New interpretation of chiral doublet bands by a pair-truncated shell model

K. Higashiyama
Department of Physics, University of Tokyo

Outline of my talk
- Introduction
- Framework of the PTSM and its application to even-even and odd-mass nuclei
- New band mechanism of doubly-odd nuclei
- Summary

In collaboration with N. Yoshinaga and K. Tanabe (Saitama university)
Introduction

Tilted Axis Cranking model
Doubly odd nuclei with triaxial deformation
Orientation of axis of rotation is tilted to three axis

Particle Rotor Model
One neutron hole and one proton particle coupled with a rigid triaxial rotor
Interpretation of chiral doublet bands

High j holes tend to align with long axis
High j particles tend to align with short axis
Triaxial core tend to align with intermediate axis

Three angular momenta construct left- and right-handed systems

Two nearly degenerate $\Delta I=1$ bands of the same parity originate from these systems
Candidates for chiral doublet bands

The \( A \sim 130 \) nuclei have several valence protons outside the closed shell \( Z=50 \) and several neutron holes with respect to the closed shell \( N=82 \).
The single particle (hole) energies for proton (neutron)

**Neutron**
- $d_{5/2}$
- $h_{11/2}$
- $s_{1/2}$
- $d_{3/2}$
- $g_{7/2}$
- $50$

**Proton**
- $s_{1/2}$
- $h_{11/2}$
- $d_{3/2}$
- $d_{5/2}$
- $g_{7/2}$
- $(2.99)$

The $0h_{11/2}$ orbital plays a Important role in the description of the chiral doublet bands.
ΔI = 1 bands have been found in many doubly-odd nuclei with mass around 130.
$\Delta I = 1$ bands have been found in many doubly-odd nuclei with mass around 130.

**Studies for $A \sim 130$ nuclei**

K. Starosta et al.,

A. A. Hecht et al.,

R. A. Bark et al.,

K. Starosta et al.,

T. Koike et al.,

G. Rainovski et al.,

A. A. Hecht et al.,

S. Zhu et al.,
Tilted axis cranking model

FIG. 3. Experimental energies vs spin for band 2 (filled symbols) and band 3 (open symbols) in $^{132}$Cs (cf. Fig. 2). The 3D TAC energies for the chiral $\pi h_{11/2} \otimes \nu h_{11/2}$ solution with no pairing included in $^{132}$Cs are shown by the solid line. The inset presents the dependence of the total routhian on the tilt angle \( \varphi \) when the pairing is included in the calculations.

G. Rainovski et al.,

<table>
<thead>
<tr>
<th>( h \omega ) (keV)</th>
<th>( \vartheta ) (deg)</th>
<th>( \varphi ) (deg)</th>
<th>( I ) (h)</th>
<th>( B(M1)/B(E2) ) (( \mu_N/e ) b)²</th>
</tr>
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<tbody>
<tr>
<td>185</td>
<td>55</td>
<td>10.3</td>
<td>10.09</td>
<td>195.1</td>
</tr>
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<td>200</td>
<td>60</td>
<td>29.2</td>
<td>11.02</td>
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<td>225</td>
<td>65</td>
<td>40.6</td>
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<td>235</td>
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<td>240</td>
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<td>250</td>
<td>65</td>
<td>45.5</td>
<td>13.57</td>
<td>9.9</td>
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</table>

TABLE II. The results of 3D TAC calculations with no pairing included for the $\pi h_{11/2} \otimes \nu h_{11/2}$ configuration in $^{132}$Cs. The deformation parameters used were \( e_2 = 0.161 \), \( e_4 = 0.003 \), and \( \gamma = 36^\circ \). These were obtained as self-consistency values at \( h \omega = 185 \text{ keV} \).

FIG. 4. (a) Measured \( B(M1; I \to I - 1)/B(E2; I \to I - 2) \) ratios for band 2 in $^{132}$Cs. (b) Measured \( B(M1; I \to I - 1)_{\text{rot}}/B(M1; I \to I - 1)_{\text{corr}} \) ratios for band 3 in $^{132}$Cs. The lines are drawn to guide the eye.
Rigid triaxial rotor + two quasi-particles

- Particle rotor model
- Phenomenological core-particle-hole coupling model

**even-even core**

![Diagram](image)

**FIG. 4.** Comparisons between energies of excited states in the yrast and γ bands in $^{130}$Ba and $^{132}$Ba with those calculated for a rigid triaxial rotor. See Table II for the model parameters. The experimental data are taken from Refs. [29] and [30] for $^{130}$Ba and $^{132}$Ba, respectively.

**FIG. 7.** Comparison between energies of excited states in the partner band in $^{132}$La with those calculated for $\pi h_{11/2} \nu^{-1} h_{11/2}$ particle-hole coupling with a rigid triaxial rotor (see Table II for the model parameters). Theoretical states are shown only when the corresponding experimental states are known.

The yrast band was built on the ground-state band of the even-even core.

The yrare band was built on the quasi-γ band of the even-even core.
Experiment
- Mass A~130: Odd-mass and doubly-odd nuclei
- Mass A~100: Odd-mass and doubly-odd nuclei
- Mass A~190: Doubly-odd nuclei

Theory
- Tilted axis cranking model: Support the chiral doublet bands
- Particle rotor model
- Phenomenological core-particle-hole coupling model:
  - One neutron hole and one proton particle coupled with a rigid triaxial rotor: Support the chiral doublet bands
- Interacting boson fermion-fermion model
The $A \sim 130$ nuclei have several valence protons outside the closed shell $Z=50$ and several neutron holes with respect to the closed shell $N=82$. 
Low-lying collective states

- Neutron
- Proton

Oblate ↔ Prolate

$\gamma$-unstable

Mean field theories are not applicable.
Exact shell model calculations are not possible.

Pair-truncated Shell Model
Pair truncated shell model

SD-pair states

**S-pair**

\[ S^\dagger = \sum_j \alpha_j A_0^{\dagger(0)}(j,j) \]

**D-pair**

\[ D_\alpha = \sum j_1 j_2 \beta_{j_1 j_2} A_M^{\dagger(2)}(j_1 j_2) \]

\[
\begin{align*}
(A_M^{\dagger(j)}(j_1 j_2) & = \sum_{J M j_1 m_1 j_2 m_2} (j_1 m_1 j_2 m_2 | JM) c_{j_1 m_1}^{\dagger} c_{j_2 m_2}^{\dagger} = [c_{j_1}^{\dagger} c_{j_2}^{\dagger}]_M^{(J)}
\end{align*}
\]

Closed shell

SD-pair state

\[ (S^\dagger)^{n_s} (D^\dagger)^{n_d} \langle - | = \left| S^{n_s} D^{n_d} J \right\rangle \]
In the PTSM, the shell-model basis is restricted to the SD subspace with angular momentum zero (S) and two (D) collective pairs.

Odd system

**SD-pair state**

\[ |S^{n_s} D^{n_d} I \rangle \]

**SD-pair+1 particle state**

\[ c_j^{\dagger} |S^{n_s} D^{n_d} I' \rangle = |j S^{n_s} D^{n_d} I \rangle \]

Basis state of even-even, odd-mass and doubly-odd nuclei

\[ |\Phi(I)\rangle = \left[ \begin{array}{c} |S_v^{n_s} D_v^{n_d} I_v \rangle \\ \vdots \\ |j_v S_v^{n_s} D_v^{n_d} I_v \rangle \end{array} \right] \otimes \left[ \begin{array}{c} |S_{\pi}^{n_s} D_{\pi}^{n_d} I_{\pi} \rangle \\ \vdots \\ |j_{\pi} S_{\pi}^{n_s} D_{\pi}^{n_d} I_{\pi} \rangle \end{array} \right]^{(I)} \]

One nucleon is included to the SD-pair states

One nucleon is included to the SD-pair states
Ba

$E$ (MeV)

$N$

PTSM

expt.

Doubly-odd nuclei

\[ ^{134}\text{La} \]

\[ (\nu h_{1/2}) \otimes (\pi h_{1/2}) \]

E (MeV)

expt. 0 1 2 3 4 5

Yrast 11\(^{+}\) 12\(^{+}\) 13\(^{+}\) 14\(^{+}\) 15\(^{+}\) 16\(^{+}\) 17\(^{+}\) 18\(^{+}\) 19\(^{+}\) 20\(^{+}\) 21\(^{+}\)

Yrare 11\(^{+}\) 12\(^{+}\) 13\(^{+}\) 14\(^{+}\) 15\(^{+}\) 16\(^{+}\) 17\(^{+}\) 18\(^{+}\) 19\(^{+}\) 20\(^{+}\) 21\(^{+}\)

PTSM 8\(^{+}\) 9\(^{+}\) 10\(^{+}\) 11\(^{+}\) 12\(^{+}\) 13\(^{+}\) 14\(^{+}\) 15\(^{+}\) 16\(^{+}\) 17\(^{+}\) 18\(^{+}\)
We analyze the behavior of three angular momenta of the unpaired neutron, unpaired proton and even-even core.
Internal structure of doublet bands

\[ \hat{j}_{\tau j} \quad (\tau = \nu \text{ or } \pi) \] : angular momenta of valence nucleons in the $0h_{11/2}$ orbital

\[ \hat{R} = \hat{J} - \hat{j}_{\nu j} - \hat{j}_{\pi j} \] : angular momentum of the core

Effective angle of two angular momenta

\[
\cos \theta_{j_{\nu j} \cdot j_{\pi j}} = \frac{\langle \Phi(J) | j_{\nu j} \cdot j_{\pi j} | \Phi(J) \rangle}{\sqrt{\langle \Phi(J) | j_{\nu j}^2 | \Phi(J) \rangle \langle \Phi(J) | j_{\pi j}^2 | \Phi(J) \rangle}}
\]

Square of core angular momentum

\[ R^2 = \langle \Phi(J) | \hat{R}^2 | \Phi(J) \rangle \]
The effective angles of two angular momenta of the neutrons and protons in the $0h_{11/2}$ orbitals calculated in the PTSM.

The squares of core angular momentum calculated in the PTSM.

The effective angles never become zero because of quantum fluctuations.
Each $\Delta I=2$ E2 band arises from the quadrupole collective excitation of the even-even part of the nucleus.

The chopsticks motion of the unpaired neutron and unpaired proton in $0h_{11/2}$ orbitals enhances the B(M1) values to be large.
The $\nu h_{11/2}$, $\pi h_{11/2}$ bands are interpreted as arising from the chopsticks configurations of the two angular momenta of the unpaired nucleons, and the quadrupole collective excitations of the even-even core.
Summary

The detailed results and their analyses are presented in