Nuclear Matrix Elements of Double Beta Decay

Probe the fundamental properties of neutrinos by using nuclei as a microscopic laboratory

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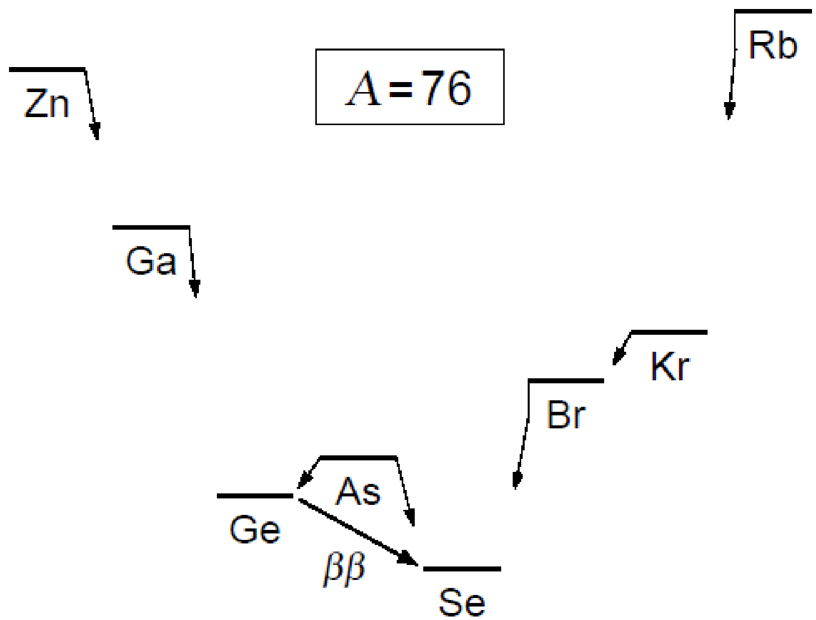
What is double beta decay?



$\beta\beta$ decay can be observed, when single β decays are forbidden.

- $\Delta Z = 2$
- Second order process of the weak interaction
- Very long half-lives

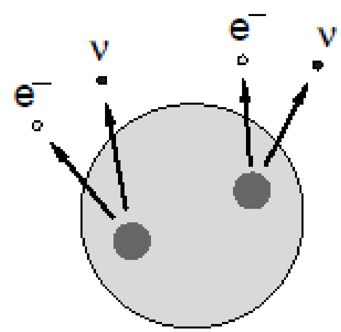
For observation, those nuclei are favored with

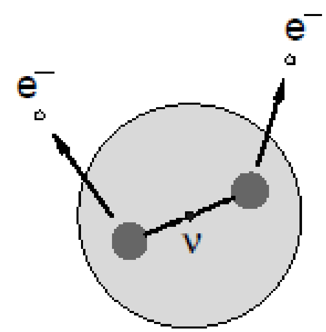
- Large Q-value
- Large abundance

Typical nuclides : ^{48}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo, ^{128}Te, ^{130}Te, ^{136}Xe, ^{150}Nd

Two Decay Modes

$$(Z,A) \rightarrow (Z+2,A) + 2e^- + 2\nu$$
$$(Z,A) \rightarrow (Z+2,A) + 2e^-$$





2ν mode

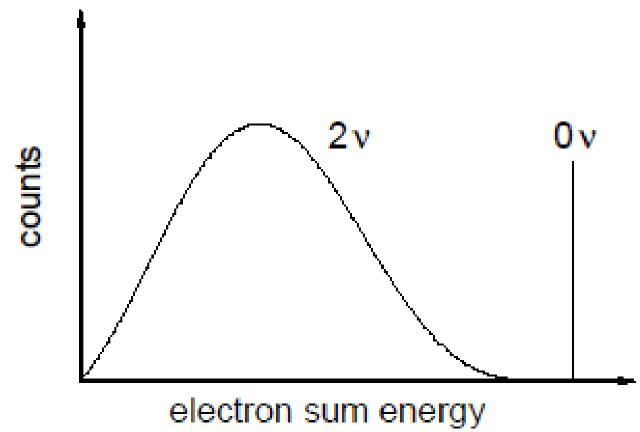
- Allowd in the Standard Model
- Observed for some 10 nuclides
- Shortest half-life
  $T_{1/2} = 10^{19}$ y

0ν mode

- Forbidden in the SM
- No observations so far

Measurements of Half-Lives

1. **Geochemical Method**
   eg.: ^{128}Te ^{128}Xe (10^7) in rocks of
   1.3 billion years old
   Cannot distinguish 0ν and 2ν decays
2. **Radiochemical Method** (^{238}U)
3. **Direct method**
   0ν and 2ν decays can be separated by sum energy spectrum of two electrons.
   1. Measurement of electron energies only (*eg*. ^{76}Ge)
   2. Measurement of electron energies and momenta (angular correlations)



Conditions for $0\nu\beta\beta$ decay

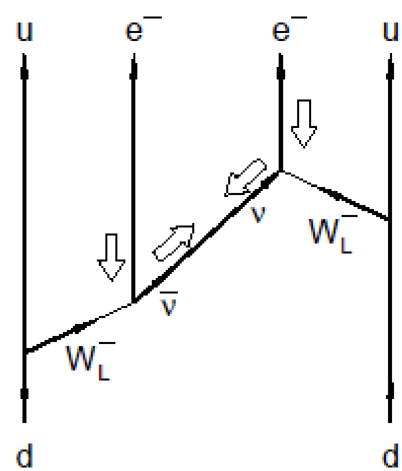
- **Condition 1 : Neutrinos are Majorana particles.**

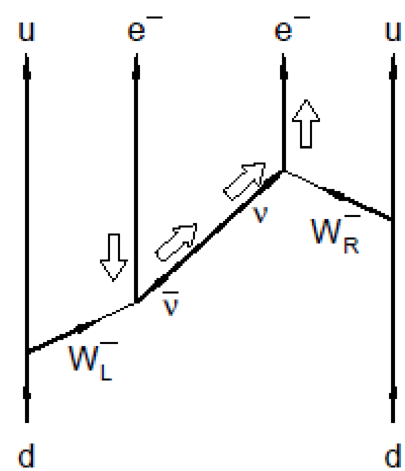
  The (anti-)neutrino emitted by a nucleon is identical to the neutrino to be absorbed by another nucleon in the nucleus.

- **Condition 2 : Neutrinos have finite masses.**

  The (anti-)neutrino is **emitted in left-handed** by the V-A interaction, but has **a mixture of right-handed component** due to the finite mass, and then can be absorbed by another nucleon via the V-A interaction.

  At the same time, a possible V+A interaction contributes.

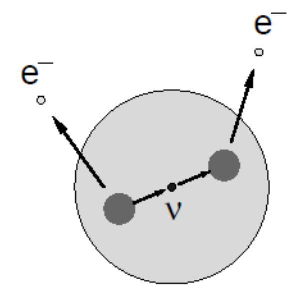




If 0νββ decay is observed

$$n \rightarrow p + e^- + \bigcirc$$

$$\bigcirc + n \rightarrow p + e^-$$



1. **Neutrinos are Majorana particles** : $\overline{\nu} = \nu$

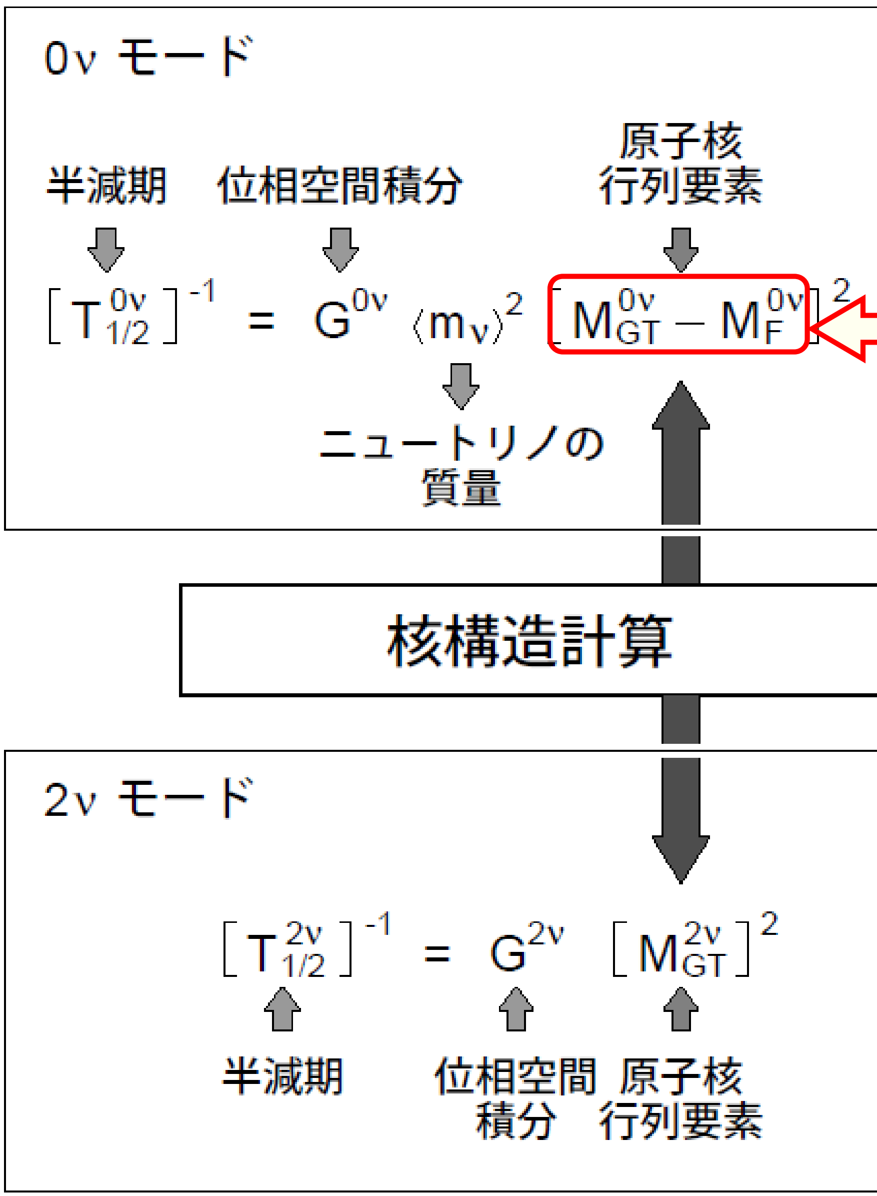
   Double beta decay is, at present, the only practical method to distinguish between **Dirac** neutrinos and **Majorana** neutrinos.

2. **Neutrinos have finite masses.**

   already found by observation of atmospheric, solar, and reactor neutrinos

3. **We can obtain information on the absolute mass scale of neutrinos.**

   Neutrino oscillations tell us only $\Delta m_{ab}^2 = m_a^2 - m_b^2$



Goal :

Predict 0ν decay nuclear matrix elements reliably

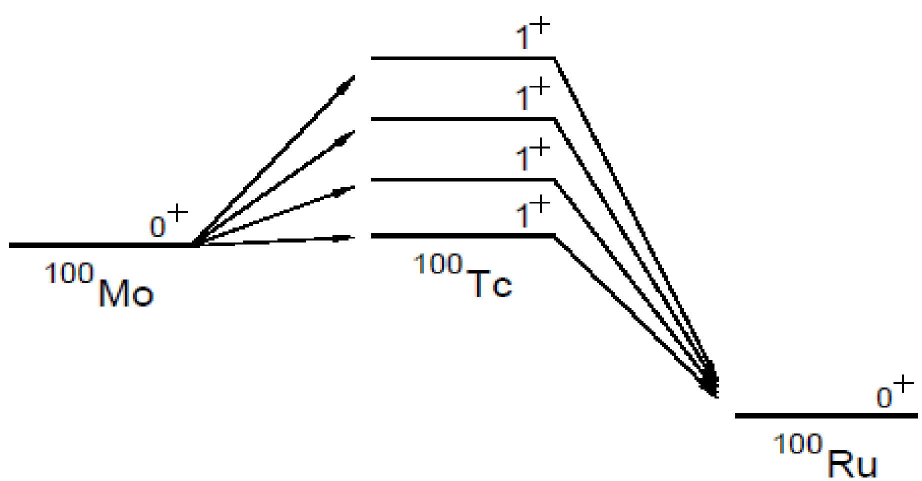
2ν decay provides a test of nuclear structure calculations by comparison of calculated and observed half-lives

But, nuclear structure calculations have poor predictive power for 2ν decay nuclear matrix elements.

Nuclear Matrix Element of 2ν decay mode

- Second-order perturbation of the weak interaction
- Successive Gamow-Teller transitions
  through virtual intermediate 1^+ states

$$M_{\mathrm{GT}}^{2\nu} = \sum_m \frac{\langle 0_f^+ \| t_- \boldsymbol{\sigma} \| 1_m^+ \rangle \langle 1_m^+ \| t_- \boldsymbol{\sigma} \| 0_i^+ \rangle}{E_m - (M_i + M_f)/2}$$



Nuclear Matrix Elements of 0ν decay mode (1)

- **Neutrino Potential** due to the exchange of a neutrino between two nucleons

$$V(|\boldsymbol{r}_1 - \boldsymbol{r}_2|) \sim \int \mathrm{d}\boldsymbol{q} \, \frac{\exp[\, i\, \boldsymbol{q} \cdot (\, \boldsymbol{r}_1 - \boldsymbol{r}_2 \,)\,]}{\omega\, [\, E_i - (\omega + m_e + E_a)\,]} \qquad \omega = \sqrt{\boldsymbol{q}^2 + m_\nu^2}$$

  - Integral over neutrino momenta
  - Contribution to the nuclear matrix elements via q ~ 100 MeV/c
  - Long-range Yukawa-type potential
    - Corresponds to the exchange of a particle with the mass of around 10 MeV
    - Independent of the neutrino mass

Nuclear Matrix Elements of 0ν decay mode (2)

- **double *Gamow-Teller* matrix element**

$$
\begin{aligned}
M_{\mathsf{T}}^{\nu} &= g_A^2 \int_0^\infty q^2 \, \mathrm{d}q \, v(q) \\
&\times \sum_{LJ^\pi m} \langle \, 0_f^+ \, \| \, \mathcal{F}_{1LJ} \, \| \, J_m^\pi \, \rangle \, \langle \, J_m^\pi \, \| \, \mathcal{F}_{1LJ} \, \| \, 0_i^+ \, \rangle
\end{aligned}
$$

$$
\mathcal{F}_{1LJ} = t_- \, [\, \boldsymbol{\sigma} \otimes \boldsymbol{Y}_L(\hat{\boldsymbol{r}}) \,]_J \, j_L(qr) \qquad J^\pi \neq 0^+
$$

- **double *Fermi* matrix element**

$$
\begin{aligned}
M_{\mathsf{F}}^{0\nu} &= (g_V/g_A)^2 \int_0^\infty q^2 \, \mathrm{d}q \, v(q) \\
&\times \sum_{J^\pi m} \langle \, 0_f^+ \, \| \, \mathcal{F}_{0JJ} \, \| \, J_m^\pi \, \rangle \, \langle \, J_m^\pi \, \| \, \mathcal{F}_{0JJ} \, \| \, 0_i^+ \, \rangle
\end{aligned}
$$

$$
\mathcal{F}_{0JJ} = t_- \, \boldsymbol{Y}_J(\hat{\boldsymbol{r}}) \, j_J(qr) \qquad J^\pi = 0^+, 1^-, 2^+, 3^-, \cdots
$$

QRPA Models

Quasiparticle **R**andom **P**hase **A**pproximation

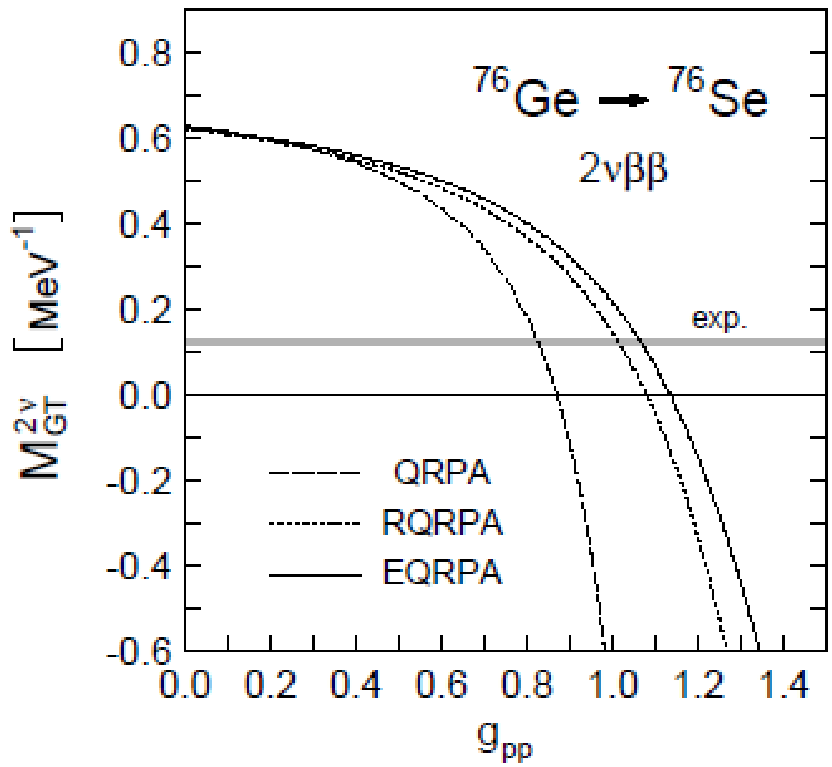
- RPA calculation in a quasiparticle basis
  - Pairing correlations in like-nucleon systems by BCS
  - Proton-Neutron correlations by RPA
- Much improvement has been achieved:
  - Commutators of nucleon pair creation and annihilation operators
    renormalized QRPA
  - Ikeda sum rule for Gamow-Teller transition strengths
    self-consistent QRPA

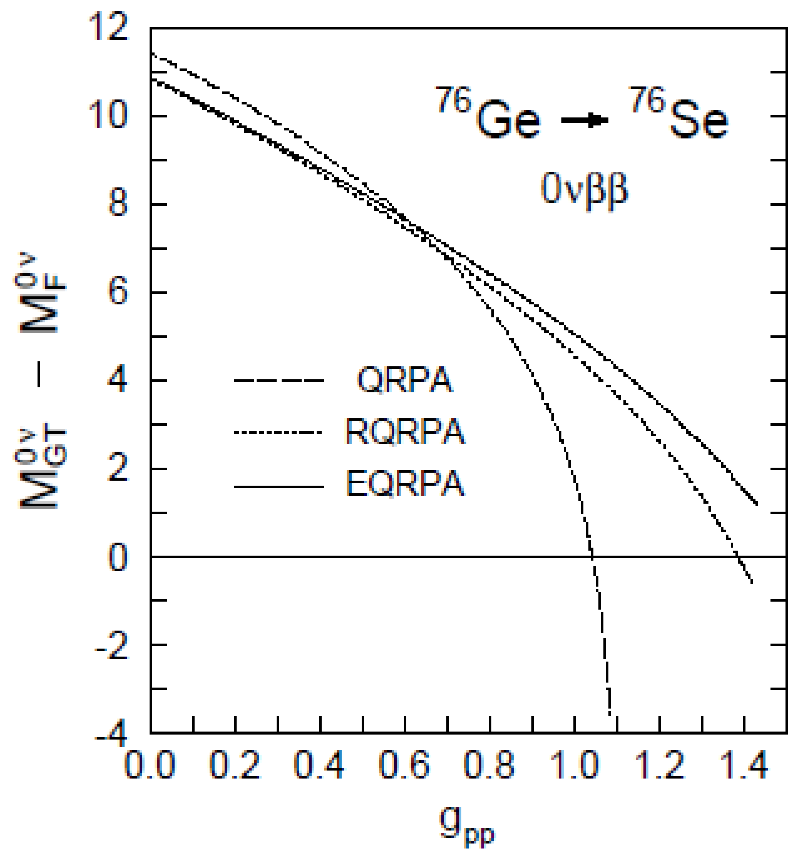
- Predictive power for 2ν decay nuclear matrix elements is still poor.

QRPA Calculations

$$\langle j_p j_n | V | j'_p j'_n \rangle_J \Rightarrow g_{pp} \langle j_p j_n | V | j'_p j'_n \rangle_J$$

by hand





g_{pp} enhances spin-isospin ground-state correlations.

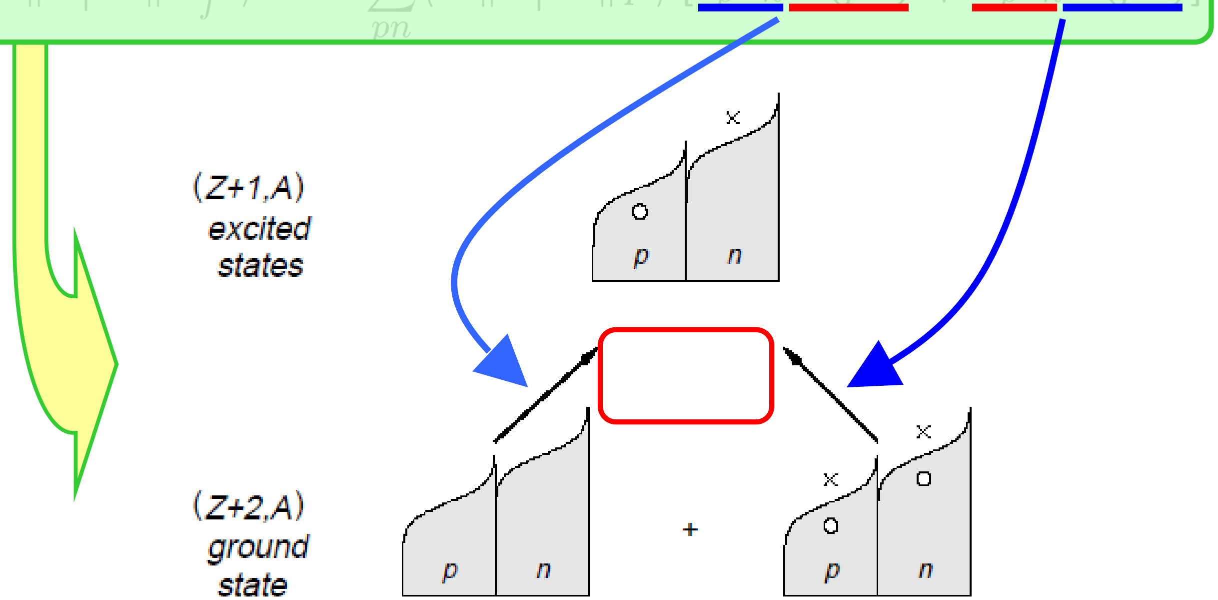
Suppression of GT Transitions

Strongly attractive proton-neutron interaction in $J^{\pi} = 1^{+}$

- Enhances spin-isospin ground-state correlations
- Enhances backward-going amplitudes, which interfere destructively with the forward-going amplitudes

$$\langle 1^{+} \| t_{-}\boldsymbol{\sigma} \| 0_{i}^{+} \rangle = \sum_{pn} \langle p \| t_{-}\boldsymbol{\sigma} \| n \rangle \left[u_p v_n X(pn) + v_p u_n Y(pn) \right]$$

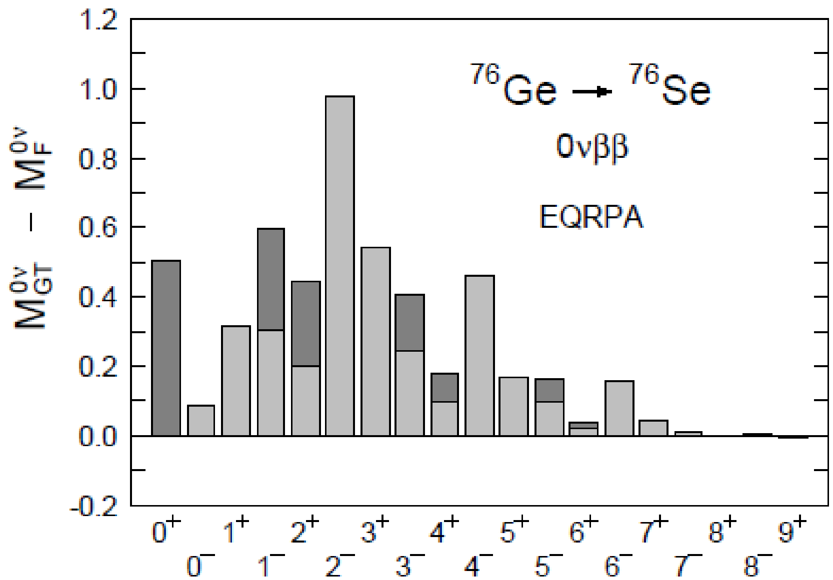
$$\langle 1^{+} \| t_{+}\boldsymbol{\sigma} \| 0_{f}^{+} \rangle = \sum_{pn} \langle n \| t_{+}\boldsymbol{\sigma} \| p \rangle \left[v_p u_n X(pn) + u_p v_n Y(pn) \right]$$



Multipole Decomposition 0ν decay NME

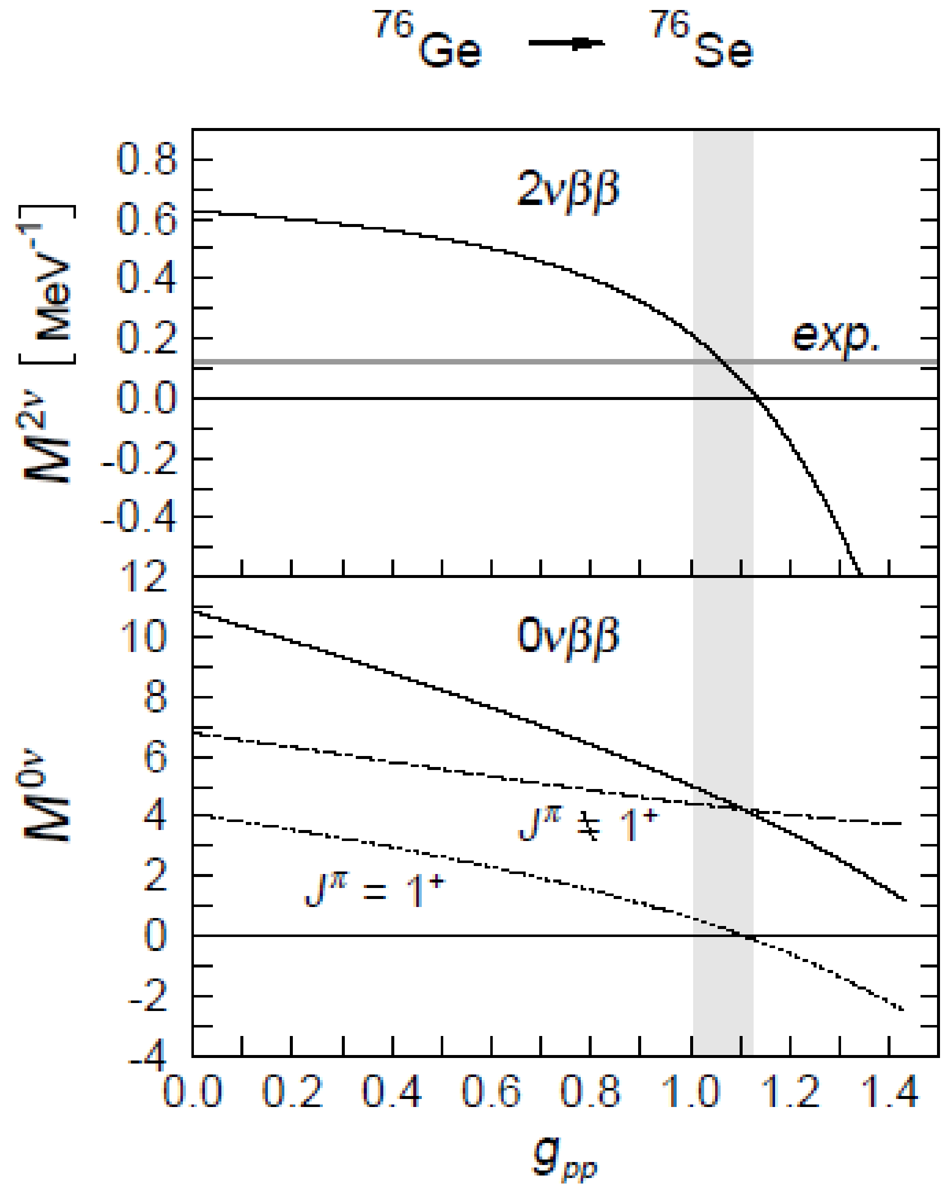
Components through nuclear intermediate states with various spin-parities J^{π} have the same sign, indicating that the NMEs are not so much sensitive to nuclear structure.

Sizable contributions through intermediate states with L = 0, 1, 2, 3. This requires a large model space consisting of two major shells.



The most reliable prediction of 0ν decay NME by QRPA models

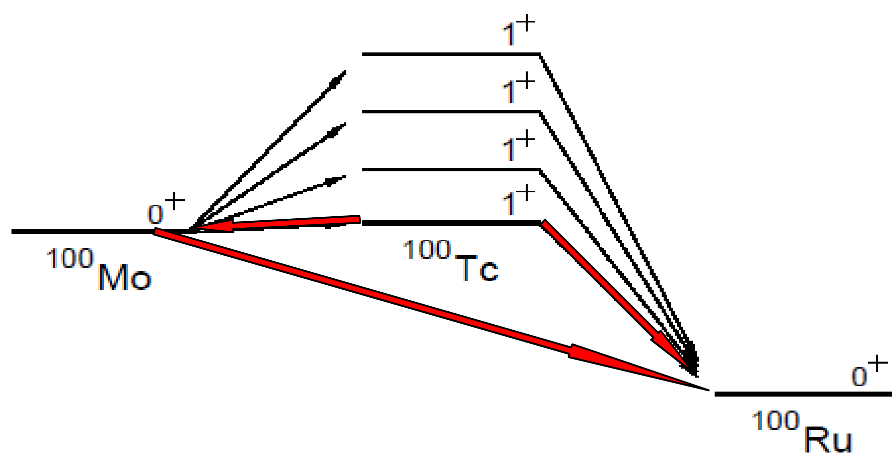
Fix g_{pp} by 2ν decay NME deduced from experimental half-lives



Problem of QRPA Calculations

Cannot reproduce three Gamow-Teller type transitions among three ground states

- B(GT−) ^{100}Tc(g.s.1^+) → ^{100}Ru(g.s.0^+)
- B(GT+) ^{100}Tc(g.s.1^+) → ^{100}Mo(g.s.0^+)
- $M^{2\nu}$ ^{100}Mo(g.s.0^+) → ^{100}Ru(g.s.0^+)



Lanczos Method

Generating linear combinations of shell-model basis states :

$$|\Phi_j^{\mathrm{L}}\rangle = \sum_{i=1}^{n} v_{ij}\,|\Phi_i^{\mathrm{SM}}\rangle$$

$$|\boldsymbol{v}_j\rangle = [v_{1j}, v_{2j}, \cdots, v_{nj}]^T$$

Lanczos procedure

Step 1 : $|\boldsymbol{w}_k\rangle = H\,|\boldsymbol{v}_k\rangle - \beta_{k-1}\,|\boldsymbol{v}_{k-1}\rangle$

Step 2 : $\alpha_k = \langle \boldsymbol{v}_k\,|\,\boldsymbol{w}_k\rangle$

Step 3 : $|\boldsymbol{u}_k\rangle = |\boldsymbol{w}_k\rangle - \alpha_k\,|\boldsymbol{v}_k\rangle$

Step 4 : $\beta_k = \sqrt{\langle \boldsymbol{u}_k\,|\,\boldsymbol{u}_k\rangle}$

Step 5 : $|\boldsymbol{v}_{k+1} = \beta_k^{-1}\,|\boldsymbol{u}_k\rangle$

Hamiltonian is tridiagonal in the Lanczos basis.

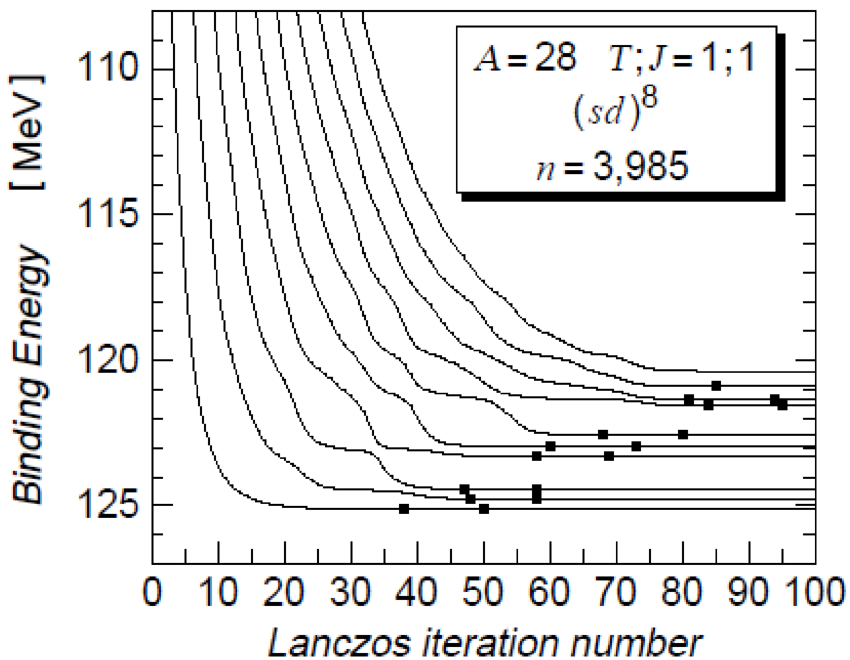
$$H_N = \begin{bmatrix} \alpha_1 & \beta_1 & & & & & \\ \beta_1 & \alpha_2 & \beta_2 & & & 0 & \\ & \beta_2 & \alpha_3 & \beta_3 & & & \\ & & \ddots & \ddots & \ddots & & \\ & & & \ddots & \ddots & \ddots & \\ & 0 & & & \beta_{N-2} & \alpha_{N-1} & \beta_{N-1} \\ & & & & & \beta_{N-1} & \alpha_N \end{bmatrix}$$

Energy eigenstates are described in a very small number of Lanczos basis states.

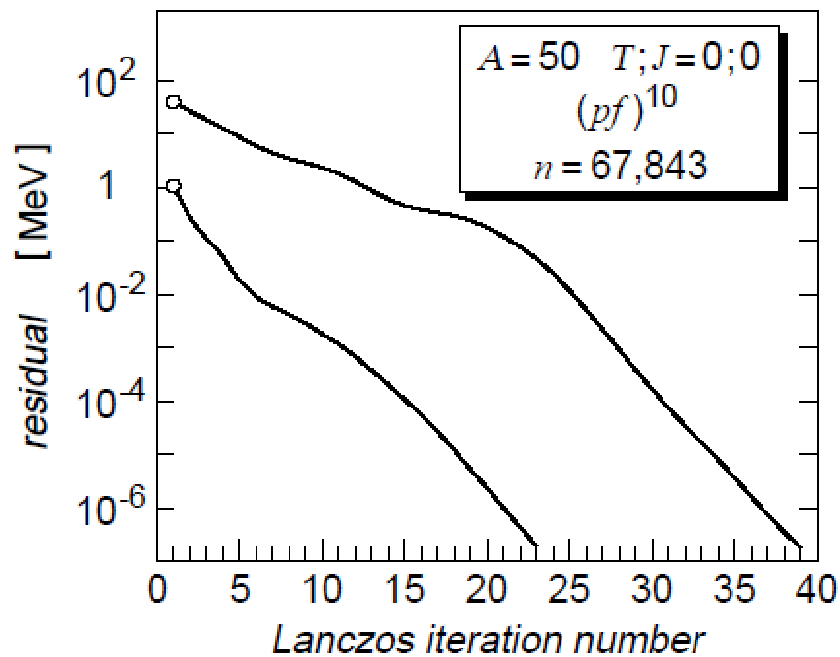
$$N \ll n$$

Energy Eigenvalues

Convergence from the lowest-lying states



A better trial vector leads to a faster convergence.



Both calculations in *TJ*-scheme

Resolvent Operator

2ν mode nuclear matrix element

$$M_{GT}^{2\nu} = \langle 0_f^+ | \mathcal{O} \frac{1}{H-E} \mathcal{O} | 0_i^+ \rangle \qquad \mathcal{O} = t_- \boldsymbol{\sigma}$$

Application of Lanczos method

$$| \boldsymbol{v}_1 \rangle = \mathcal{S}^{-1/2} \mathcal{O} | 0_i^+ \rangle \qquad \mathcal{S} = \langle 0_i^+ | \mathcal{O}^\dagger \mathcal{O} | 0_i^+ \rangle$$

$$\frac{1}{H-E} | \boldsymbol{v}_1 \rangle = \sum_{k=1}^{N} c_k | \boldsymbol{v}_k \rangle$$

$$c_k = f_1 \prod_{j=2}^{k} \left(-\beta_{j-1} f_j \right)$$

$$f_j = \cfrac{1}{\alpha'_j - \cfrac{\beta_j^2}{\alpha'_{j+1} - \cfrac{\beta_{j+1}^2}{\alpha'_{j+2} - \cfrac{\beta_{j+2}^2}{\alpha'_{j+3} - \cdots}}}}$$

$$\alpha'_j = \alpha_j - E$$

QRPA Model? or Shell Model?

QRPA model

- ***Much too simplifed***
- ***Two-major-shell calc.***
- ***Easy to handle***
- ***Crucial assumption of***
  $Q_\omega^\dagger$ ***by 2qp operators***

Shell model

- ***Various correlations***
- ***One-major-shell calc.***
  - ***Good for 2$\nu\beta\beta$***
  - ***Insufficient model space for 0$\nu\beta\beta$***

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Quasiparticle Shell Model