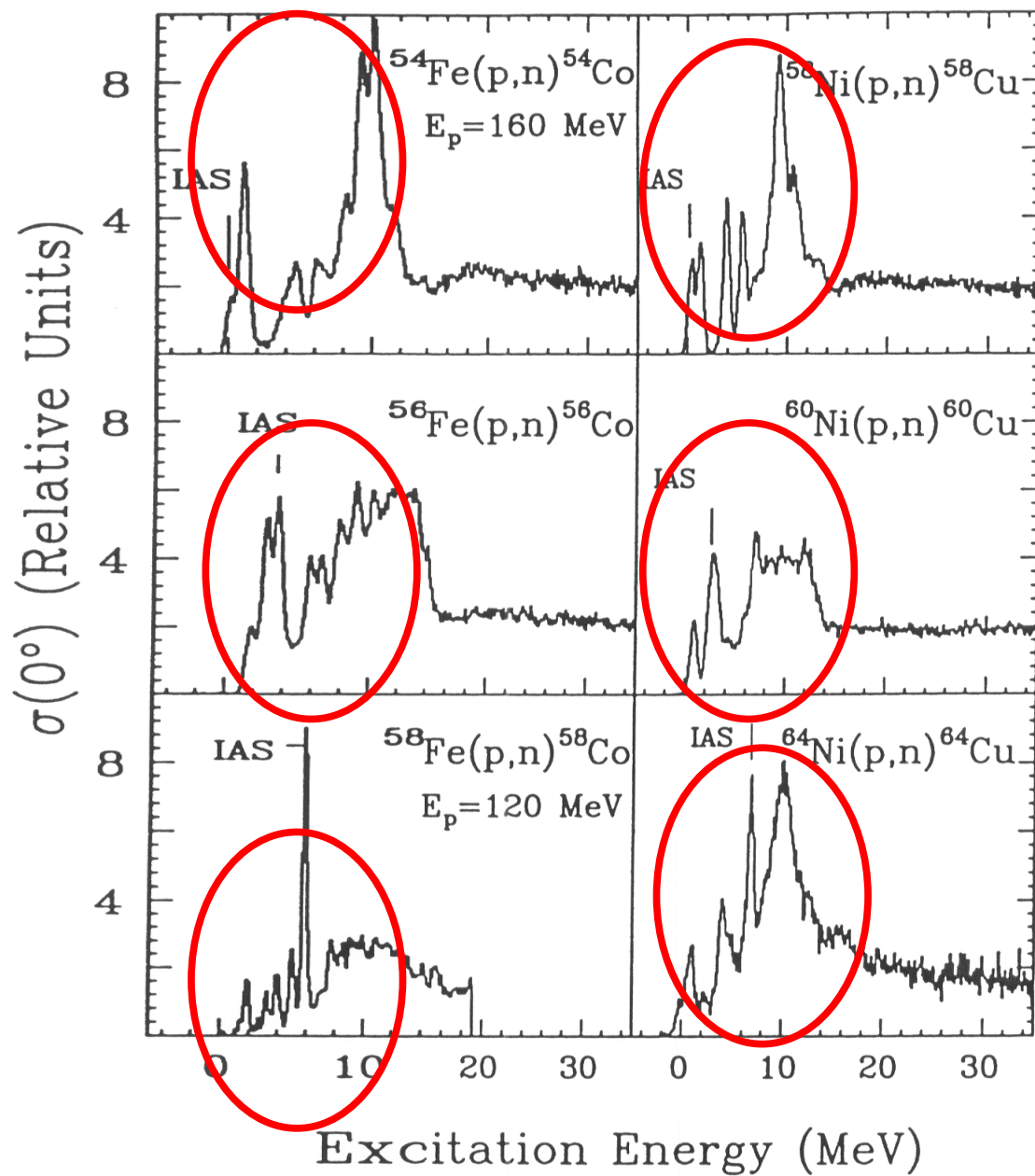


$^{28}\text{Si}$ : CE  
Reactions  
(II)





# $(p, n)$ spectra for Fe and Ni Isotopes





# $(p, n)$ spectra for $A > 90$ Nuclei

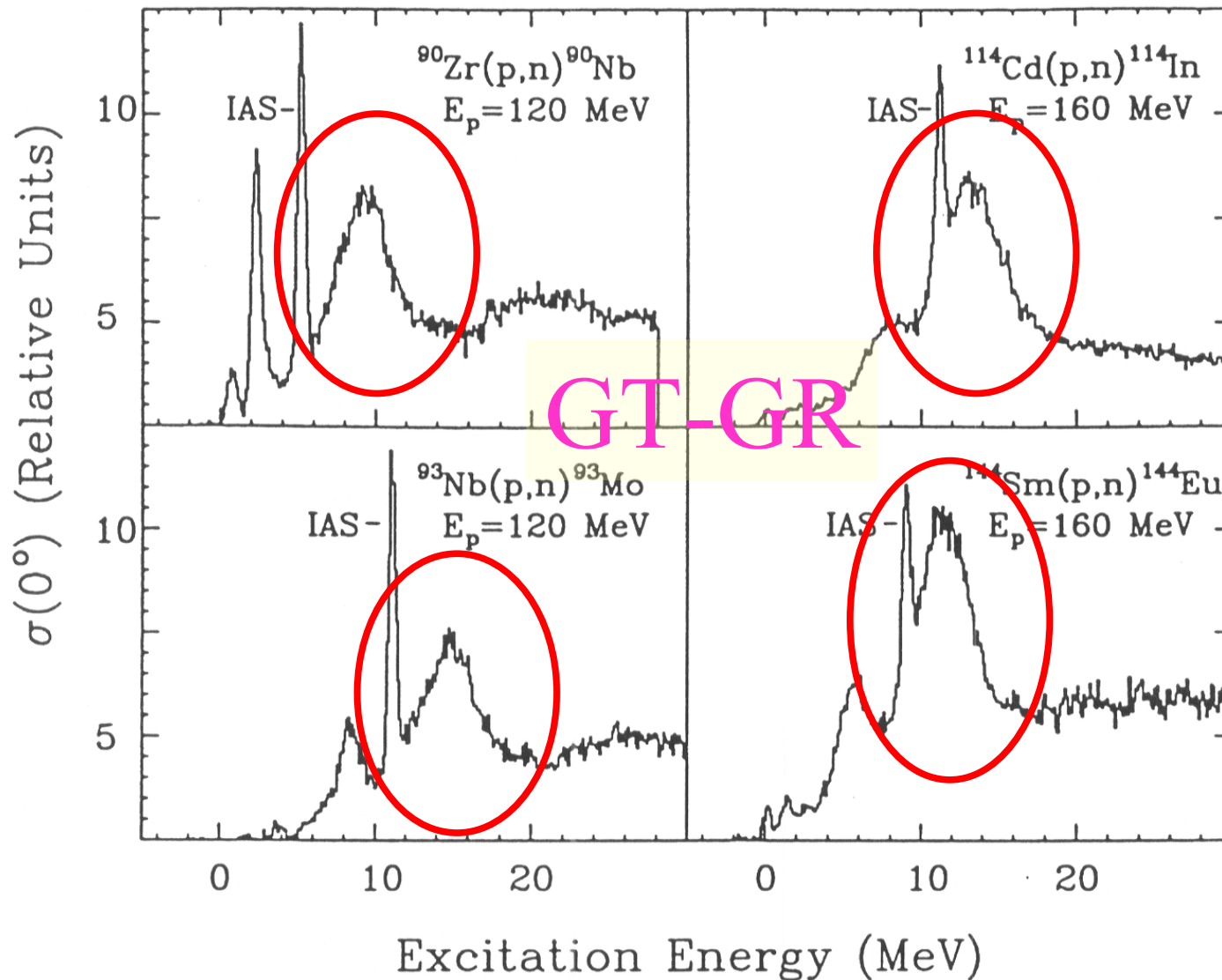


Figure 10 Zero-degree  $(p, n)$  spectra for medium  $A$ -mass nuclei at the indicated incident energies.

# B(GT) derivation

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☆ magnetic spectrometer, matching techniques

"proportionality" : good ( $B(\text{GT}) > 0.03$ )

⇒ Breakthrough against "Energy resolution limitation"

⇒ Reliable  $B(\text{GT})$  values for individual transitions

# Comparison of (p, n) and (<sup>3</sup>He, t) spectra

Counts

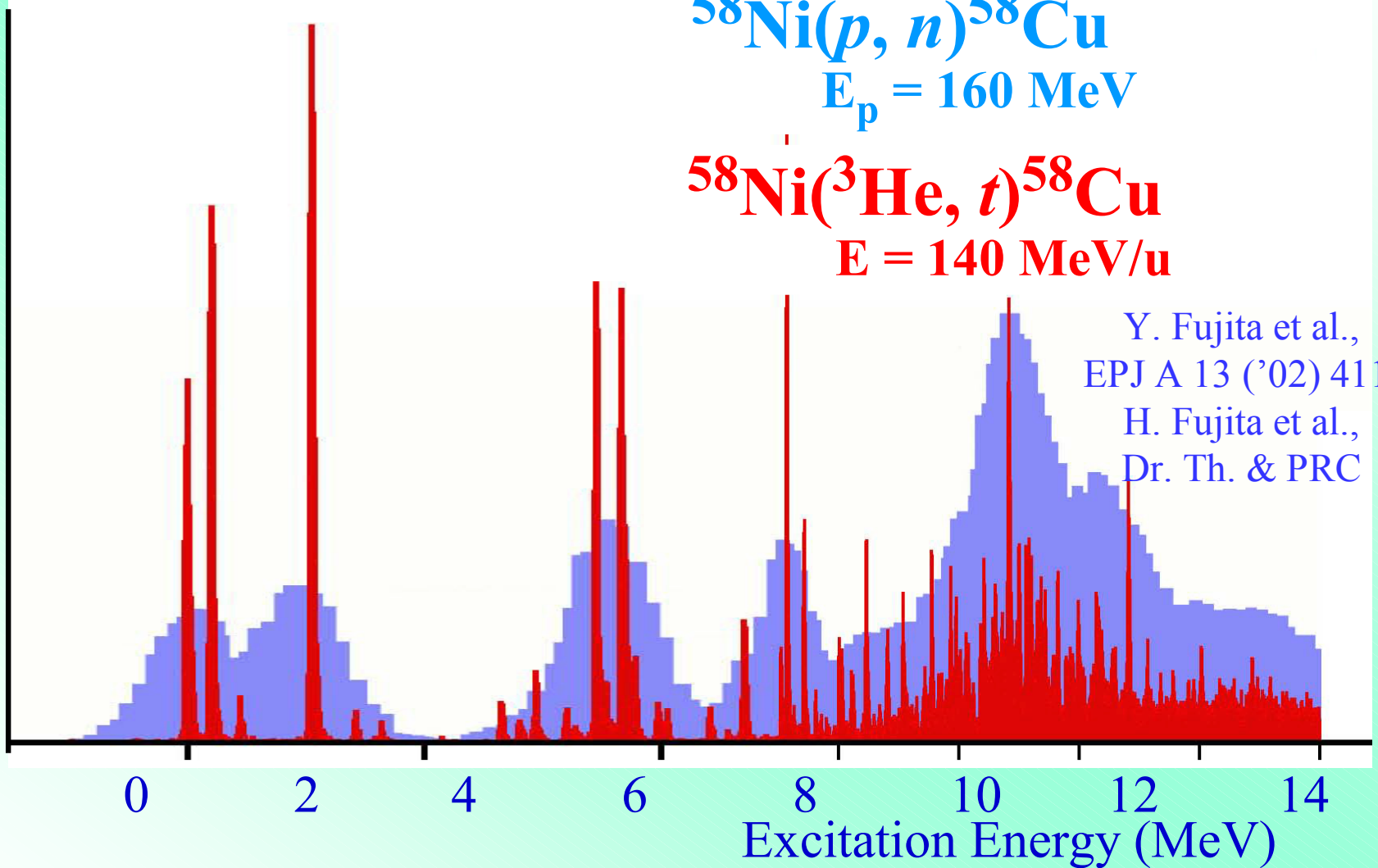
**$^{58}\text{Ni}(p, n)^{58}\text{Cu}$**

**$E_p = 160 \text{ MeV}$**

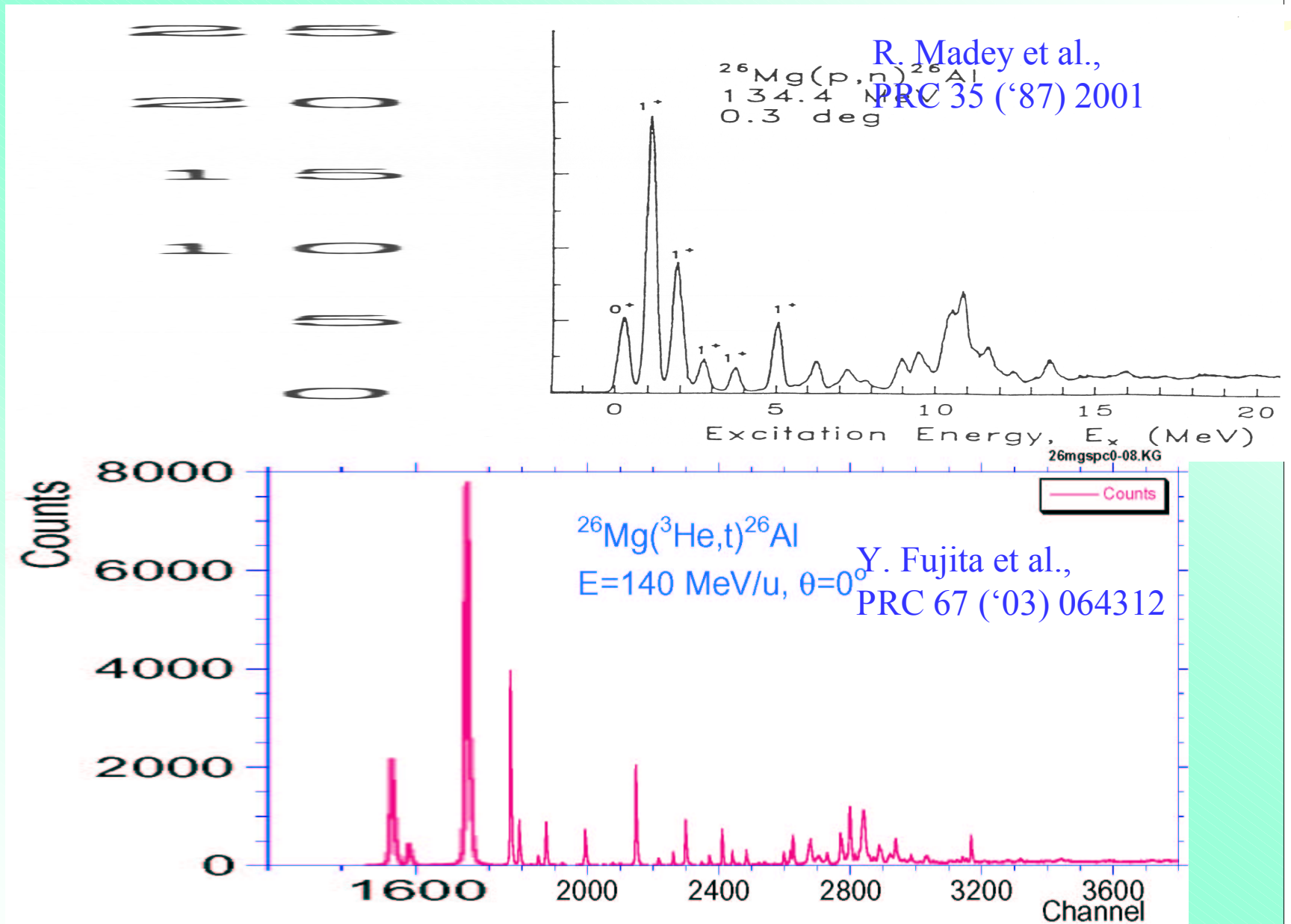
**$^{58}\text{Ni}(^3\text{He}, t)^{58}\text{Cu}$**

**$E = 140 \text{ MeV/u}$**

Y. Fujita et al.,  
EPJ A 13 ('02) 411.  
H. Fujita et al.,  
Dr. Th. & PRC



# $^{26}\text{Mg}(p, n)^{26}\text{Al}$ & $^{26}\text{Mg}(^3\text{He}, t)^{26}\text{Al}$ spectra

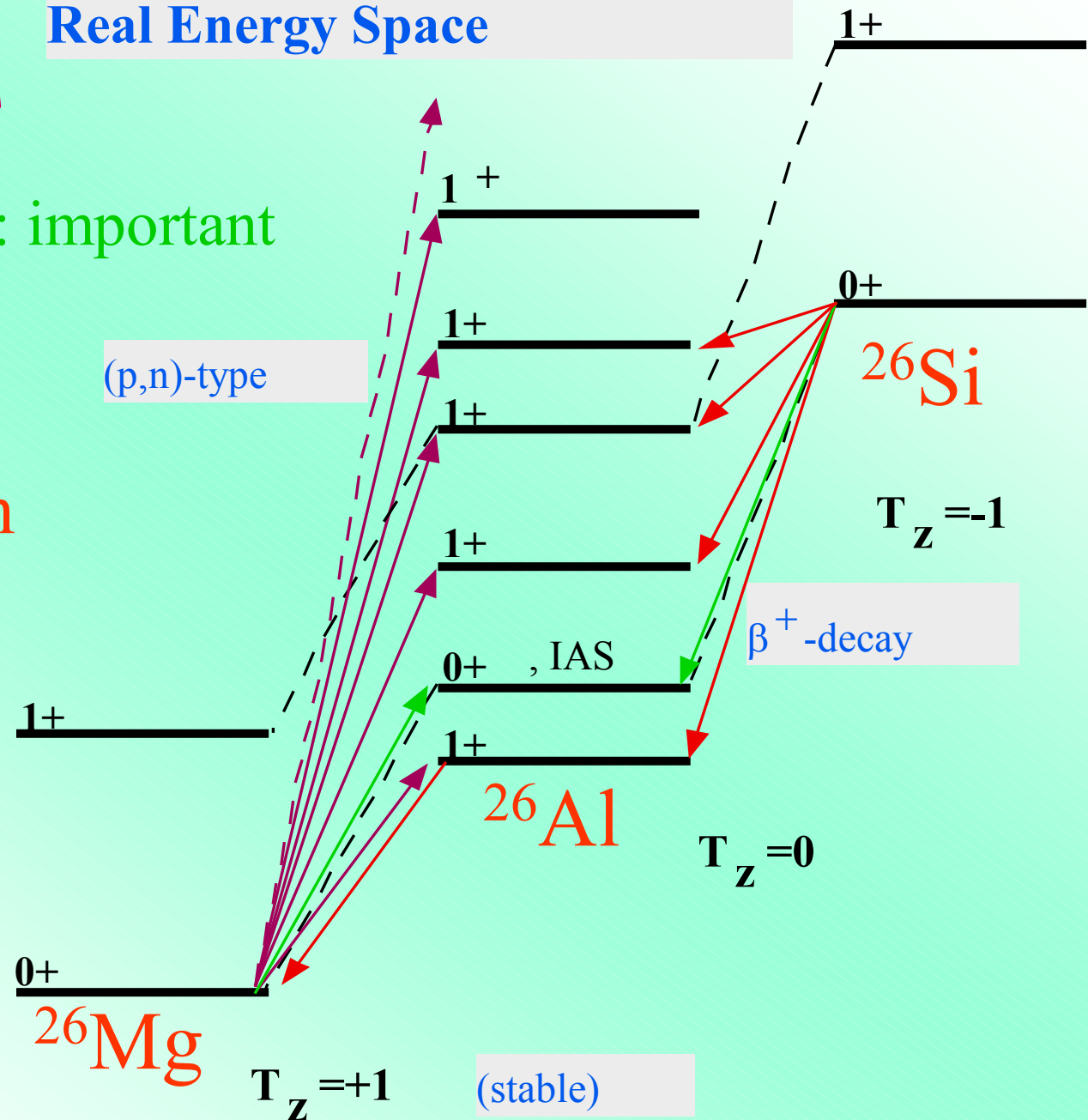


# T=1 system

Coulomb Energy: important

A=26 system

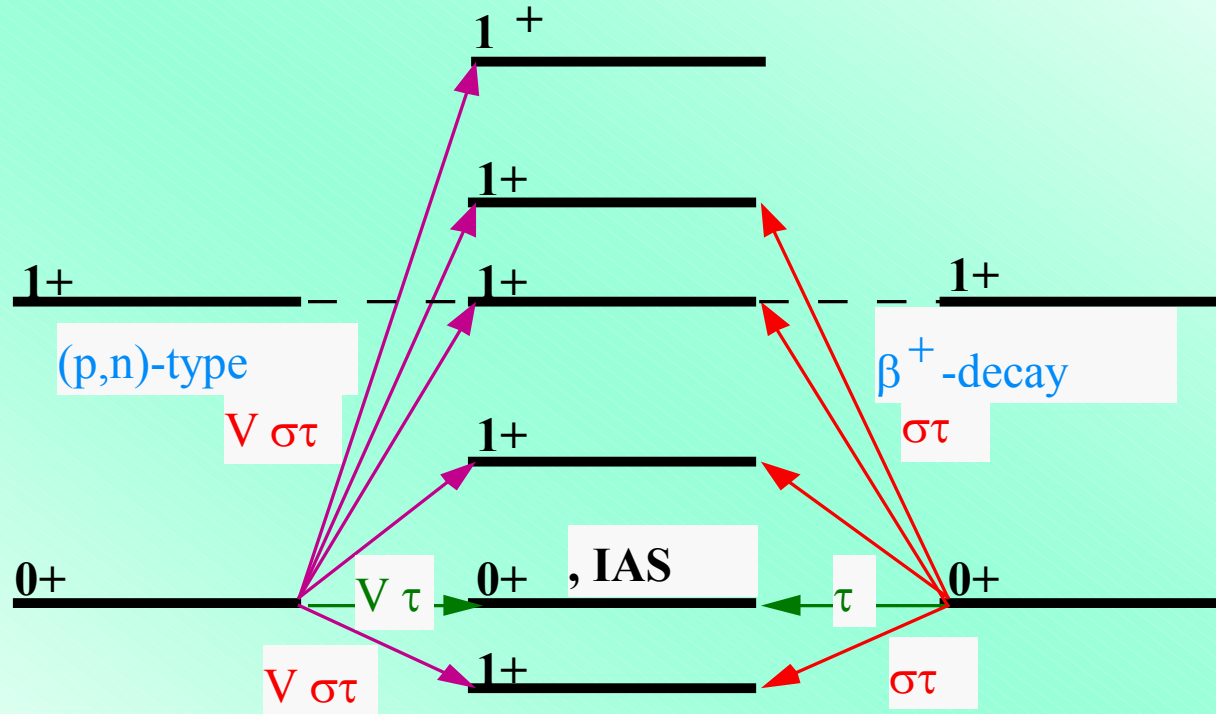
## Real Energy Space



# T=1 symmetry : Structures & Transitions

$$T_Z = +1 \quad \longrightarrow \quad T_Z = 0 \quad \longleftarrow \quad T_Z = -1$$

(in isospin symmetry space\*)



$T_Z = +1$

$^{26}\text{Mg}$

Z=12, N=14

$T_Z = 0$

$^{26}\text{Al}$

Z=13, N=13

$T_Z = -1$

$^{26}\text{Si}$

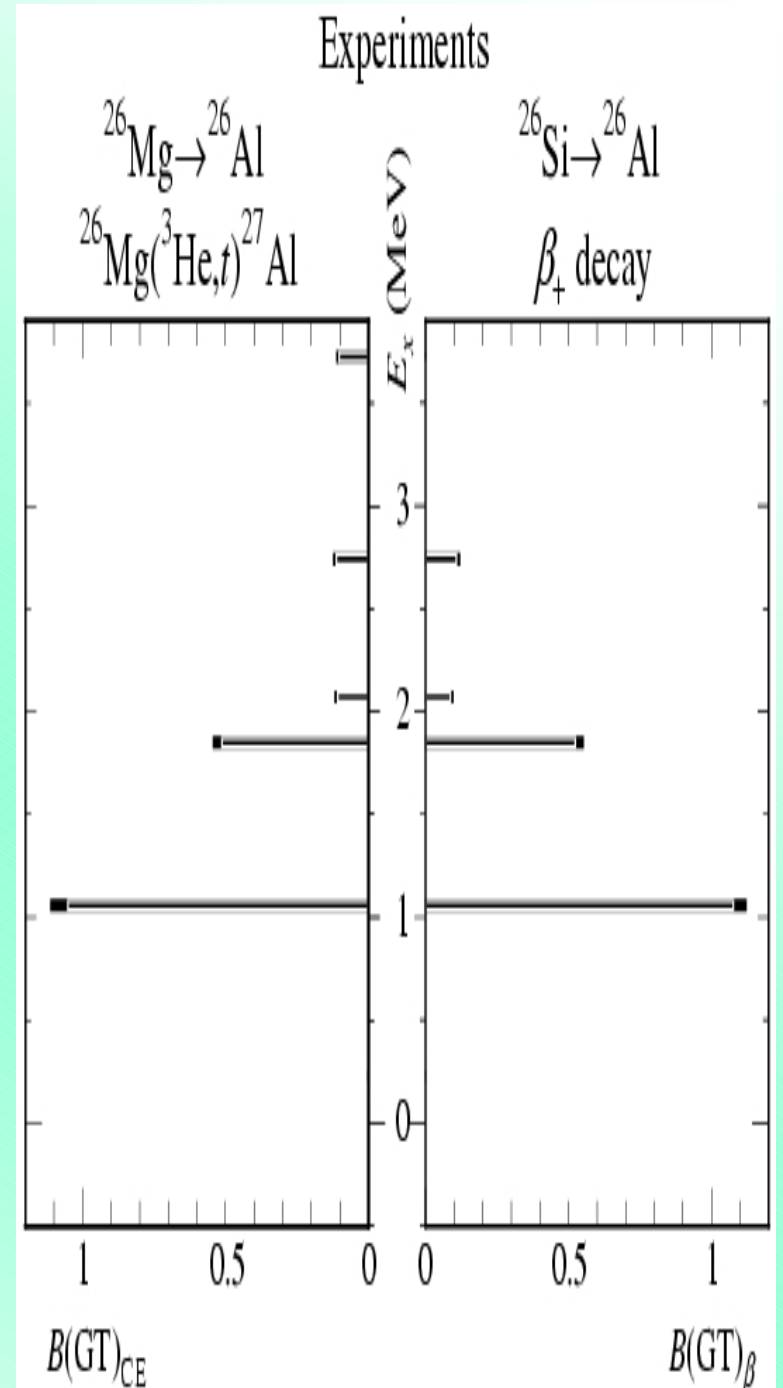
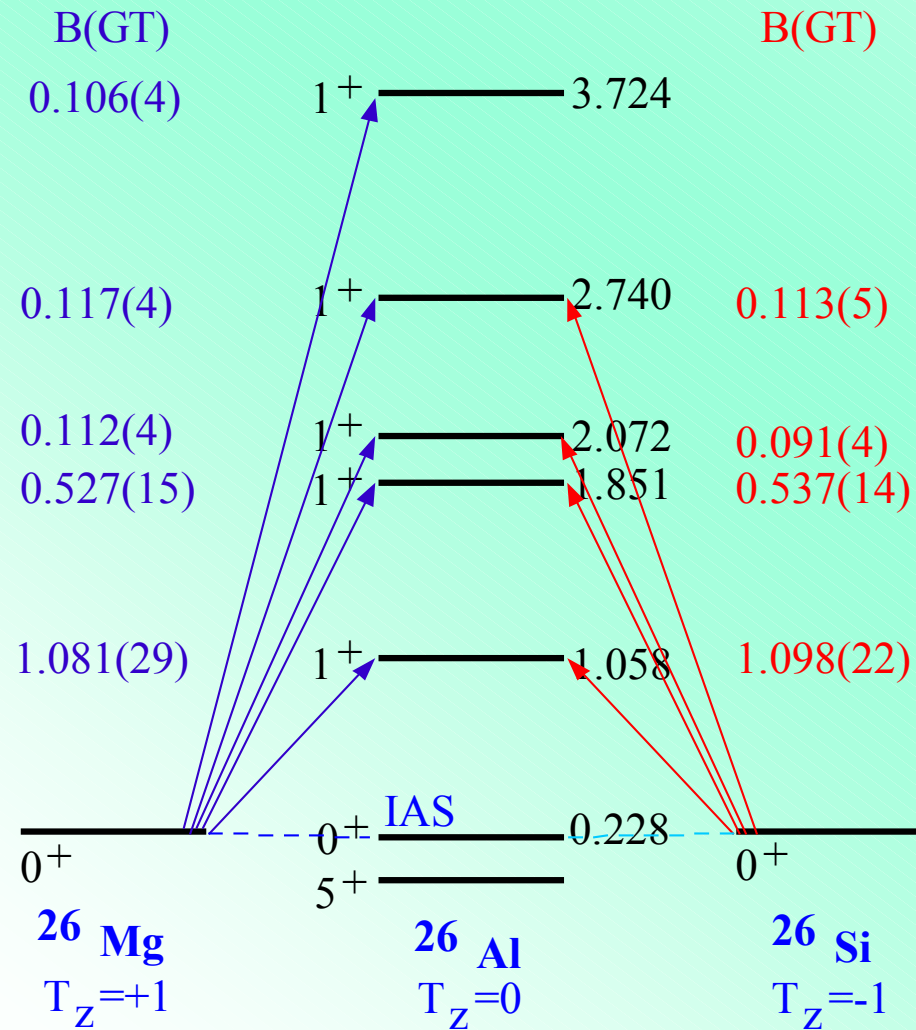
Z=14, N=12



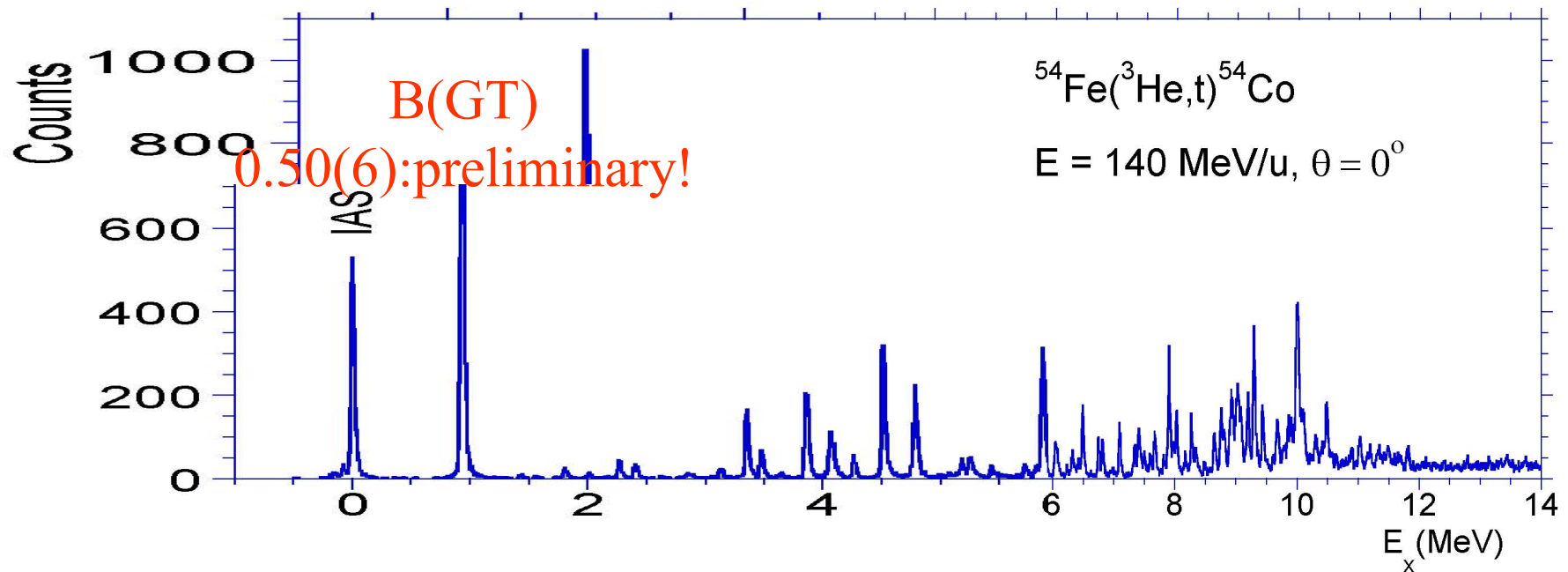
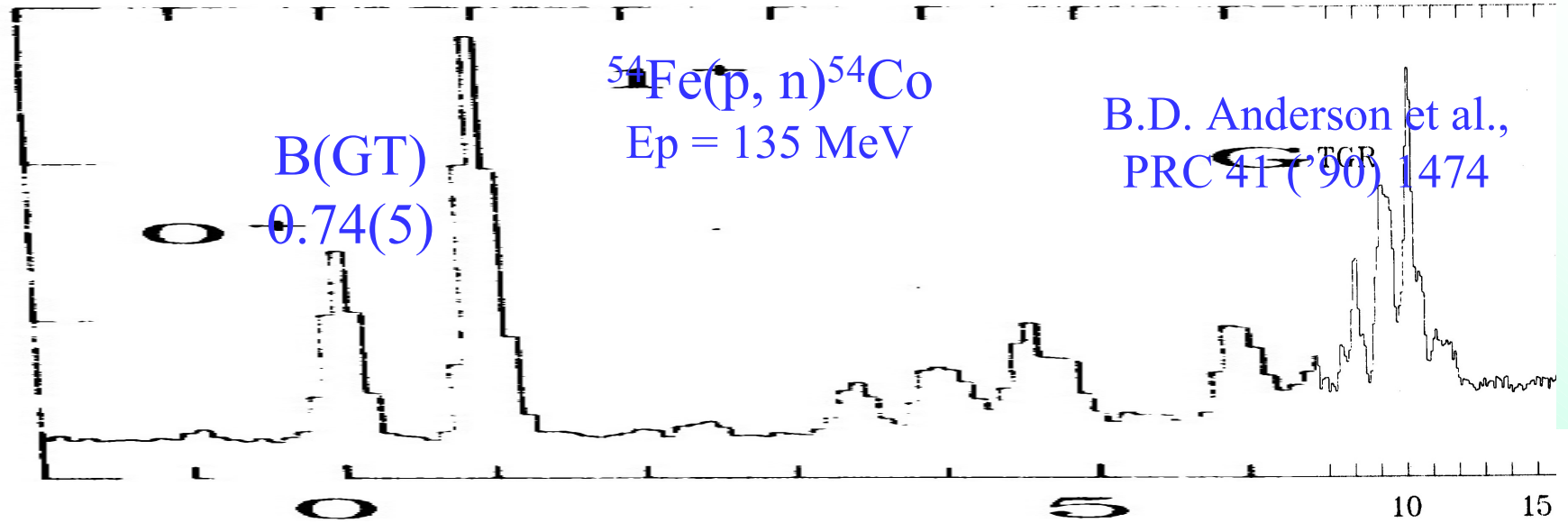
# B(GT) values from Symmetry Transitions (A=26)

from ( $^3\text{He},t$ )

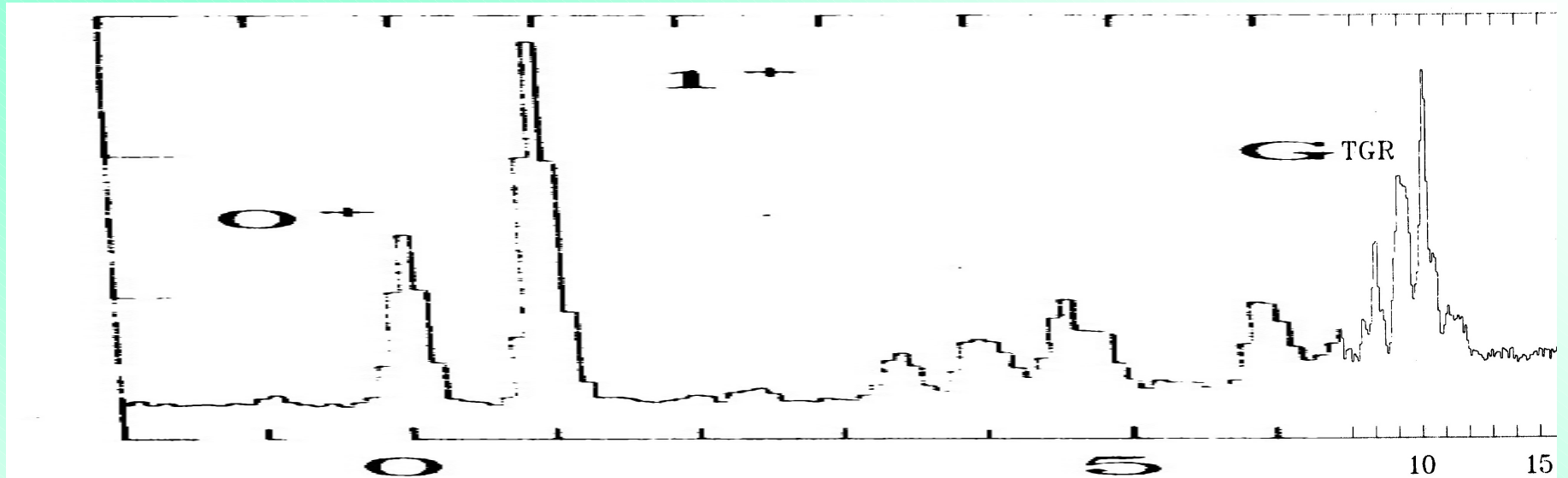
from  $\beta$ -decay



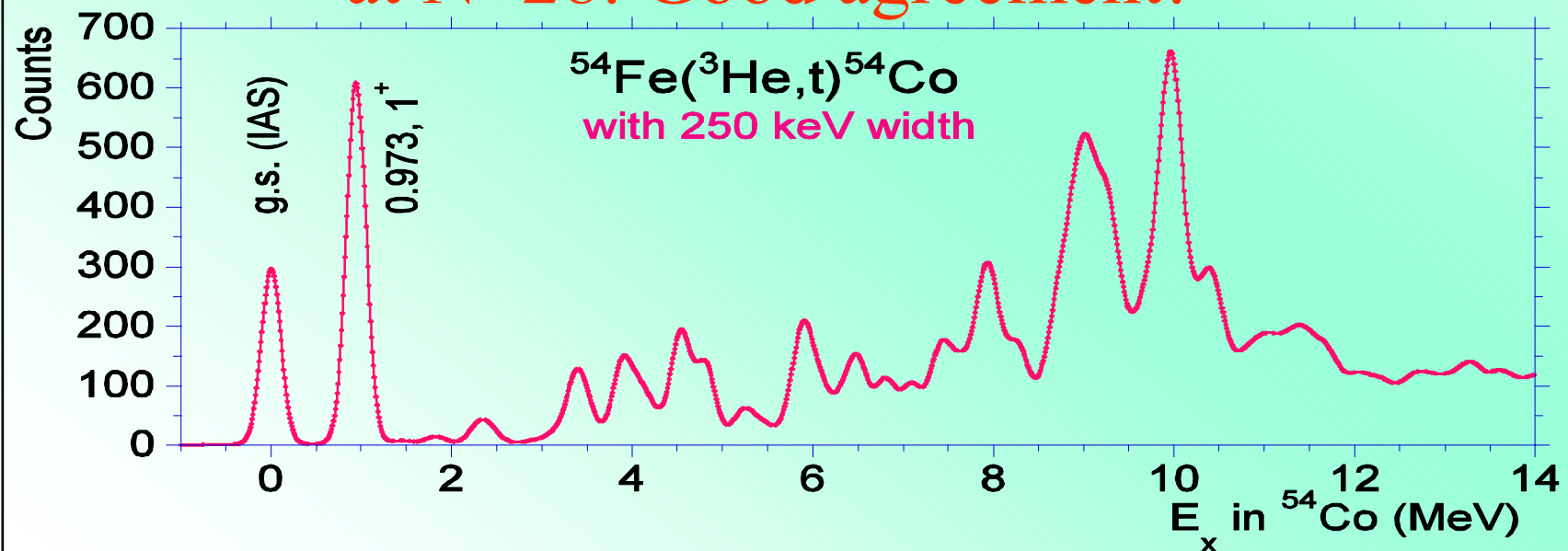
# $^{54}\text{Fe}(p,n)$ & $^{54}\text{Fe}(^3\text{He},t)$



# $^{54}\text{Fe}(p,n)$ & $^{54}\text{Fe}(^3\text{He},t)$ +width



at N=28: Good agreement!



# $^{56}\text{Fe}$ target: $(^3\text{He}, t)$ vs. $(p, n)$

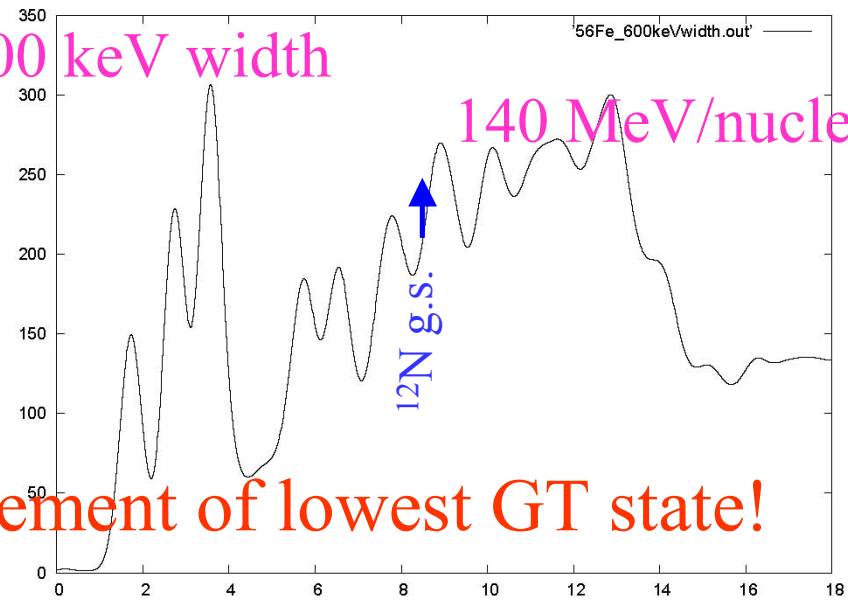
$(^3\text{He}, t)$

1.7 MeV GT state  
enhanced!

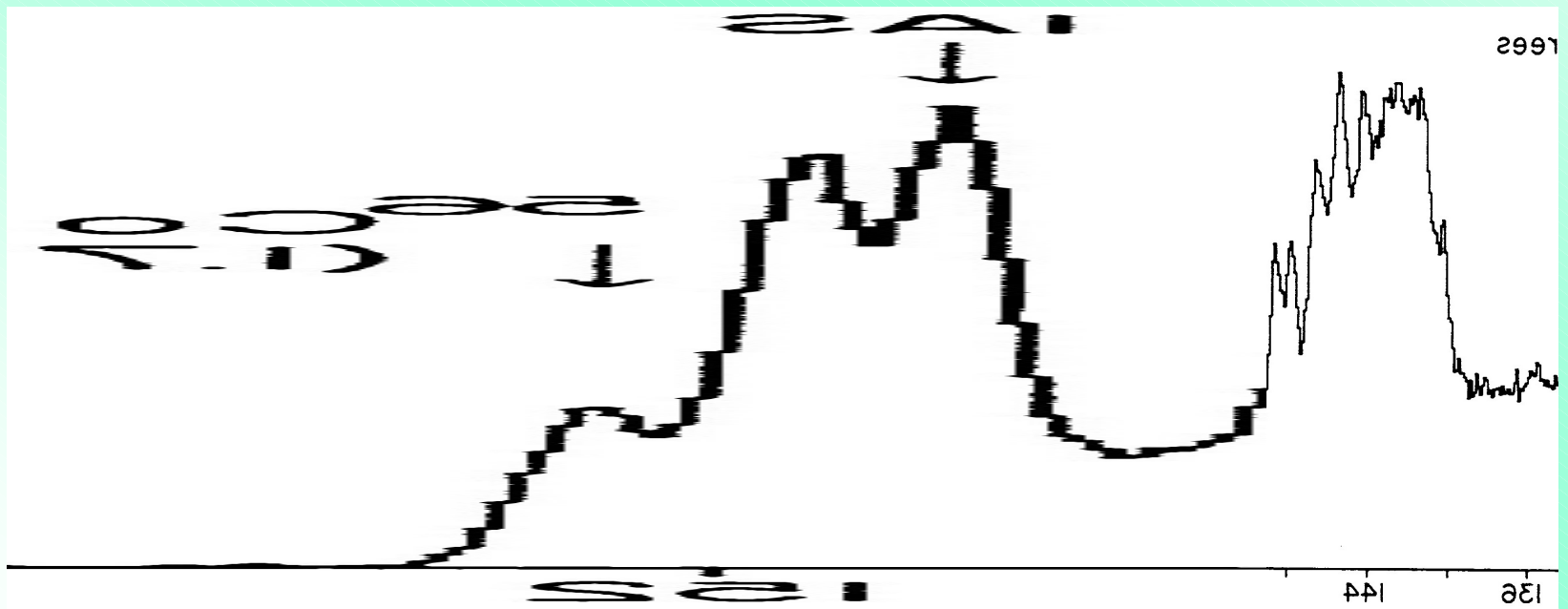


at  $N=30$ , exceptional enhancement of lowest GT state!

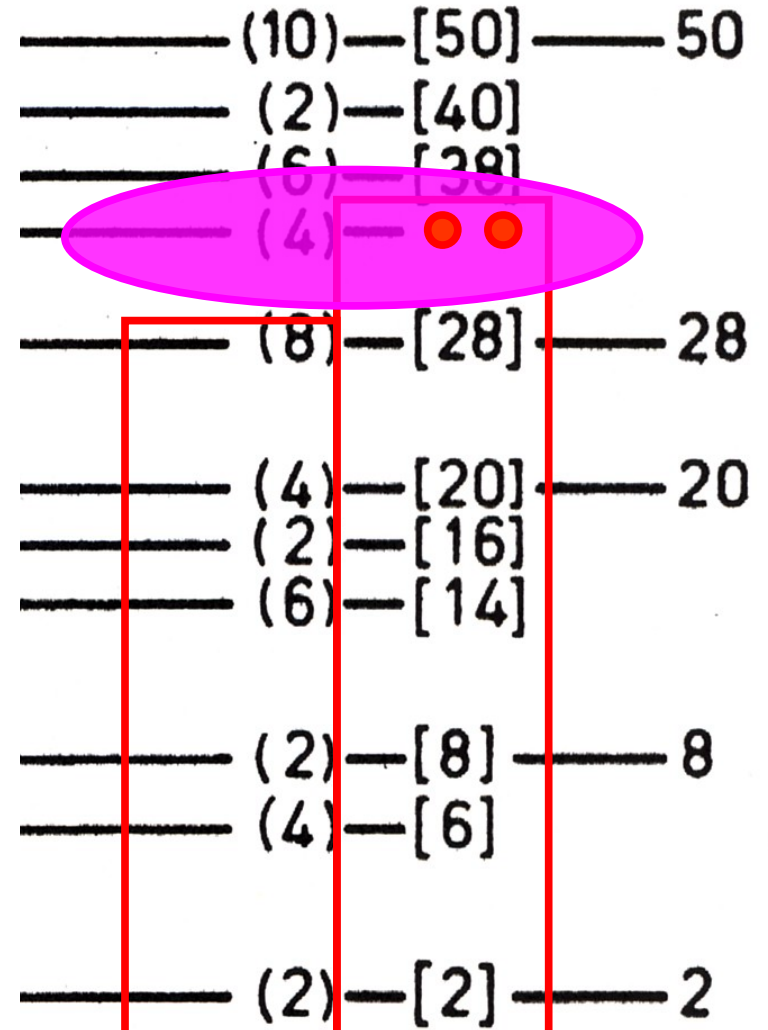
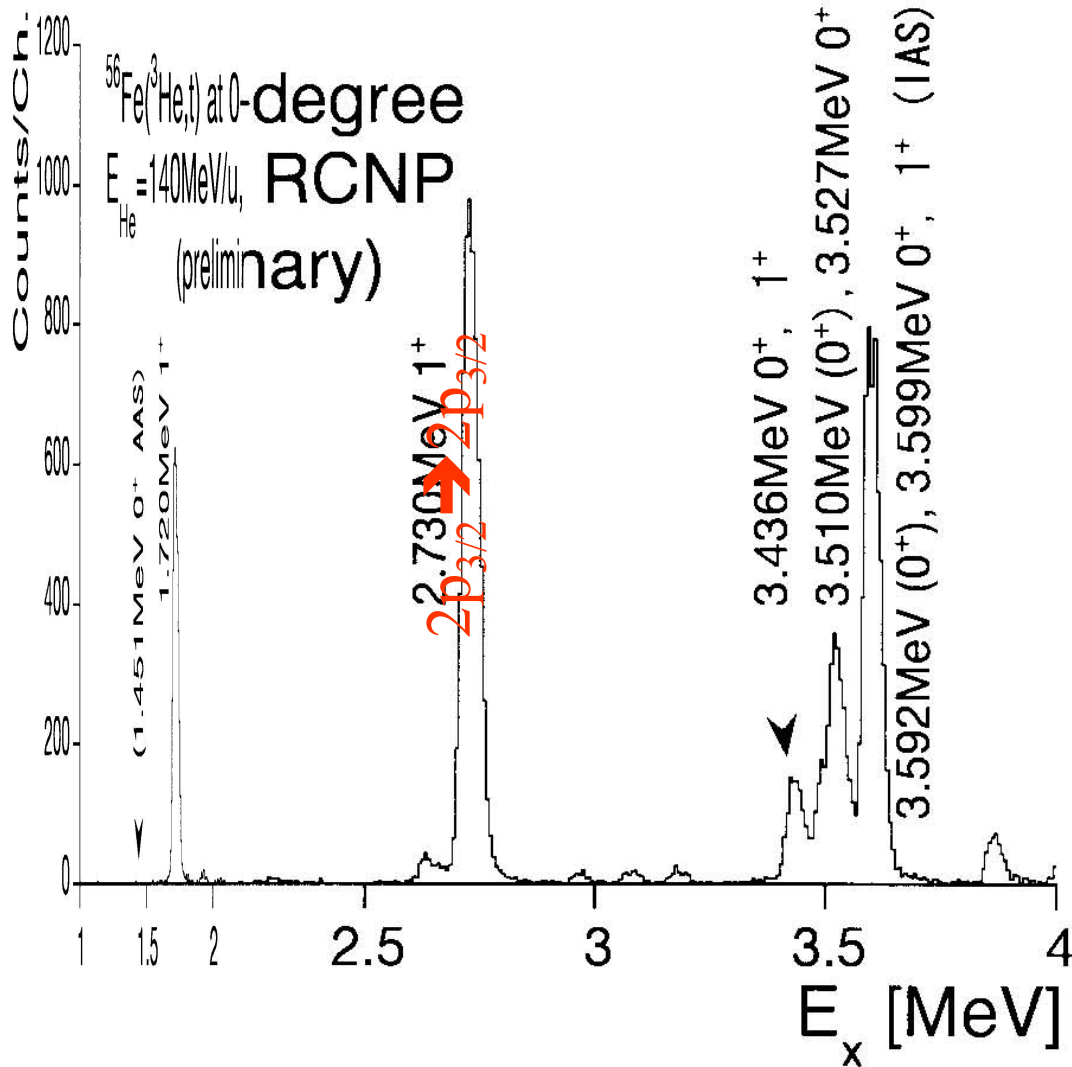
with 600 keV width



$(p, n)$

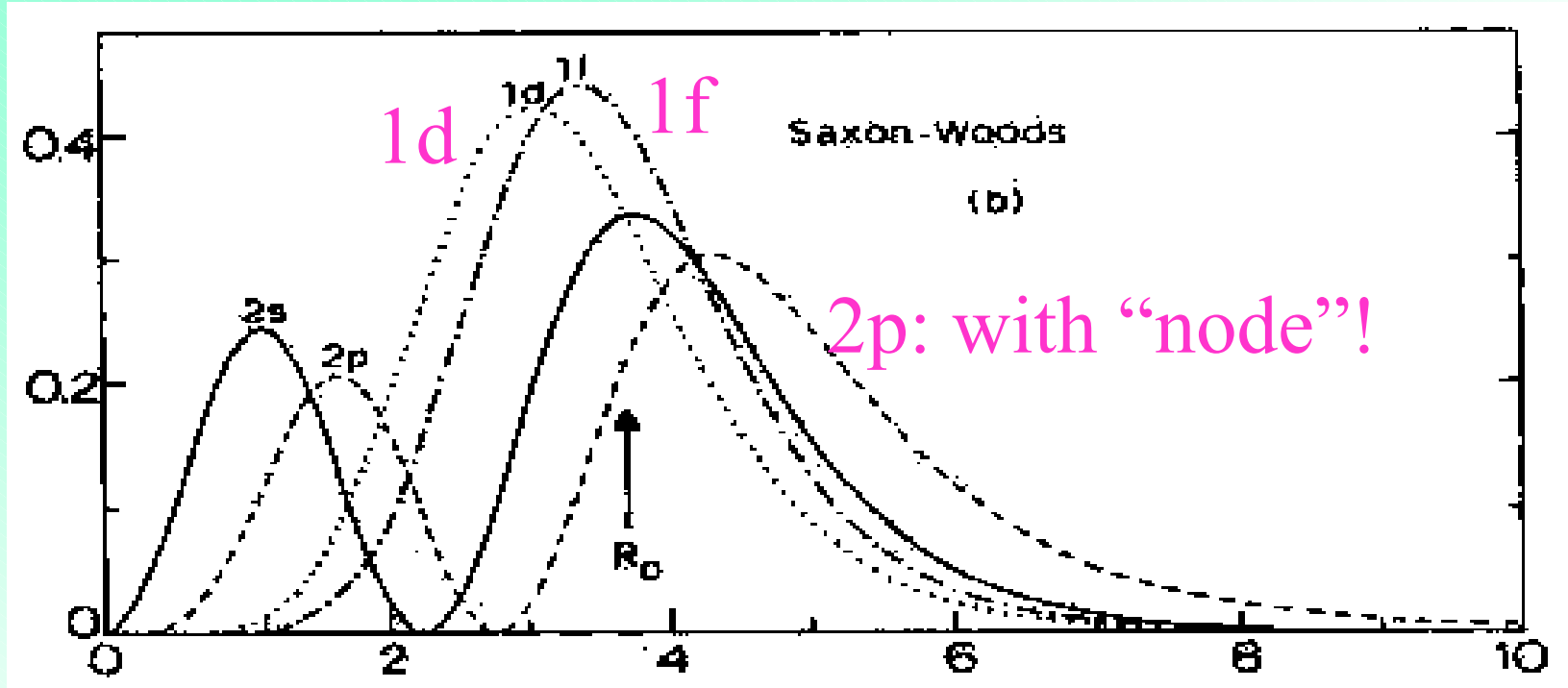


# $^{56}\text{Fe}(^3\text{He},t)$ : low-lying GT states



Z=26 N=30

# Wave function (radial direction)



$^{56}\text{Fe} \rightarrow ^{56}\text{Co}$   
GT transition

lowest  
low-lying  
higher Ex

$2p_{3/2} \rightarrow 2p_{3/2}$   
 $2p_{3/2} \rightarrow 2p_{1/2}$   
 $1f_{7/2} \rightarrow 1f_{5/2}$

Due to “surface nature” of ( $^3\text{He}, t$ ), reaction strength may be affected by the presence of the node.



# ?? Proportionality ??

\*proportionality has been most well studied in ( $^3\text{He},t$ )

Universal proportionality  $\times$  ← for all mass  $A$

Specific proportionality  $\circ$  ← for each  $A$

(of the order of a few - 10%)

Larger uncertainties of the proportionality:

1) for the  $j_{<} \rightarrow j_{<}$  transitions (weak!)

(ex.  $p_{1/2} \rightarrow p_{1/2}$ ,  $d_{3/2} \rightarrow d_{3/2}$ )

2) when a w.f. has a node

(ex.  $2s_{1/2} \rightarrow 2s_{1/2}$ ,  $2p_{3/2} \rightarrow 2p_{3/2}$ )

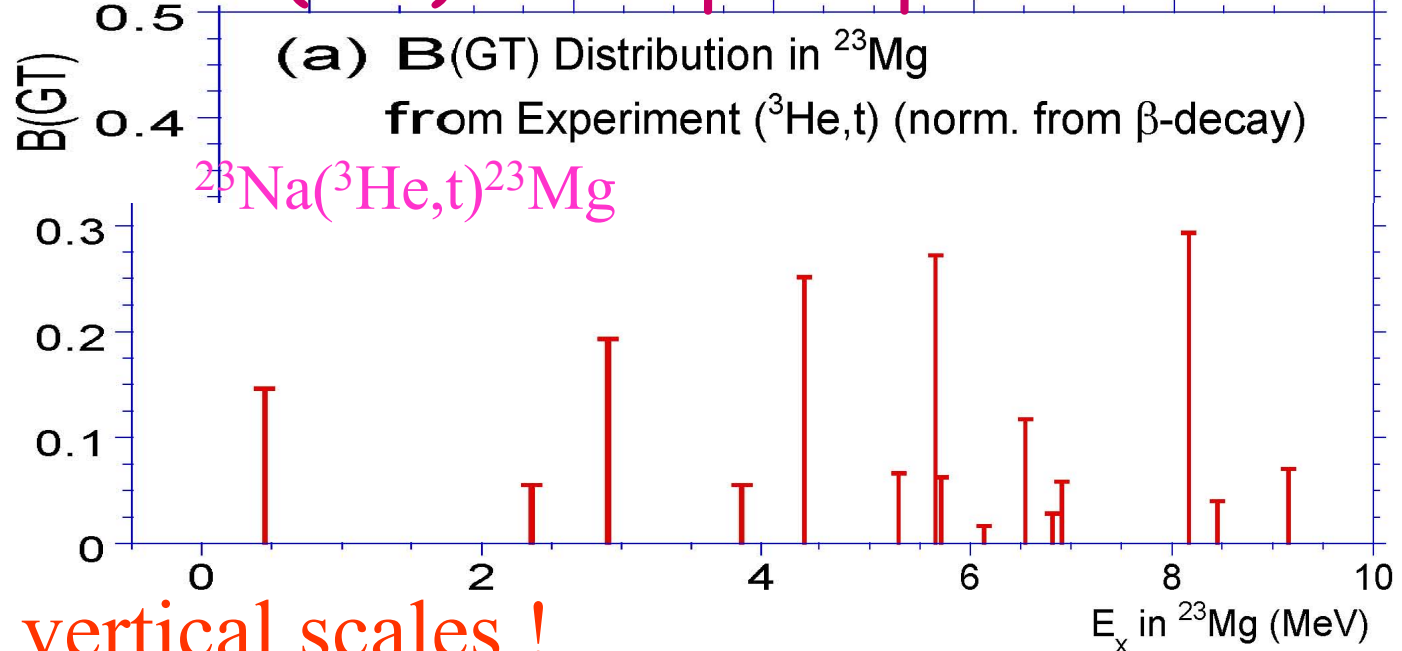
at  $N=18$

at  $N=30$

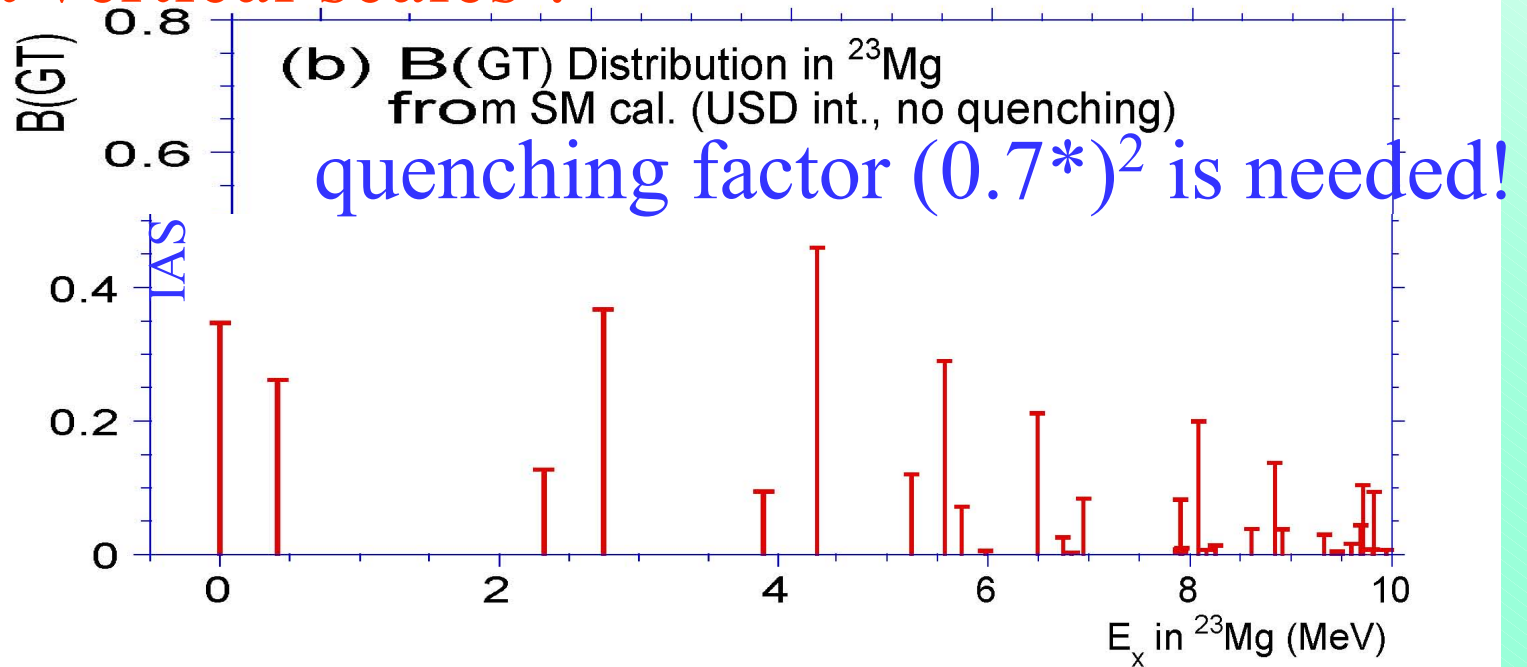
\*\*\*at the *sd*-shell region\*\*\*

\*\*  $A=23$   $T=1/2 \rightarrow 1/2$  transitions

# A=23 B(GT) : Comp. Exp. & SM

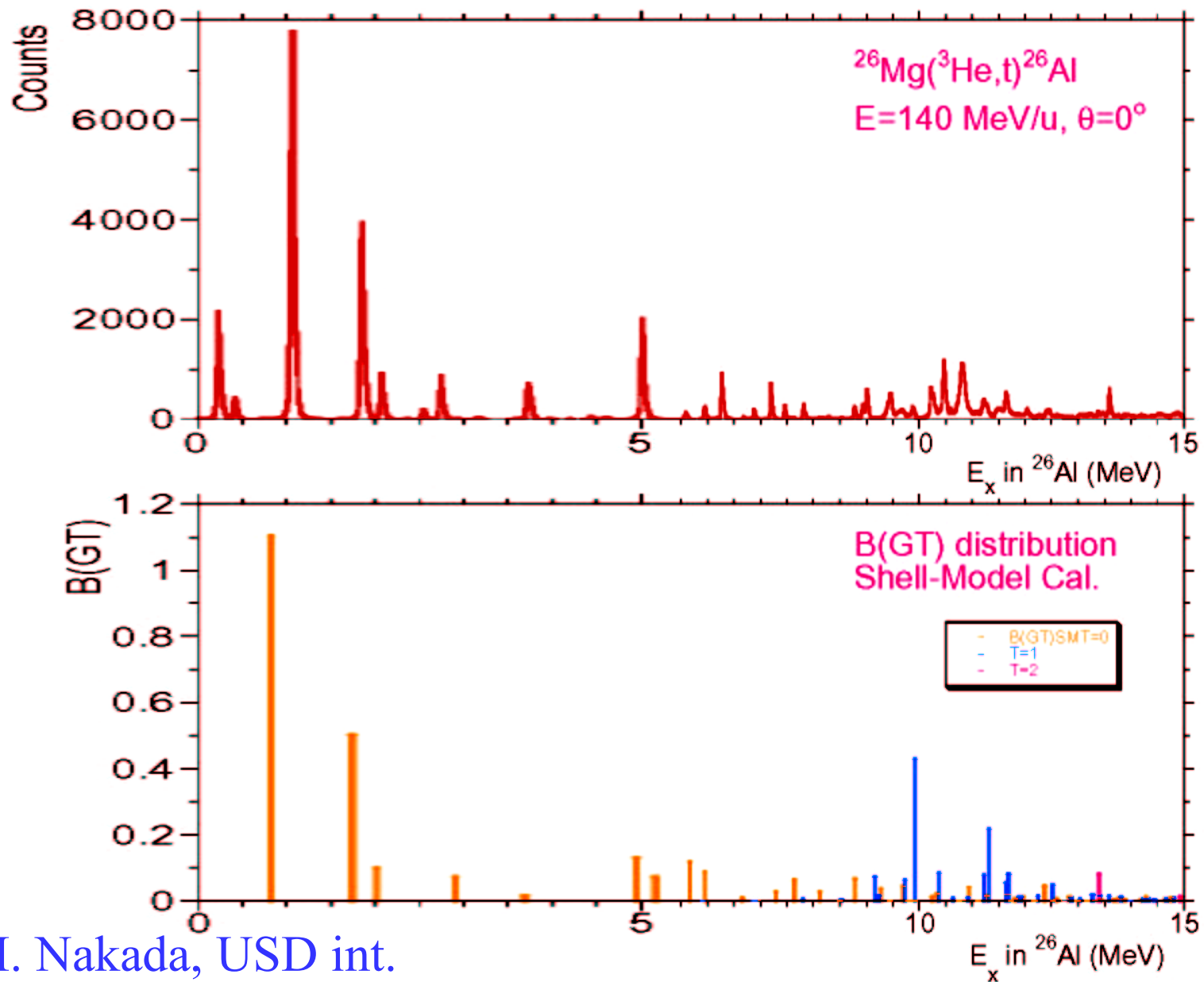


different vertical scales !



**\*\***  $A=26$   $T=1 \rightarrow 0$  transitions

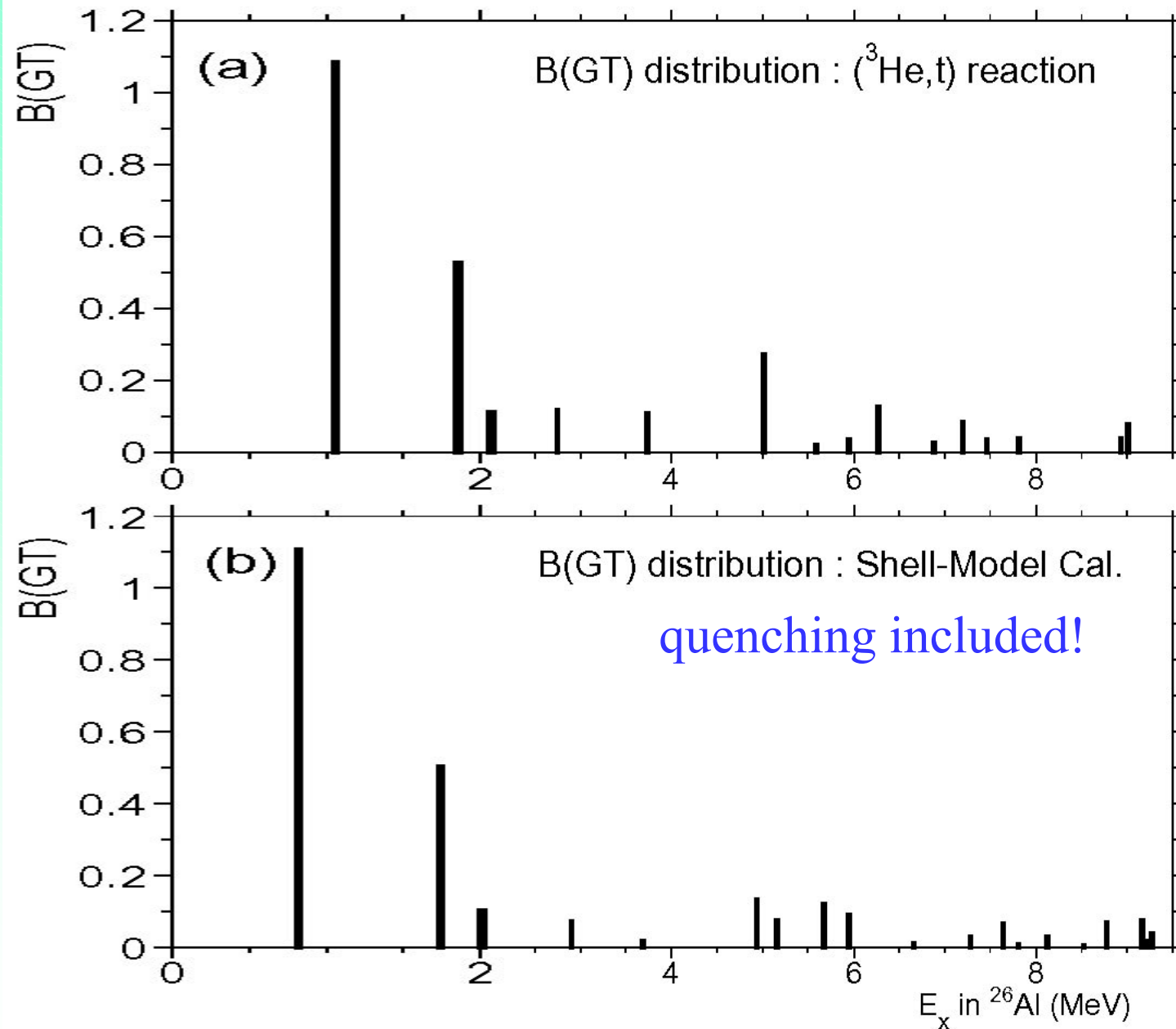
# $^{26}\text{Mg}(^3\text{He},t)$ spectrum and SM cal. B(GT)



by H. Nakada, USD int.

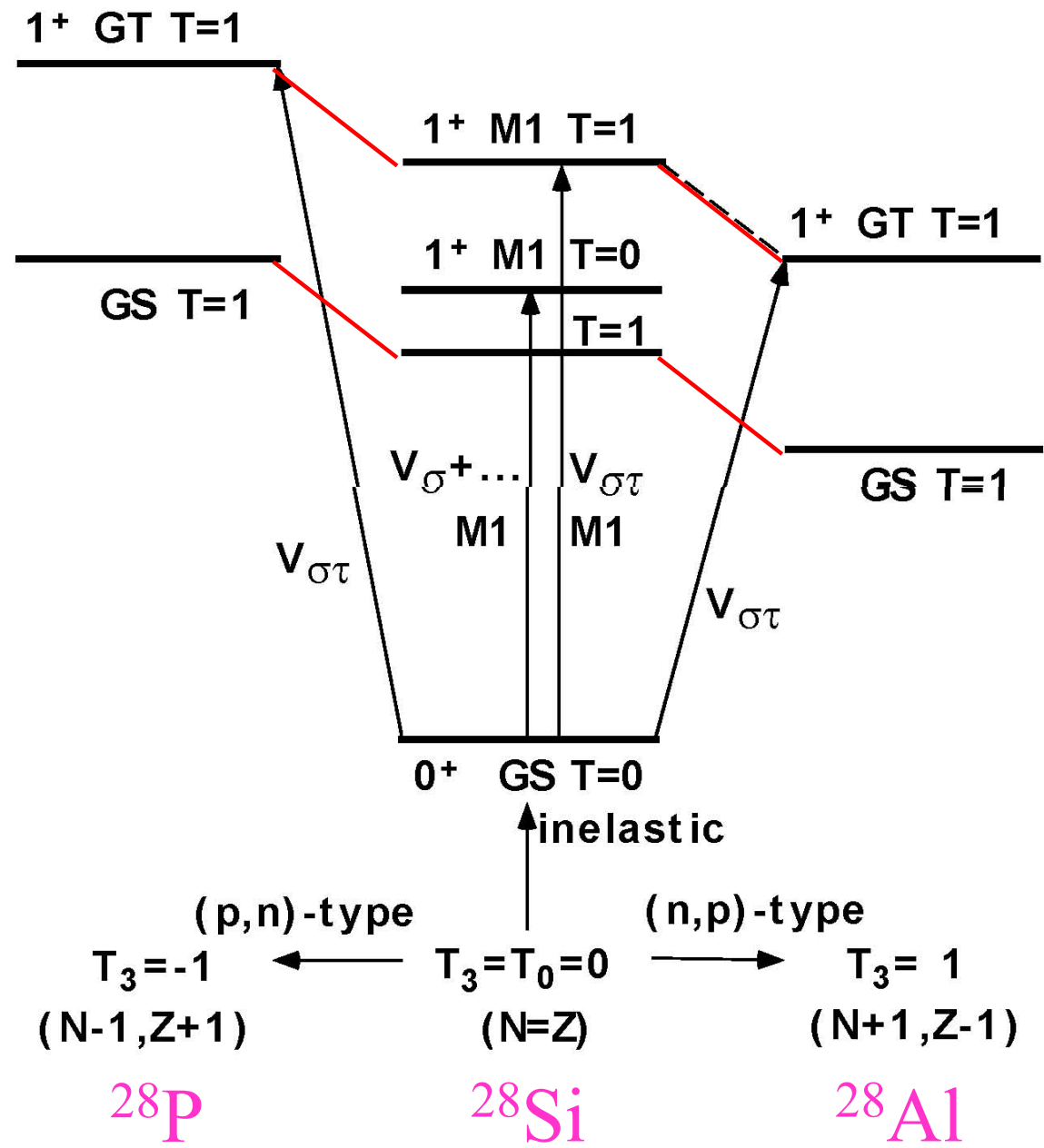


# B(GT) distributions (low-lying): Exp. & SM

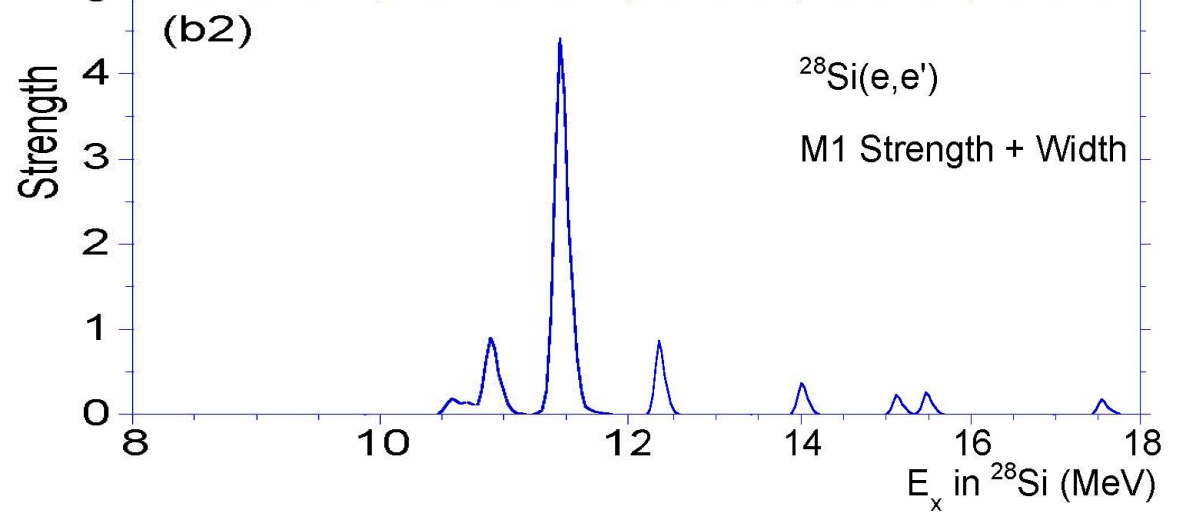
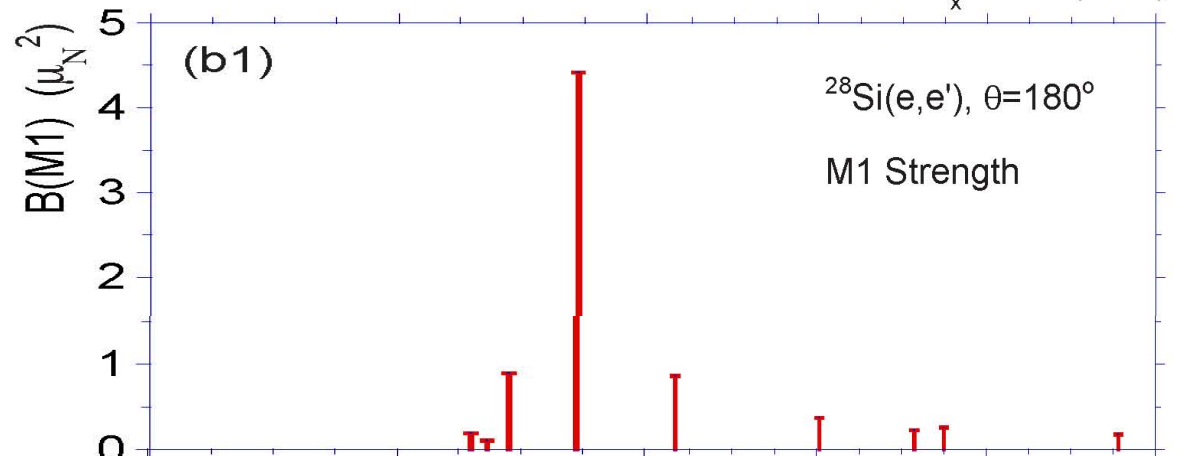
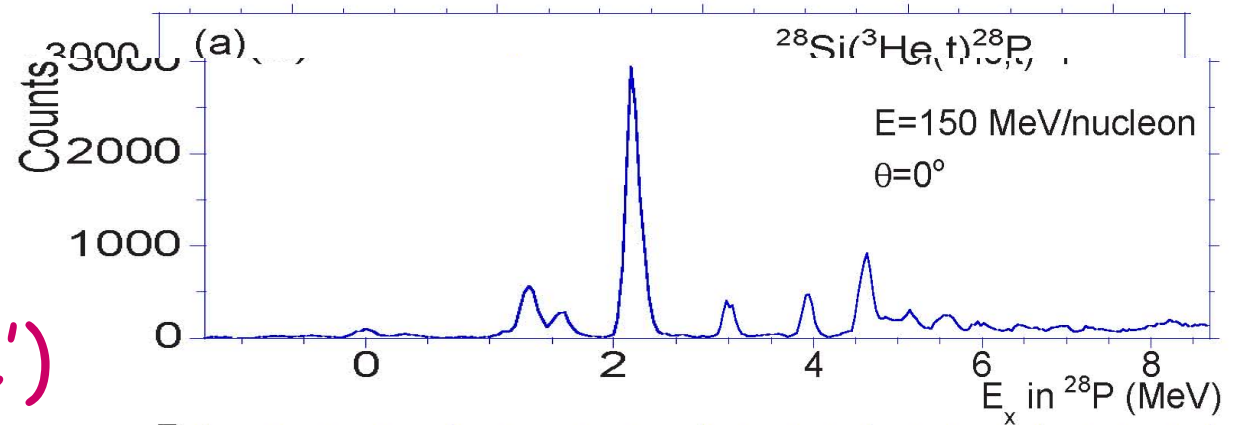


**\*\***  $A=28$   $T=0 \rightarrow 1$  transitions

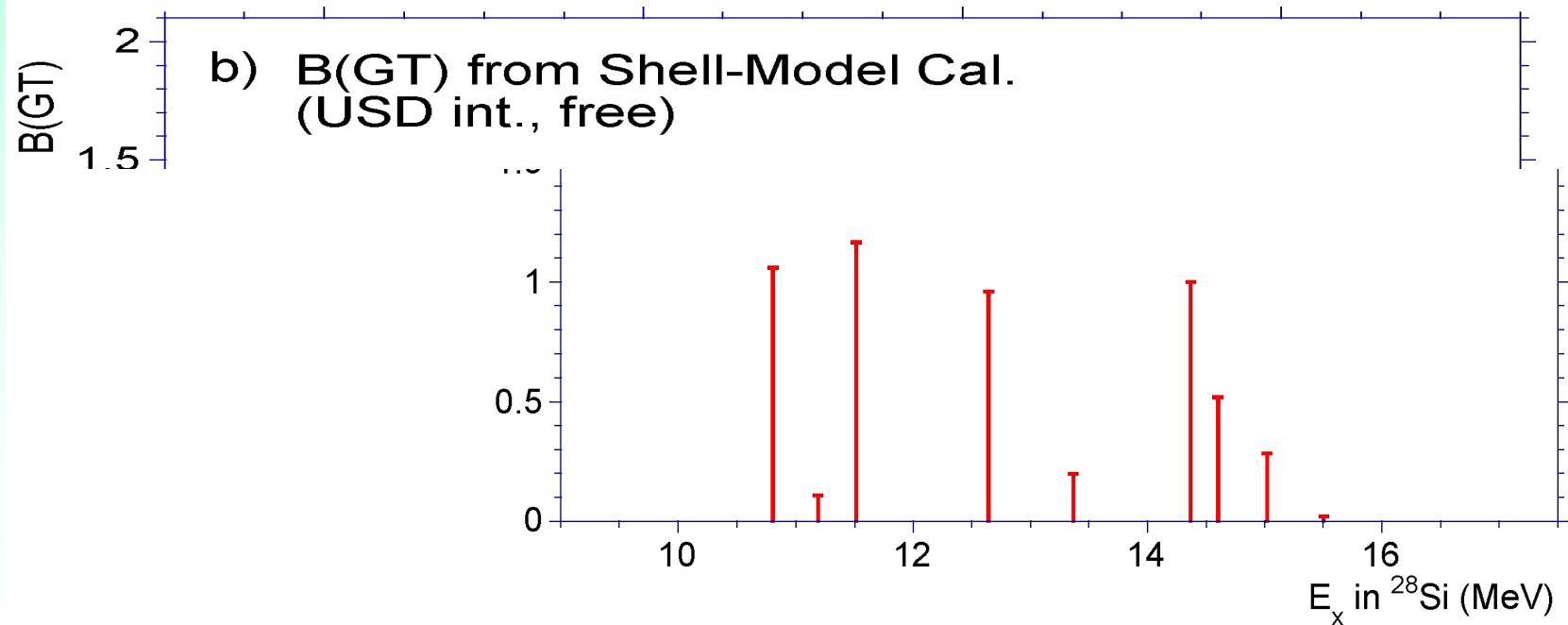
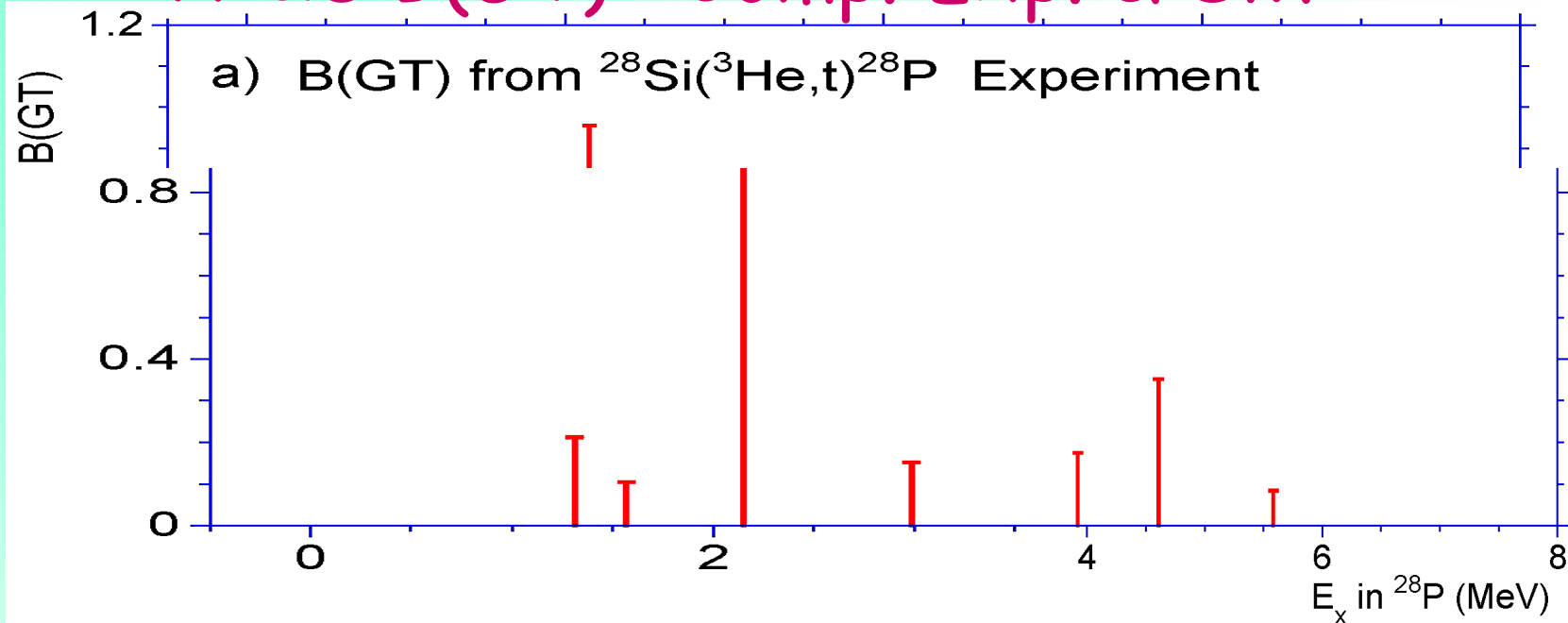
# Isospin Structure of T=0 Nucleus $^{28}\text{Si}$



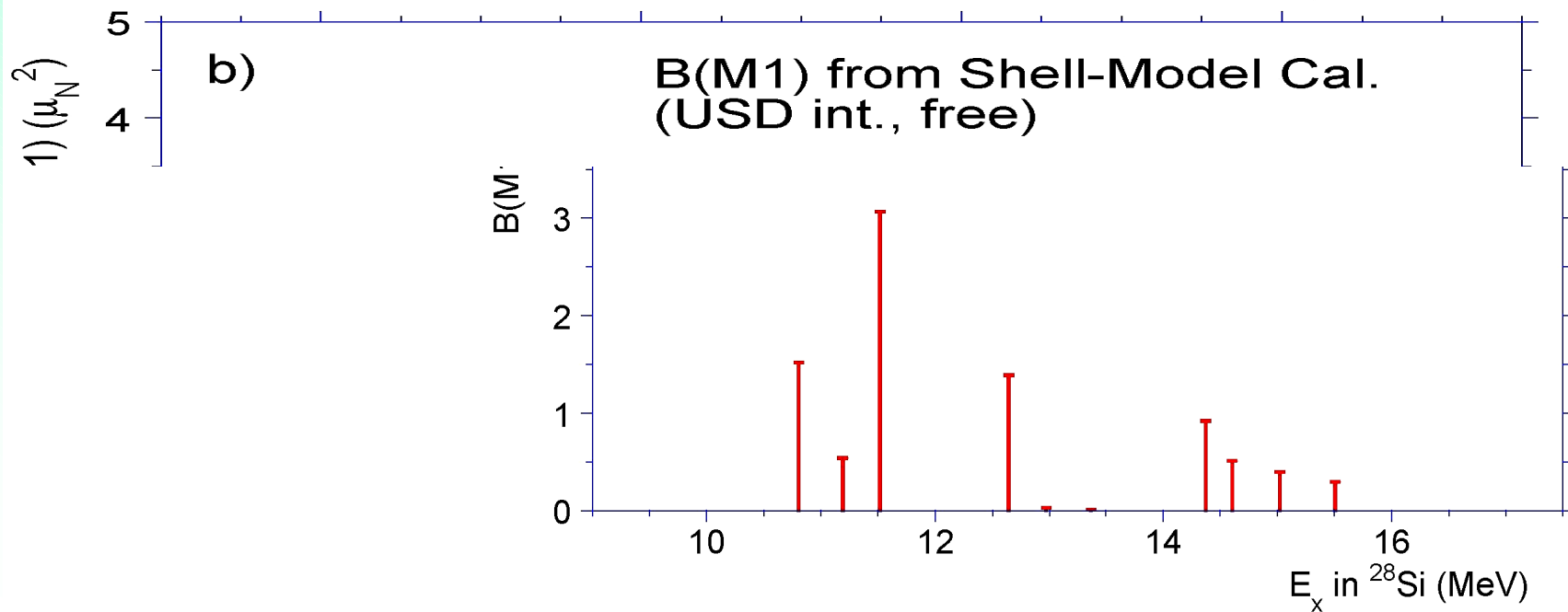
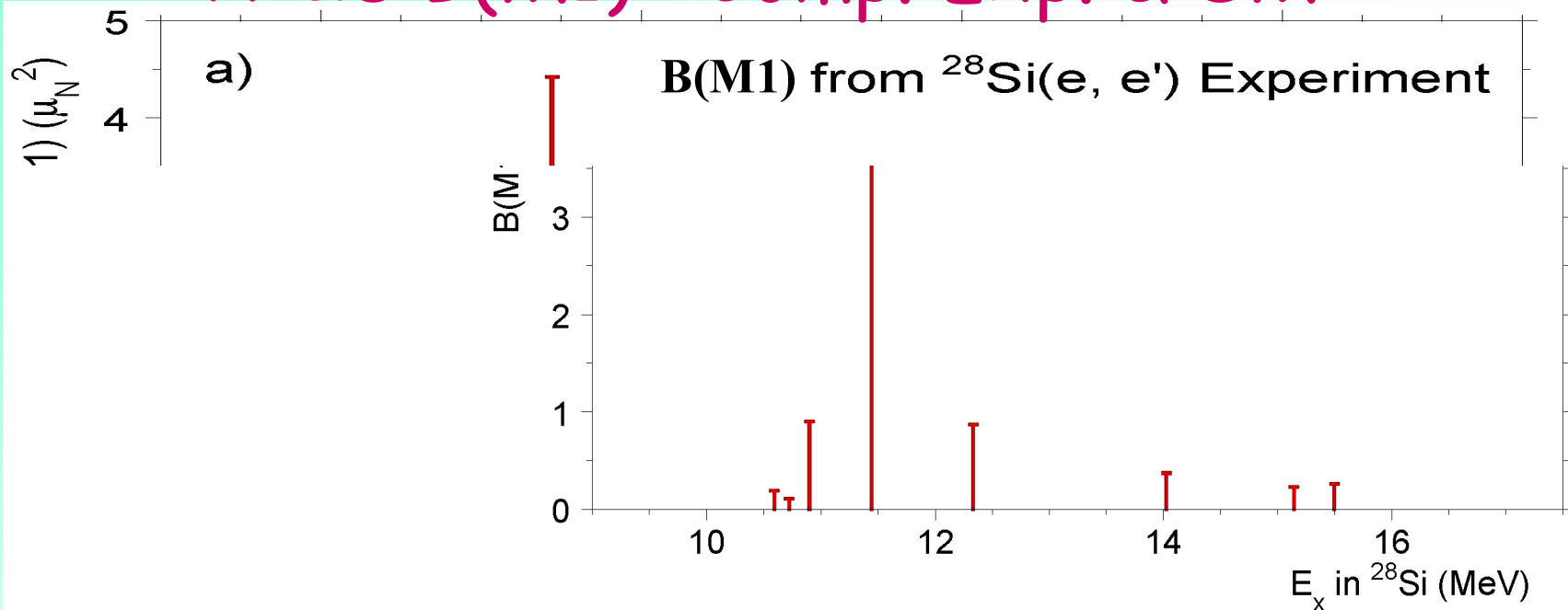
$^{28}\text{Si}$  Comp:  
( $^3\text{He}, t$ ) & ( $e, e'$ )



# A=28 B(GT) : Comp. Exp. & SM



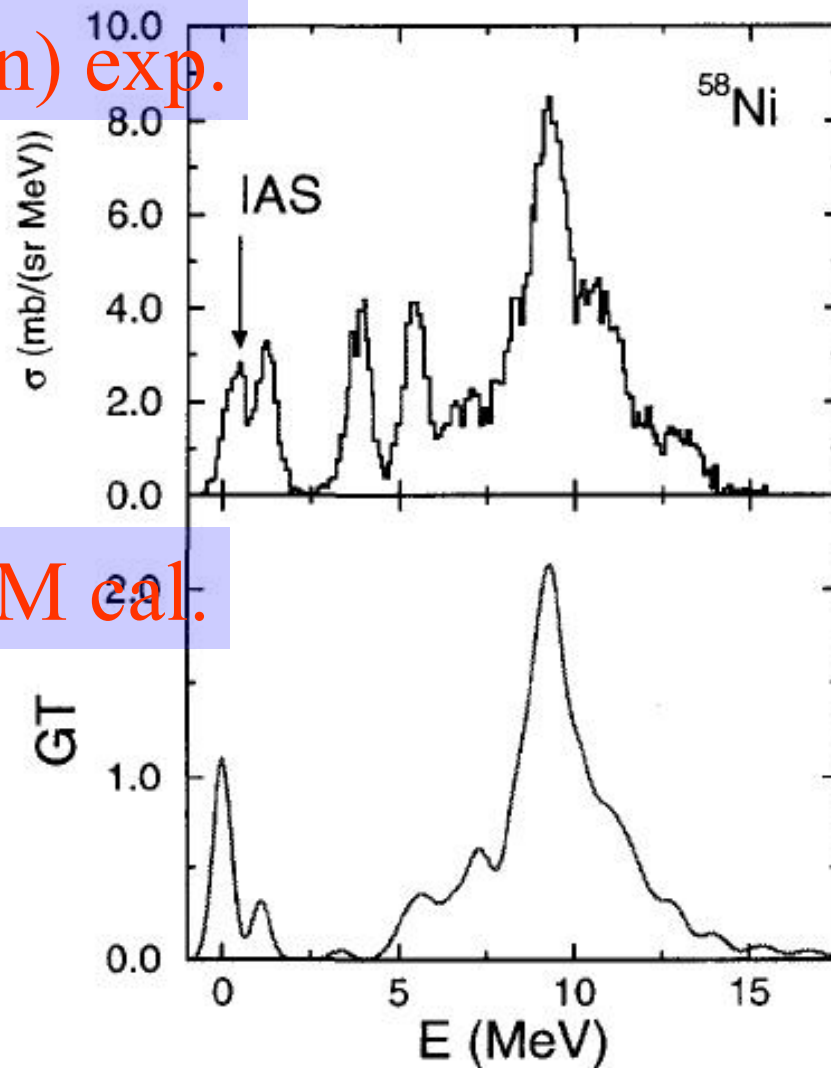
# A=28 B(M1) : Comp. Exp. & SM



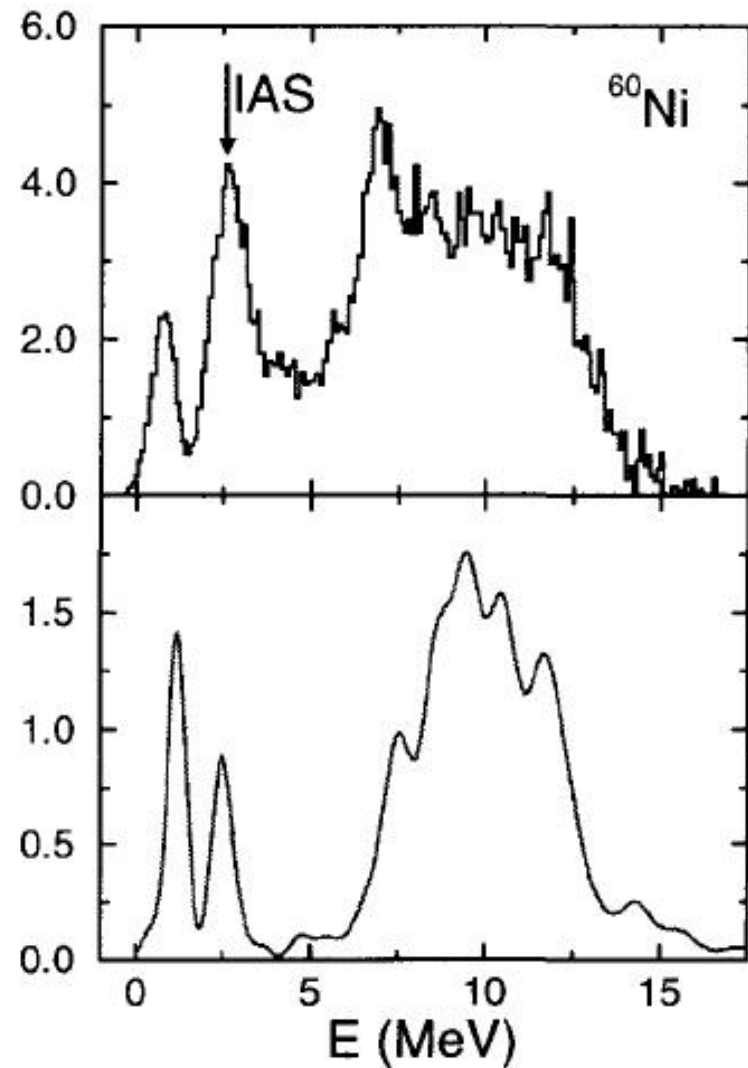
\*\*\*at the Ni region of  
*fp* -shell\*\*\*

# SM-cal: GT- from Ni isotopes

(p,n) exp.



SM cal.



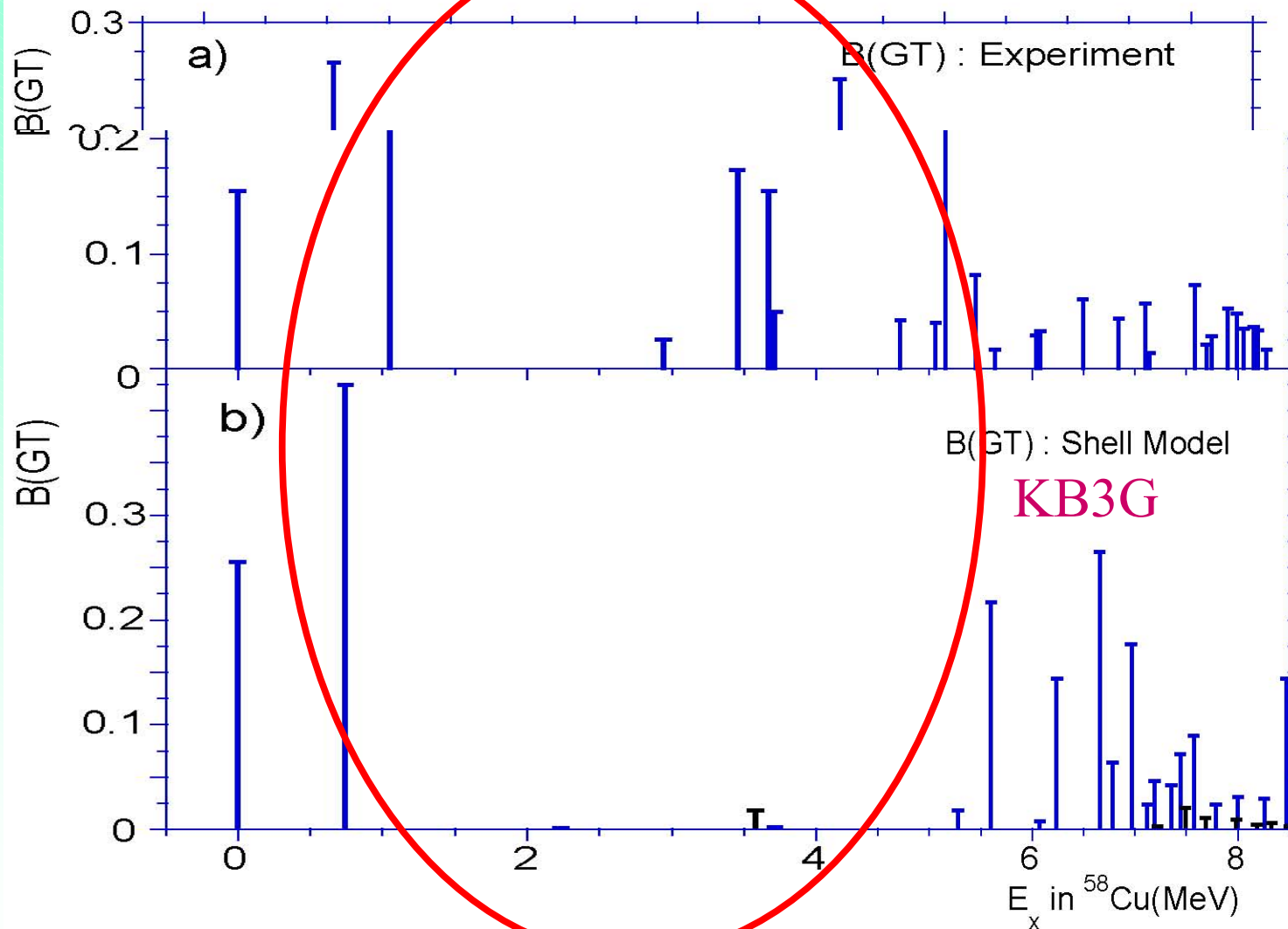
E. Caurier et al., NPA653 ('99) 439

KB3G int.



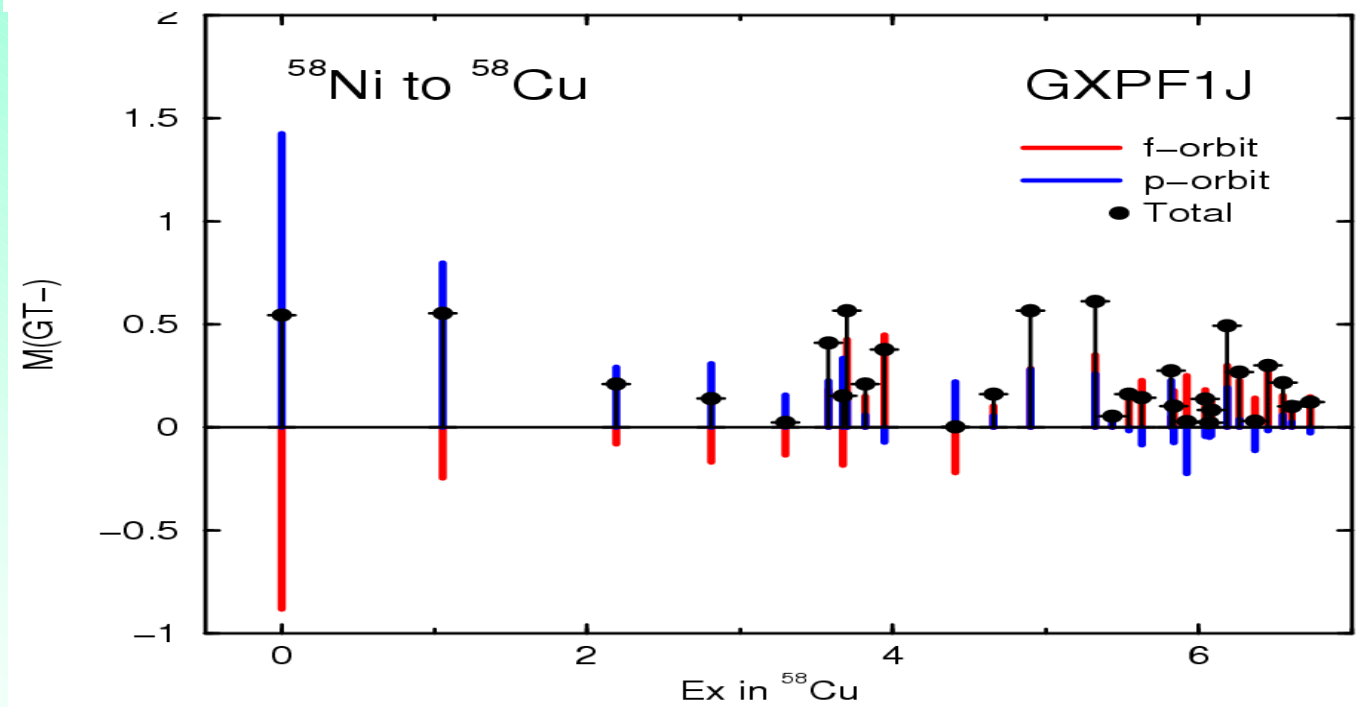
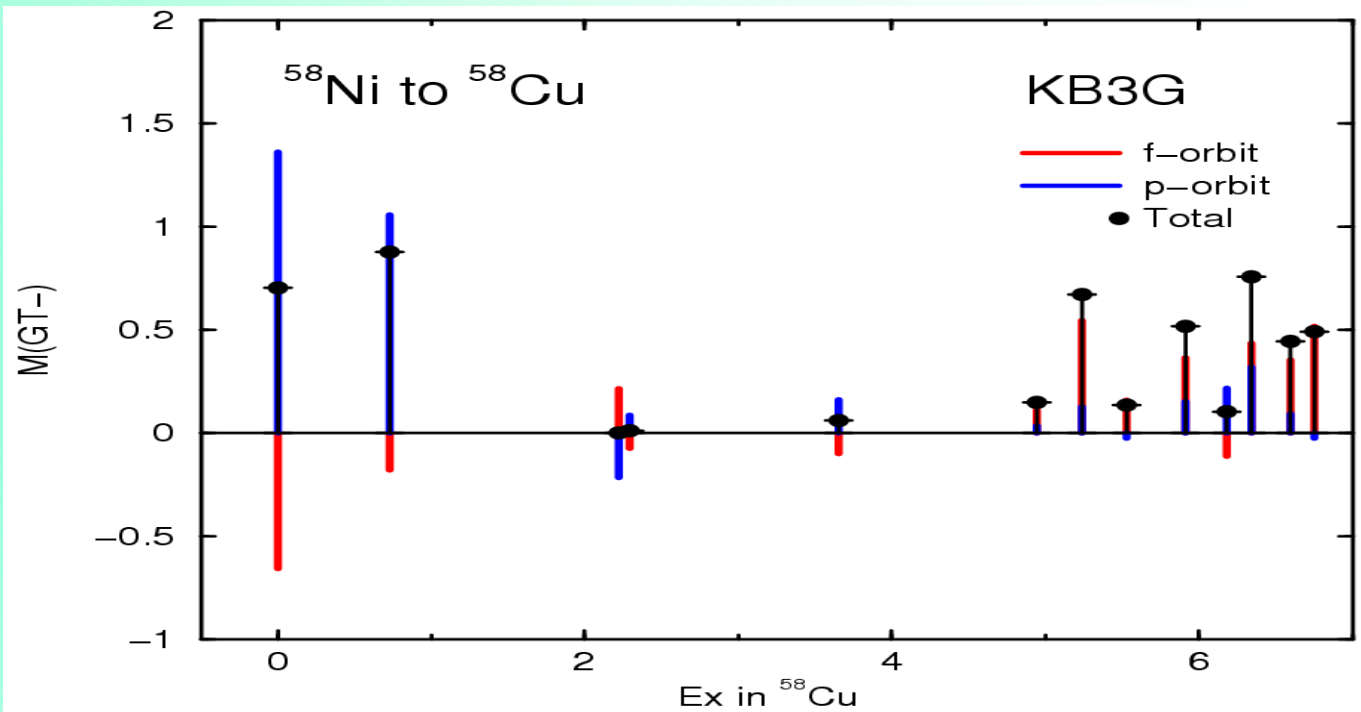
# Comparison of B(GT): Exp. & Theory

Y. Fujita et al.,  
Eur. Phys. J J 13 ('02) 411

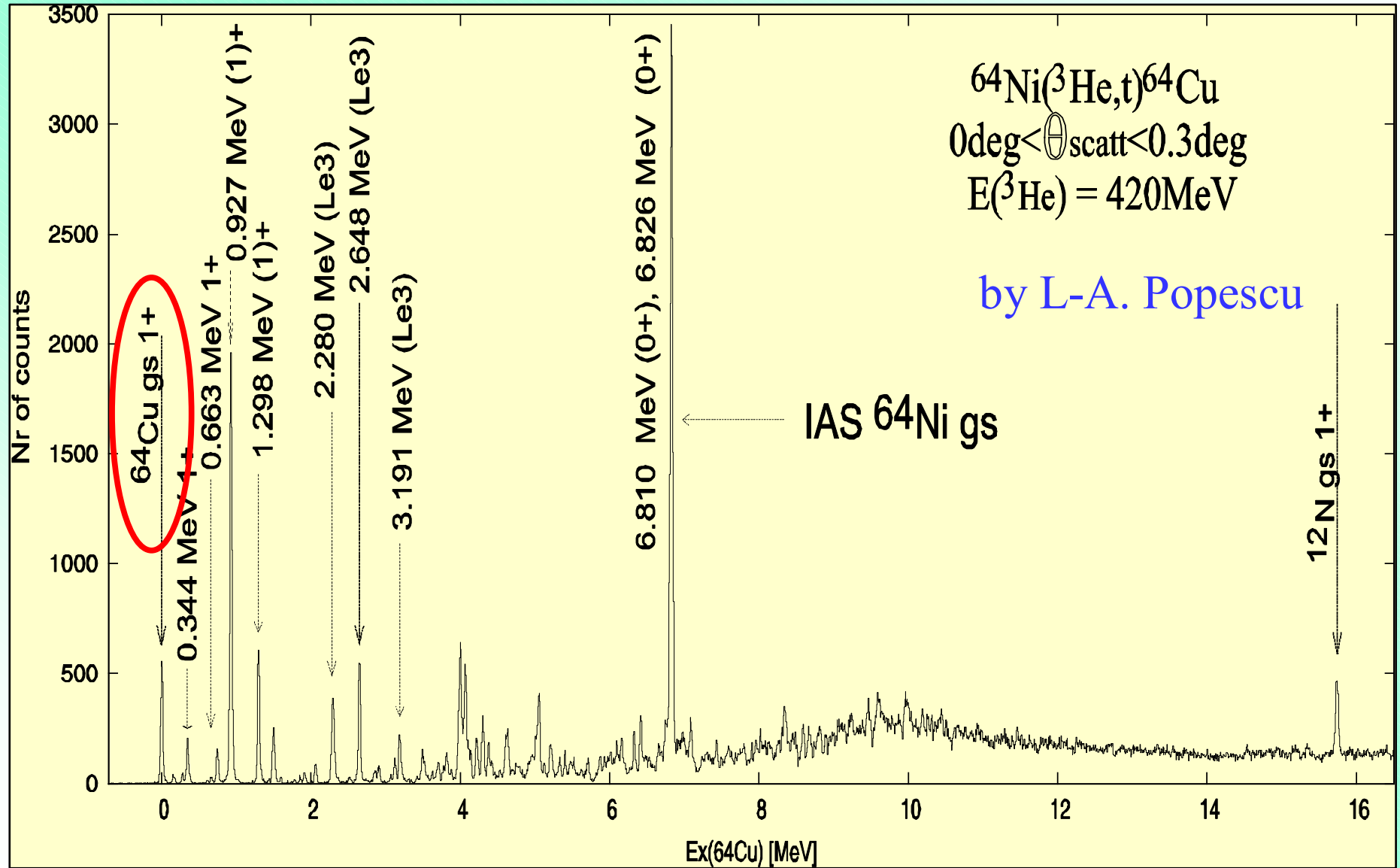


# Interaction Dependence

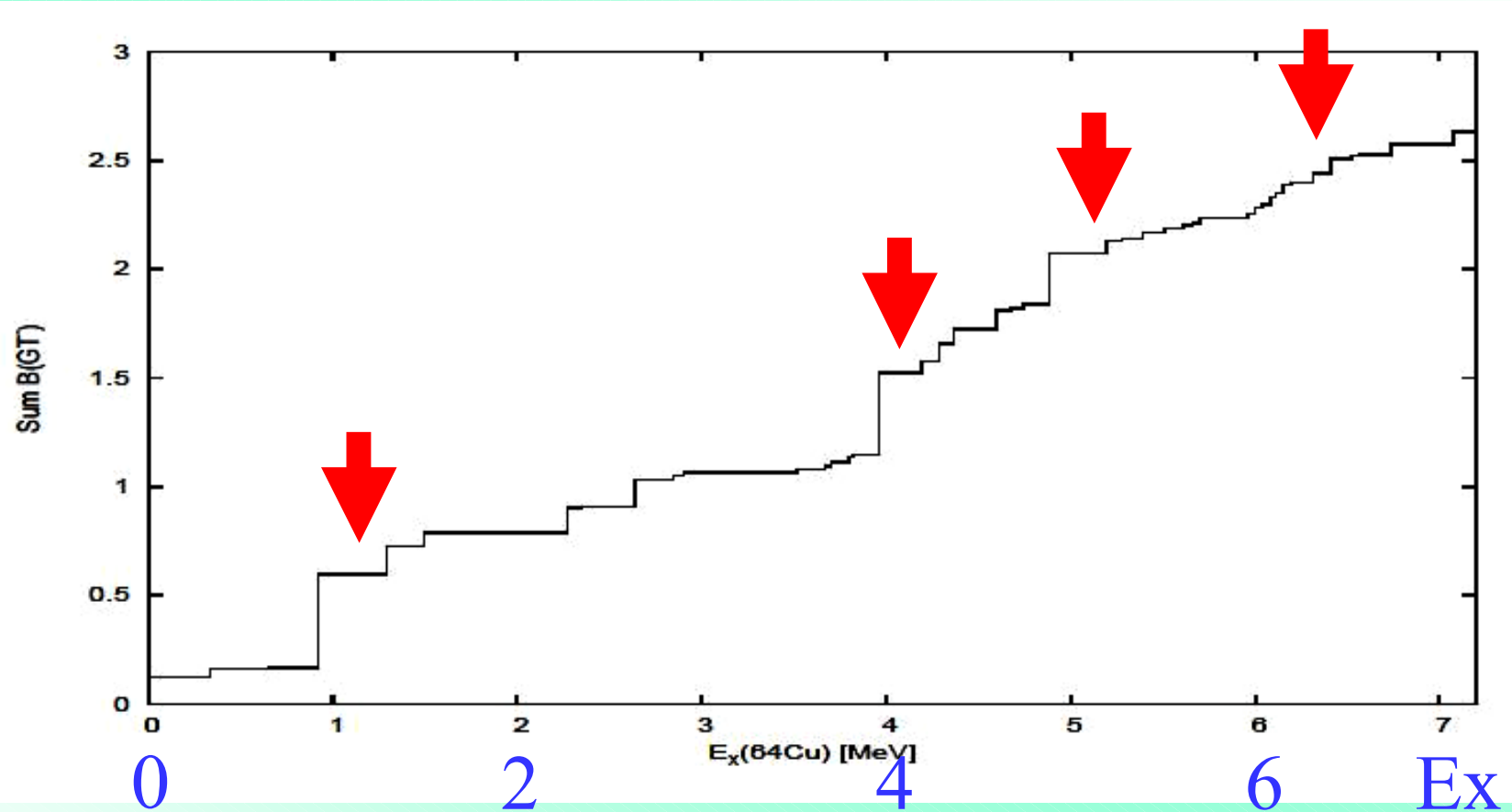
by  
M. Honma



# $^{64}\text{Ni}(^3\text{He},t)^{64}\text{Cu}$ spectrum



# B(GT) Running Sum



$$p_{3/2} \rightarrow p_{3/2}$$

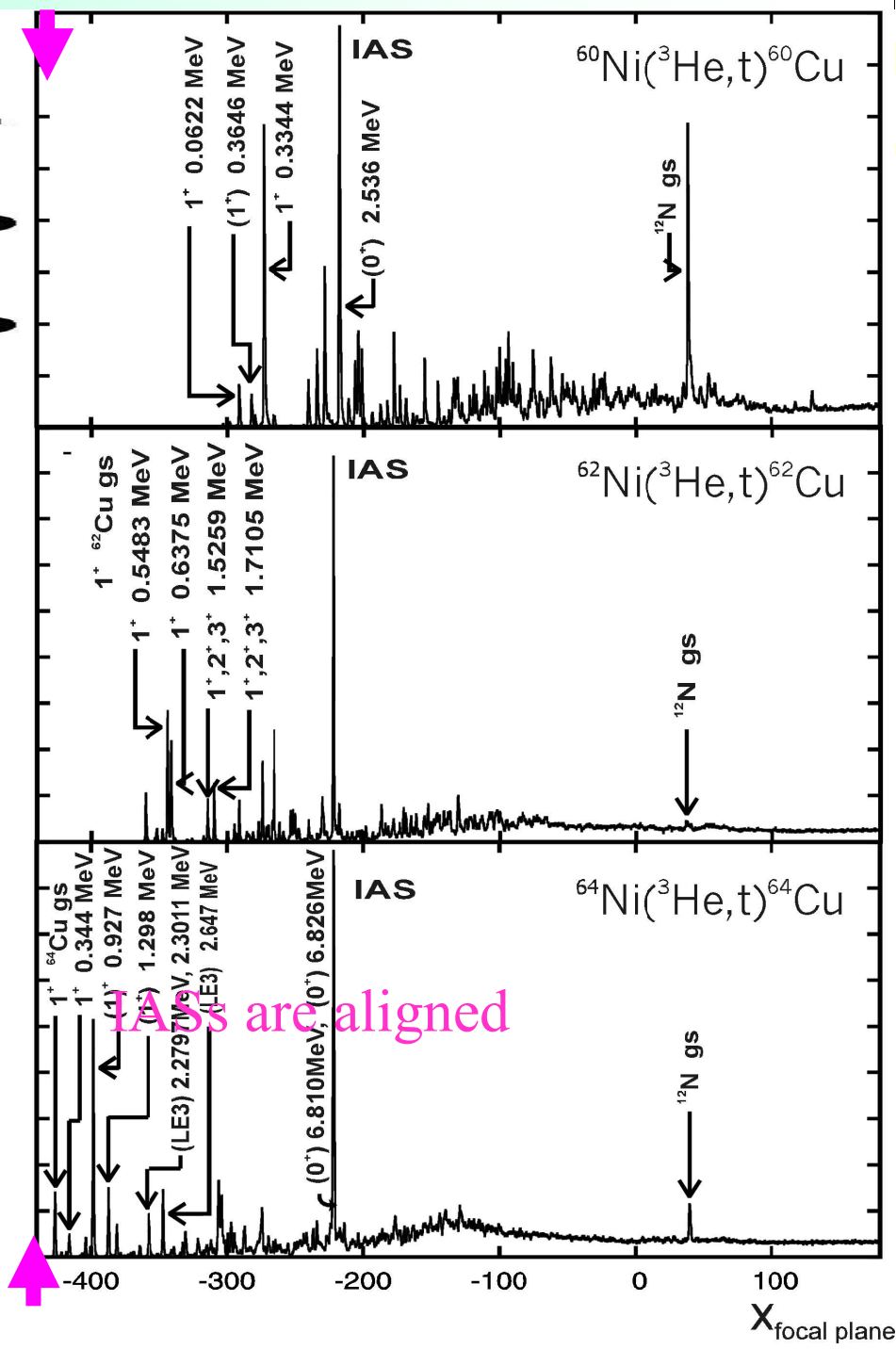
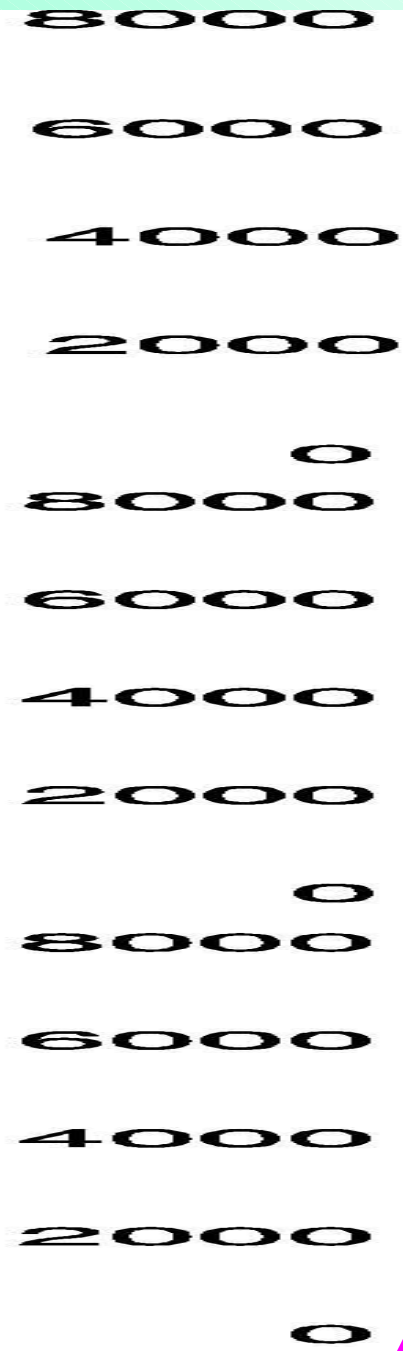
$$p_{3/2} \rightarrow p_{1/2}$$

$$f_{5/2} \rightarrow f_{5/2}$$

$$f_{7/2} \rightarrow f_{5/2}$$

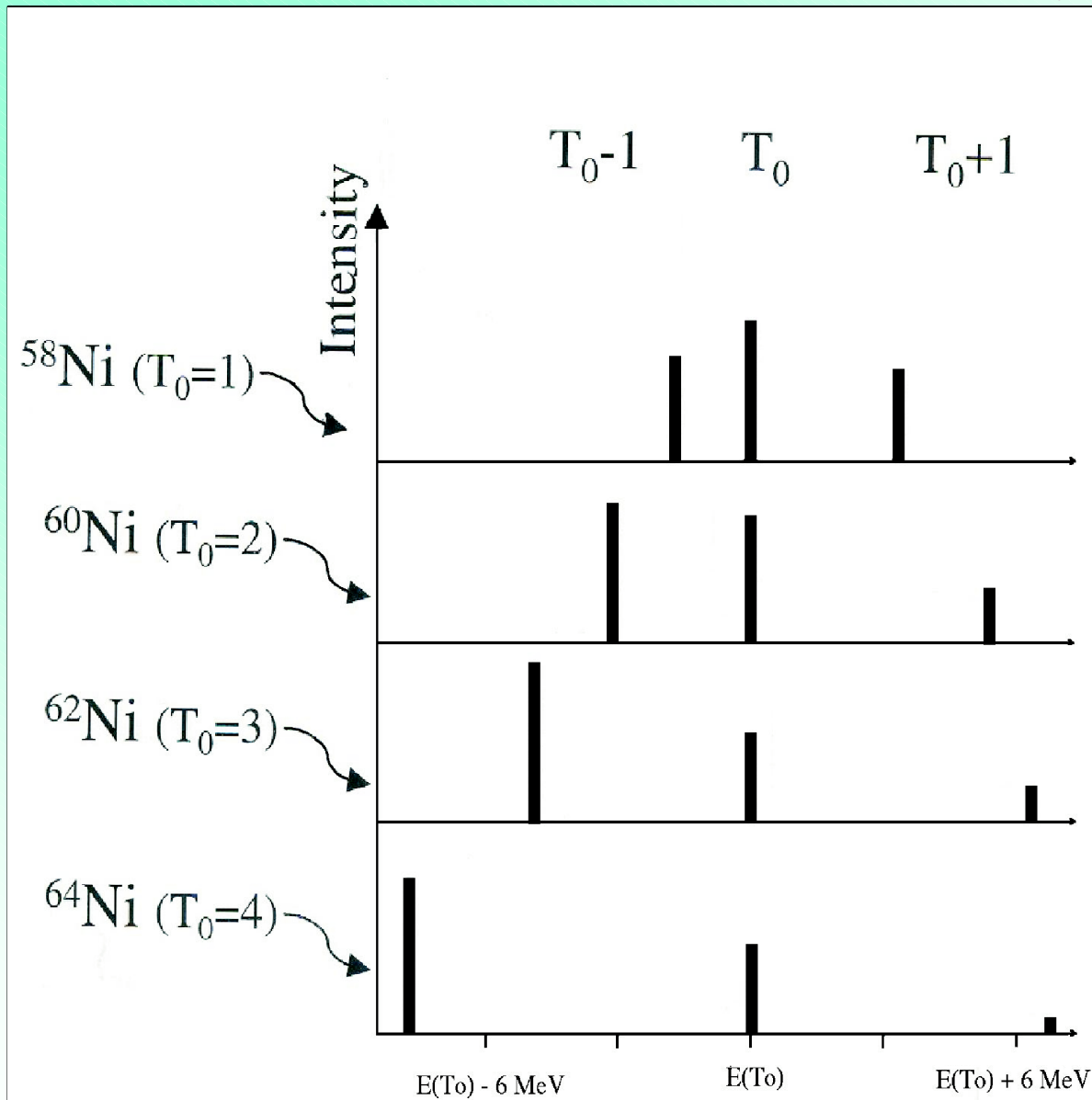
60,62,64Ni  
 (3He,t)  
 60,62,64Cu  
 Spectra

Nr. of counts



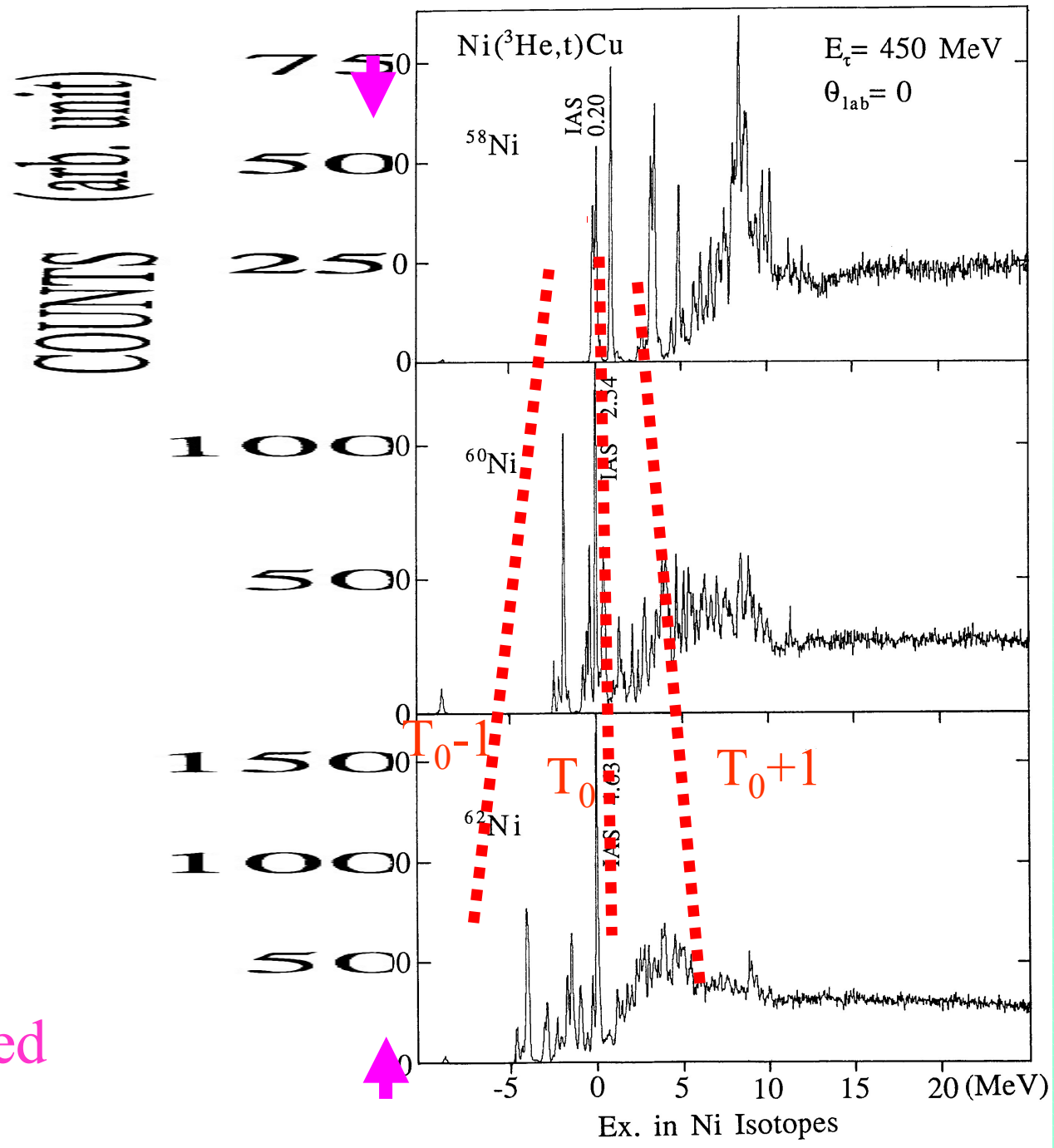
IASs are aligned

# GT transitions from Ni isotopes



Isospin  
symmetry  
energy  
&  
Ratio of  
isospin CG  
coefficients

# Ni isotopes

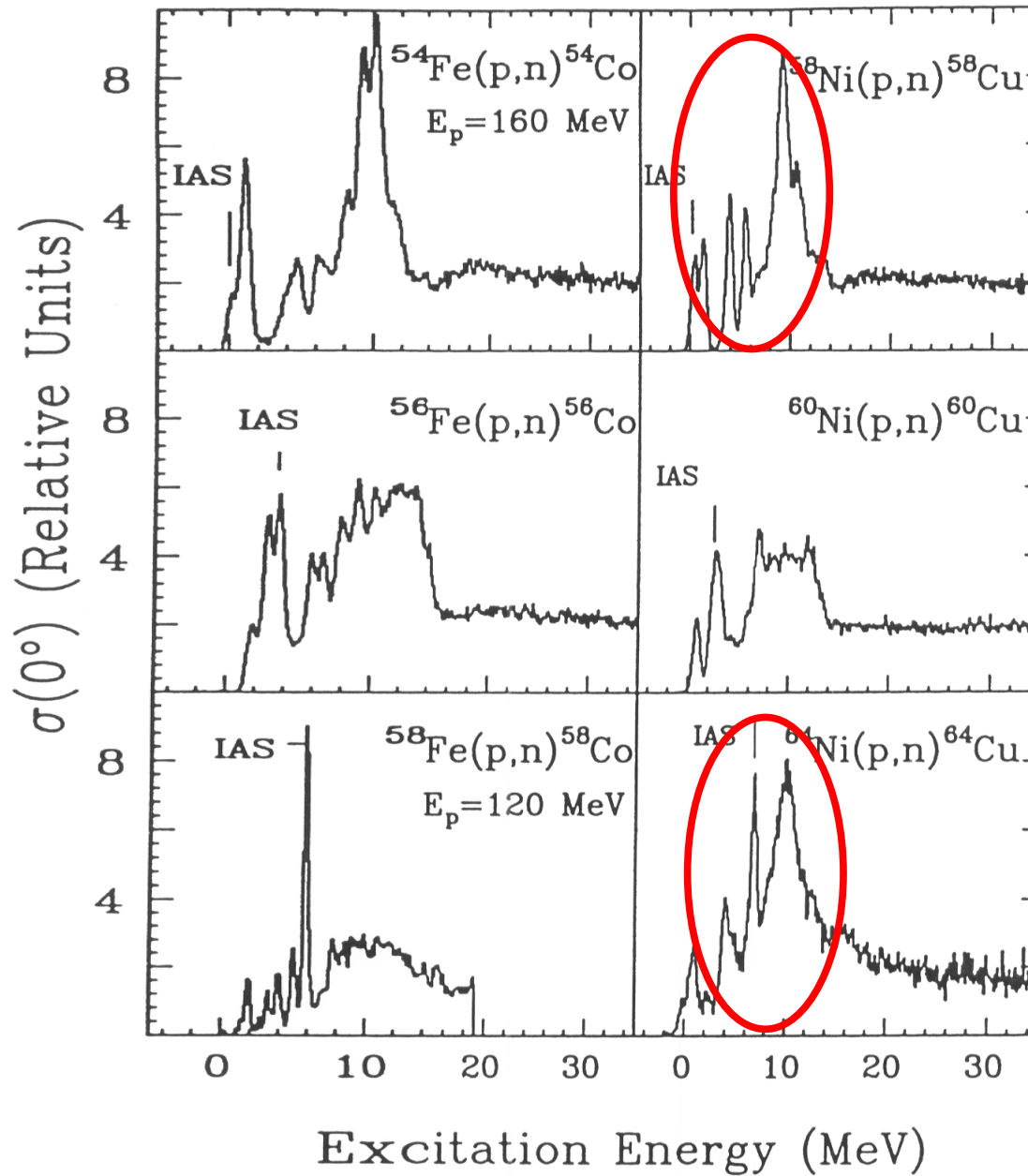


IASs are aligned





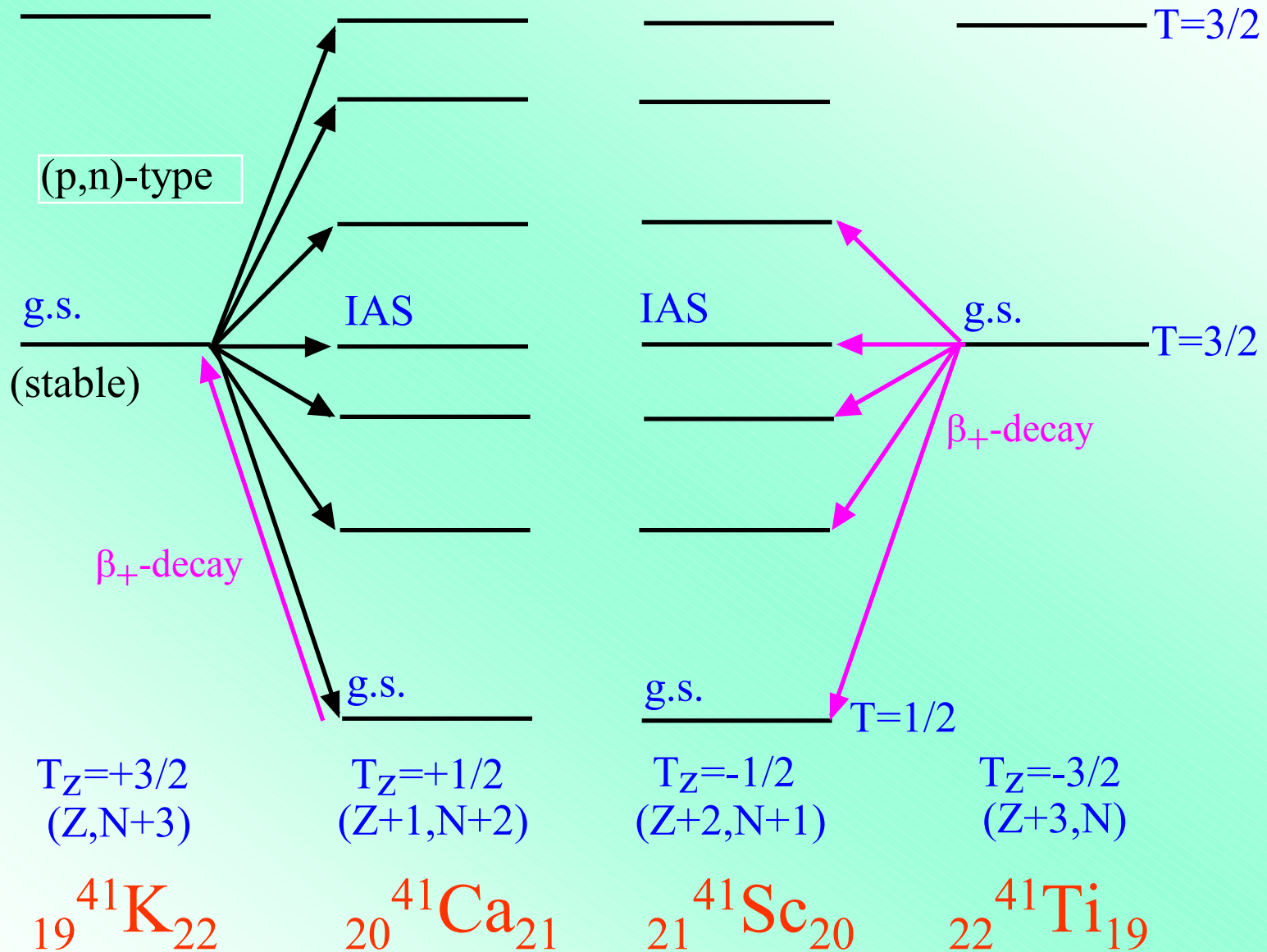
# $(p, n)$ spectra for Fe and Ni Isotopes



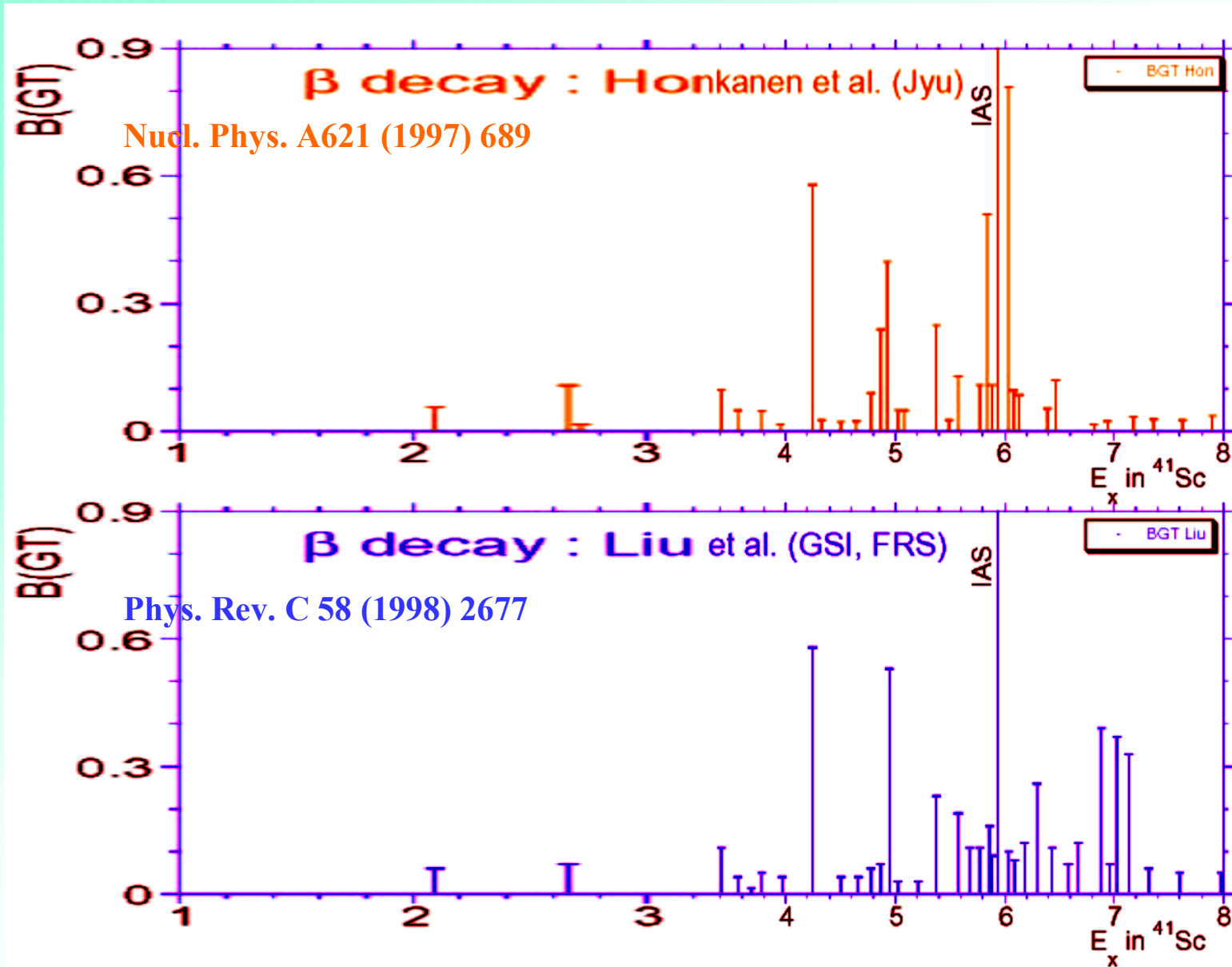
\*\*\*at the lower part of  
*fp* -shell region\*\*\*

**\*\*Critical case:  $A=41$ ,  
 $T_z=3/2 \rightarrow 1/2$  transitions**

# T=3/2 system: structures & transitions

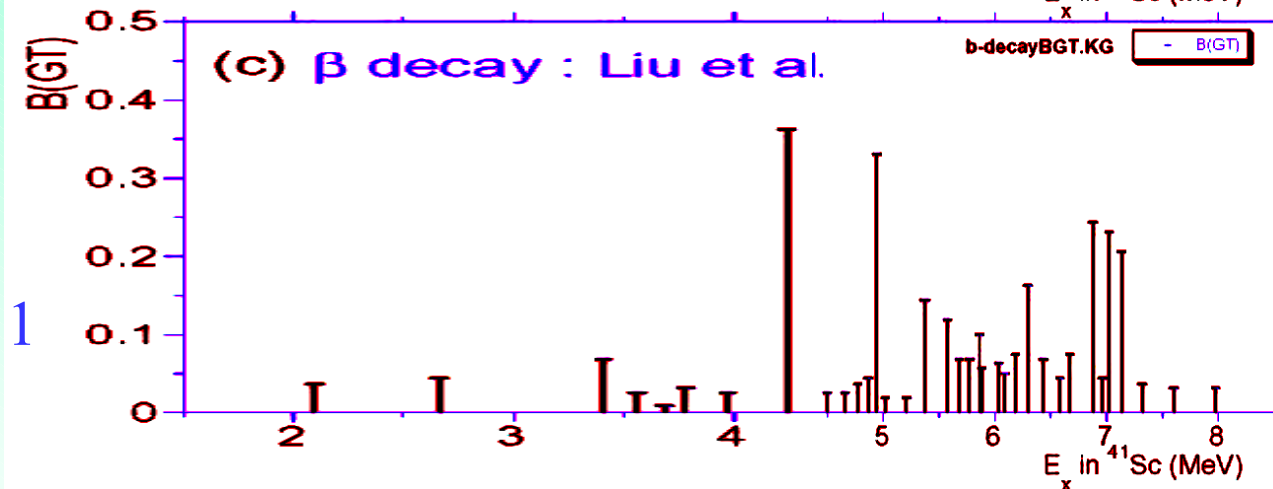
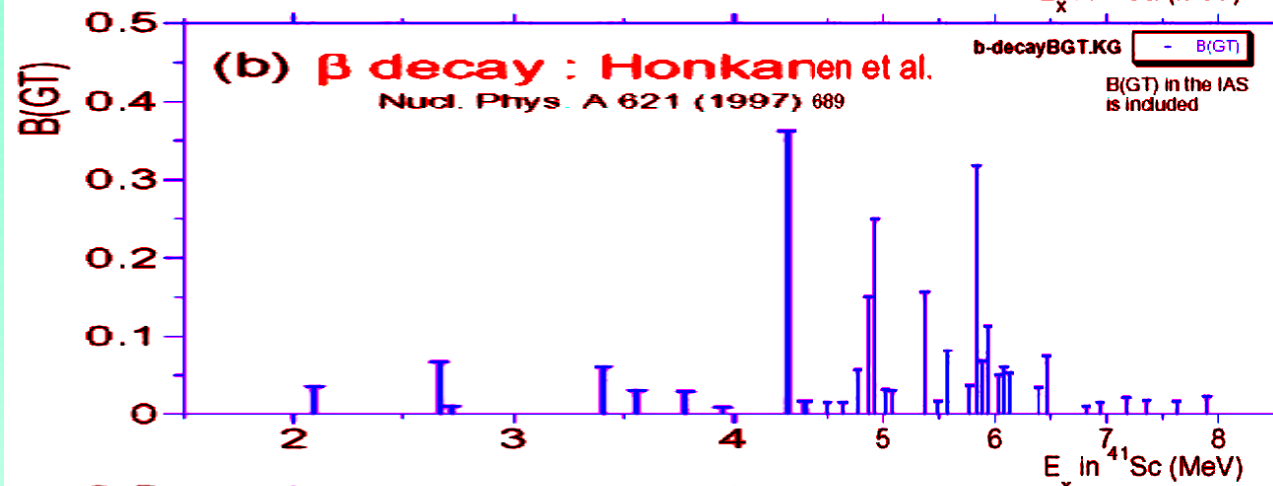
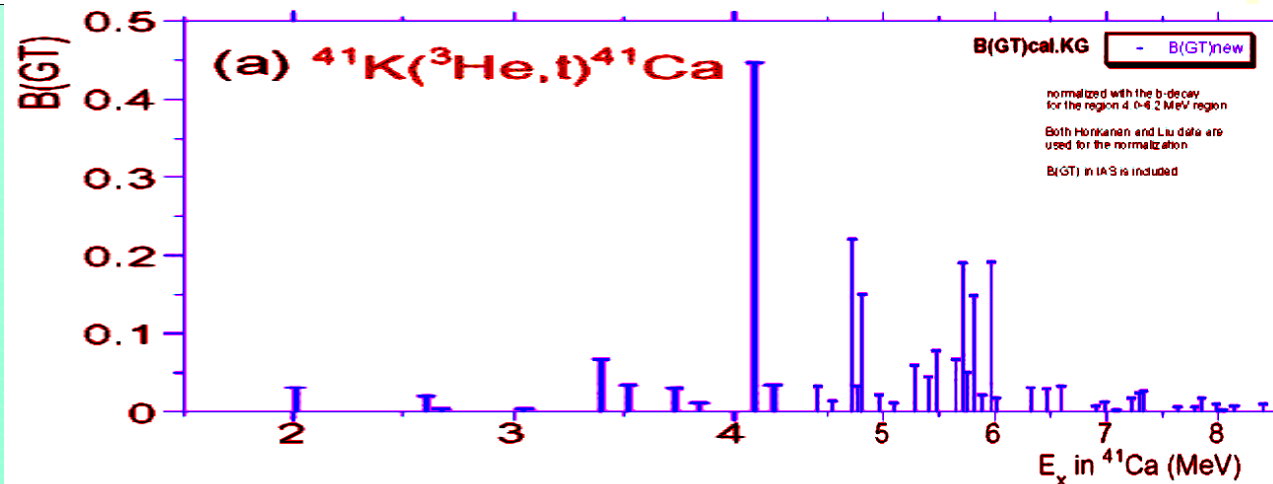


# Comparison of two $\beta$ -decay results



Comparison:  
 $^{41}\text{K}(^3\text{He},t)^{41}\text{Ca}$   
 &  
 $^{41}\text{Ti}$   $\beta$ -decay

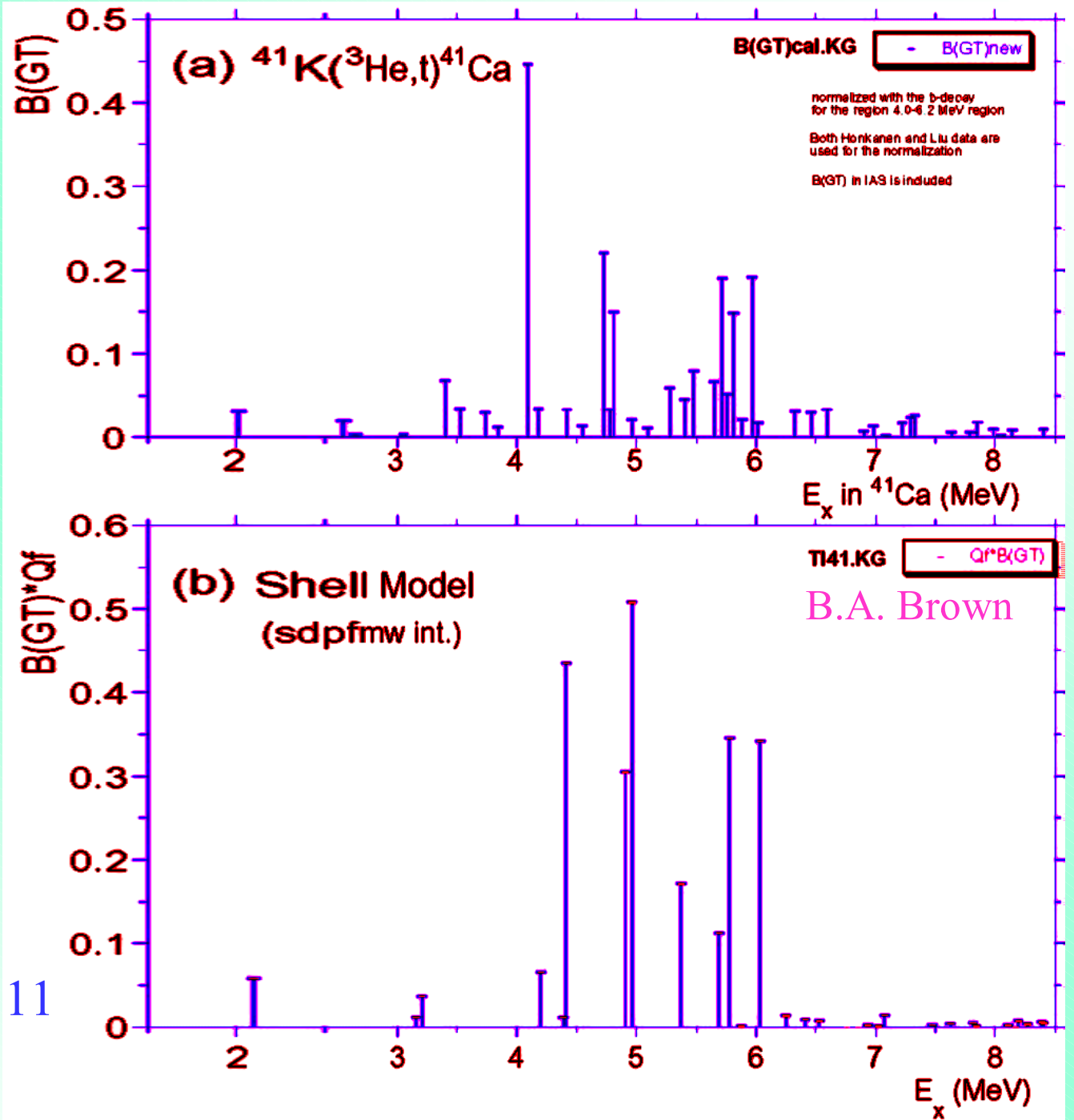
Y. Fujita et al.,  
 PRC 70 ('04) 054311



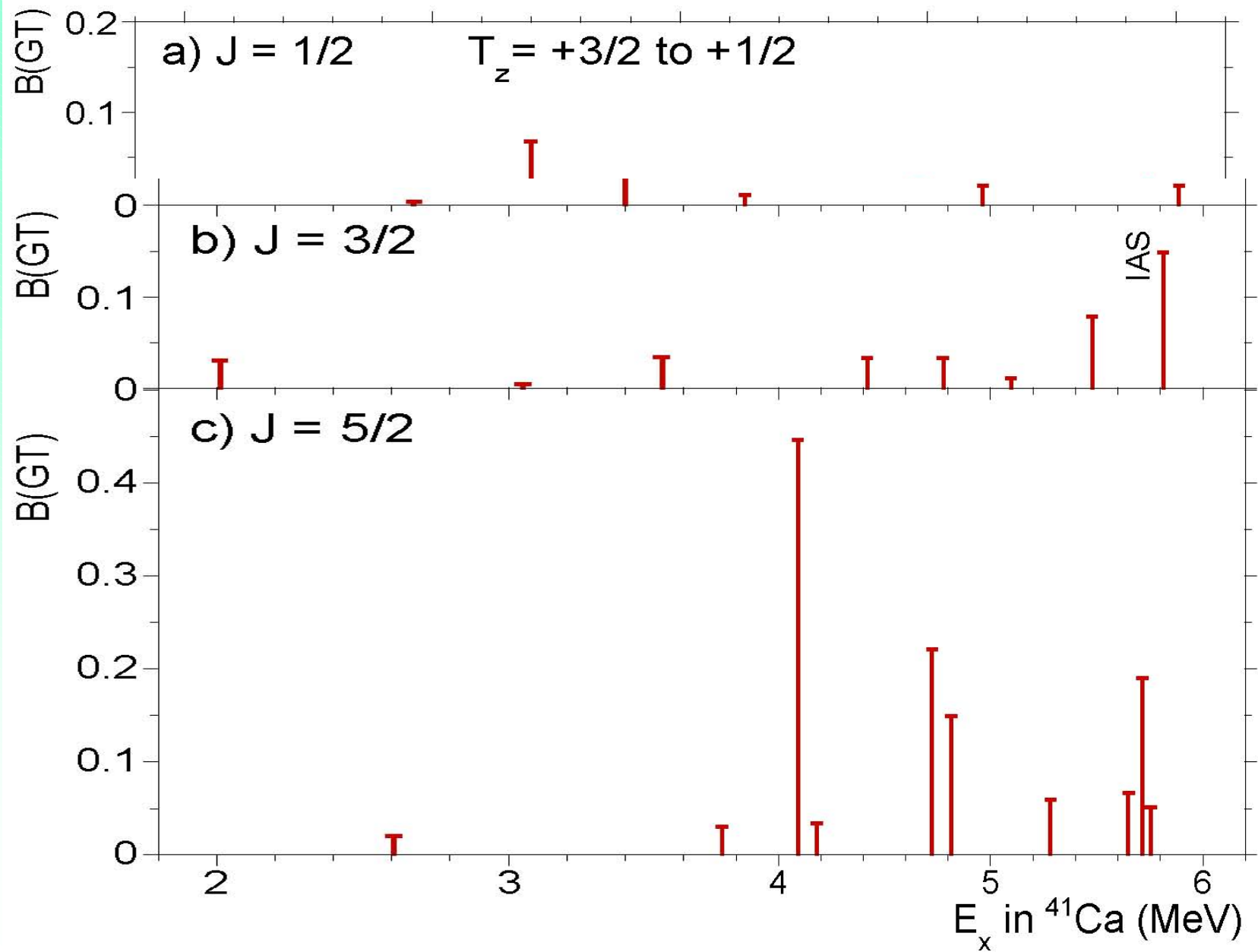


# Comparison: $^{41}\text{K}(^3\text{He},t)^{41}\text{Ca}$ & Shell Model

Y. Fujita et al.,  
 PRC 70 ('04) 054311



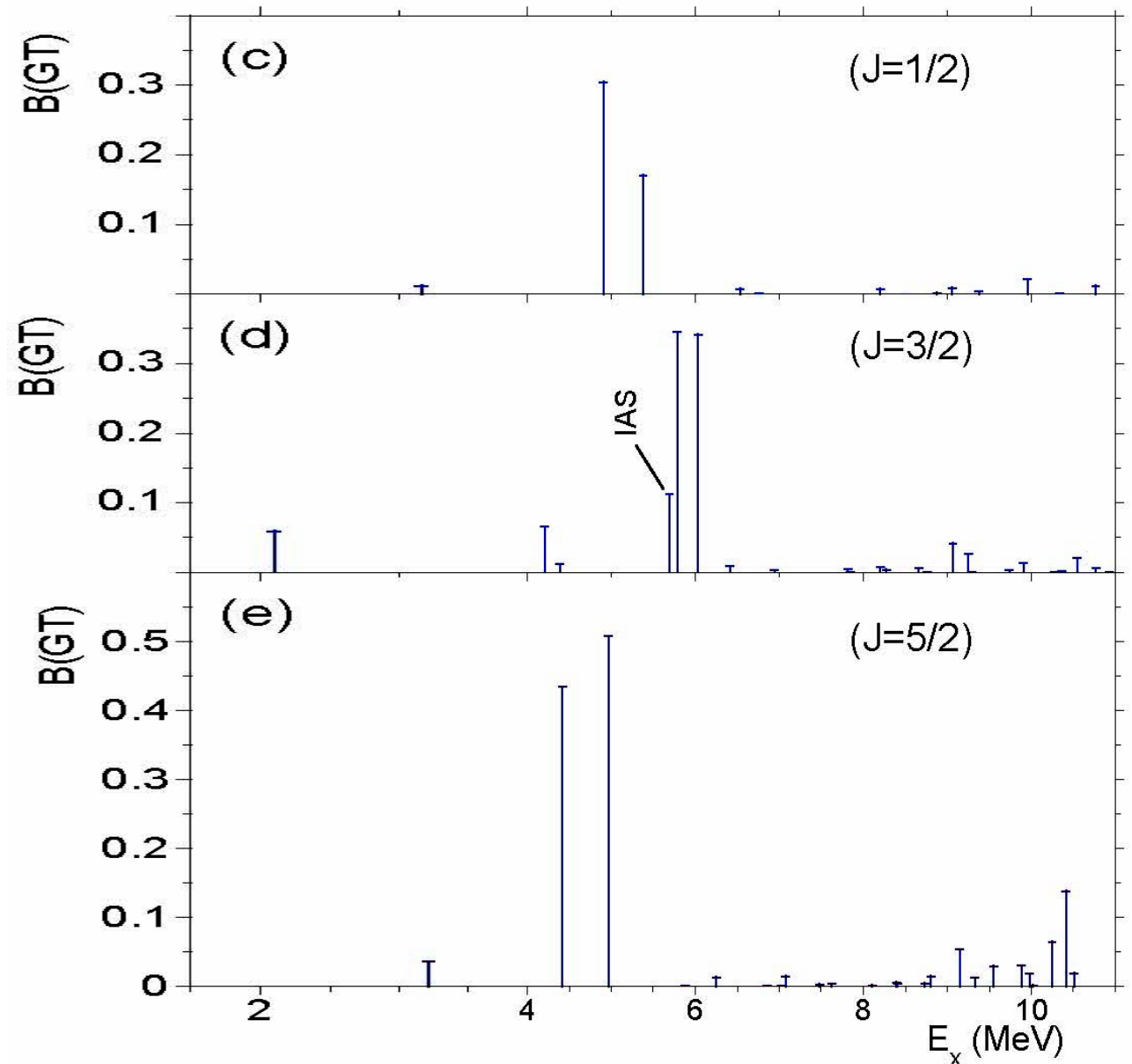
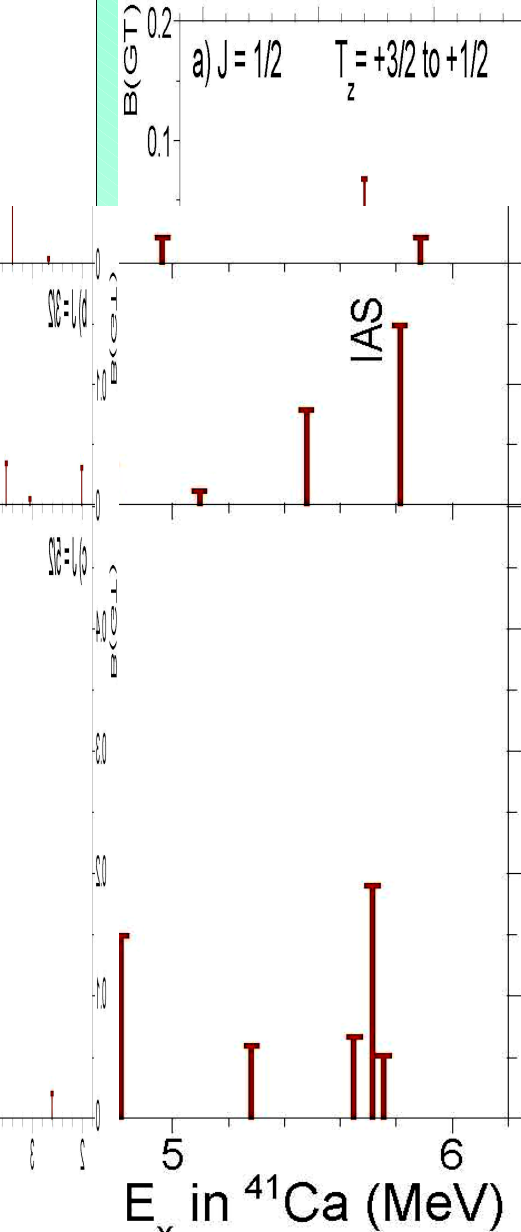
# $(^3\text{He},t) B(\text{GT}) : J = 1/2, 3/2, 5/2$



# Exp. & SM $B(GT)$ : $J = 1/2, 3/2, 5/2$

Exp.

SM cal.

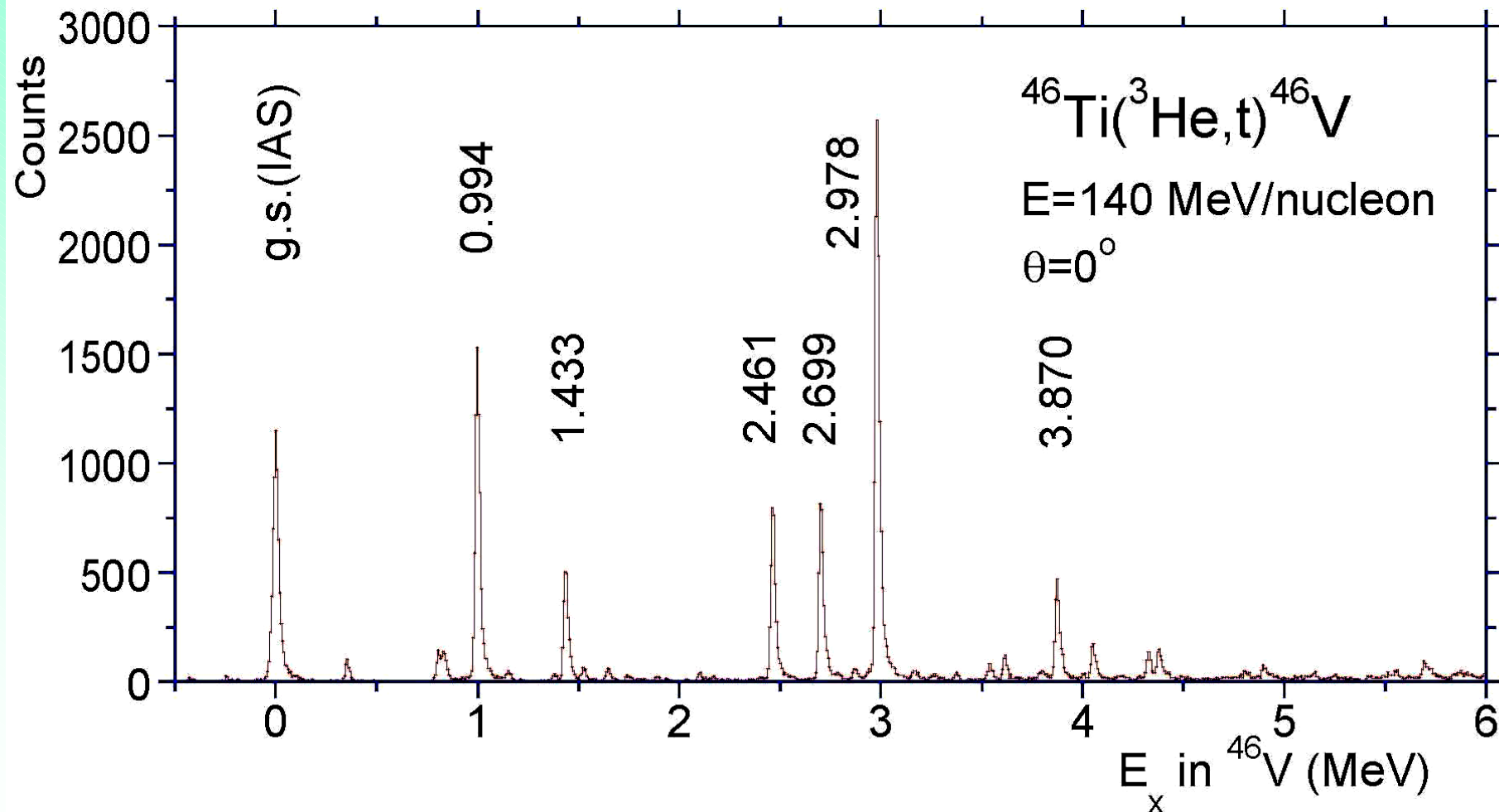


\*\*  $A=46$ ,  $T=0$ , 1 system

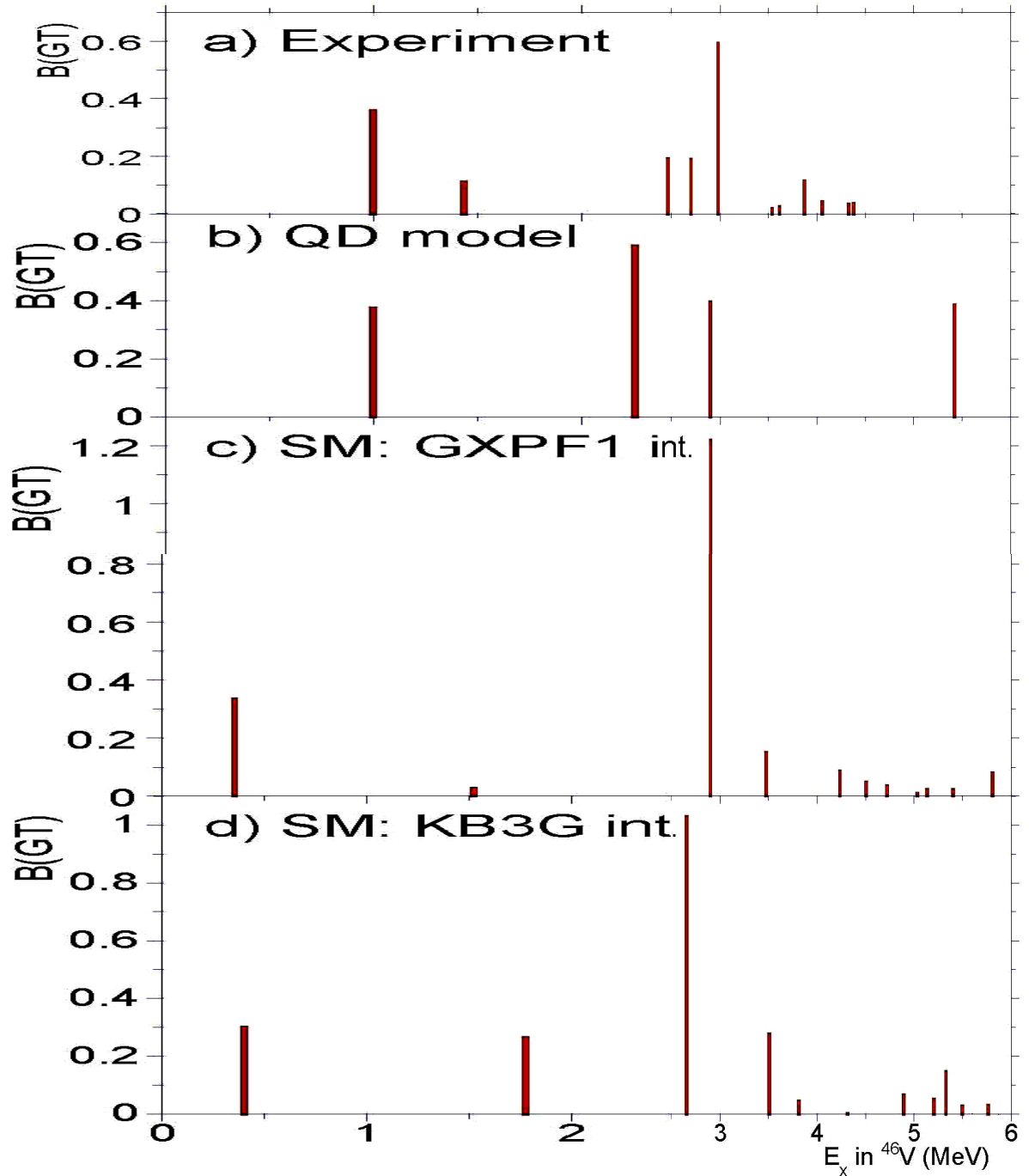
# $^{46}\text{Ti}(^3\text{He},t)$

T. Adachi et al.

PRC in press

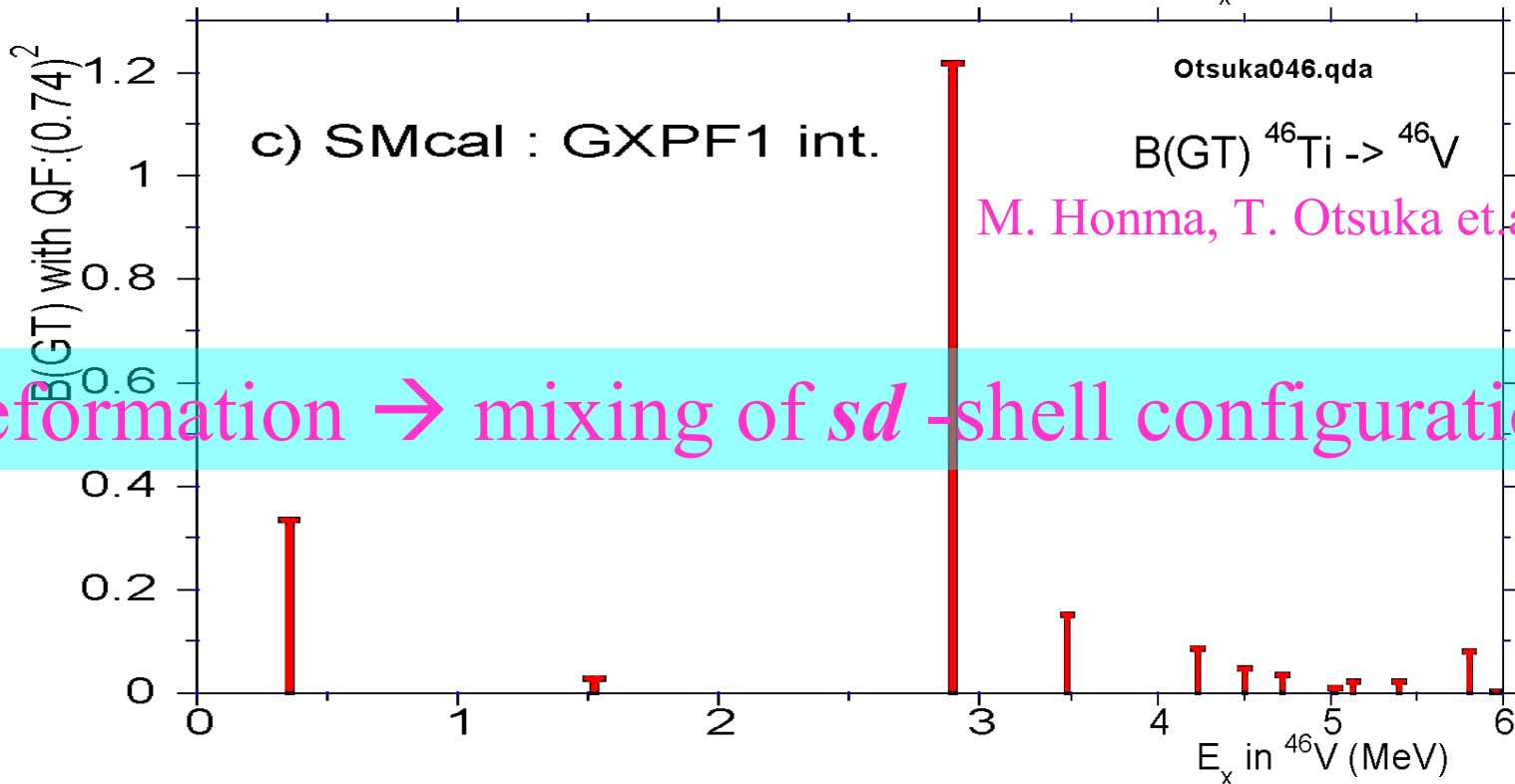
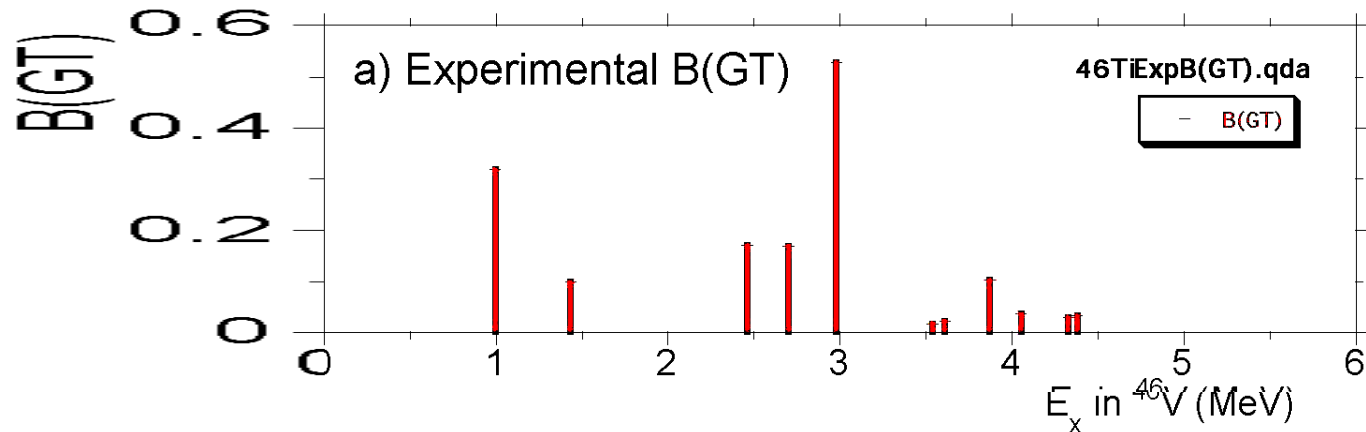


# B(GT) Experiment vs Calculations





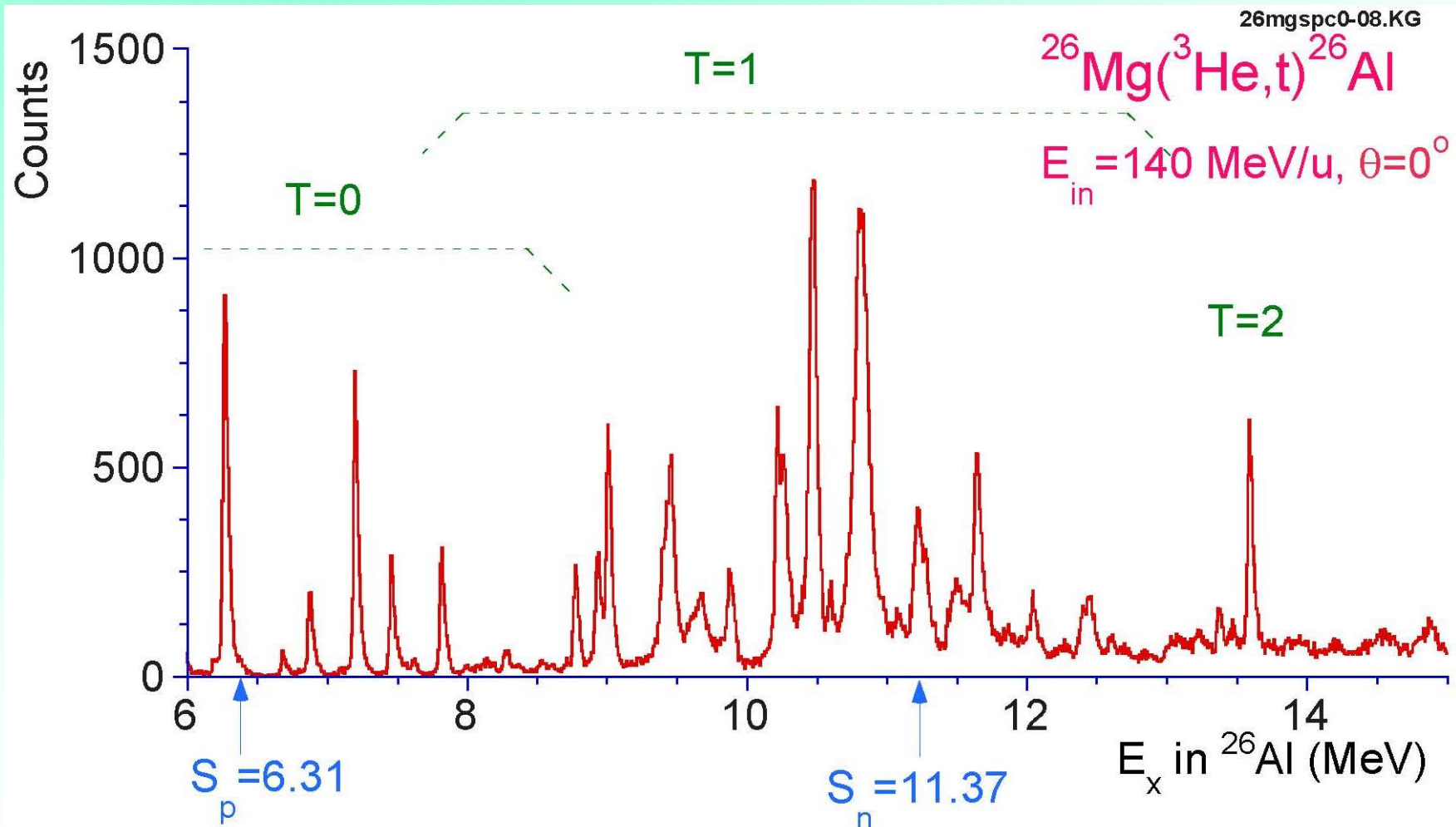
# B(GT) : Exp. vs. SM



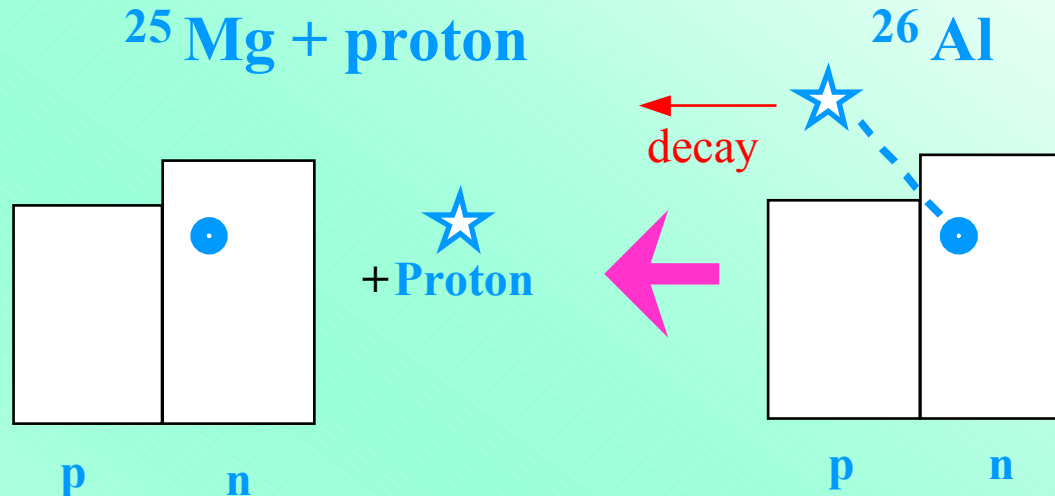
deformation  $\rightarrow$  mixing of *sd*-shell configurations

**\*\*"Width" represents a lot !**

# $^{26}\text{Mg}(^3\text{He},t)$ spectra



# Importance of Isospin : $p$ -decay of $^{26}\text{Al}$



$$T_Z : 1/2 + (-1/2) = 0$$

$$T : 1/2 + 1/2 = 0 \text{ or } 1$$

$$3/2 + 1/2 = 1 \text{ or } 2$$

# $S_p$  (p-sep. energy) in  $^{26}\text{Al}$  : 6.31 MeV

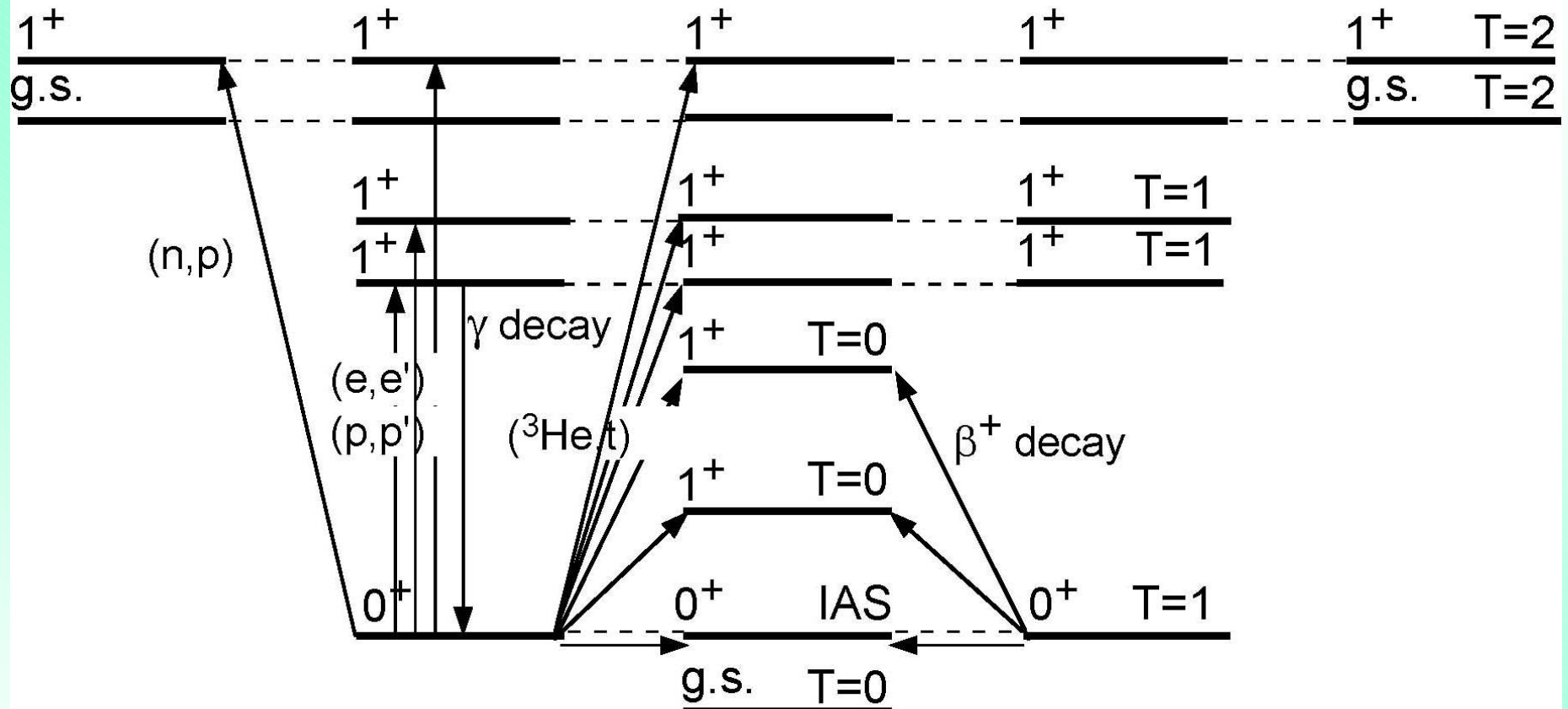
# $T=3/2$  state in  $^{25}\text{Mg}$  :  $E_X > 7.79$  MeV

➔ effective  $S_p$  in  $^{26}\text{Al}$

for  $T = 0, 1$  states :  $E_X = 6.31$  MeV

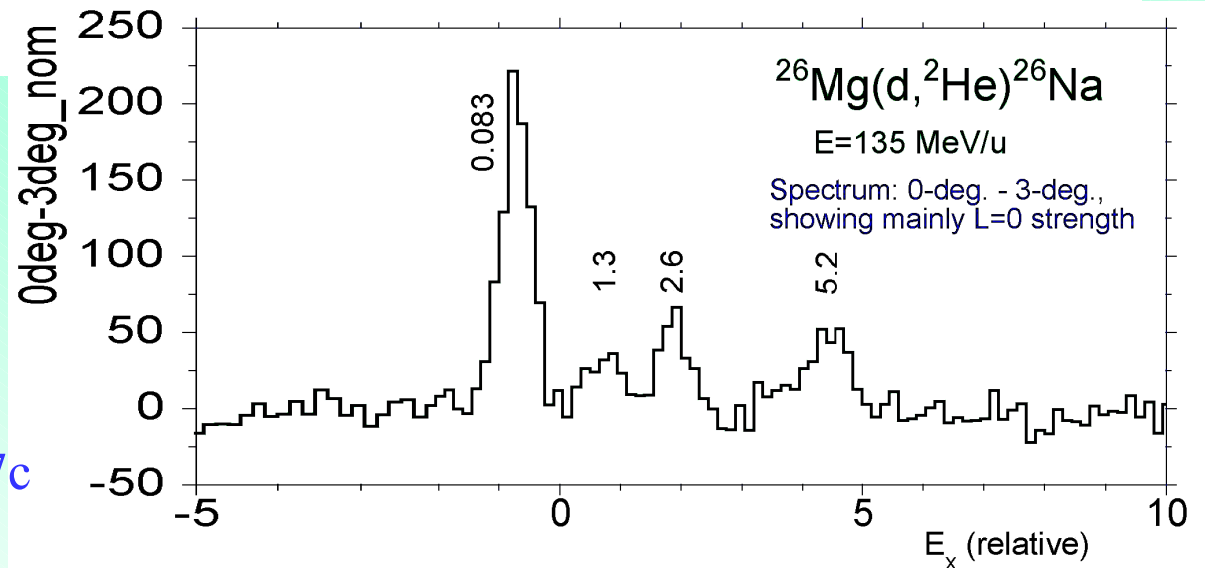
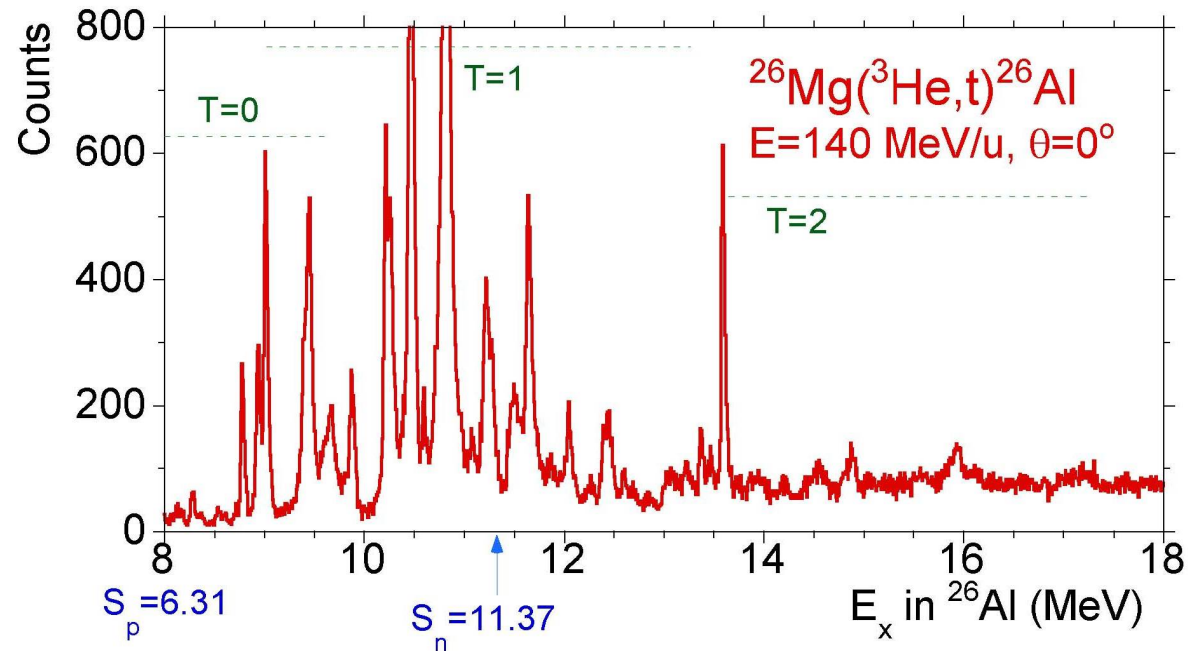
for  $T = 2$  states :  $E_X = 14.1$  MeV

# Analog Relationship ( $T=0,1,2$ )



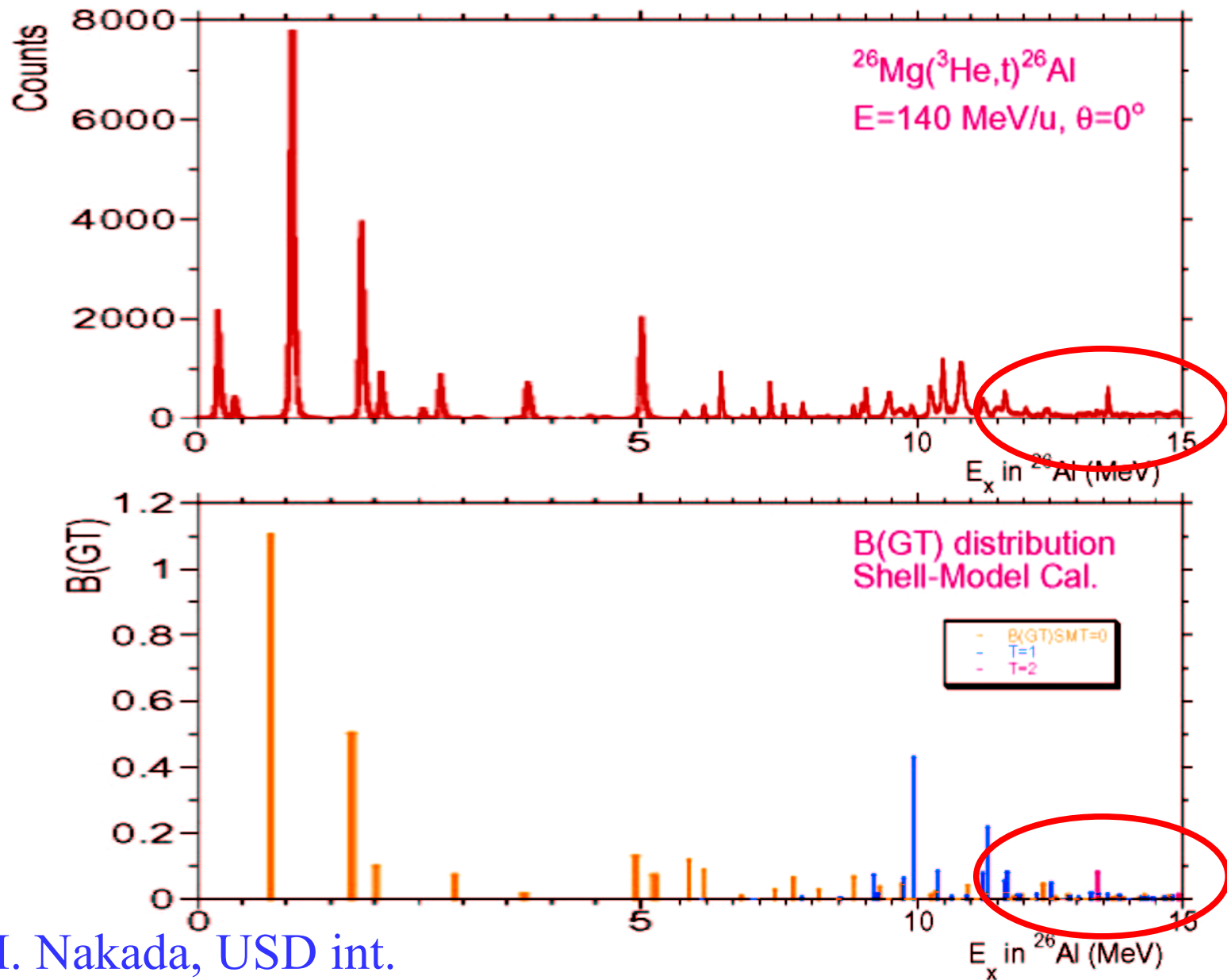
$^{26}\text{Na}$	$^{26}\text{Mg}$	$^{26}\text{Al}$	$^{26}\text{Si}$	$^{26}\text{P}$
$^{58}\text{Co}$	$^{58}\text{Ni}$	$^{58}\text{Cu}$	$^{58}\text{Zn}$	$^{58}\text{Ga}$
$T_z = +2$	$T_z = +1$	$T_z = 0$	$T_z = -1$	$T_z = -2$

# Comparison: $^{26}\text{Mg}(^3\text{He},t)^{26}\text{Al}$ and $^{26}\text{Mg}(d,^2\text{He})^{26}\text{Na}$



T. Niizeki et al.,  
 N.P. A577 (1994) 37c

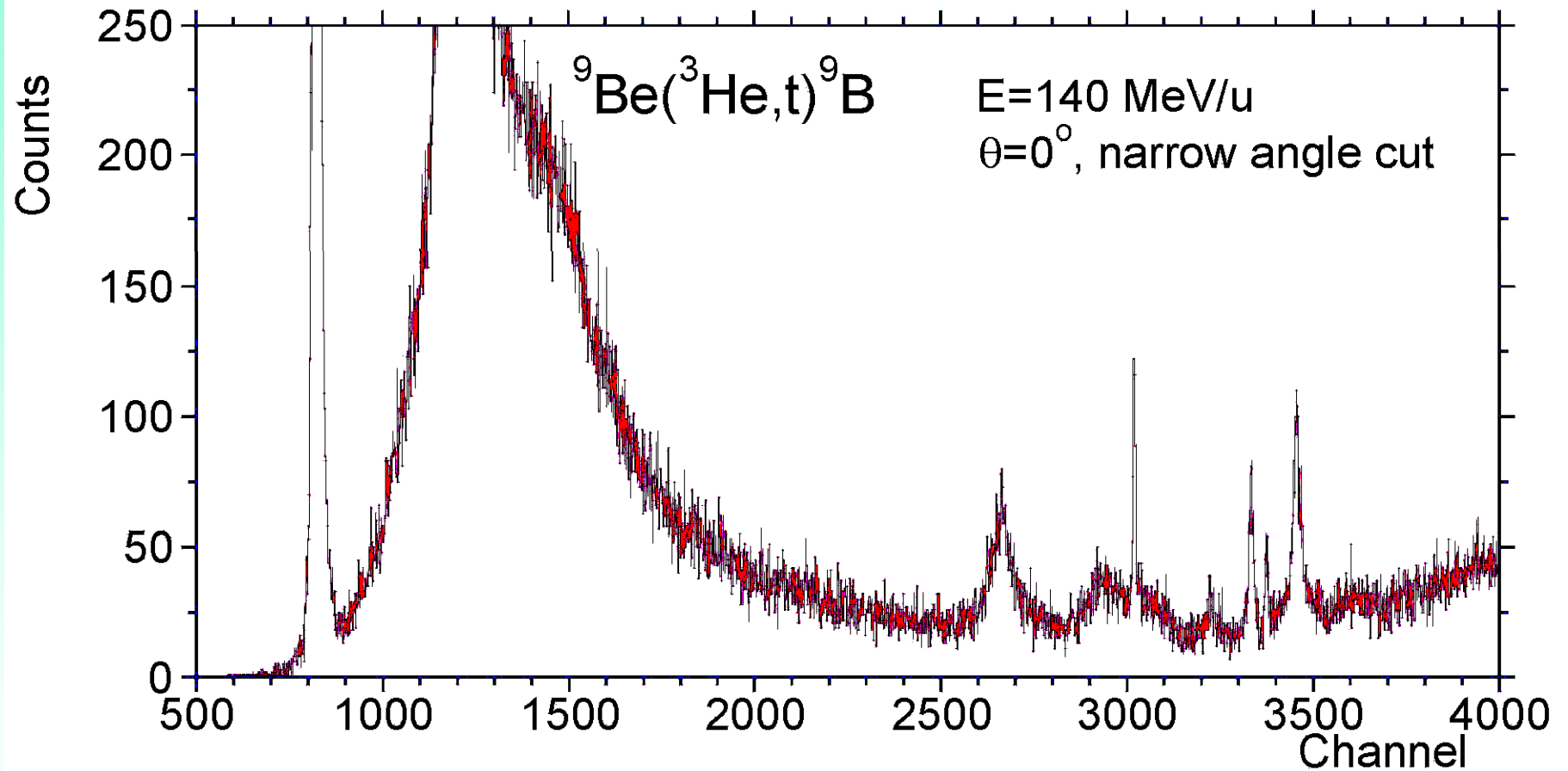
# $^{26}\text{Mg}(^3\text{He},t)$ spectrum and SM cal. B(GT)



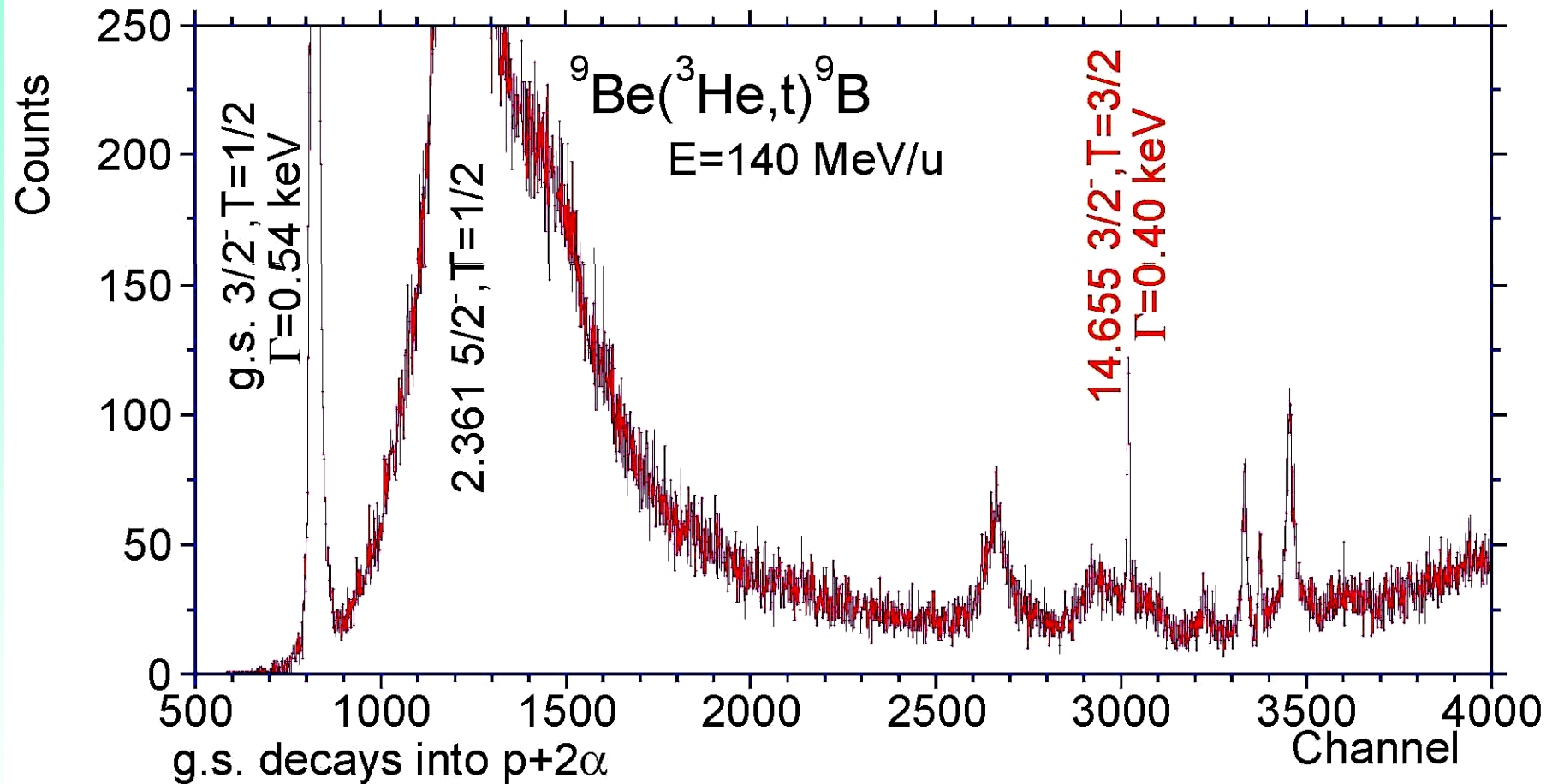
by H. Nakada, USD int.



# ${}^9\text{Be}({}^3\text{He},t){}^9\text{B}$ spectrum (at various scales)



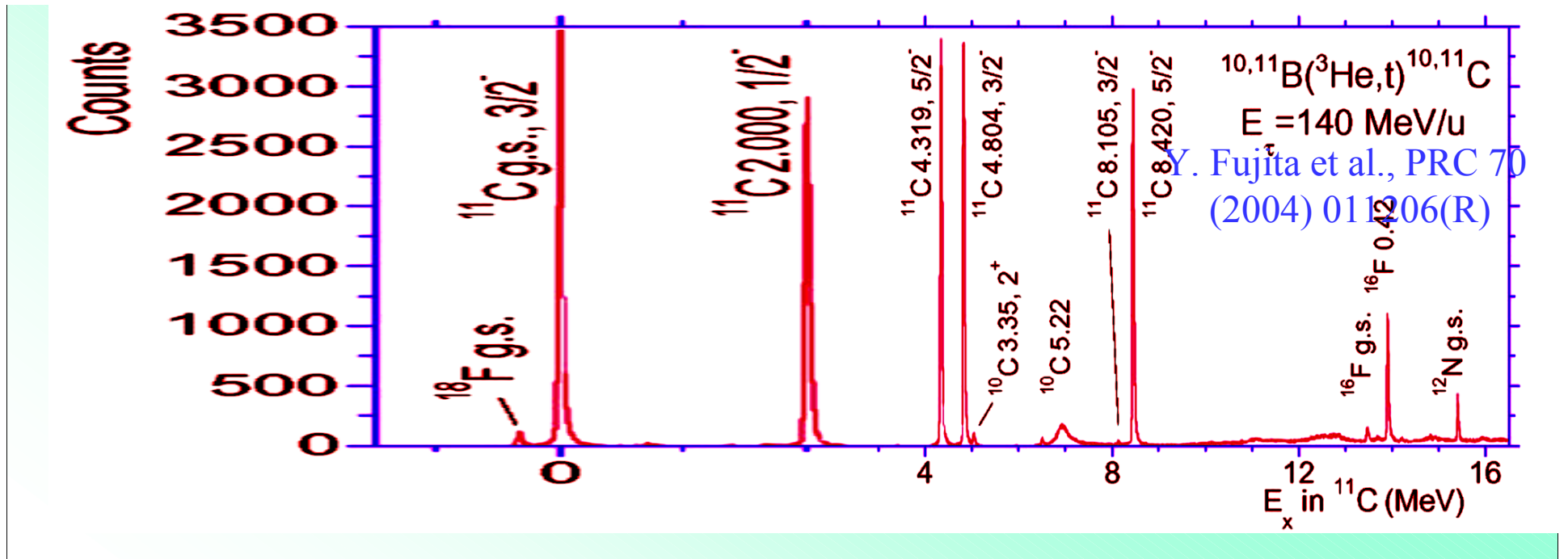
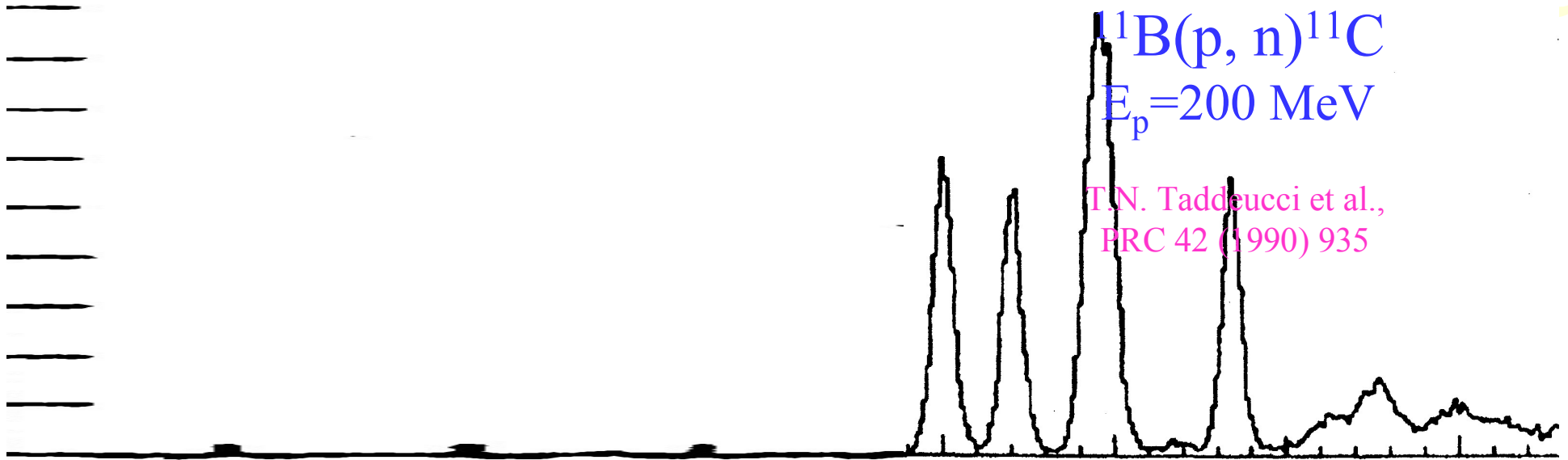
# ${}^9\text{Be}({}^3\text{He},t){}^9\text{B}$ spectrum (II)



**Isospin selection rule prohibits  
proton decay of  $T=3/2$  state!**

**\*\*Limitation of the Space**

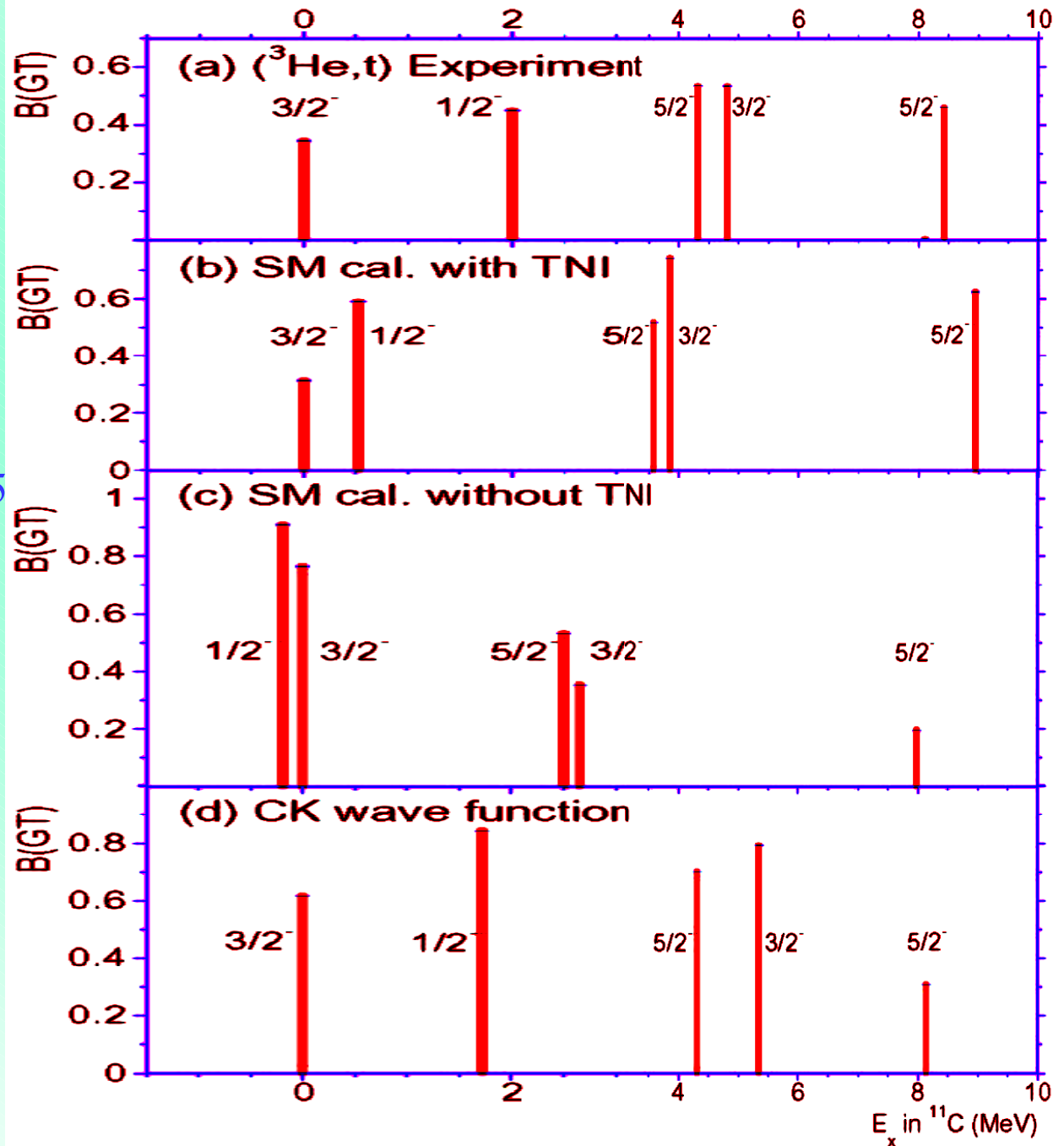
# (p,n) and ( $^3\text{He},t$ ) Spectra on $^{11}\text{B}$



# Comparison: $^{11}\text{B}(^3\text{He},t)^{11}\text{C}$ & Shell Models

non-core SMcal:  
 By Navratil & Ormand  
 Phys. Rev. C 68 ('03)034305

Y. Fujita et al., PRC 70  
 (2004) 011206(R)



# Summary

\*Observed strengths are well reproduced  
(by including the recommended  
“quenching factor”)

\*Fragmentation of strengths:

A=23, 26 *very good !*

A=28 *not so well*

A=41 *sd & f should be included*

A=46 *deformed: fp and sd are needed*

A=58 *GXPf1 reproduced well !*

\*Widths of states: more sophisticated !?