Shell model approach to $N \approx Z$ medium-heavy nuclei using extended P+QQ interaction

M. Hasegawa

Topics:

Structure of $N \approx Z$ medium-heavy nuclei,

Large-scale shell model calculations,

Extended P+QQ interaction (EPQQ).

based on the collaboration

with K. Kaneko and T. Mizusaki

1. At the end of the 20th century

--- Shell model study of $f_{7/2}$-shell nuclei ---

SM had succeeded for sd shell nuclei
using *realistic* effective interaction USD

B.A. Brown & B.H. Wildenthal, Ann. Rev. Nucl. Part. Sci., 38(1988), 29

- *realistic* effective interaction for pf shell

  KB3 A. Poves & A.P. Zuker, Phys. Rep., 70(1981), 235

  FPD6 W.A. Richter, … B.A. Brown, Nucl. Phys. 523(1991), 325

- Caurier's calculation code

  → full pf shell model calculations
  for $A = 48$, odd nuclei $A = 47{,}49$, …

  E. Caurier, A.P. Zuker, A. Poves, G. M., Phys. Rev. C, 50(1994), 225

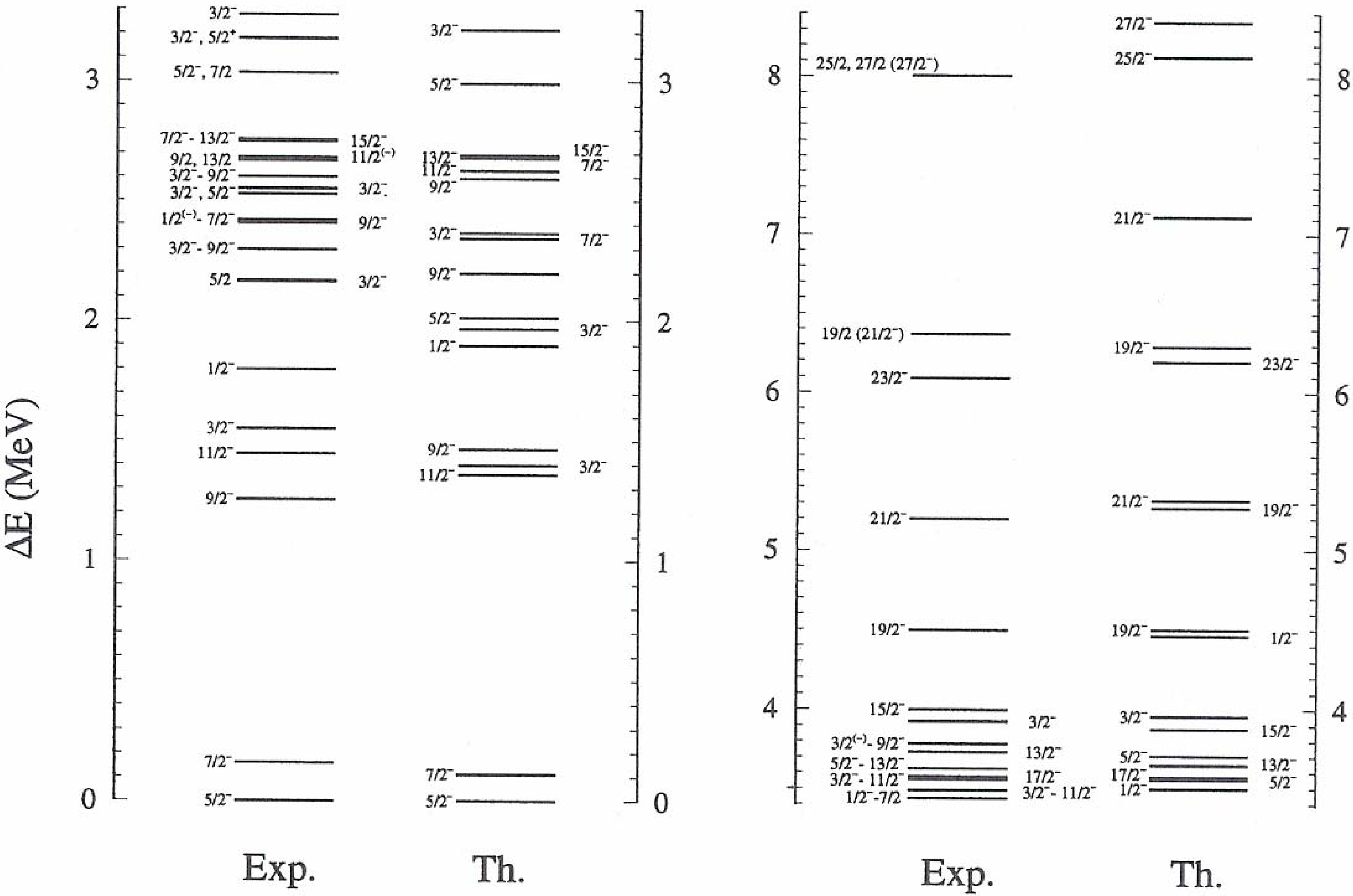


FIG. 3. Theoretical and experimental energy levels of ^{47}Ti.

G. Martinez-Pinedo, A. Zuker, A. Poves, E. Caurier, Phys. Rev. C 55(1997), 187.

Better effective interaction was required.
for heavier pf shell nuclei
$\rightarrow$ KB3G, applied to $A = 50, 51, 52$

....

A. Poves, et al.,nucl-th/0012077.

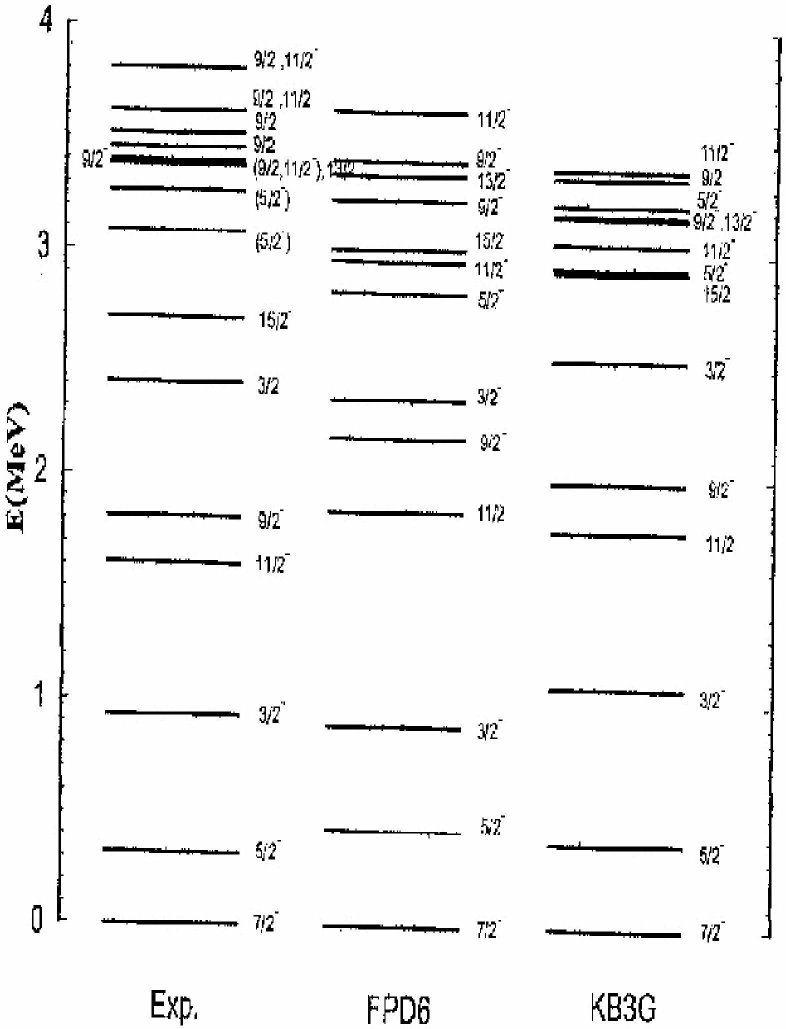


Fig. 9. Energy levels of ^{51}V.

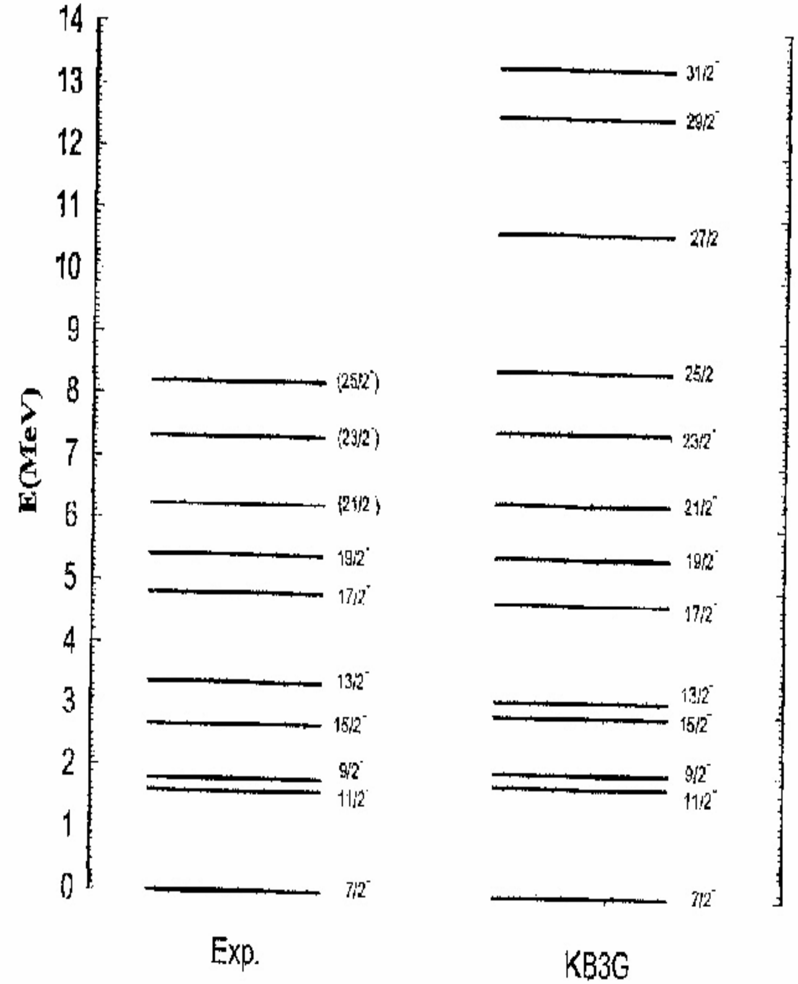


Fig. 8. Yrast band of ^{51}V.

- Monte Carlo shell model
  a. S.E. Koonin, D.J.Dean, and K. Langange, Phys. Rep., 278(1997), 2.
  b. M. Honma, T. Mizusaki, and T. Otsuka, Phys. Rev. Lett., 77(1996), 3315.

→ Shape coexistence in ^{56}Ni.

T. Mizusaki, T. Otsuka, Y. Utsuno, M. Honma, T. Sebe,
P. R. C., 59(1999), R1846.

········> 2000

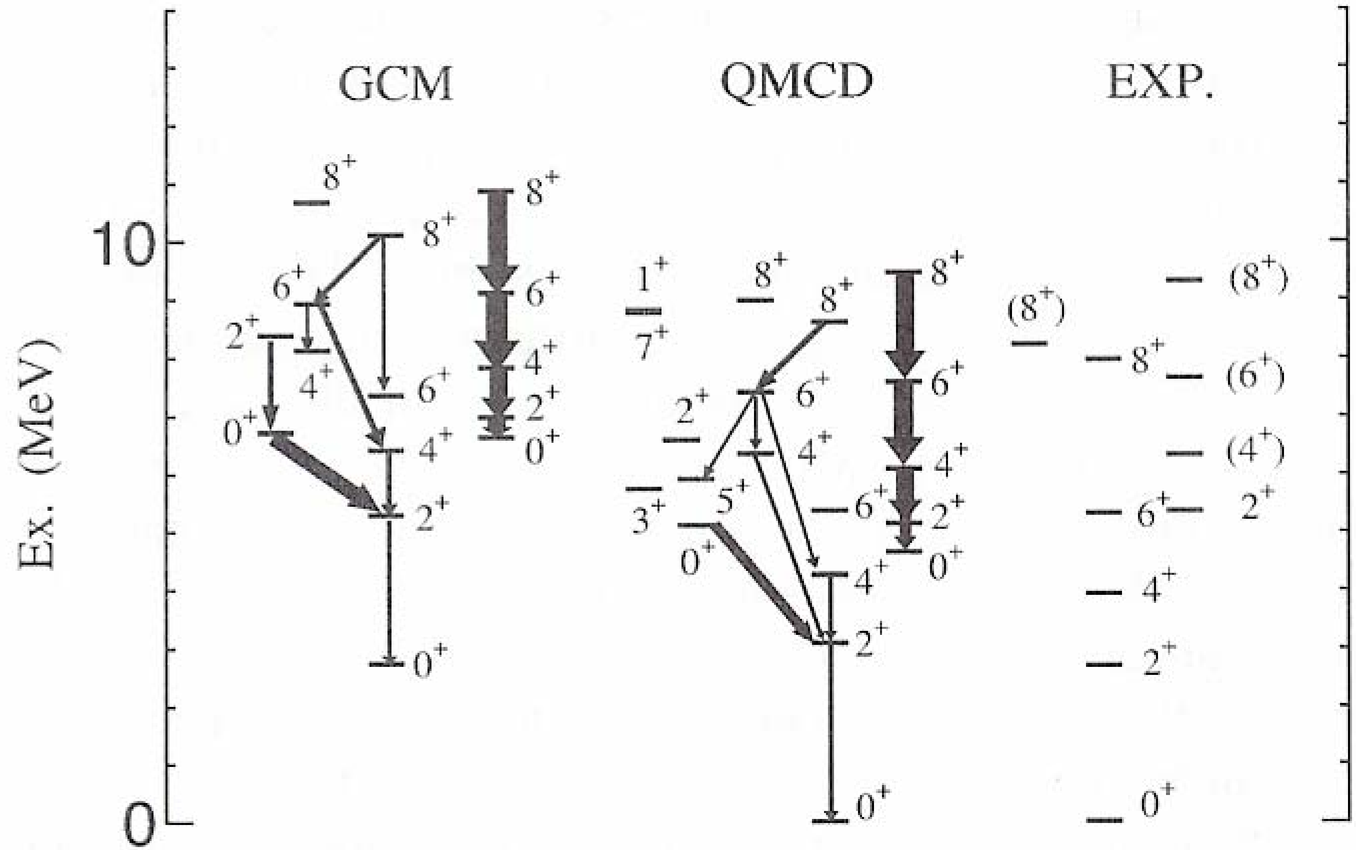


FIG. 2. Experimental levels [1,12] of ^{56}Ni compared with QMCD and GCM results with a FPD6 Hamiltonian. The GCM energies are shown relative to the QMCD ground state. The $B(E2{:}(J+1 \text{ or } 2)\rightarrow J^+)$ values, which are larger than 100 e^2 fm^4, are indicated by the width of the arrows.

T. Mizusaki, T. Otsuka, Y. Utsuno, M. Honma, T. Sebe, P. R. C., 59(1999), R1846.

2. Various collective models and our approach

- Collective models

  RPA

  IBM

  Mean field approximations

  HF, HF+RPA

  HFB

  using P+QQ force,

  Skyrme force, etc.

- VAMPIR — A. Petrovici, *et al*., Nucl. Phys. A483(1988), 317.

  Projected SM — Y. Sun, nucl-th/0211043.

  K. Hara & S. Iwasaki, Nucl. Phys. A332(1979), 61.

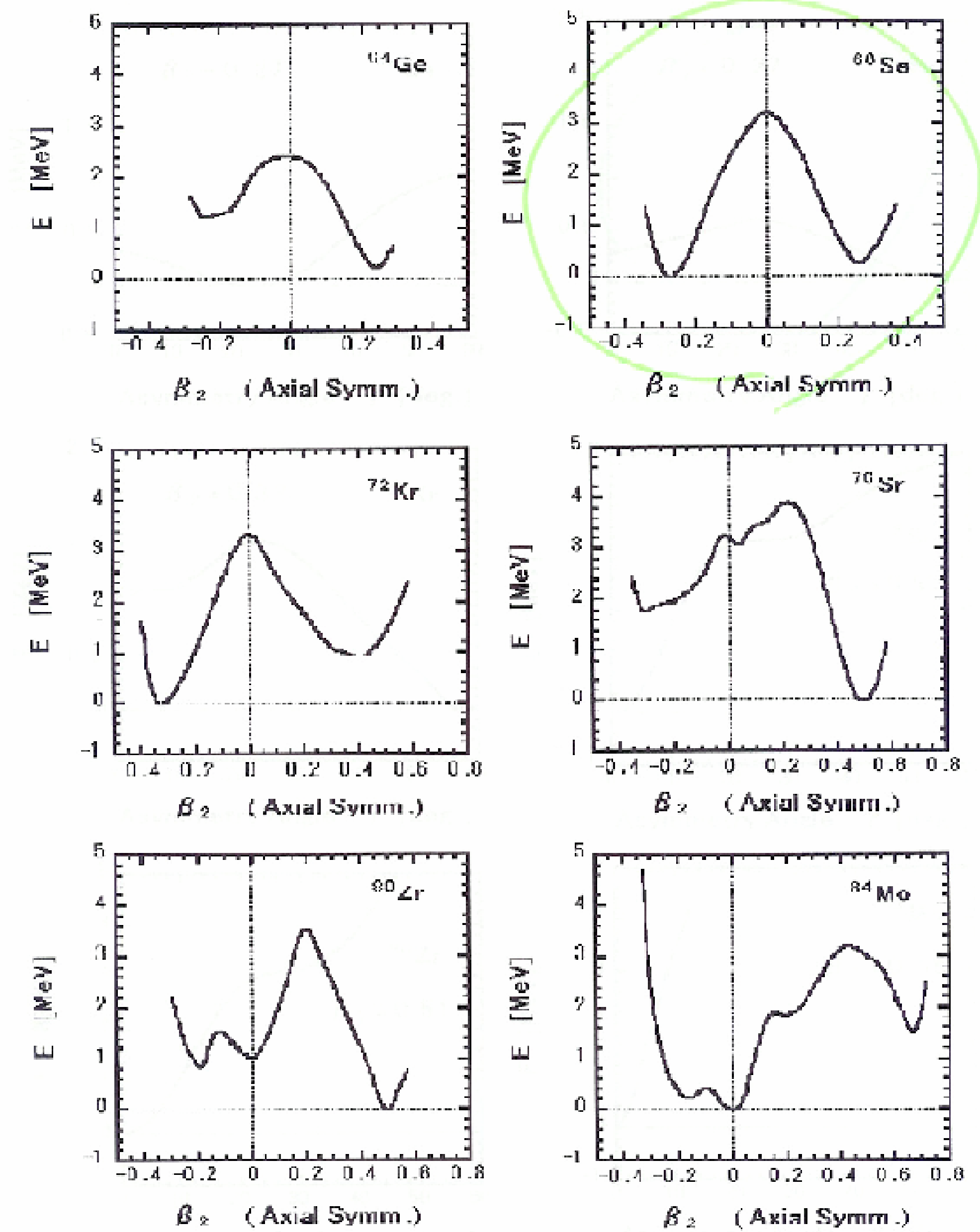


Fig. 1. Potential-energy curves calculated by the constrained Skyrme–HFB method for ^{64}Ge, ^{68}S
^{72}Kr, ^{76}Sr, ^{80}Zr and ^{84}Mo are drawn as functions of the quadrupole deformation parameter β_2. Th
SIII interaction is used for the particle–hole channel, while the density-dependent pairing interactic
with $V_0 = -1000.0$ MeV fm^3 and $\rho_c = 0.16$ fm^{-3} is used for the particle–particle channel.

M. Yamagami,
K. Matsuyanagi, M. Matsuo,
Nucl. Phys. A693(2001), 579

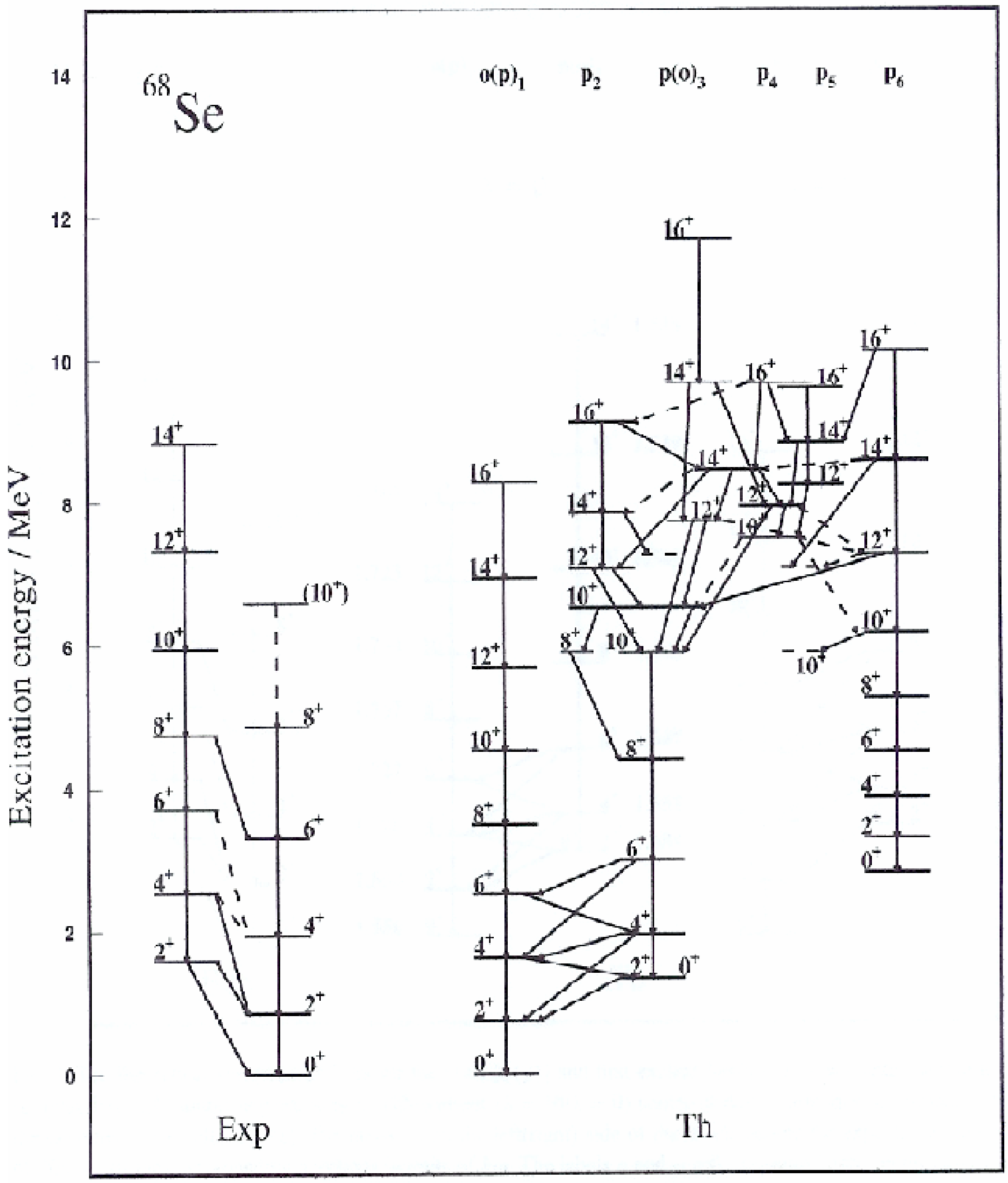


Fig. 2. The alignment plot for some states calculated in ^{68}Se.

A. Petrovici, K.W. Scmid,
A. Faessler,
Nucl. Phys. A710(2002), 246.

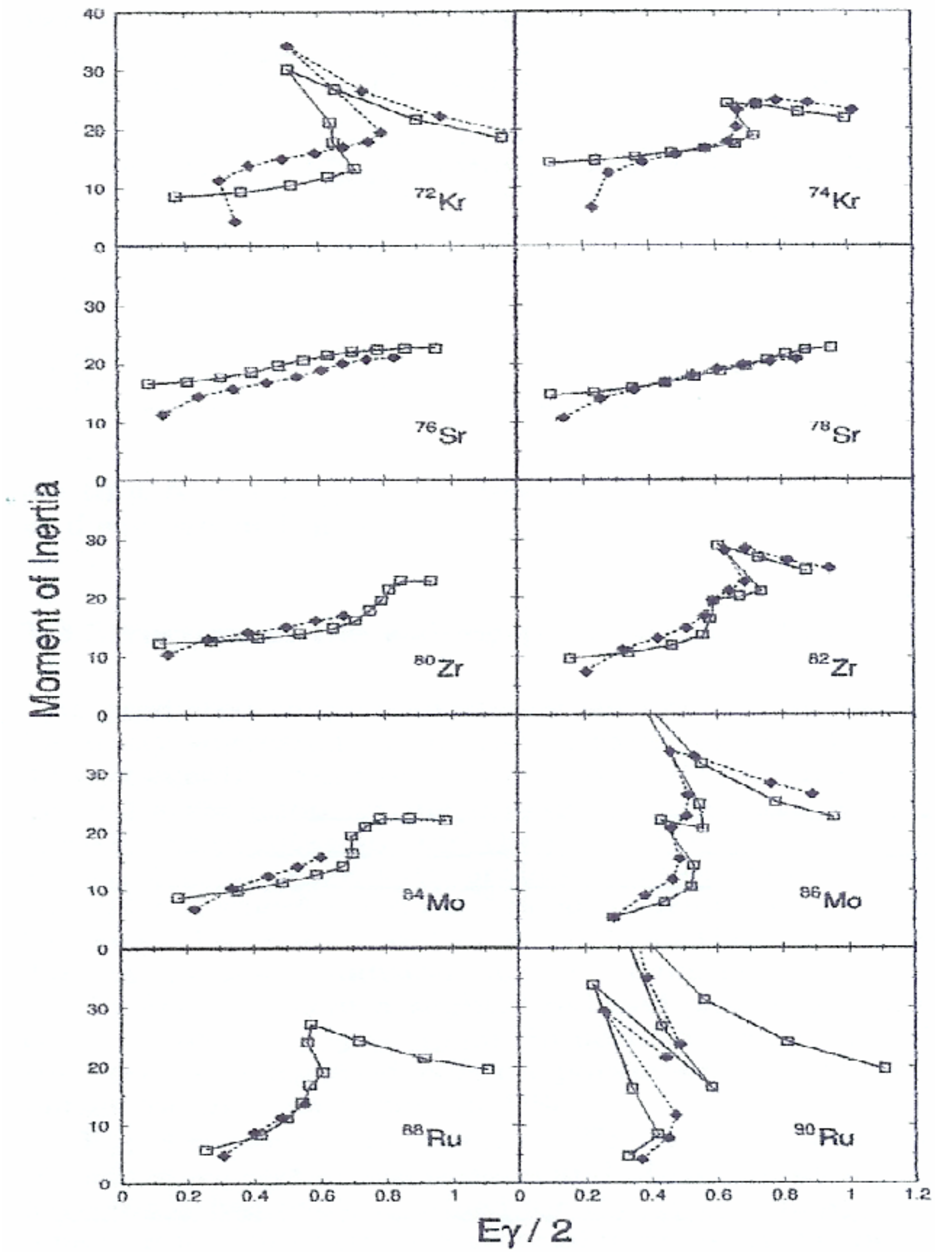


Fig. 2. Comparison of the calculated yrast energy levels with known data for $N = Z$ and $N = Z + 2$ nuclei in the plots of moment of inertia $\Im^{(1)}(I) = (2I-1)/E_\gamma(I \to I-2)$ vs. rotational frequency $\omega(I) = E_\gamma(I \to I-2)/2$.

Y. Sun, nucl-th/0211043.

We investigated T=0 and T=1 pairing correlations

by means of the shell model with P+QQ.

- Important T=0 monopole field

$$V_{\pi\nu}^{T=0} = -k^0 \sum_{a\le b} \sum_{JM} A^{+}_{JMT=0}(ab) A_{JMT=0}(ab)$$

can explain the binding energy.

M. Hasegawa & K. Kaneko, Phys. Rev. C, 59(1999), 1449.

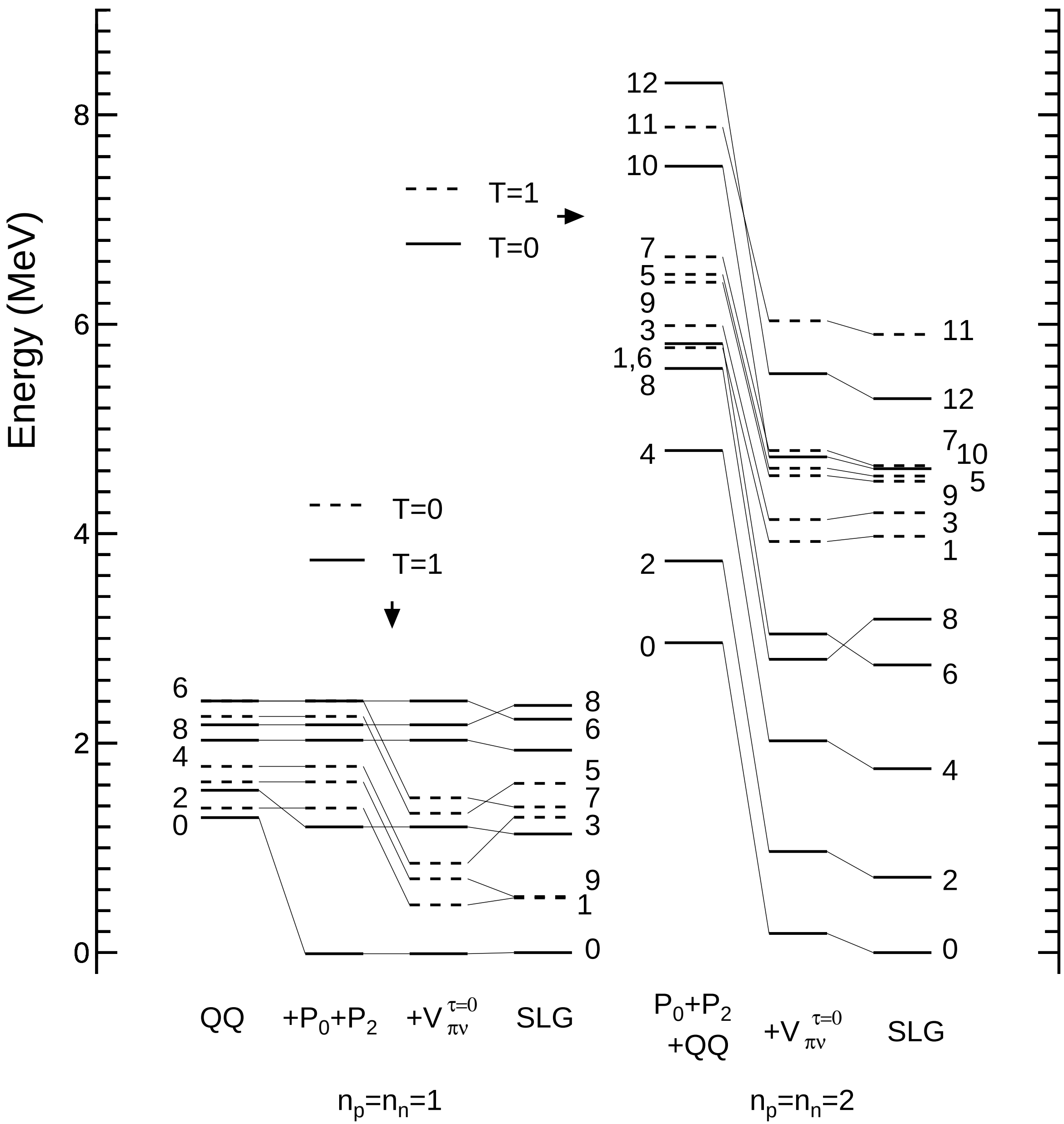
adding quadrupole pairing force, etc.,

and T=0 and T=1 monopole corrections,

- Extended P+QQ int. is more than a toy interaction.

→ successfully describes $f_{7/2}$-shell nuclei.

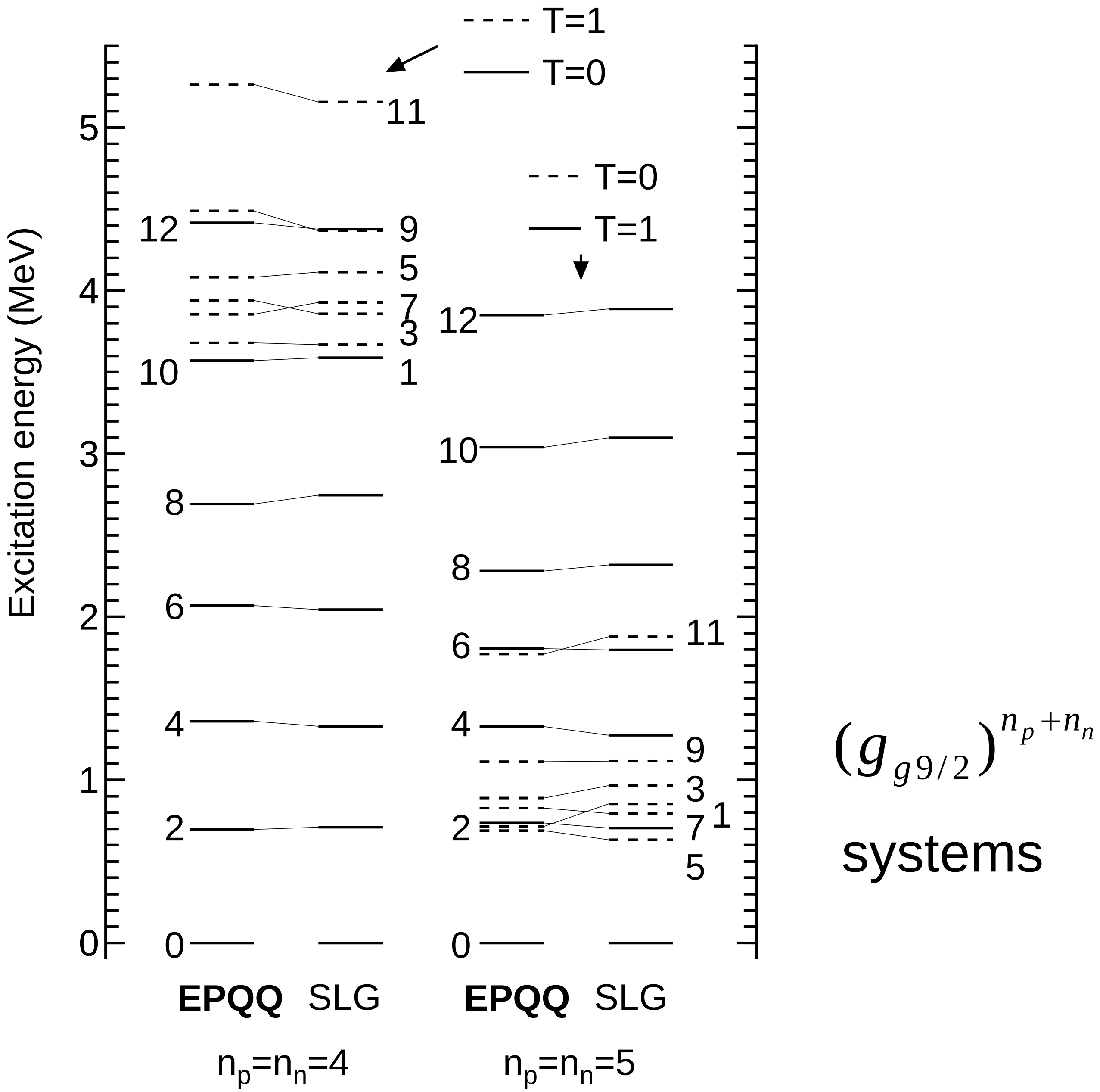
M. Hasegawa & K. Kaneko, N. P. A, 674(2000), 411; 688(2001), 765.



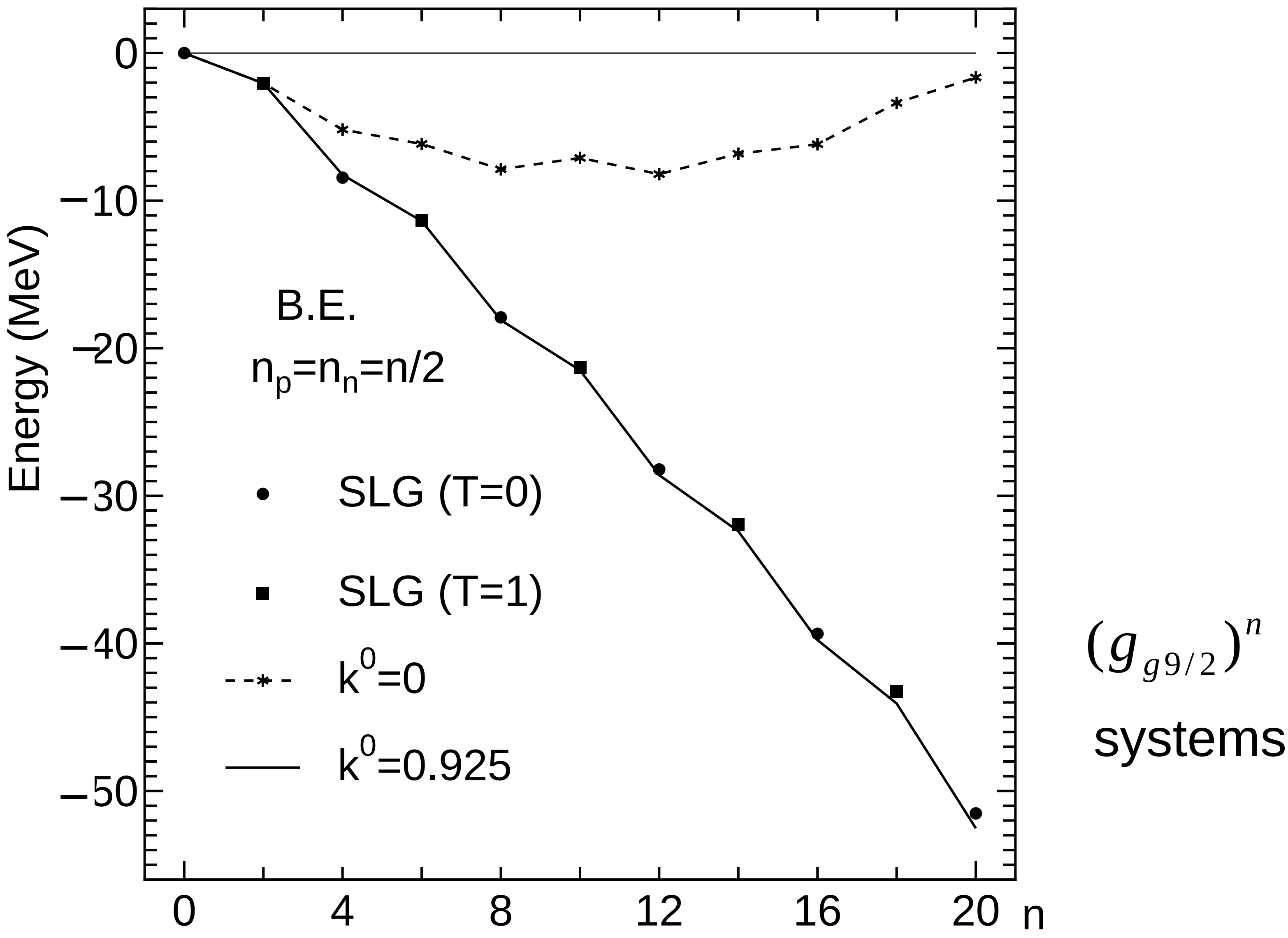
$$V_{\pi\nu}^{T=0} = -\frac{1}{2} k^0 \times \{ \frac{n_\nu}{2} (\frac{n_\nu}{2} + 1) - \hat{T}^2 \}$$

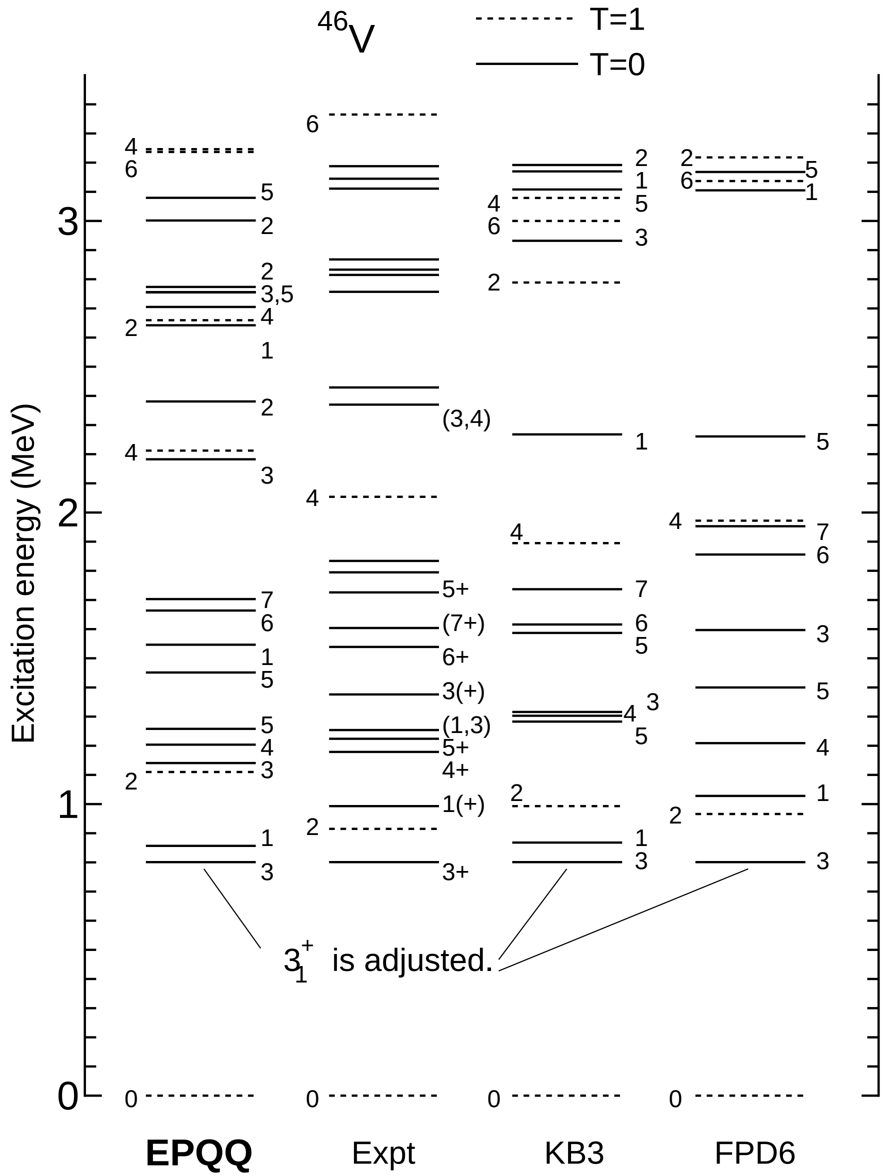
$(g_{g9/2})^{n_p+n_n}$

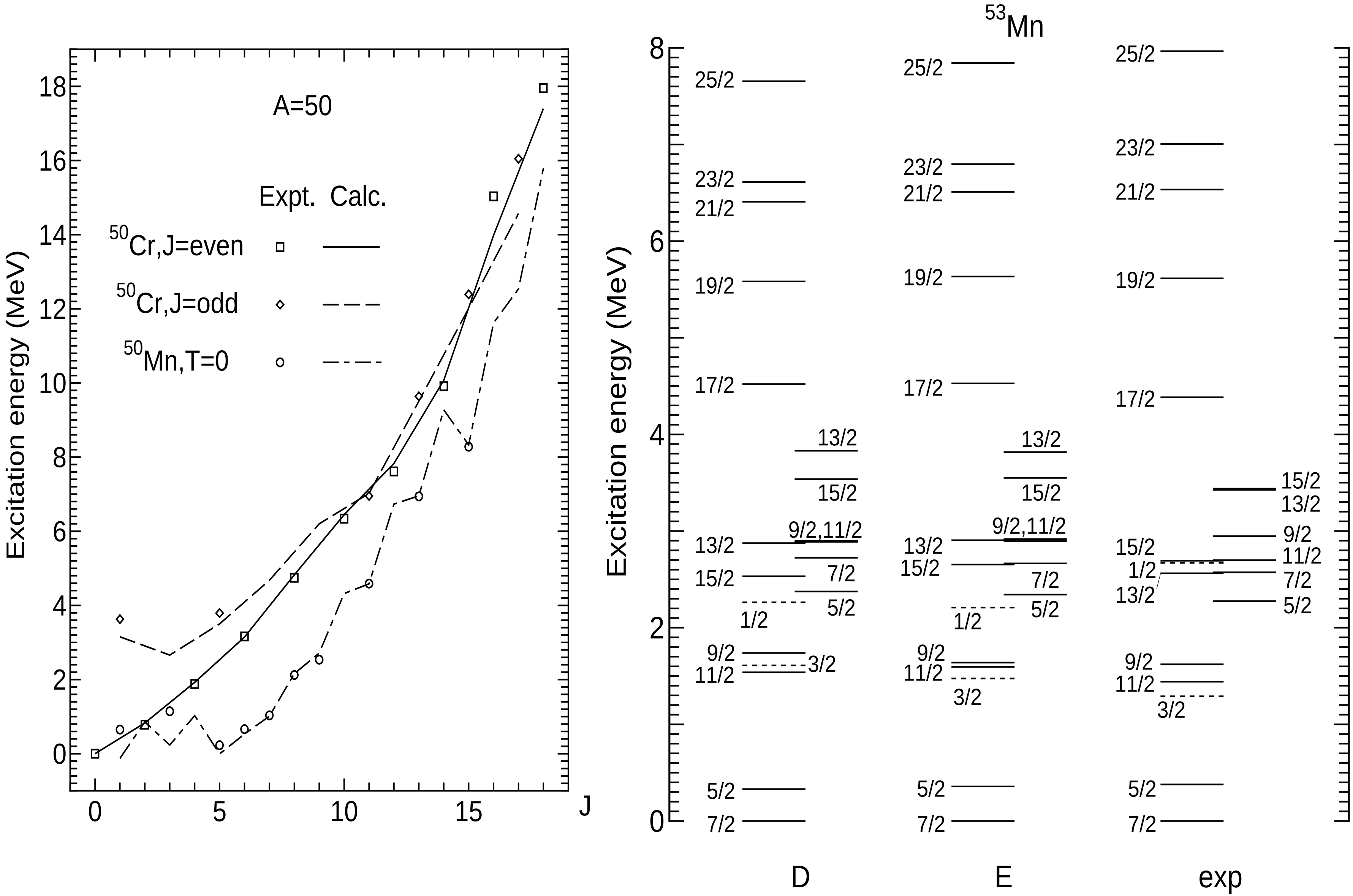
systems



T=1
T=0
T=0
T=1
Excitation energy (MeV)
0
1
2
3
4
5
0
2
4
6
8
10
12
1
3
7
5
9
11
0
2
4
6
8
10
12
5
7
1
3
9
11
EPQQ
SLG
EPQQ
SLG
$n_p=n_n=4$
$n_p=n_n=5$
$(g_{g9/2})^{n_p+n_n}$
systems


Energy (MeV)
0
−10
−20
−30
−40
−50
0
4
8
12
16
20
n
B.E.
n_p=n_n=n/2
SLG (T=0)
SLG (T=1)
k^0=0
k^0=0.925
(g_g9/2)^n
systems


46V
T=1
T=0
Excitation energy (MeV)
0
1
2
3
EPQQ
Expt
KB3
FPD6
(3,4)
5+
(7+)
6+
3(+)
(1,3)
5+
4+
1(+)
3+
3+ 1 is adjusted.



A=50
Expt. Calc.
50Cr,J=even
50Cr,J=odd
50Mn,T=0
Excitation energy (MeV)
J
53Mn
Excitation energy (MeV)
D
E
exp

3. About effective interaction

- Realistic effective interactions ～ EPQQ

$$H \approx H_{sp} + V_{\pi\nu}^{T=0} + \Delta V_{mc}$$

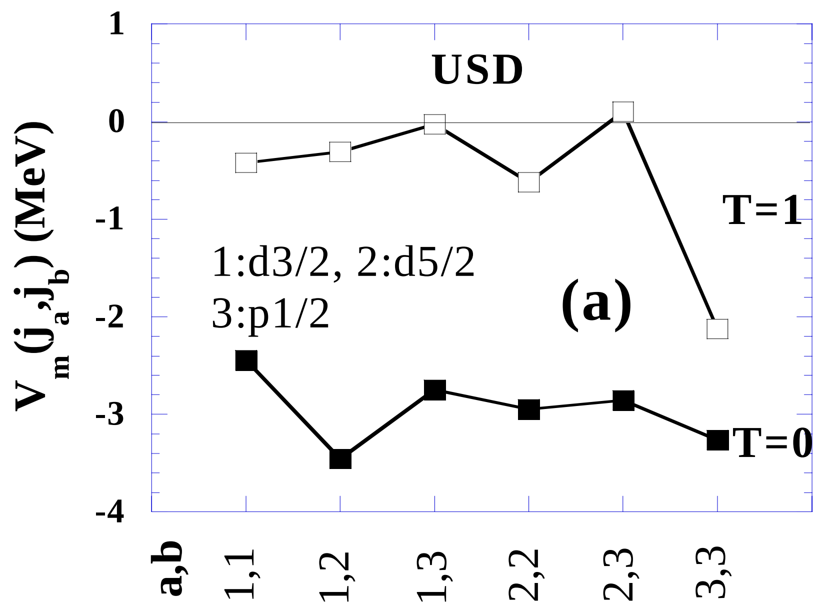
$$+V(P_0) + V(P_2) + V(QQ) + V(OO) \ldots.$$

pointed out by A.P. Zuker, M. Dufour, .. P. R. C., 54(1996), 1641.

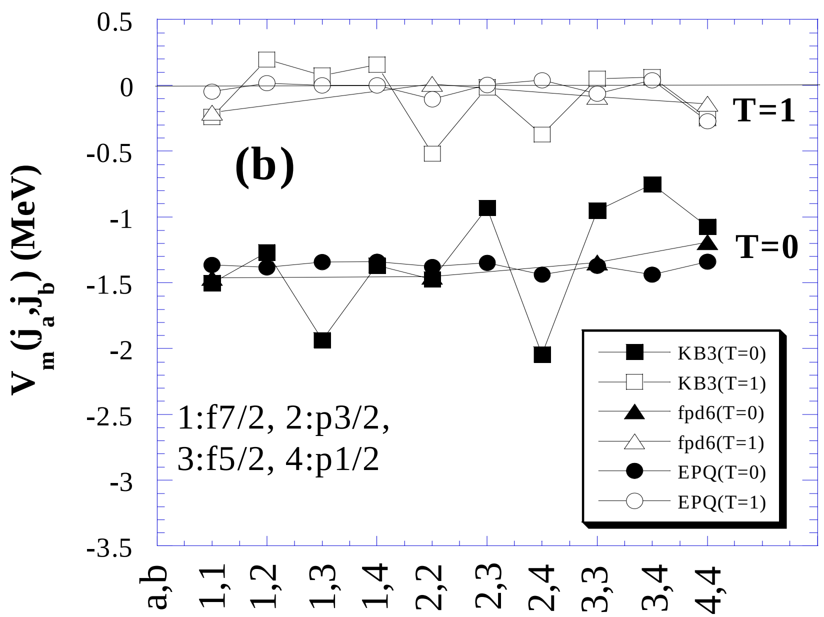
J. Duflo, .. P. R. C., 59(1999), R2347.

Our calculations proved this concretely.

→ Shell model with EPQQ



$$V_{\pi\nu}^{T=0} = -k^0 \sum_{a\le b}\sum_{JM} \times A^{+}_{JMT=0}(ab)A_{JMT=0}(ab)$$



K. Kaneko, M. Hasegawa,
Prog. Theor. Phys. 106(2001),1179.

- Better effective interactions are desired.

GXPF1 by M. Honma *et al*., Phys. Rev. C, 65(2002), 061301

There are many interesting phenomena in pf-shell nuclei, which could be investigated by using GXPF1 int.

Study heavier nuclei as well as light ones!

4. Our work

Large-scale shell model calculations with EPQQ

- Mizusaki's calculation code

T. Mizusaki, RIKEN Accel. Prog. Rep., 33(2000), 14.

- Physics: $N \approx Z$ nuclei with $A = 60\text{–}100$

When Z, N, and J increase,
structure changes rapidly,
change in shape, shape coexistence,
particle alignments, …

^{64}Ge: quadrupole and octupole corr.

- Observed heaviest $N=Z$ nucleus ^{88}Ru

  $(p_{3/2}, f_{5/2}, p_{1/2}, g_{9/2})^{-12}$

  full SM cal. with maximum dimension
  1.4×10^8

  explains difference of backbending
  between ^{88}Ru and ^{90}Ru,
  and gives good prediction for ^{89}Ru.

  Advantages:

  describes *odd* and even-even nuclei
  using the same parameters,
  better than the mean field approx,
  etc.

- back to Ge isotopes

^{68}Ge

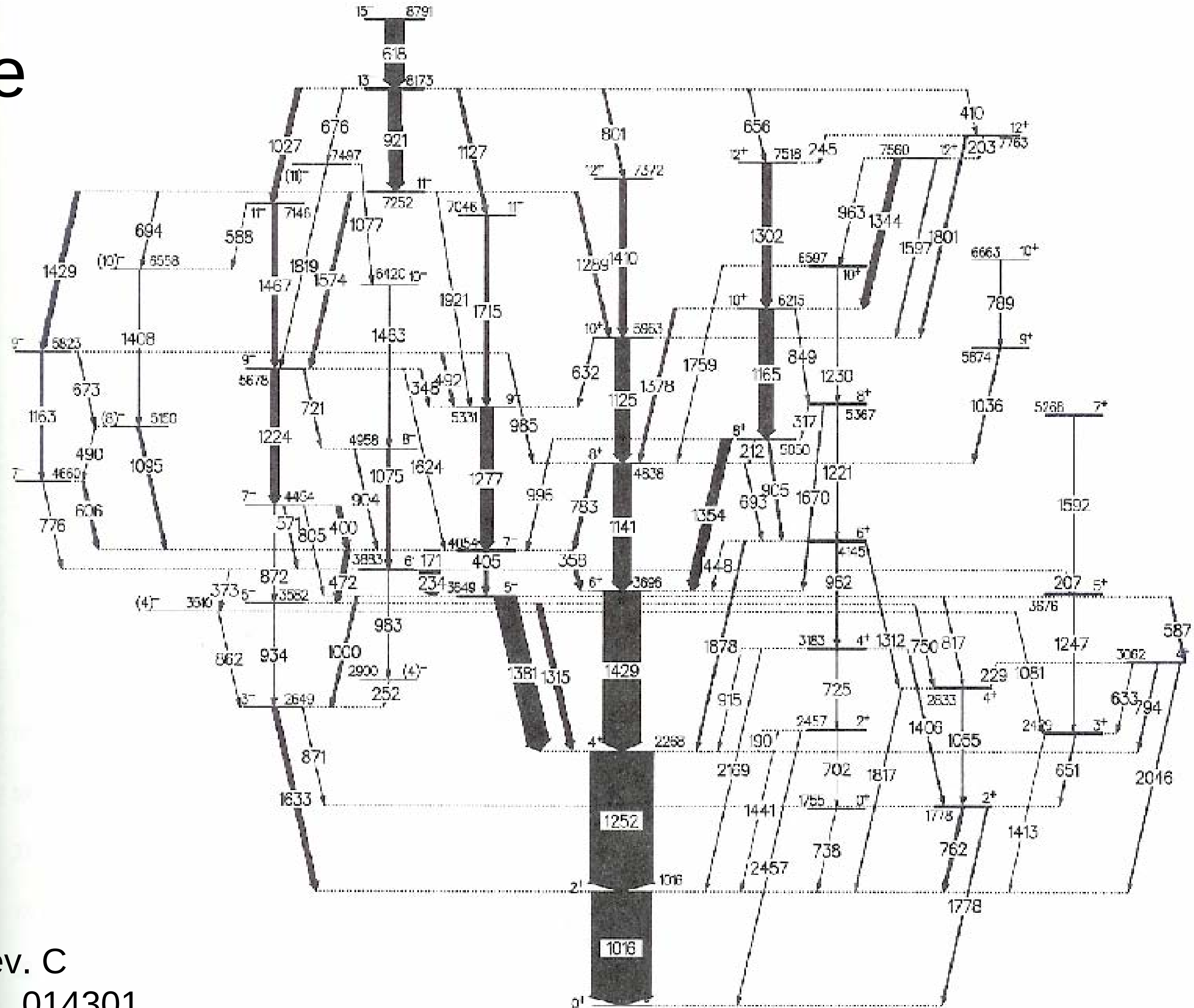
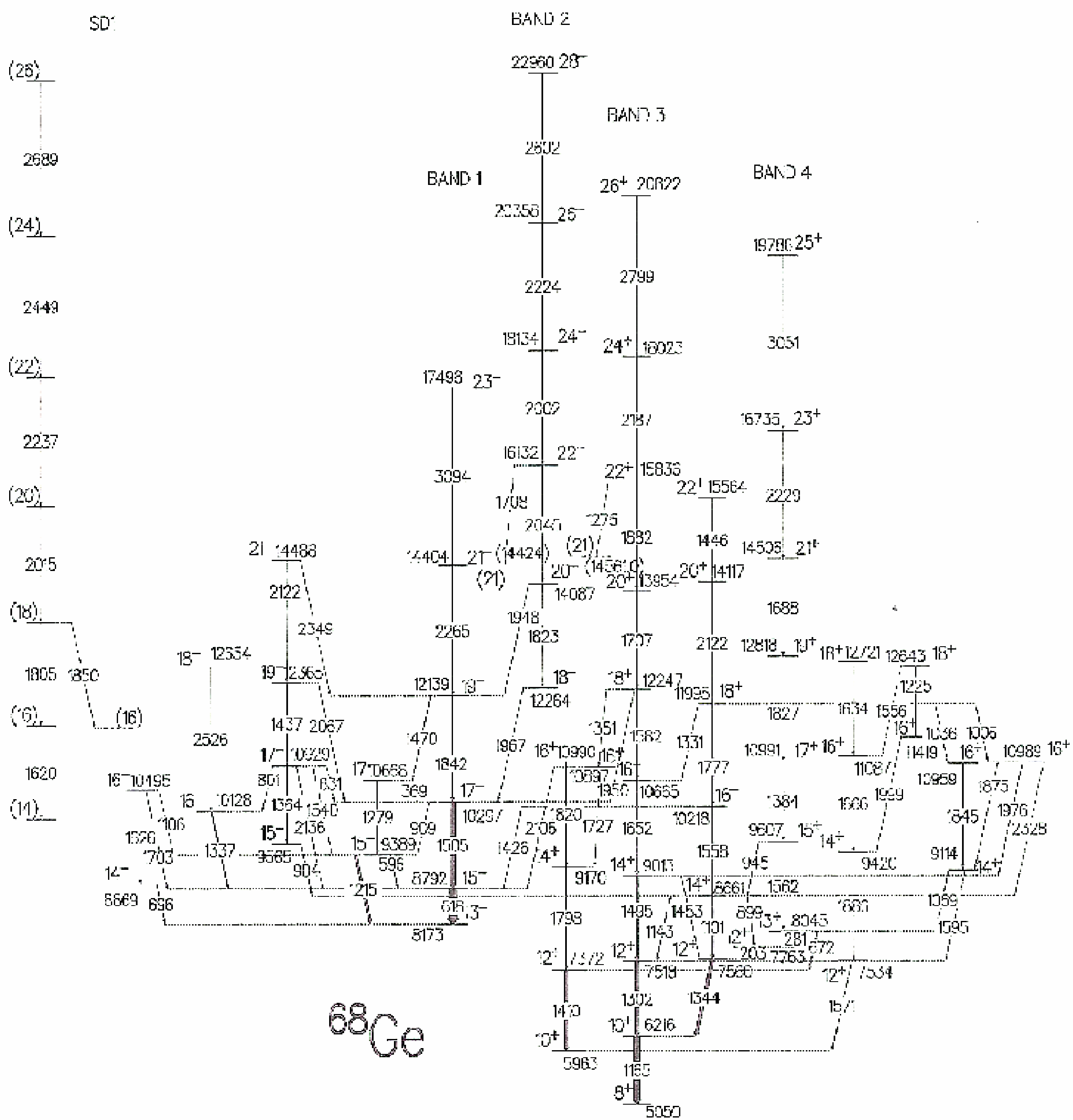
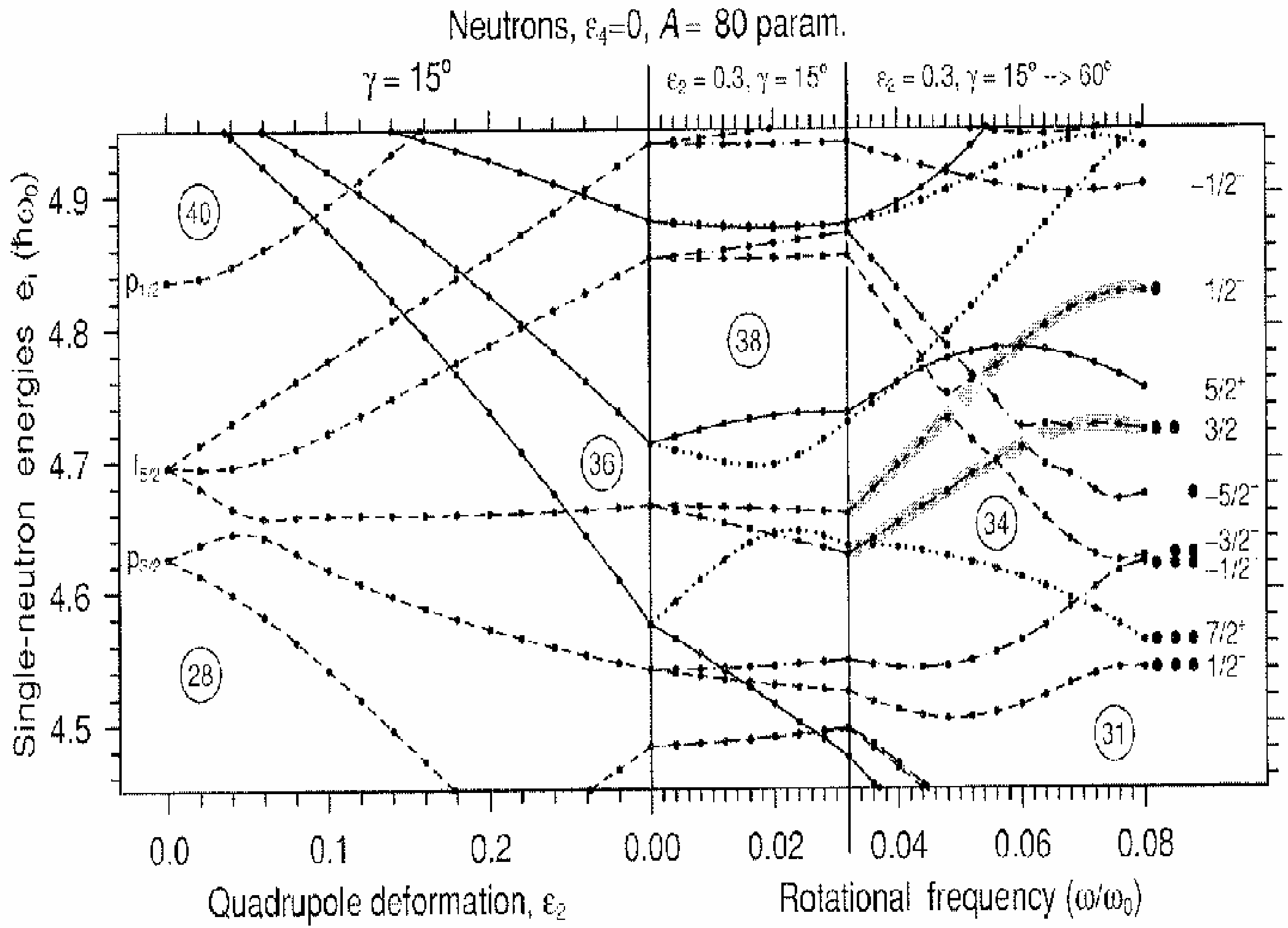


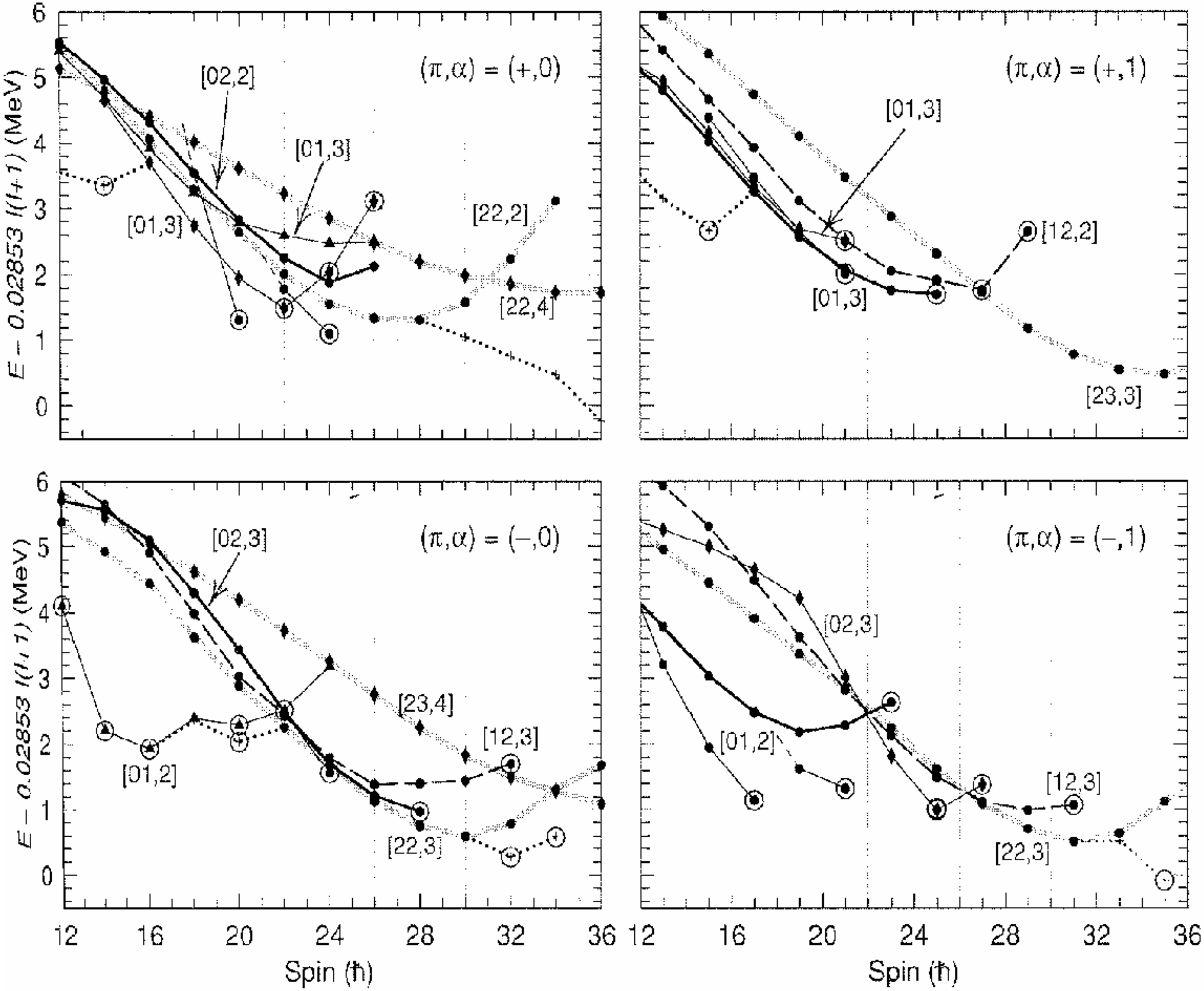
FIG. 2. Low-spin level scheme for ^{68}Ge from the present experiment.

D. Ward,
Phys. Rev. C
63(2001), 014301.

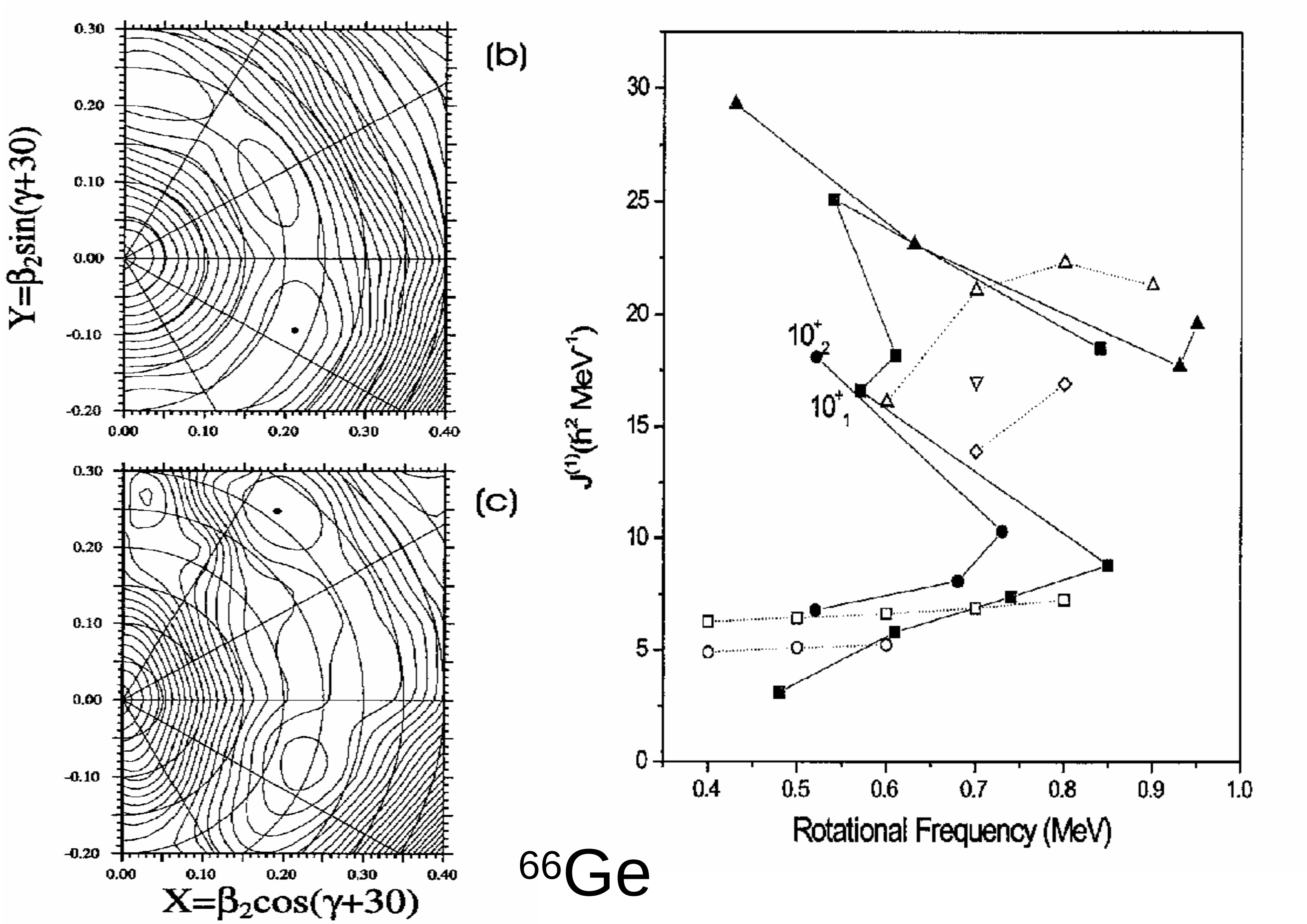

SD1
BAND 1
BAND 2
BAND 3
BAND 4
68Ge



Neutrons, ε4=0, A = 80 param.
γ = 15°
ε2 = 0.3, γ = 15°
ε2 = 0.3, γ = 15° --> 60°
Single-neutron energies ei (ħω0)
4.9
4.8
4.7
4.6
4.5
p1/2
f5/2
p3/2
40
38
36
34
28
31
-1/2⁻
1/2⁻
5/2⁺
3/2
-5/2⁻
-3/2⁻
-1/2
7/2⁺
1/2⁻
0.0
0.1
0.2
0.00
0.02
0.04
0.06
0.08
Quadrupole deformation, ε2
Rotational frequency (ω/ω0)



(π,α) = (+,0)
(π,α) = (+,1)
(π,α) = (–,0)
(π,α) = (–,1)
[02,2]
[01,3]
[01,3]
[22,2]
[22,4]
[01,3]
[12,2]
[01,3]
[23,3]
[02,3]
[23,4]
[12,3]
[01,2]
[22,3]
[02,3]
[01,2]
[12,3]
[22,3]
E – 0.02853 I(I+1) (MeV)
Spin (ħ)



^{66}Ge

E.A. Stefanova, Phys. Rev. C 67(2003), 054319.

- back to Ge isotopes

  Parameter search using EPQQ

  including s. p. energies.

  reproduces well

  a lot of energy levels, different bands,

  B(E2) values of collective bands, Q moments.

  Advantages: strict wave functions

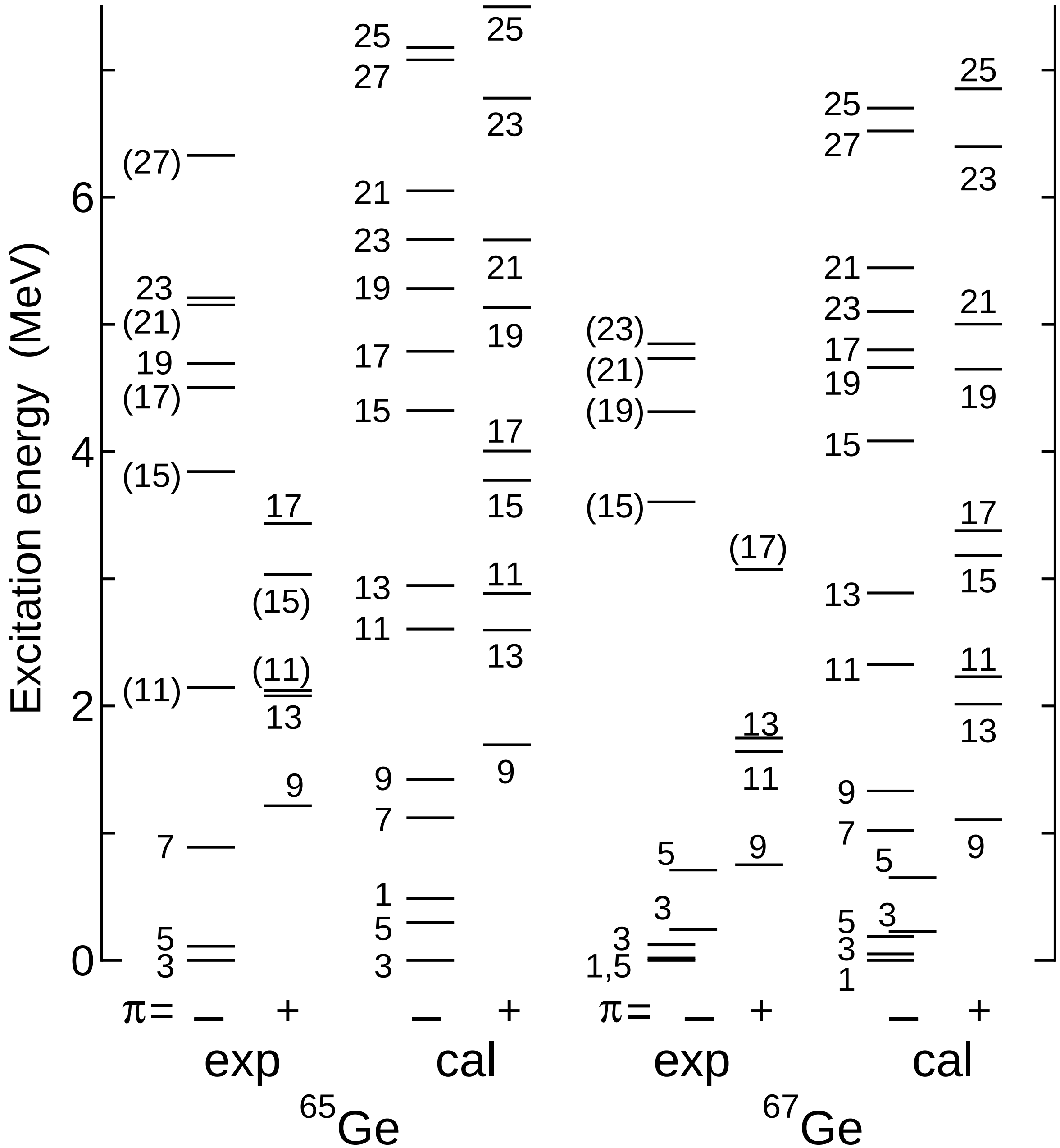
  cal. of $\langle j_a \rangle, \langle t_a \rangle, \langle n_a^{\pi} \rangle, \langle n_a^{\nu} \rangle, \ldots$

  → T=0 proton-neutron alignment

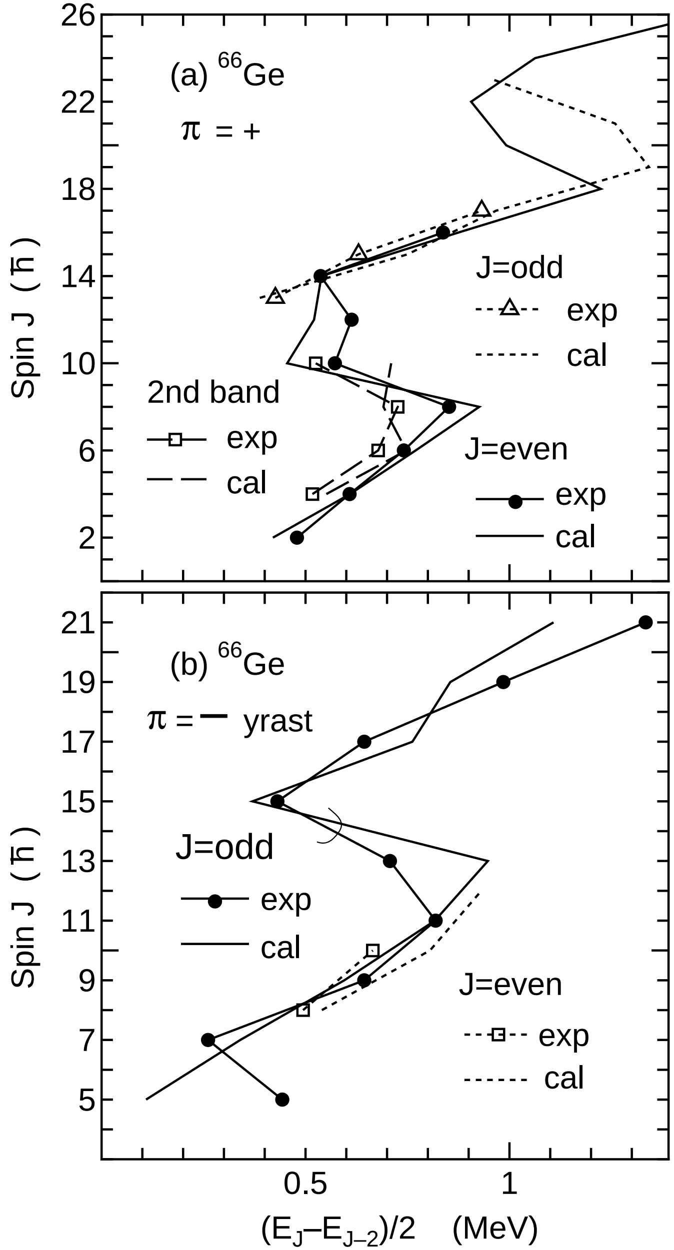
  in even-even, odd, and odd-odd nuclei,

  successive alignments with increasing J .

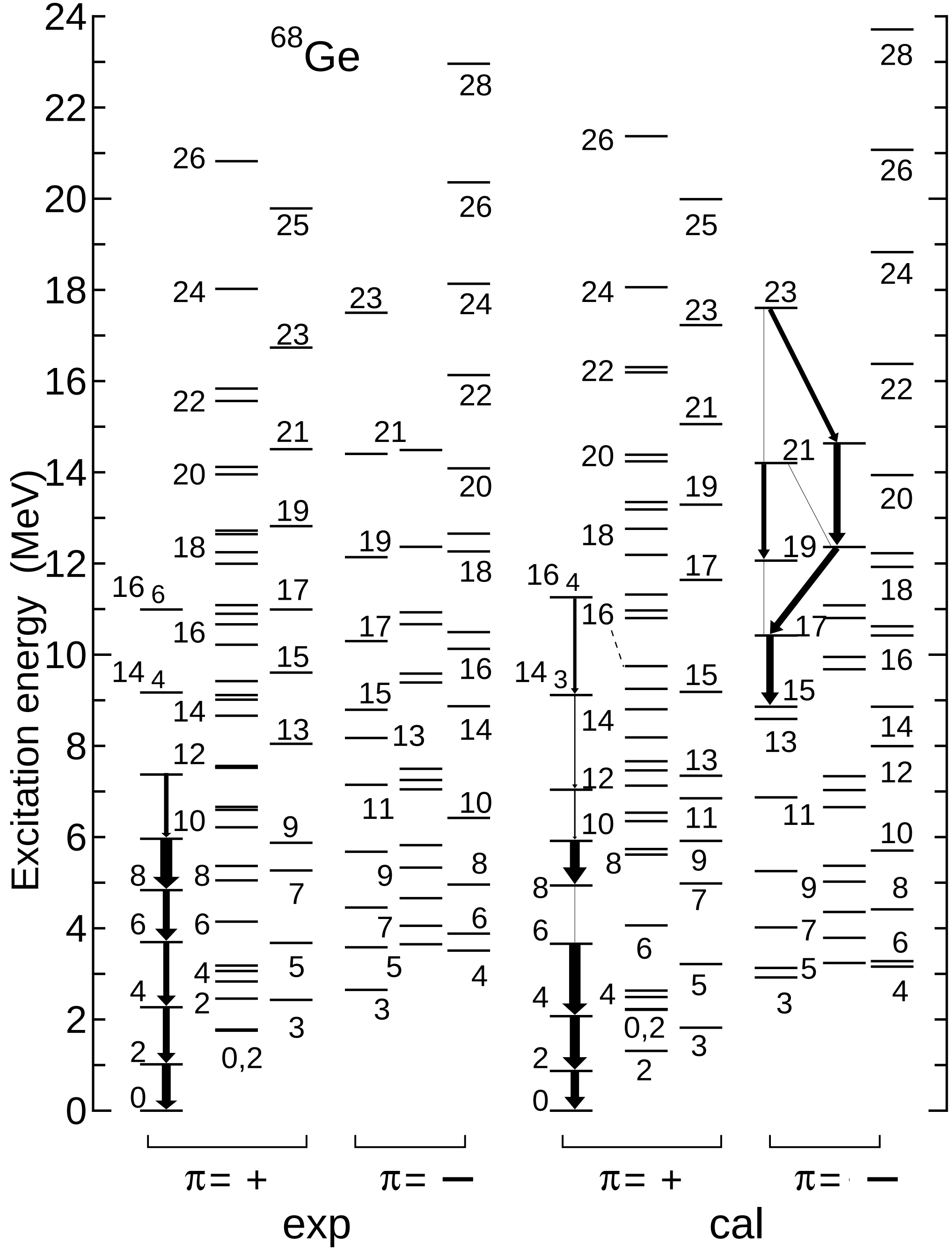
M. H., K. Kaneko, and T. Mizusaki, P. R. C, 70(2004), 031301(R),
71(2005), 044301.



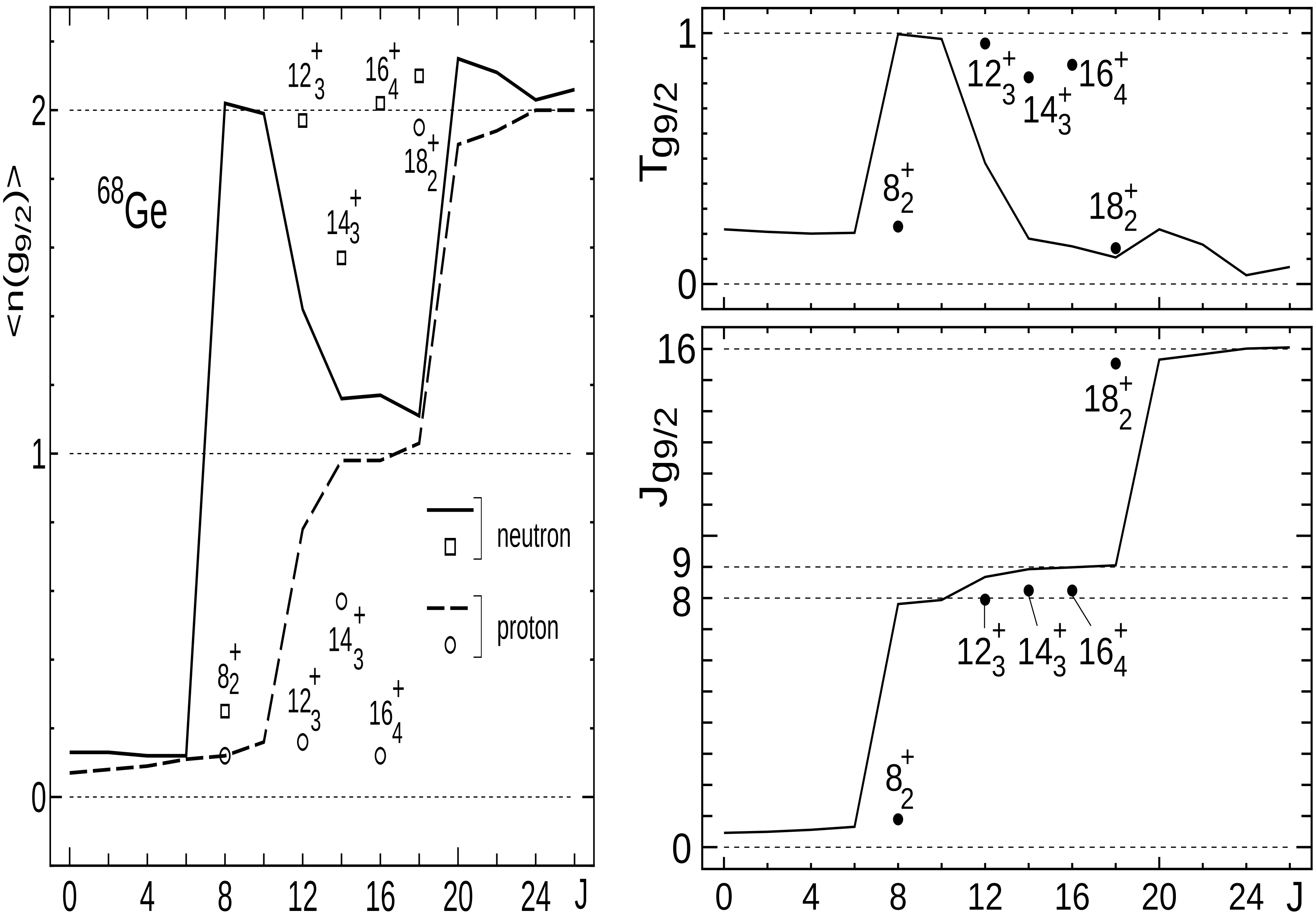
Excitation energy (MeV)
0
2
4
6
π=
−
+
exp
cal
^{65}Ge
^{67}Ge
(27)
23
(21)
19
(17)
(15)
(11)
7
5
3
17
(15)
(11)
13
9
25
27
21
23
19
17
15
13
11
9
7
1
5
3
(23)
(21)
(19)
(15)
(17)
1,5



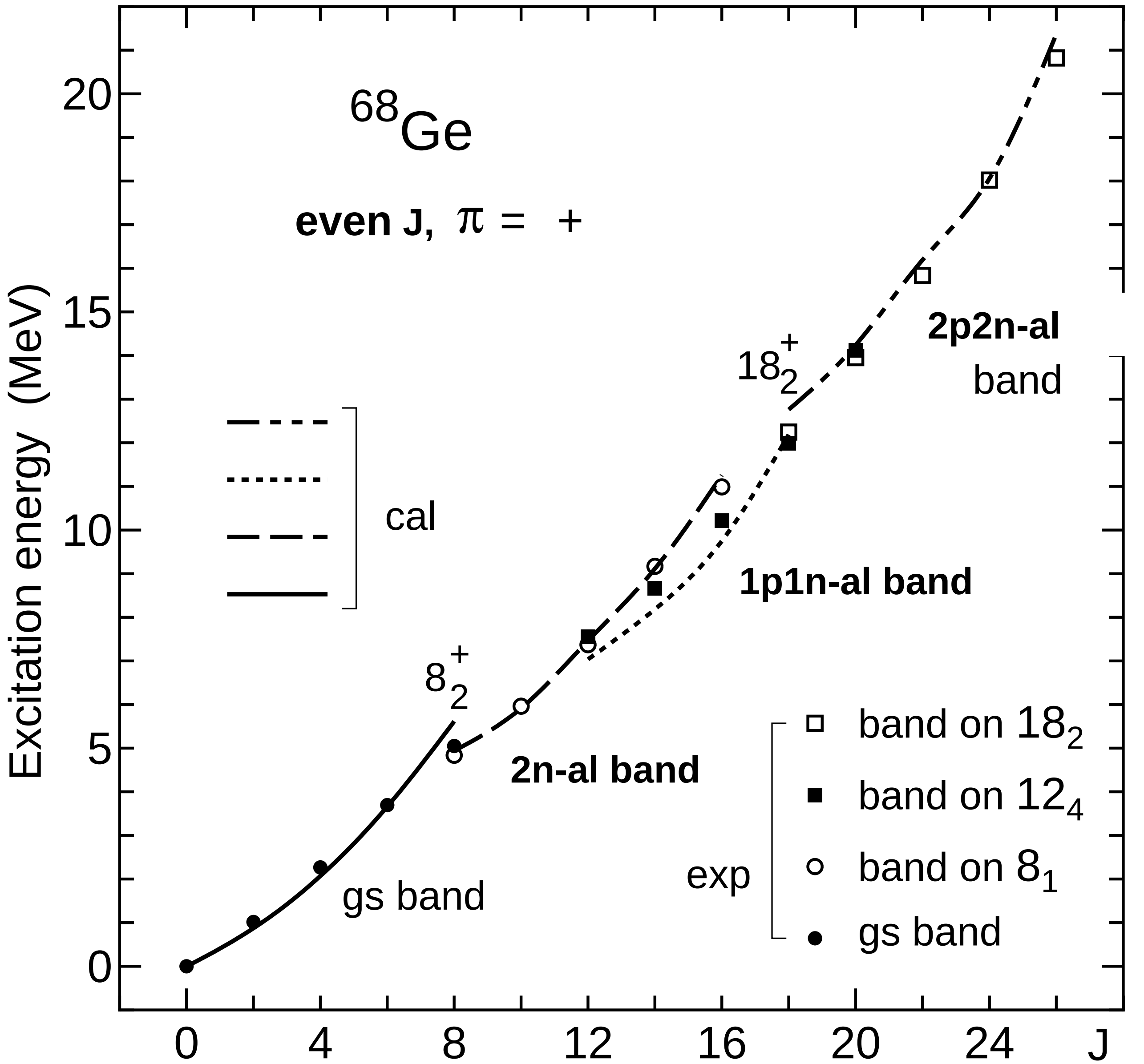
(a) ^{66}Ge
π = +
J=odd
exp
cal
2nd band
exp
cal
J=even
exp
cal
Spin J (ħ)
(b) ^{66}Ge
π = − yrast
J=odd
exp
cal
J=even
exp
cal
Spin J (ħ)
0.5
1
$(E_J - E_{J-2})/2$ (MeV)



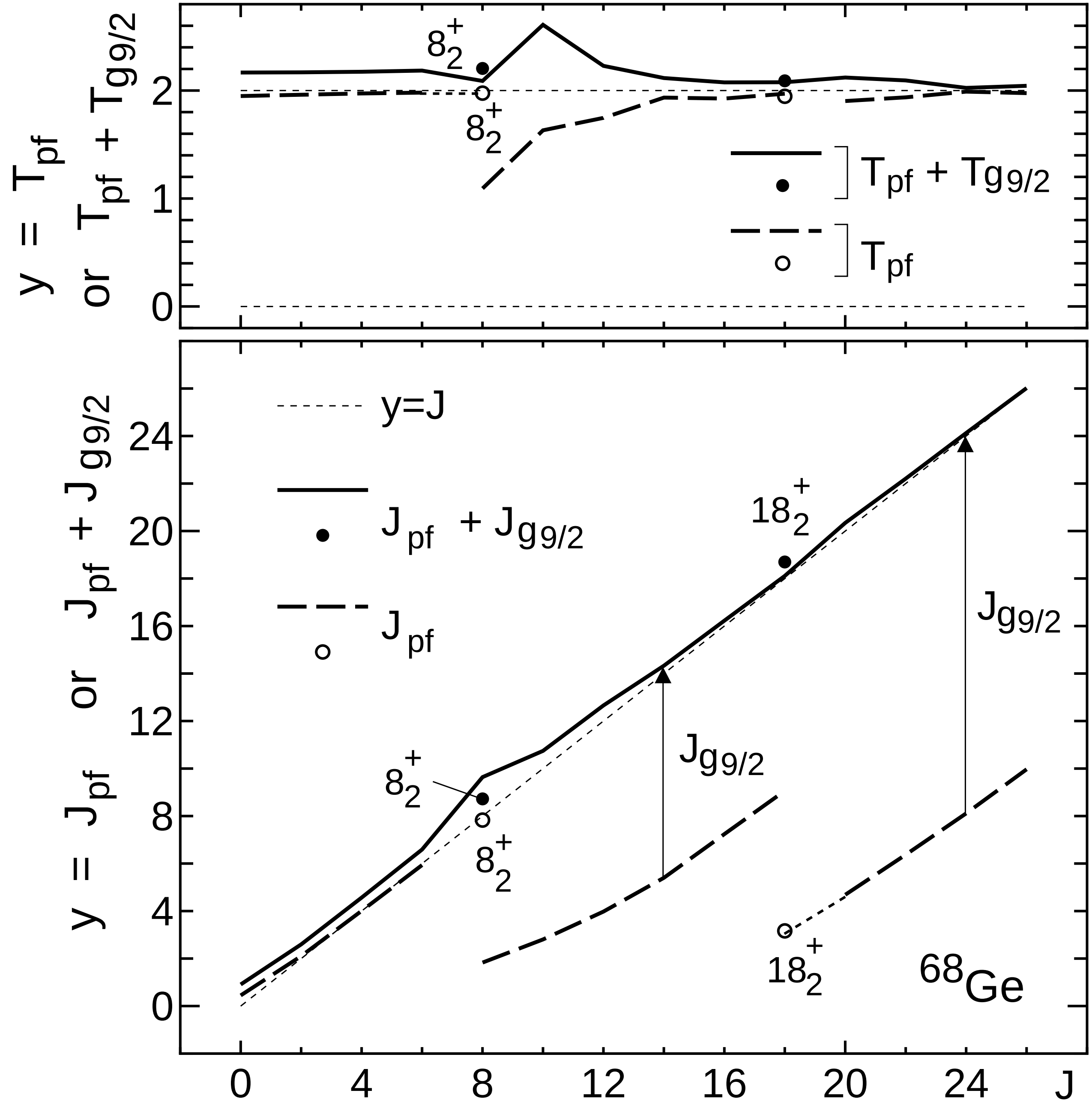
68Ge
Excitation energy (MeV)
0
2
4
6
8
10
12
14
16
18
20
22
24
π= +
π= −
π= +
π= −
exp
cal



<n(g9/2)>
68Ge
12+3
16+4
18+2
14+3
8+2
neutron
proton
Tg9/2
Jg9/2
0
4
8
12
16
20
24
J
1
2
9



68Ge
even J, π = +
Excitation energy (MeV)
J
0
4
8
12
16
20
24
0
5
10
15
20
cal
exp
band on 182
band on 124
band on 81
gs band
gs band
2n-al band
1p1n-al band
2p2n-al
band
8+2
18+2



y = T_pf or T_pf + Tg9/2
8_2^+
8_2^+
T_pf + Tg9/2
T_pf
y = J_pf or J_pf + J_g9/2
y=J
J_pf + J_g9/2
J_pf
18_2^+
J_g9/2
J_g9/2
8_2^+
8_2^+
18_2^+
68Ge
J

- Shape transition in ^{60}Zn, ^{64}Ge, ^{68}Se, (^{72}Kr)
  prolate, → oblate(coexisting prolate) → triaxial

  K. Kaneko *et al*, Phys. Rev. C, 70(2004)., 051301.

  using the successful Hamiltonian,
  constrained HF calculation.

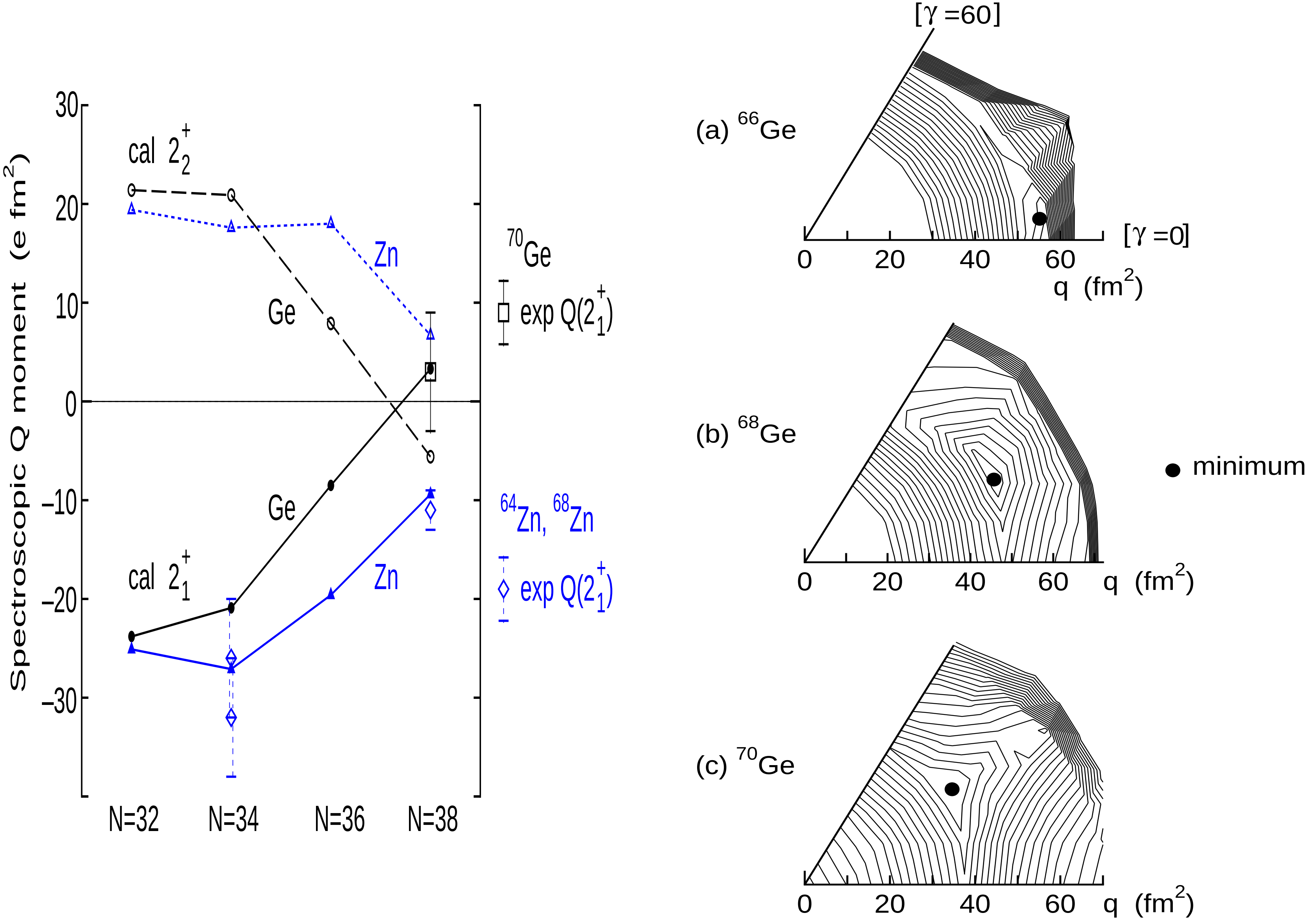
  T. Mizusaki *et al*., P. R. C, 59(1999). 1846(R).

- Isomeric states in odd-odd ^{66}As, and ^{67}As

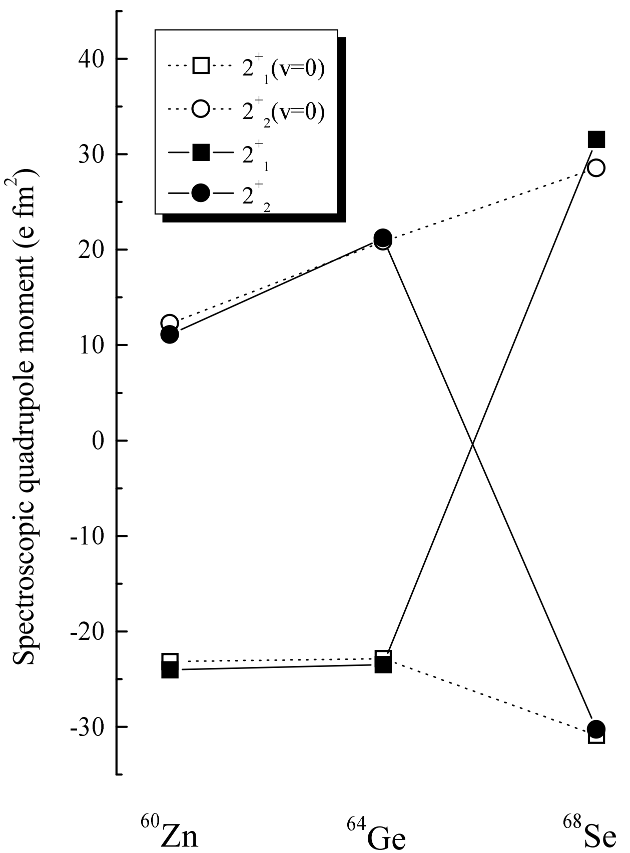
  Phys. Lett. B, 617(2005), 150.

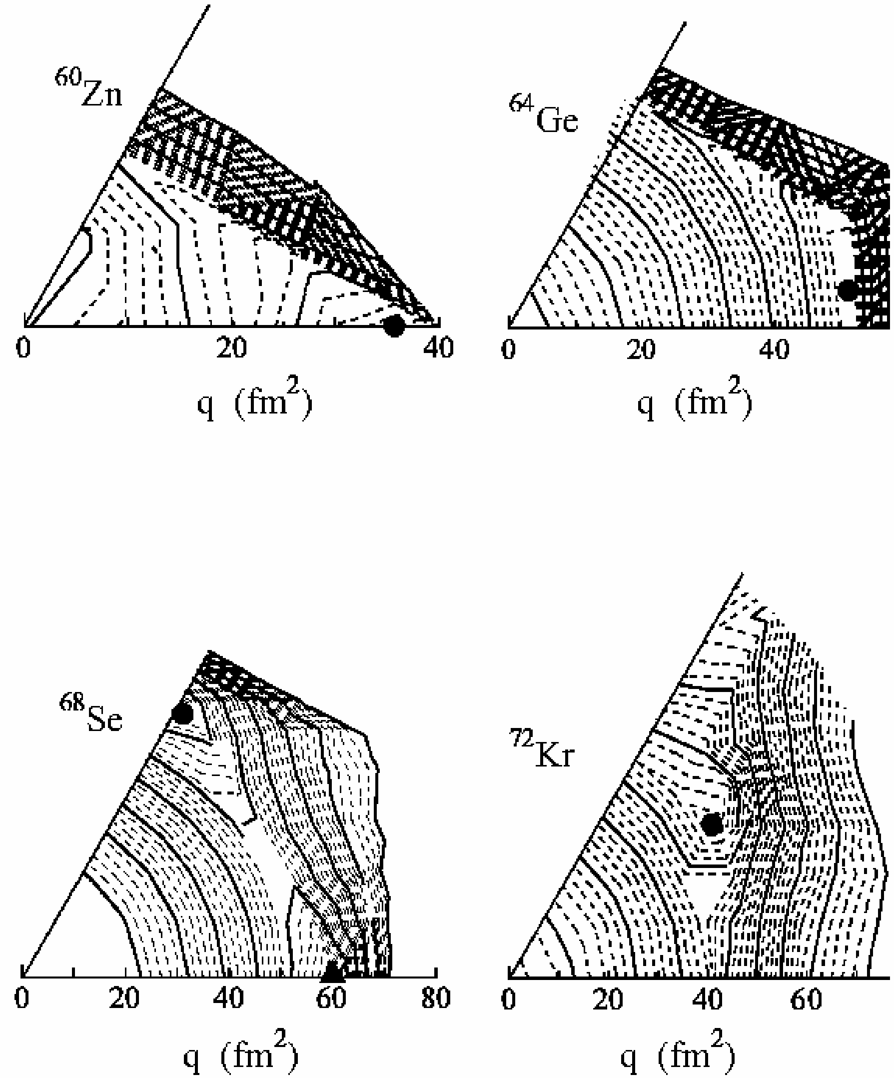
- detailed structure of odd nucleus ^{69}As

  Phys. Rev. C, 72(2005), 064320.

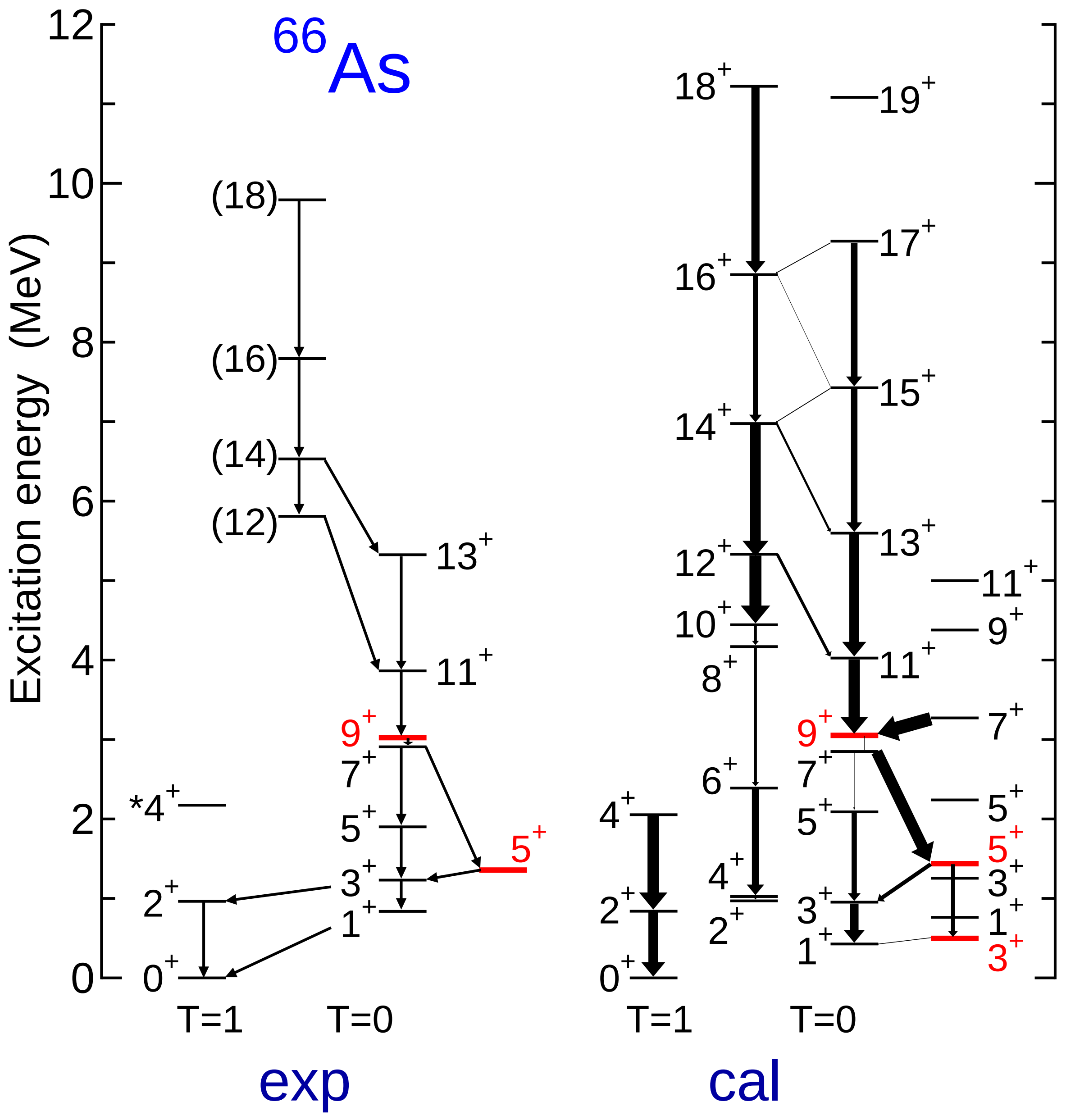


Spectroscopic Q moment (e fm^2)
30
20
10
0
−10
−20
−30
N=32
N=34
N=36
N=38
cal 2_2^+
Ge
Zn
cal 2_1^+
Ge
Zn
^{70}Ge
exp Q(2_1^+)
^{64}Zn, ^{68}Zn
exp Q(2_1^+)
[γ =60]
[γ =0]
(a) ^{66}Ge
(b) ^{68}Ge
(c) ^{70}Ge
0
20
40
60
q (fm^2)
minimum





K. Kaneko, M. Hasegawa, T. Mizusaki,
Phys. Rev. C., 70(2004), 051301.



66As
Excitation energy (MeV)
0
2
4
6
8
10
12
(18)
(16)
(14)
(12)
13+
11+
9+
7+
5+
3+
1+
*4+
2+
0+
5+
18+
16+
14+
12+
10+
8+
6+
4+
2+
19+
17+
15+
13+
11+
9+
7+
5+
3+
1+
11+
9+
7+
5+
5+
3+
1+
3+
4+
2+
0+
T=1
T=0
T=1
T=0
exp
cal

^{69}As

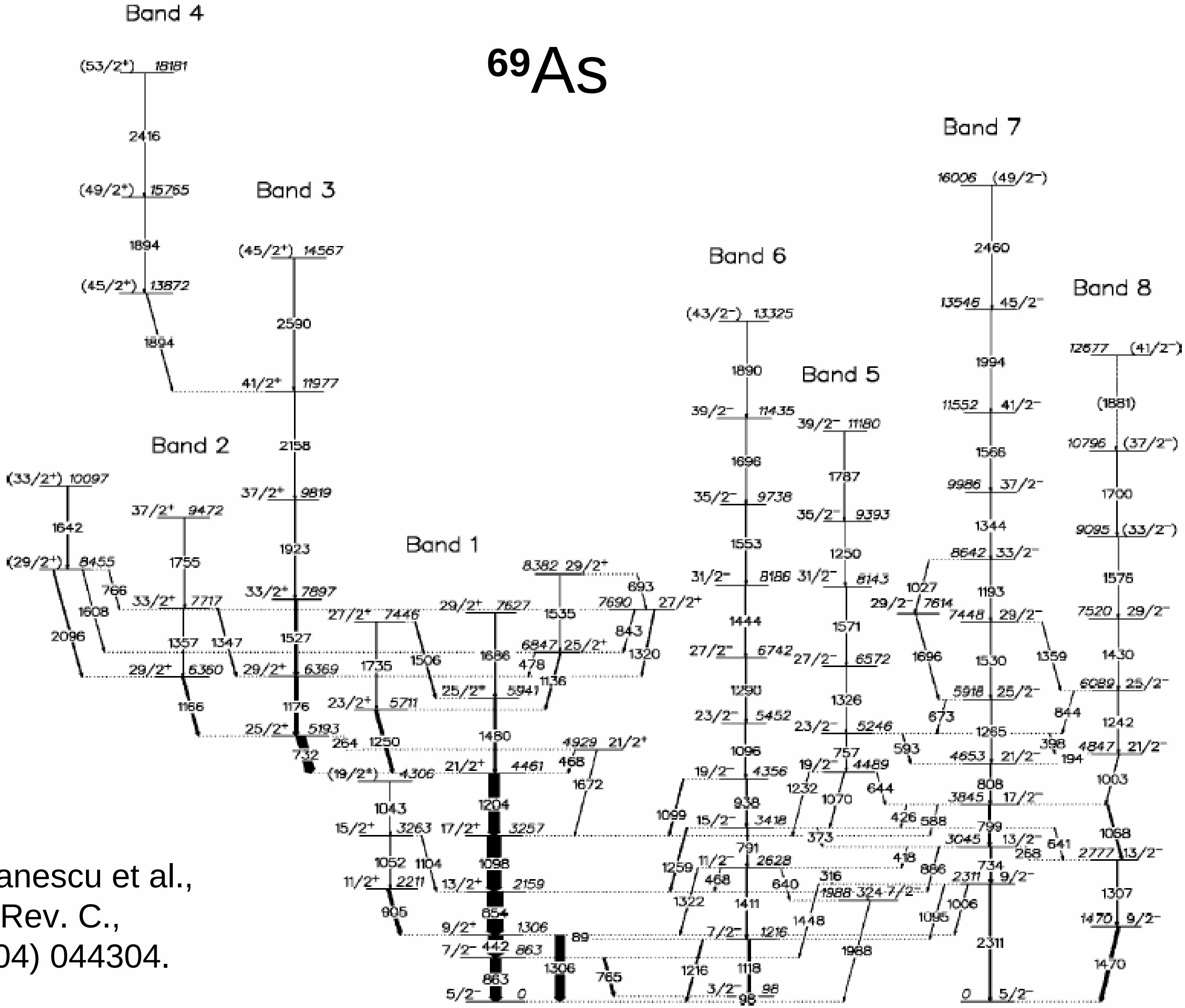
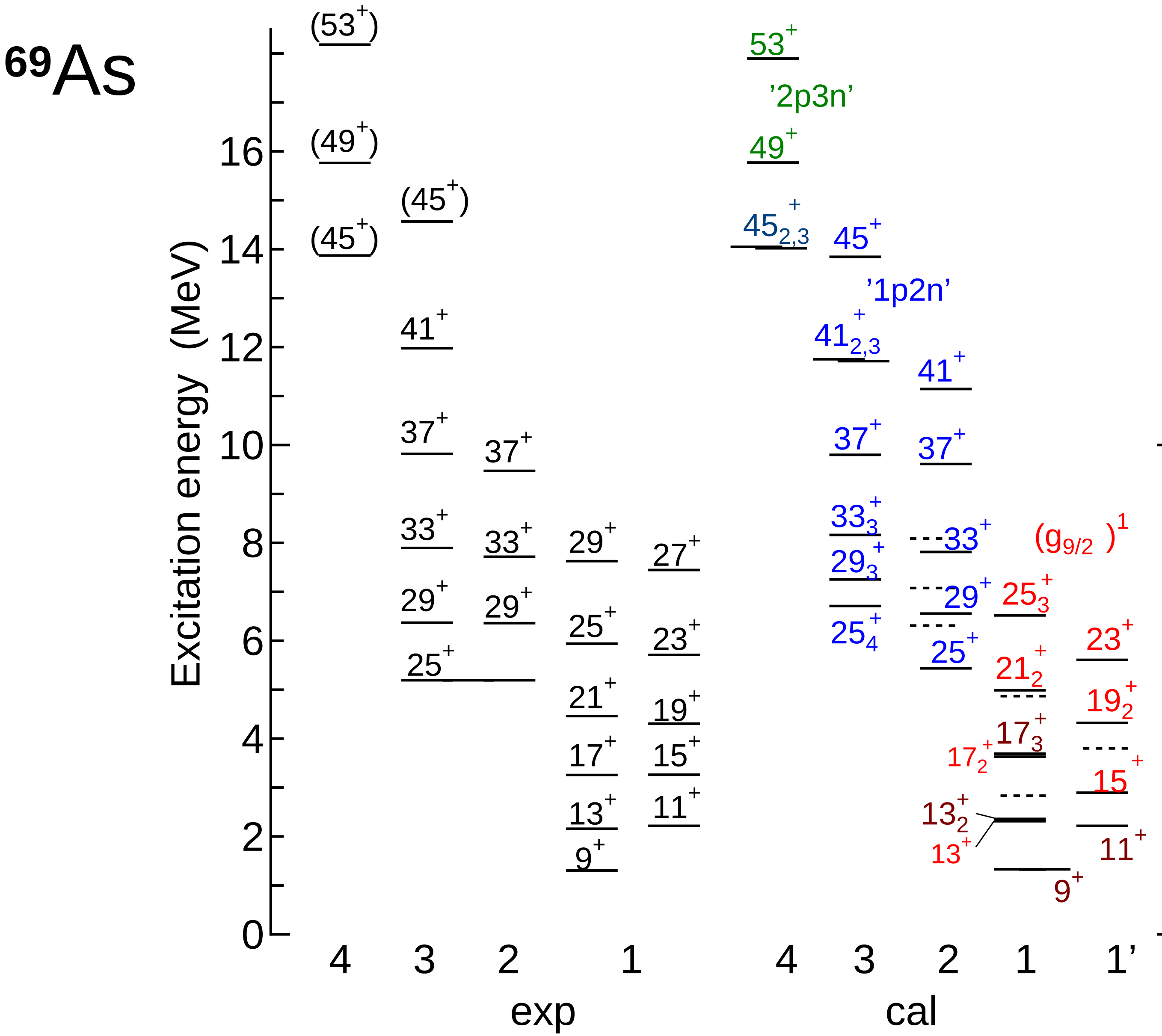
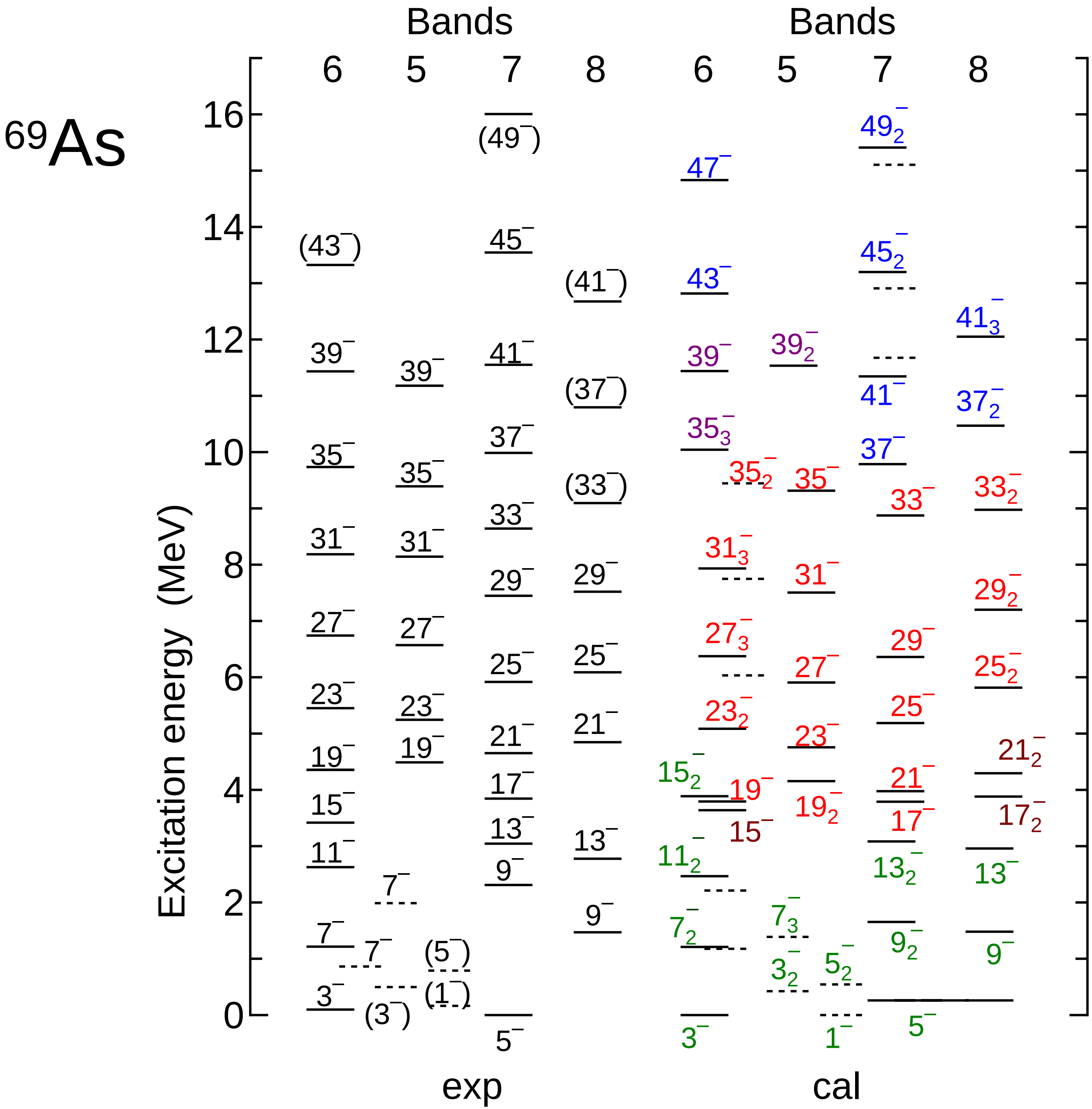


FIG. 3. High-spin states in ^{69}As obtained from the present analysis.

I. Stefanescu et al., Phys. Rev. C., 79(2004) 044304.



69As
Excitation energy (MeV)
0
2
4
6
8
10
12
14
16
(53+)
(49+)
(45+)
(45+)
41+
37+
33+
29+
25+
37+
33+
29+
29+
25+
21+
17+
13+
9+
27+
23+
19+
15+
11+
53+
'2p3n'
49+
45 2,3 +
45+
'1p2n'
41 2,3 +
41+
37+
37+
33 3 +
33+
29 3 +
29+
25 4 +
25+
(g9/2)1
25 3 +
21 2 +
17 3 +
17 2 +
13 2 +
13+
9+
23+
19 2 +
15+
11+
4
3
2
1
4
3
2
1
1'
exp
cal



69As
Bands
6 5 7 8
Bands
6 5 7 8
Excitation energy (MeV)
0 2 4 6 8 10 12 14 16
(43−) 39− 35− 31− 27− 23− 19− 15− 11− 7− 3−
39− 35− 31− 27− 23− 19− 7− 7− (3−)
(5−) (1−)
(49−) 45− 41− 37− 33− 29− 25− 21− 17− 13− 9− 5−
(41−) (37−) (33−) 29− 25− 21− 13− 9−
47− 43− 39− 35₃− 35₂− 31₃− 27₃− 23₂− 19− 15− 15₂− 11₂− 7₂− 3−
39₂− 35− 31− 27− 23− 19₂− 7₃− 3₂− 5₂− 1−
49₂− 45₂− 41− 37− 33− 29− 25− 21− 17− 13₂− 9₂− 5−
41₃− 37₂− 33₂− 29₂− 25₂− 21₂− 17₂− 13− 9−
exp
cal

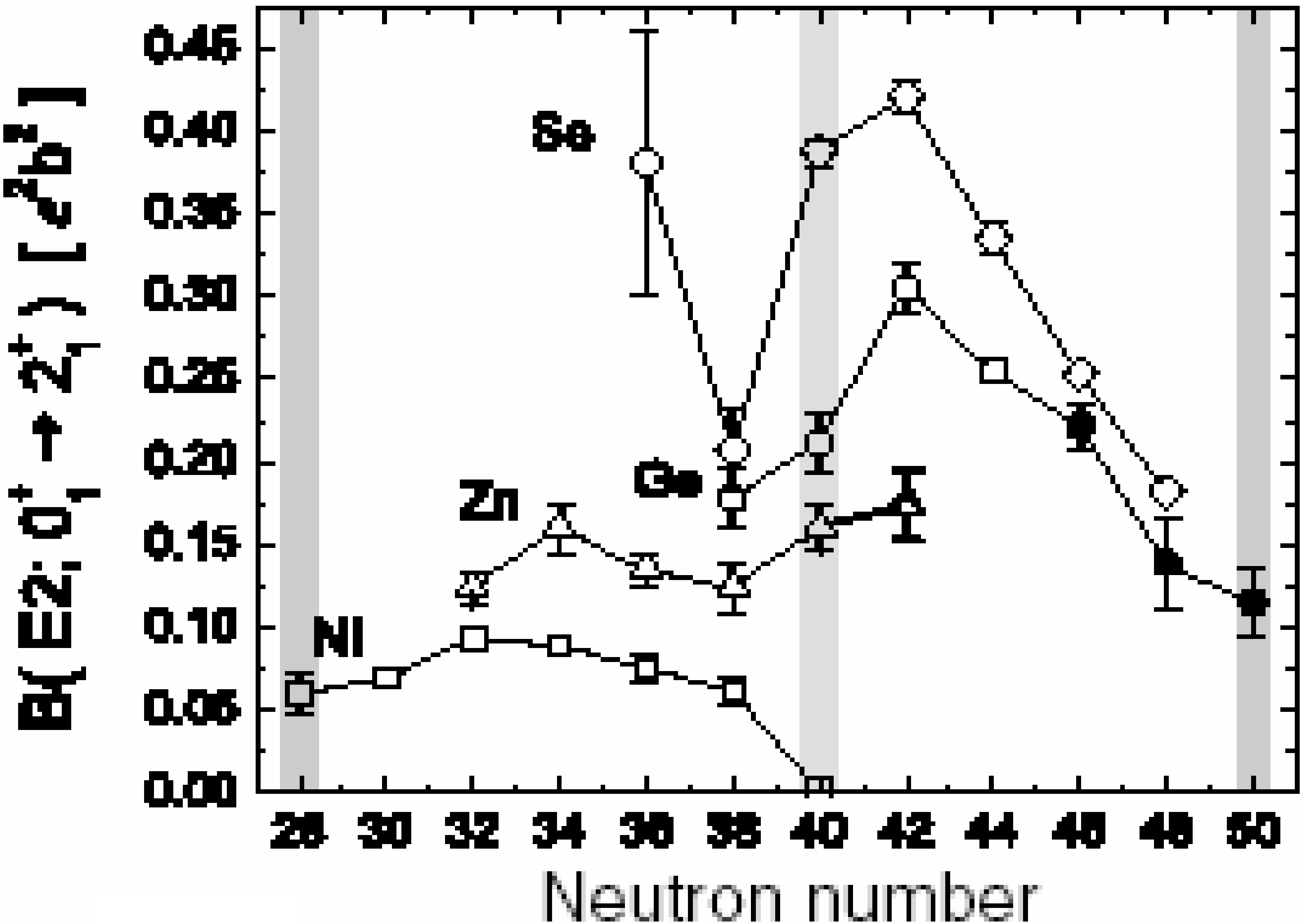
4. Problems

- Present EPQQ fits for a narrow region $A=60-70$.
  desired a better fit for $N=Z$ nuclei,

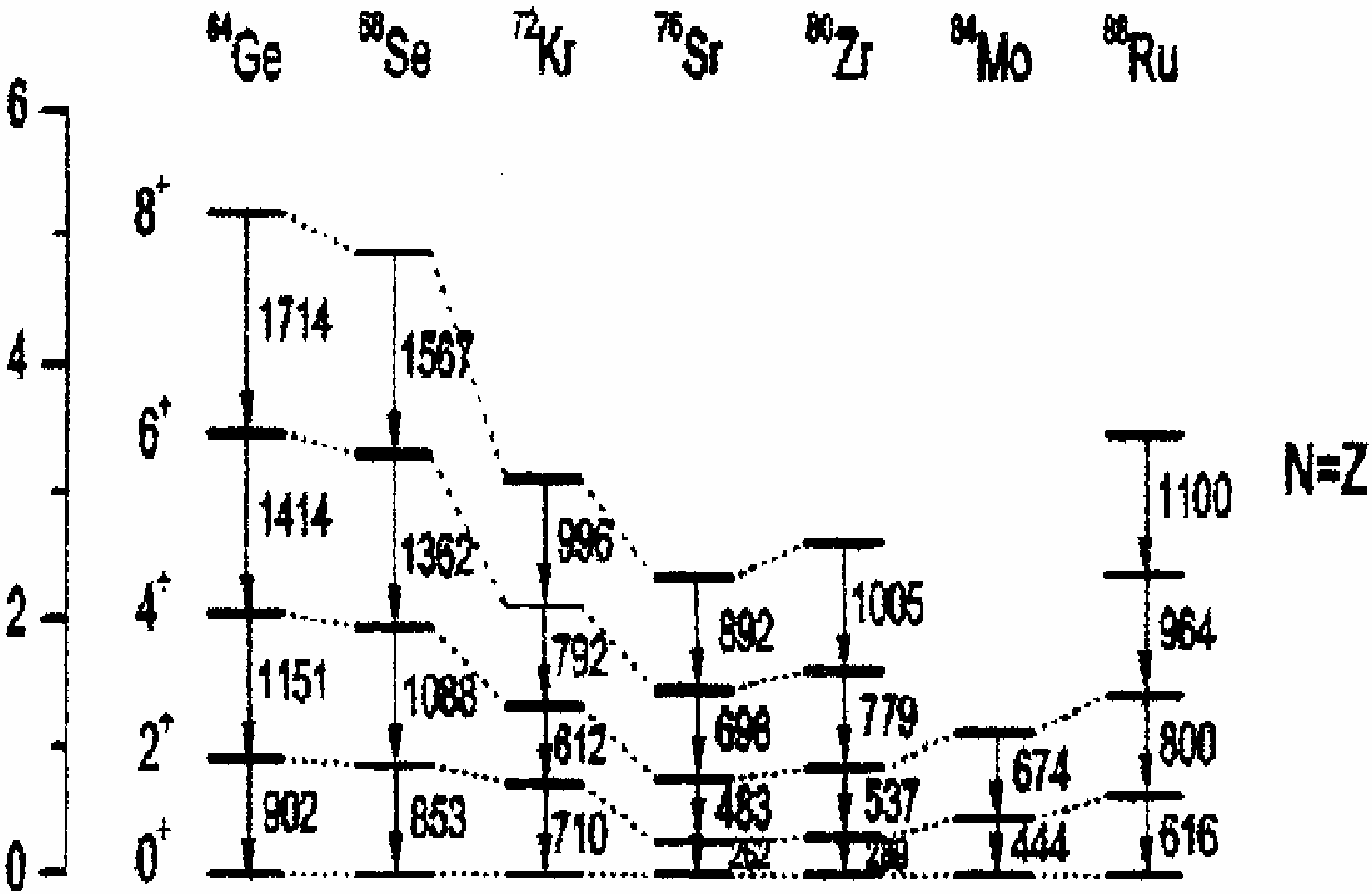
  - Lowering of $g_{9/2}$ near $N,Z=40$?
  - Drastic change at $N=40$ in Ge isotopes.
  - Drastic change from ^{68}Se to ^{72}Kr.

- $N>Z$ nuclei,
- many interesting phenomena,

open for shell model.



E. Padilla-Rodal et al., P. R. L., 94(2005), 122591.



M.Marginean et al., P. R. C., 63(2001), 031303.