Shell model approach to $N\approx Z$ medium-heavy nuclei using extended P+QQ interaction

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Topics:
Structure of $N\approx Z$ medium-heavy nuclei,
Large-scale shell model calculations,
Extended P+QQ interaction (EPQQ).

based on the collaboration
with K. Kaneko and T. Mizusaki
1. At the end of the 20th century
   --- Shell model study of $f_{7/2}$-shell nuclei ---

   SM had succeeded for sd shell nuclei
   using \textit{realistic} effective interaction USD


   \begin{itemize}
   \item \textit{realistic} effective interaction for pf shell
     \begin{itemize}
     \item KB3  A. Poves & A.P. Zuker, Phys. Rep., 70(1981), 235
     \item FPD6  W.A. Richter, … B.A. Brown, Nucl. Phys. 523(1991), 325
     \end{itemize}
   \end{itemize}

   \begin{itemize}
   \item Caurier’s calculation code
     \rightarrow \text{full pf shell model calculations}
     \begin{itemize}
     \item for $A = 48$, odd nuclei $A = 47, 49$, …
     \end{itemize}
   \end{itemize}

FIG. 3. Theoretical and experimental energy levels of $^{47}$Ti.

G. Martinez-Pinedo, A. Zuker, A. Poves, E. Caurier,
Better effective interaction was required for heavier pf shell nuclei → KB3G, applied to \( A = 50,51,52 \)
Fig. 8. Yrast band of $^{51}$V.

Fig. 9. Energy levels of $^{51}$V.
● **Monte Carlo** shell model


→ **Shape coexistence in** $^{56}\text{Ni}$.

T. Mizusaki, T. Otsuka, Y. Utsuno, M. Honma, T. Sebe,
P. R. C., 59(1999), R1846.

..........> 2000
FIG. 2. Experimental levels [1,12] of $^{56}$Ni compared with QMCD and GCM results with a FPD6 Hamiltonian. The GCM energies are shown relative to the QMCD ground state. The $B(E2: (J+1 \text{ or } 2) \rightarrow J^+)$ values, which are larger than $100 \ E^2 \ fm^4$, are indicated by the width of the arrows.

2. Various collective models and our approach

● Collective models
  RPA
  IBM
  Mean field approximations
  HF, HF+RPA
  HFB
  using P+QQ force,
  Skyrme force, etc.

● VAMPIR

● Projected SM
  Y. Sun, nucl-th/0211043.

Fig. 1. Potential-energy curves calculated by the conventional Skyrme–HFB method for $^{64}\text{Ge}$, $^{68}\text{Sr}$, $^{72}\text{Kr}$, $^{76}\text{Sr}$, $^{80}\text{Kr}$, and $^{84}\text{Sr}$ are drawn as functions of the quadrupole deformation parameter $\beta_2$. The STI interaction is used for the particle–hole channel, while the density-dependent pairing interaction with $V_0 = 1000.0 \text{ MeV fm}^3$ and $\alpha = 0.16 \text{ fm}^{-1}$ is used for the particle–particle channel.

Fig. 2. The alignment plot for some states calculated in $^{68}$Se.
Fig. 2. Comparison of the calculated yrast energy levels with known data for $N = Z$ and $N = Z + 2$ nuclei in the plots of moment of inertia $\mathcal{I}^{(1)}(I) = (2I + 1)/E_\gamma(I \rightarrow I - 2)$ vs. rotational frequency $\omega(I) = E_\gamma(I \rightarrow I - 2)/2$. 

Y. Sun, nucl-th/0211043.
We investigated T=0 and T=1 pairing correlations by means of the shell model with P+QQ.

- **Important T=0 monopole field**

\[ V_{\pi\nu}^{T=0} = -k^0 \sum_{a \leq b} \sum_{JM} A_{JM\pi\nu T=0}^{+}(ab)A_{JM\pi\nu T=0}(ab) \]

...can explain the binding energy.


- adding quadrupole pairing force, etc.,
- and T=0 and T=1 monopole corrections,

- **Extended P+QQ int. is more than a toy interaction.**

→ successfully describes \( f_{7/2} \)-shell nuclei.

$V_{\pi\nu}^{T=0} = -\frac{1}{2} k^0$

$x \left\{ \frac{n_{\nu}}{2} \left( \frac{n_{\nu}}{2} + 1 \right) \right\} - \widehat{T}^2$

$(g_{g9/2}^{n_p+n_n})$ systems
\begin{align*}
\left( g_{g9/2} \right)^{n_p+n_n} 
\end{align*}

 systems
B.E. $n_p=n_n=n/2$

- SLG (T=0)
- SLG (T=1)
- $k^0=0$
- $k^0=0.925$

$(g_{9/2})^n$ systems
$^{46}\text{V}$

$T=1$

$T=0$

Excitation energy (MeV)

3+

The 3$^+$ is adjusted.

EPQQ

Expt KB3 FPD6
3. About effective interaction

- Realistic effective interactions $\sim$ EPQQ

$$H \approx H_{sp} + V_{\pi V}^{T=0} + \Delta V_{mc}$$

$$+ V(P_0) + V(P_2) + V(QQ) + V(OO)\ldots$$

pointed out by A.P. Zuker, M. Dufour, P. R. C., 54(1996), 1641.

J. Duflo, P. R. C., 59(1999), R2347.

Our calculations proved this concretely.

→ Shell model with EPQQ
\[ V_{\pi\nu}^{T=0} = -k^0 \sum_{a \leq b} \sum_{JM} A_{JMT=0}^+(ab) A_{JMT=0}^-(ab) \]

Better effective interactions are desired.


There are many interesting phenomena in pf-shell nuclei, which could be investigated by using GXPF1 int.

Study heavier nuclei as well as light ones!
4. Our work

Large-scale shell model calculations with EPQQ

- Mizusaki’s calculation code

- **Physics:** $N \approx Z$ nuclei with $A = 60 - 100$
  When $Z$, $N$, and $J$ increase, structure changes rapidly,
  change in shape, shape coexistence, particle alignments, ...

$^{64}$Ge: quadrupole and octupole corr.
- Observed heaviest $N=Z$ nucleus $^{88}\text{Ru}$
  \[
  (p_{3/2}, f_{5/2}, p_{1/2}, g_{9/2})^{-12}
  \]
  full SM cal. with maximum dimension
  \[1.4 \times 10^8\]
  explains difference of backbending
  between $^{88}\text{Ru}$ and $^{90}\text{Ru}$,
  and gives good prediction for $^{89}\text{Ru}$.

**Advantages:**
- describes *odd* and even-even nuclei
  using the same parameters,
  better than the mean field approx, etc.
back to Ge isotopes
FIG. 2. Low-spin level scheme for $^{68}\text{Ge}$ from the present experiment.
BAND STRUCTURE OF $^{68}$Ge

The diagrams show the band structure for different angular momenta $(\pi,\alpha) = (\pm,0)$ and $(\pm,1)$. Each graph plots the energy $E - 0.02853 (\hbar^2/2I)$ (MeV) against spin $\hbar$. The bands are labeled with quantum numbers such as [02,2], [01,3], [22,2], [22,4], [01,3], [23,3], [12,2], [01,2], and [12,3]. The bands represent different quantum states and their energies are marked with distinct symbols for different quantum numbers.
back to Ge isotopes
Parameter search using EPQQ
including s. p. energies.
reproduces well
a lot of energy levels, different bands,
B(E2) values of collective bands, Q moments.

Advantages: strict wave functions
cal. of $\langle j_a \rangle, \langle t_a \rangle, \langle n_\pi^a \rangle, \langle n_\nu^a \rangle, \ldots$

→ $T=0$ proton-neutron alignment
in even-even, odd, and odd-odd nuclei,
Successive alignments with increasing $J$.

M. H., K. Kaneko, and T. Mizusaki, P. R. C, 70(2004), 031301(R),
71(2005), 044301.
<table>
<thead>
<tr>
<th>Excitation energy (MeV)</th>
<th>$^{65}\text{Ge}$</th>
<th>$^{67}\text{Ge}$</th>
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$^{65}\text{Ge}$ and $^{67}\text{Ge}$ are nuclear isotopes, with $\pi$ representing the parity and $\text{cal}$ and $\text{exp}$ denoting calculated and experimental data, respectively.
(a) $^{66}$Ge
\[ \pi = + \]

(b) $^{66}$Ge
\[ \pi = \text{ yrast} \]
$^{68}\text{Ge}$

\[ \langle n_{g_{9/2}} \rangle \]

- Neutron
- Proton

\[ 12^+_3, 16^+_4, 18^+_2, 8^+_2, 14^+_3, 12^+_3, 14^+_3, 16^+_4 \]
$^{68}$Ge

even $J$, $\pi = +$

- gs band
- $2n$-$\alpha$ band
- $1p1n$-$\alpha$ band
- $2p2n$-$\alpha$ band

Excitation energy (MeV)

- cal
- $8^+_2$
- $18^+_2$
- $2n$-$\alpha$ band
- $1p1n$-$\alpha$ band
- $18^+_2$ band on $18^+_2$
- band on $12_4$
- band on $8_1$
- gs band

$J$
• **Shape transition in** $^{60}$Zn, $^{64}$Ge, $^{68}$Se, ($^{72}$Kr)
prolate, $\rightarrow$ oblate (coexisting prolate) $\rightarrow$ triaxial


  using the successful Hamiltonian, 
constrained HF calculation.


• **Isomeric states in odd-odd** $^{66}$As, and $^{67}$As


• **detailed structure of odd nucleus** $^{69}$As

I. Stefanescu et al.,
Phys. Rev. C.,

FIG. 3. High-spin states in $^{69}$As obtained from the present analysis.
<table>
<thead>
<tr>
<th>Excitation energy (MeV)</th>
<th>Bands</th>
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**69 As**
4. Problems

- Present EPQQ fits for a narrow region $A=60-70$. Desired a better fit for $N=Z$ nuclei,
  - Lowering of $g_{9/2}$ near $N,Z=40$?
  - Drastic change at $N=40$ in Ge isotopes.
  - Drastic change from $^{68}$Se to $^{72}$Kr.
- $N>Z$ nuclei,
- many interesting phenomena,

  *open for shell model.*
E. Padilla-Rodal et al., P. R. L., 94(2005), 122591.
M. Marginean et al., P. R. C., 63(2001), 031303.