Dyson Boson Mapping and Shell-Model Calculations for Even-Even Nuclei

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Past, Present and Future of Shell Model 2006 CNS

1. Introduction

The shell-model Hilbert space is in general very large

⇒
- Truncation of the shell-model space
  to an appropriate subspace with a small dimension
- Truncation of the degrees of freedom
  to a small number of the shell-model space

The problems in this subject are as follows:

(1) What are the relevant collective degrees of freedom?

(2) What is the method suitable for the truncation of the degrees of freedom?

(3) How can we renormalize the coupling effects from
the non-collective degrees of freedom?

We have found a very useful method to solve
all of these problems using the Dyson boson mapping theory.

1. Introduction

- *Why Dyson boson mapping ?*

Full details the Dyson boson mapping theory and the relating references are summarized in:
K. Takada, Prog.Theor.Phys.Suppl.141(2001)179

A good point : Finiteness of the boson expansion

~~A weak point~~ : Non-hermiticity of the boson Hamiltonian

⇨ Takada's hermitian method

And, great discovery

Renormalization of the effects from the non-collective part

Takada and Yasumoto NP A706 (2002) p.365

⇩

Dyson boson mapping is very suitable !

2. Formulation —*Isospin formalism*—

- 2.1 *Shell-model Hamiltonian*

$$H = H_0 + H_{\rm int}$$

$$H_0 = \sum_{\alpha} \epsilon(\alpha)\, c^\dagger_\alpha c_\alpha$$

$$H_{\rm int} = \sum_{abcd} \sum_{JM} \sum_{TT_z} G_{JT}(abcd) A^\dagger_{JMTT_z}(ab) A_{JMTT_z}(cd)$$

Nucleon pair operators

$$A^\dagger_{JMTT_z}(ab) = \sqrt{\tfrac{1}{2}} \sum_{m_a m_b} \sum_{\tau_a \tau_b} \langle j_a m_a j_b m_b | JM \rangle \, \langle {}^1\!/\!_2\, \tau_a\, {}^1\!/\!_2\, \tau_b | TT_z \rangle \, c^\dagger_\alpha c^\dagger_\beta$$

$$B^\dagger_{JMTT_z}(ab) = \sum_{m_a m_b} \sum_{\tau_a \tau_b} \langle j_a m_a j_b m_b | JM \rangle \, \langle {}^1\!/\!_2\, \tau_a\, {}^1\!/\!_2\, \tau_b | TT_z \rangle \, c^\dagger_\alpha \tilde{c}_\beta$$

$$\tilde{c}_\beta = (-)^{j_b - m_b} (-)^{1/2 - \tau_b} c_{-\beta} \qquad -\beta = (n_b, l_b, j_b, -m_b, -\tau_b)$$

2. Formulation －*Isospin formalism*－

- 2.2 *Tamm－Dancoff phonon*

$$X^{(i)\dagger}_{l_i m_i t_i z_i} = \sum_{ab} \psi_i(ab) A^{\dagger}_{l_i m_i t_i z_i}(ab)$$

Eigenvalue equation : $[H, X^{(i)\dagger}_{l_i m_i t_i z_i}]|0\rangle = E^{(i)} X^{(i)\dagger}_{l_i m_i t_i z_i}|0\rangle$

$$(\epsilon_a + \epsilon_b)\psi_i(ab) + \sum_{a'b'} G_{l_i t_i}(aba'b')\psi_i(a'b') = E^{(i)}\psi_i(ab)$$

We get the lowest-energy TD phonons.

The collective phonons : $X^{(i)\dagger}_{l_i m_i t_i z_i}$

2. Formulation －*Isospin formalism*－

Closed algebra and coefficients

$$[X^{(i)}_{l_i m_i t_i z_i}, X^{(j)\dagger}_{l_j m_j t_j z_j}] = \delta_{ij}\delta_{m_i m_j}\delta_{z_i z_j} - 2\,\widehat{l_i}\widehat{l_j}\widehat{t_i}\widehat{t_j}(-)^{l_i+m_j+t_i+z_j}\sum_{abc}\psi_i(ab)\psi_j(ca)$$
$$\times\sum_{JT}(-)^{J+T}\begin{Bmatrix} l_i & l_j & J \\ j_c & j_b & j_a \end{Bmatrix}\begin{Bmatrix} t_i & t_j & T \\ 1/2 & 1/2 & 1/2 \end{Bmatrix}$$
$$\times\langle l_i m_i l_j - m_j|JM\rangle\,\langle t_i z_i t_j - z_j|TT_z\rangle\, B_{JMTT_z}(bc)$$
$$[X^{(i)}_{l_i m_i t_i t_{iz}}, B_{JMTT_z}(ab)] = \sum_j\sum_{m_j z_j} D^{(ij)}_{JT}(ab)\langle JMl_im_i|l_jm_j\rangle\,\langle TT_zt_iz_i|t_jz_j\rangle\, X^{(j)}_{l_jm_jt_jz_j}$$
$$[X^{(i)}_{l_i m_i t_i z_i}, [X^{(j)}_{l_j m_j t_j z_j}, X^{(k)\dagger}_{l_k m_k t_k z_k}]] = -2\sum_l\sum_{m_l z_l}\sum_{LT} C^{(ijkl)}_{LT}\langle l_im_il_jm_j|LM\rangle\,\langle l_km_kl_lm_l|LM\rangle$$
$$\times\langle t_iz_it_jz_j|TT_z\rangle\,\langle t_kz_kt_lz_l|TT_z\rangle\, X^{(l)}_{l_lm_lt_lz_l}$$

SO(2N)
Lie algebra

$$D^{(ij)}_{JT}(ab) = 2\widehat{J}\widehat{l_i}\widehat{T}\widehat{t_i}\sum_c(-)^{j_a+j_c+l_j+t_j}\psi_i(bc)\psi_j(ac)\times\begin{Bmatrix} l_i & l_j & J \\ j_a & j_b & j_c \end{Bmatrix}\begin{Bmatrix} t_i & t_j & T \\ 1/2 & 1/2 & 1/2 \end{Bmatrix}$$
$$C^{(ijkl)}_{LT} = 2\widehat{l_i}\widehat{l_j}\widehat{l_k}\widehat{l_l}\widehat{t_i}\widehat{t_j}\widehat{t_k}\widehat{t_l}\sum_{abcd}\psi_i(ab)\psi_j(cd)\psi_k(ac)\psi_l(bd)\times\begin{Bmatrix} j_a & j_b & l_i \\ j_c & j_d & l_j \\ l_k & l_l & L \end{Bmatrix}\begin{Bmatrix} 1/2 & 1/2 & t_i \\ 1/2 & 1/2 & t_j \\ t_k & t_l & T \end{Bmatrix}$$
$$\widehat{l} = \sqrt{2l+1}$$

2. Formulation —*Isospin formalism*—

- 2.3 *Separation of collective and non-collective Hamiltonian*

Nucleon pair operator

$$X^{(i)\dagger}_{l_i m_i t_i z_i} = \sum_{ab} \psi_i(ab) A^\dagger_{l_i m_i t_i z_i}(ab)$$

$$\begin{aligned} A^\dagger_{JMTT_z}(ab) &= \sum_i \delta_{l_i J}\delta_{m_i M}\delta_{t_i T}\delta_{z_i T_z}\psi_i(ab) X^{(i)\dagger}_{l_i m_i t_i z_i} \\ &= \sum_{i=\mathrm{col}} \delta_{l_i J}\delta_{m_i M}\delta_{t_i T}\delta_{z_i T_z}\psi_i(ab) X^{(i)\dagger}_{l_i m_i t_i z_i} \\ &\quad + \sum_{a'b'} \left\{ \delta_{aa'}\delta_{bb'} - \sum_{i=\mathrm{col}} \delta_{l_i J}\delta_{t_i T}\psi_i(ab)\psi_i(a'b') \right\} A^\dagger_{JMTT_z}(a'b') \end{aligned}$$

$$\therefore \; A^\dagger_{JMTT_z}(ab) = \sum_{a'b'} \left\{ \Psi^{(\mathrm{c})}_{JT}(ab;a'b') + \Psi^{(\mathrm{n})}_{JT}(ab;a'b') \right\} A^\dagger_{JMTT_z}(a'b')$$

$$\Psi^{(\mathrm{c})}_{JT}(ab;a'b') = \sum_{i=\mathrm{col}} \delta_{l_i J}\delta_{t_i T}\psi_i(ab)\psi_i(a'b')$$

$$\Psi^{(\mathrm{n})}_{JT}(ab;a'b') = \frac{1}{2}\left\{ \delta_{aa'}\delta_{bb'} + (-)^{j_a+j_b-J-T}\delta_{ab'}\delta_{ba'} \right\} - \Psi^{(\mathrm{c})}_{JT}(ab;a'b')$$

2. Formulation －*Isospin formalism*－

Separation of Shell model Hamiltonian

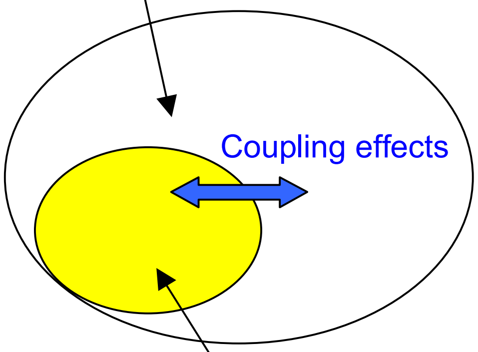
$$H = H_0 + H_{\rm int}$$

$$H_{\rm int} = H_{\rm int}^{\rm (c)} + H_{\rm int}^{\rm (n)}$$

$$\begin{cases} H_{\rm int}^{\rm (c)} = \sum_{i,j={\rm col}} \delta_{m_i m_j} \delta_{z_i z_j} F(ij)\, X^{(i)\dagger}_{l_i m_i t_i z_i} X^{(j)}_{l_j m_j t_j z_j} \\ H_{\rm int}^{\rm (n)} = \sum_{JT} \sum_{abcd} G^{\rm (n)}_{JT}(abcd) \sum_{MT_z} A^\dagger_{JMTT_z}(ab) A_{JMTT_z}(cd) \end{cases}$$

$$F(ij) = \begin{cases} \sum_{abcd} G_{l_i t_i}(abcd)\psi_i(ab)\psi_j(cd), & \text{if } l_i = l_j \text{ and } t_i = t_j \\ 0, & \text{otherwise} \end{cases}$$

$$G^{\rm (n)}_{JT}(abcd) = \sum_{a'b'c'd'} G_{JT}(a'b'c'd')\Big\{ \Psi^{\rm (n)}_{JT}(a'b';ab)\Psi^{\rm (n)}_{JT}(c'd';cd) + \Psi^{\rm (n)}_{JT}(a'b';ab)\Psi^{\rm (c)}_{JT}(c'd';cd) + \Psi^{\rm (c)}_{JT}(a'b';ab)\Psi^{\rm (n)}_{JT}(c'd';cd) \Big\}$$



2. Formulation －*Isospin formalism*－

- 2.4 *Dyson boson mapping*

TD phonon operators $\{X^{(i)\dagger}_{l_i m_i t_i z_i}\}$

↓

Dyson boson images $(X^{(i)\dagger}_{l_i m_i t_i z_i})_{\rm D}$

Closed-algebra approximation

Full details the Dyson boson mapping theory and the relating references are summarized in: K. Takada, Prog.Theor.Phys.Suppl.141(2001)179

$$(X^{(i)\dagger}_{l_i m_i t_i z_i})_{\rm D} = \boldsymbol{b}^{(i)\dagger}_{l_i m_i t_i z_i} - (\hat{l}_i \hat{t}_i)^{-1} \sum_{jkl} \sum_{LT} (-)^{l_i+t_i+l_l+t_l-L-T} \hat{L}\hat{T} C^{(jkil)}_{LT} [[\boldsymbol{b}^{(j)\dagger}_{l_j t_j} \boldsymbol{b}^{(k)\dagger}_{l_k t_k}]_{LT} \tilde{\boldsymbol{b}}^{(l)}_{l_l t_l}]_{l_i m_i t_i z_i}$$

$$(X^{(i)}_{l_i m_i t_i z_i})_{\rm D} = \boldsymbol{b}^{(i)}_{l_i m_i t_i z_i}$$

$$(A^{\dagger}_{l_i m_i t_i z_i}(ab))_{\rm D} = \sum_{j(l_j=l_i, t_j=t_i)} \psi_j(ab) (X^{(j)\dagger}_{l_j m_j t_j z_j})_{\rm D}$$

$$(A_{l_i m_i t_i z_i}(ab))_{\rm D} = \sum_{j(l_j=l_i, t_j=t_i)} \psi_j(ab) (X^{(j)}_{l_j m_j t_j z_j})_{\rm D}$$

$$(B_{LMTT_z}(ab))_{\rm D} = (\hat{L}\hat{T})^{-1} \sum_{ij} \hat{l}_j \hat{t}_j D^{(ij)}_{LT}(ab) (-)^{L+T-M-T_z} [\boldsymbol{b}^{(i)\dagger}_{l_i t_i} \tilde{\boldsymbol{b}}^{(j)}_{l_j t_j}]_{L-MT-T_z}$$

$$(B^{\dagger}_{LMTT_z}(ab))_{\rm D} = (\hat{L}\hat{T})^{-1} \sum_{ij} (-)^{l_i+t_i+l_j+t_j+L+T} \hat{l}_j \hat{t}_j D^{(ij)}_{LT}(ab) [\boldsymbol{b}^{(j)\dagger}_{l_j t_j} \tilde{\boldsymbol{b}}^{(i)}_{l_i t_i}]_{LMTT_z}$$

2. Formulation －*Isospin formalism*－

Commutation relations of the boson operator $\boldsymbol{b}^{(i)\dagger}_{l_i m_i t_i z_i}$

$$\boldsymbol{b}^{(i)\dagger}_{l_i m_i t_i z_i} = \sqrt{\tfrac{1}{2}} \sum_{ab} \psi_i(ab) \sum_{m_a m_b} \sum_{\tau_a \tau_b} \langle j_a m_a j_b m_b | l_i m_i \rangle \, \langle 1/2 \, \tau_a \, 1/2 \, \tau_b | t_i z_i \rangle \, \boldsymbol{b}^\dagger_{\alpha\beta}$$

Symmetry property

$$\boldsymbol{b}^\dagger_{\alpha\beta} = -\boldsymbol{b}^\dagger_{\beta\alpha}$$

Commutation relations

$$[\, \boldsymbol{b}_{\alpha\beta}, \boldsymbol{b}^\dagger_{\gamma\delta} \,] = \delta_{\alpha\gamma}\delta_{\beta\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma}, \quad [\, \boldsymbol{b}_{\alpha\beta}, \boldsymbol{b}_{\gamma\delta} \,] = [\, \boldsymbol{b}^\dagger_{\alpha\beta}, \boldsymbol{b}^\dagger_{\gamma\delta} \,] = 0$$

2. Formulation －*Isospin formalism*－

- 2.5 *Dyson boson Hamiltonian*

$$\boldsymbol{H} = (H_0)_{\rm D} + (H^{\rm (c)}_{\rm int})_{\rm D} + (H^{\rm (n)}_{\rm int})_{\rm D} = \boldsymbol{H}_0 + \boldsymbol{H}^{\rm (c)}_{\rm int} + \boldsymbol{H}^{\rm (n)}_{\rm int}$$

$$\boldsymbol{H}_0 = \sum_{ij} \delta_{m_i m_j} \delta_{z_i z_j} E_0(ij)\, \boldsymbol{b}^{(i)\dagger}_{l_i m_i t_i z_i} \boldsymbol{b}^{(j)}_{l_j m_j t_j z_j}$$

$$\left\{\begin{aligned}
\boldsymbol{H}^{\rm (c)}_{\rm int} &= \sum_{i,j={\rm col}} \delta_{m_i m_j} \delta_{z_i z_j} F(ij)\, \boldsymbol{b}^{(i)\dagger}_{l_i m_i t_i z_i} \boldsymbol{b}^{(j)}_{l_j m_j t_j z_j} \\
&\quad - \sum_{i,i'={\rm col}} \sum_{jkl} \sum_{JT} \sum_{MT_z} F(ii')\, C^{(jkil)}_{JT}\, [\boldsymbol{b}^{(j)\dagger}_{l_j t_j} \boldsymbol{b}^{(k)\dagger}_{l_k t_k}]_{JMTT_z} [\boldsymbol{b}^{(i')}_{l_{i'} t_{i'}} \boldsymbol{b}^{(l)}_{l_l t_l}]_{JMTT_z} \\
\boldsymbol{H}^{\rm (n)}_{\rm int} &= \sum_{ij} \delta_{m_i m_j} \delta_{z_i z_j} \{\widetilde{F}^{\rm (n)}_1(ij) + \widetilde{F}^{\rm (n)}_2(ij)\}\, \boldsymbol{b}^{(i)\dagger}_{l_i m_i t_i z_i} \boldsymbol{b}^{(j)}_{l_j m_j t_j z_j} \\
&\quad + \sum_{ijkl} \sum_{J'T'} \widetilde{F}^{\rm (n)}_{J'T'}(ijkl) \sum_{M'T'_z} [\boldsymbol{b}^{(j)\dagger}_{l_j t_j} \boldsymbol{b}^{(k)\dagger}_{l_k t_k}]_{J'M'T'T'_z} [\boldsymbol{b}^{(l)}_{l_l t_l} \boldsymbol{b}^{(i)}_{l_i t_i}]_{J'M'T'T'_z}
\end{aligned}\right.$$

2. Formulation ー*Isospin formalism*ー

Coefficients in Dyson boson Hamiltonian

$$E_0(ij) = \begin{cases} 2\sum_{ab} \epsilon(a)\,\psi_i(ab)\,\psi_j(ab), & \text{if } l_i = l_j \text{ and } t_i = t_j \\ 0, & \text{otherwise} \end{cases}$$

$$\widetilde{F}_1^{(\mathrm{n})}(ij) = \begin{cases} \sum_{ab}\sum_{JT} \frac{(2J+1)(2T+1)}{2(2j_a+1)} G_{JT}^{(\mathrm{n})}(abab) \sum_c \psi_i(ac)\,\psi_j(ac), & \text{if } l_i = l_j \text{ and } t_i = t_j \\ 0, & \text{otherwise} \end{cases}$$

$$\widetilde{F}_2^{(\mathrm{n})}(ij) = \begin{cases} -\sum_{abcd}\sum_{JT} \widetilde{G}_{JT}^{(\mathrm{n})}(abcd) \sum_k D_{JT}^{(ki)}(ad) D_{JT}^{(kj)}(cb), & \text{if } l_i = l_j \text{ and } t_i = t_j \\ 0, & \text{otherwise} \end{cases}$$

$$\widetilde{F}_{J'T'}^{(\mathrm{n})}(ijkl) = -\sum_{abcd}\sum_{JT} \widetilde{G}_{JT}^{(\mathrm{n})}(abcd)\,(-)^{l_j+l_k-J'+t_j+t_k-T'} \times \hat{l}_j \hat{t}_j \hat{l}_l \hat{t}_l \begin{Bmatrix} l_i & l_j & J \\ l_k & l_l & J' \end{Bmatrix} \begin{Bmatrix} t_i & t_j & T \\ t_k & t_l & T' \end{Bmatrix} D_{JT}^{(ij)}(ad) D_{JT}^{(kl)}(cb)$$

where

$$\widetilde{G}_{JT}^{(\mathrm{n})}(abcd) = \frac{1}{2}\sum_{J'T'} G_{J'T'}^{(\mathrm{n})}(abcd) \times (2J'+1)(2T'+1) \begin{Bmatrix} j_a & j_b & J' \\ j_c & j_d & J \end{Bmatrix} \begin{Bmatrix} 1/2 & 1/2 & T' \\ 1/2 & 1/2 & T \end{Bmatrix}$$

3. Numerical analyses and discussion

- Akiyama, Arima and Sebe, *Nuclear Physics* A138(1969), p.273

^{22}Ne and ^{24}Mg in sd-shell

Effective two-body interaction of Gaussian and Yukawa type forces

$$V(r) = (V_{13}P_{13} + V_{31}P_{31} + V_{11}P_{11} + V_{33}P_{33})\, f(r)$$

$$f(r) = \begin{cases} e^{-(\mu r)^2} & \text{for the Gaussian type force} \\ e^{-\mu r}/(\mu r) & \text{for the Yukawa type force} \end{cases}$$

Range of the interaction $\lambda = \frac{\nu}{\sqrt{2}\mu}$

Size parameter $\nu = \sqrt{M\omega/\hbar}$

Effective charges

e(P) = 1.50 [e] ; e(N) = 0.50 [e]

3. Numerical analysis and discussion

The simplest type of collective boson space :

Two types of *s*-phonons and two types of *d*-phonons,

i.e. the lowest-energy TD phonons with

$(l, t, t_z) = (0, 1, -1)$
$(2, 1, -1)$ } Proton-proton TD phonon

$(0, 1, 1)$
$(2, 1, 1)$ } Neutron-neutron TD phonon

This boson space has a similarity to the IBM (Interacting Boson Model).

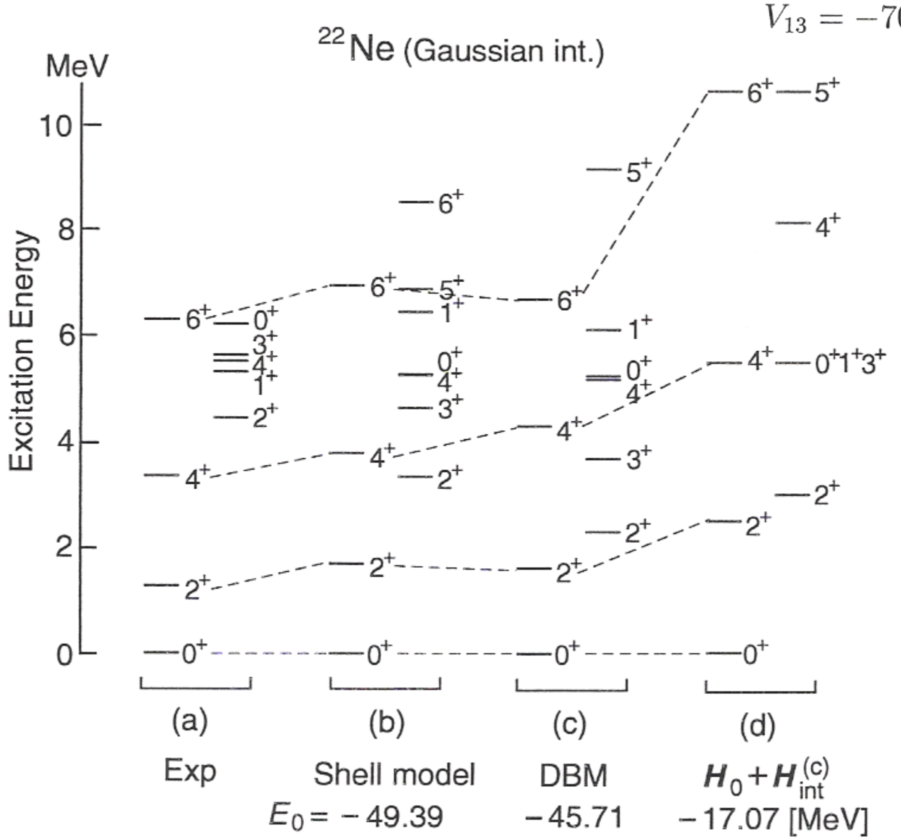
3. Numerical analyses and discussion

- Case of ^{22}Ne

$\epsilon(0d^5/_2) = 0.00\,\mathrm{MeV}$, $\epsilon(1s^1/_2) = 0.73\,\mathrm{MeV}$, $\epsilon(0d^3/_2) = 5.00\,\mathrm{MeV}$

$\lambda = 0.7$

$V_{13} = -70\,\mathrm{MeV}$, $V_{31} = -52\,\mathrm{MeV}$, $V_{11} = 0\,\mathrm{MeV}$, $V_{33} = 26\,\mathrm{MeV}$



Dimension

I^π	0^+	1^+	2^+	3^+	4^+	5^+	6^+
SM	216	534	777	798	723	525	345
DBM	4	2	7	3	4	1	1

Q-moment [e × fm^2] and B(E2) [e^2 × fm^4]

	Exp.	SM	DBM	$H_0+H_{int}^{(c)}$
$Q_2(2_1)$	-19±4	-14.70	-11.82	-12.32
$2_1 \rightarrow 0_1$	45.6±1.8	56.37	46.29	36.76
$4_1 \rightarrow 2_1$	64.5±1.1	54.60	21.15	4.27

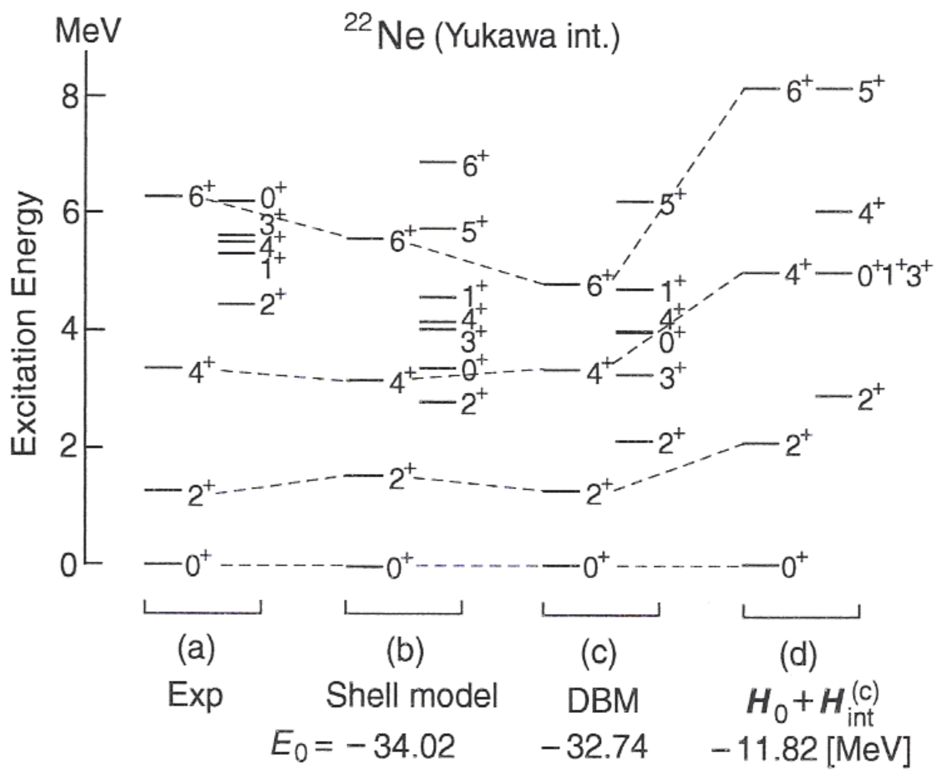
3. Numerical analyses and discussion

- Case of ^{22}Ne

$\epsilon(0d_{5/2}) = 0.00\,\mathrm{MeV},\ \ \epsilon(1s_{1/2}) = 0.80\,\mathrm{MeV},\ \ \epsilon(0d_{3/2}) = 5.00\,\mathrm{MeV}$

$\lambda = 2/3$

$V_{13} = -35\,\mathrm{MeV},\ \ V_{31} = -27\,\mathrm{MeV},\ \ V_{11} = 0\,\mathrm{MeV},\ \ V_{33} = 13.5\,\mathrm{MeV}$



Dimension

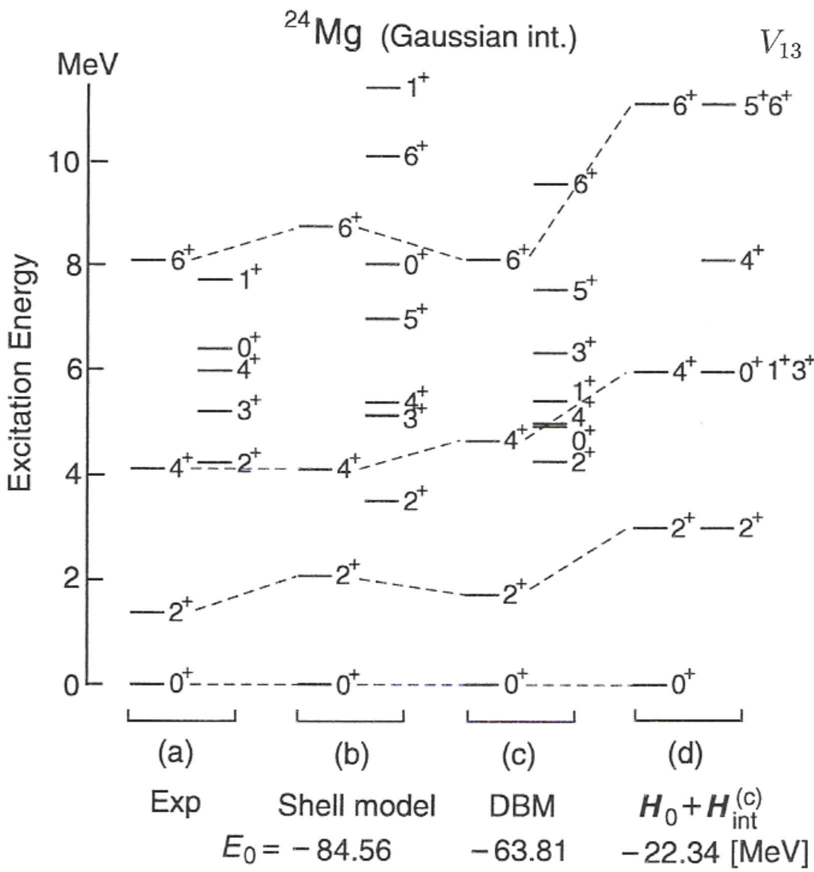
I^π	0^+	1^+	2^+	3^+	4^+	5^+	6^+
SM	216	534	777	798	723	525	345
DBM	4	2	7	3	4	1	1

Q_2-moment [e × fm^2] and B(E2) [e^2 × fm^4]

	Exp.	SM	DBM	$H_0+H_{\mathrm{int}}^{(c)}$
$Q_2(2_1)$	-19±4	-13.35	-11.47	-11.33
$2_1 \rightarrow 0_1$	45.6±1.8	53.68	43.56	35.56
$4_1 \rightarrow 2_1$	64.5±1.1	42.00	12.48	4.04

3. Numerical analyses and discussion

- Case of ^{24}Mg



$\epsilon(0d^{5}/_{2}) = 0.00\,\mathrm{MeV},\ \ \epsilon(1s^{1}/_{2}) = 0.80\,\mathrm{MeV},\ \ \epsilon(0d^{3}/_{2}) = 5.00\,\mathrm{MeV}$

$\lambda = 0.7$

$V_{13} = -70\,\mathrm{MeV},\ \ V_{31} = -52\,\mathrm{MeV},\ \ V_{11} = 0\,\mathrm{MeV},\ \ V_{33} = 26\,\mathrm{MeV}$

Dimension

I^π	0^+	1^+	2^+	3^+	4^+	5^+	6^+
SM	1161	3096	4518	4968	4734	3843	2799
DBM	9	5	17	9	13	5	5

Q-moment [e × fm^2] and B(E2) [e^2 × fm^4]

	Exp.	SM	DBM	$H_0+H_{int}^{(c)}$
$Q_2(2_1)$	-16.6±0.6	-5.73	12.93	9.00
$2_1 \to 0_1$	86.5±1.7	89.67	47.36	40.30
$4_1 \to 2_1$	156±13	42.65	46.85	30.56

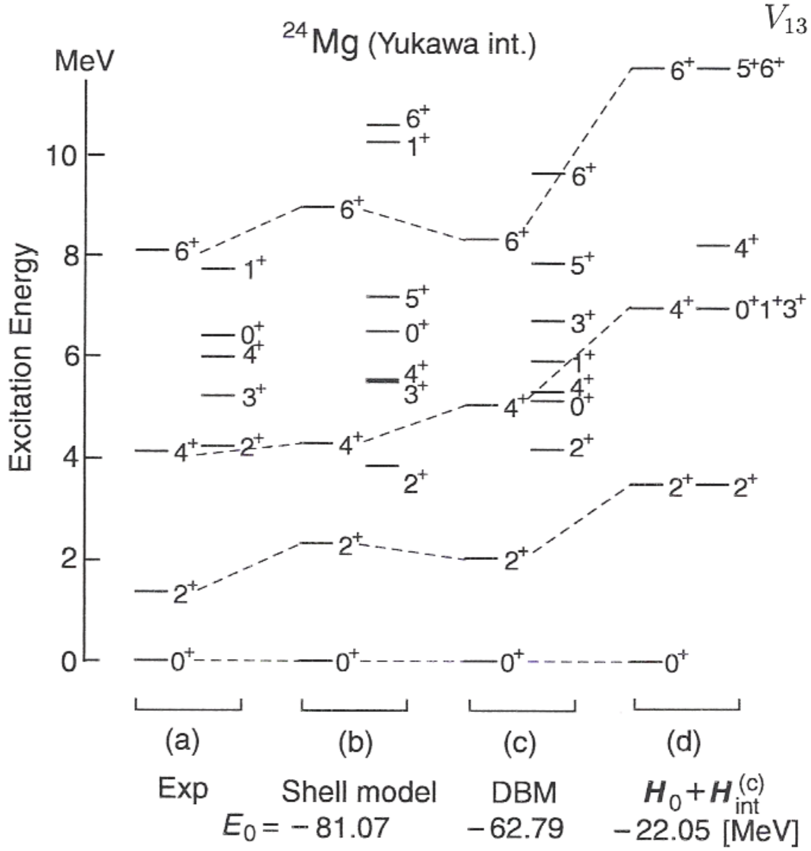
3. Numerical analyses and discussion

- Case of ^{24}Mg

$\epsilon(0d5/2) = 0.00\,\text{MeV},\ \epsilon(1s1/2) = 0.80\,\text{MeV},\ \epsilon(0d3/2) = 5.00\,\text{MeV}$

$\lambda = 2/3$

$V_{13} = -47\,\text{MeV},\ V_{31} = -36\,\text{MeV},\ V_{11} = 0\,\text{MeV},\ V_{33} = 18\,\text{MeV}$



Dimension

I^{π}	0^+	1^+	2^+	3^+	4^+	5^+	6^+
SM	1161	3096	4518	4968	4734	3843	2799
DBM	9	5	17	9	13	5	5

Q-moment [e × fm^2] and B(E2) [e^2 × fm^4]

	Exp.	SM	DBM	$H_0+H_{int}^{(c)}$
$Q_2(2_1)$	-16.6±0.6	-2.22	12.42	9.28
$2_1 \to 0_1$	86.5±1.7	86.46	45.16	17.20
$4_1 \to 2_1$	156±13	19.91	26.96	2.91

3. Numerical analyses and discussion

- Strength parameters search

Effective two-body interaction of Gaussian type forces

$$V(r) = (V_{13}P_{13} + V_{31}P_{31} + V_{11}P_{11} + V_{33}P_{33})\, f(r)$$

4 parameters

We searched the ``best fit'' by which the excitation energies of the ground band up to the 6^+ state were reproduced best.

^{22}Ne and ^{24}Mg in sd-shell ^{46}Ti in fp-shell

Range of the interaction $\lambda = 0.85$

Effective charges

e(P) = 1.50 [e] ; e(N) = 0.50 [e]

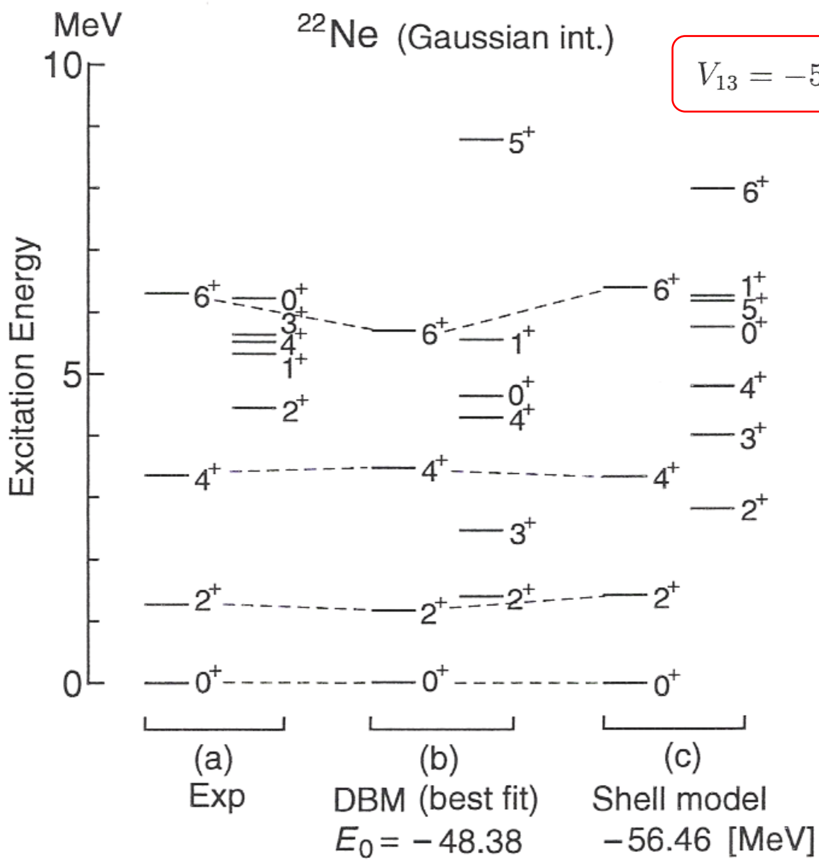
3. Numerical analyses and discussion

- Case of ^{22}Ne

$$\epsilon(0d\,5/2) = 0.00\,\mathrm{MeV},\quad \epsilon(1s\,1/2) = 0.73\,\mathrm{MeV},\quad \epsilon(0d\,3/2) = 5.00\,\mathrm{MeV}$$

Strength parameters search

$$V_{13} = -55.68\,\mathrm{MeV},\quad V_{31} = -41.37\,\mathrm{MeV},\quad V_{11} = 0.0\,\mathrm{MeV},\quad V_{33} = 20.35\,\mathrm{MeV}$$



Dimension

I^π	0^+	1^+	2^+	3^+	4^+	5^+	6^+
SM	216	534	777	798	723	525	345
DBM	4	2	7	3	4	1	1

Q-moment [e × fm^2] and B(E2) [e^2 × fm^4]

	Exp.	DBM	SM
$Q_2(2_1)$	-19±4	-12.77	-14.89
$2_1 \rightarrow 0_1$	45.6±1.8	34.50	57.92
$4_1 \rightarrow 2_1$	64.5±1.1	8.51	55.23

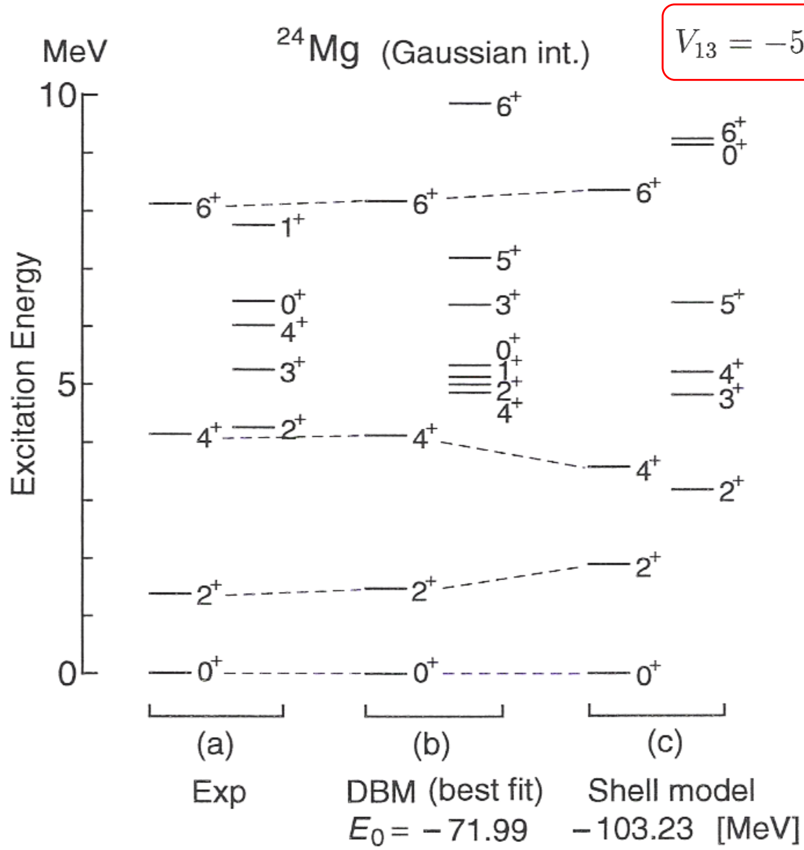
3. Numerical analyses and discussion

- Case of ^{24}Mg

$$\epsilon(0d_{5/2}) = 0.00\,\mathrm{MeV},\quad \epsilon(1s_{1/2}) = 0.80\,\mathrm{MeV},\quad \epsilon(0d_{3/2}) = 5.00\,\mathrm{MeV}$$

Strength parameters search

$$V_{13} = -59.19\,\mathrm{MeV},\quad V_{31} = -43.81\,\mathrm{MeV},\quad V_{11} = 0.0\,\mathrm{MeV},\quad V_{33} = 21.96\,\mathrm{MeV}$$



Dimension

I^π	0^+	1^+	2^+	3^+	4^+	5^+	6^+
SM	1161	3096	4518	4968	4734	3843	2799
DBM	9	5	17	9	13	5	5

Q-moment [e × fm^2] and B(E2) [e^2 × fm^4]

	Exp.	DBM	SM
$Q2(2_1)$	-16.6 ± 0.6	13.85	-3.39
$2_1 \to 0_1$	86.5 ± 1.7	49.50	89.95
$4_1 \to 2_1$	156 ± 13	44.91	33.40

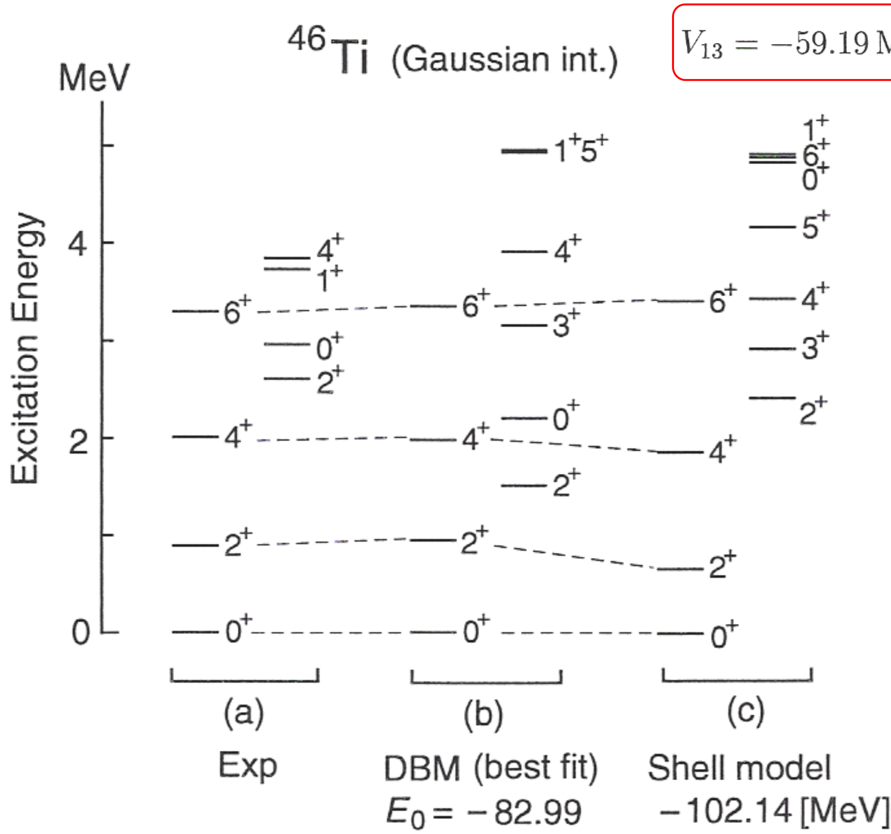
3. Numerical analyses and discussion

- Case of ^{44}Ti

$\epsilon(0f_{7/2}) = -8.3876$ MeV, $\epsilon(1p_{3/2}) = -6.4952$ MeV
$\epsilon(0f_{5/2}) = -1.8966$ MeV, $\epsilon(1p_{1/2}) = -4.4783$ MeV

Strength parameters search

$V_{13} = -59.19$ MeV, $V_{31} = -43.81$ MeV, $V_{11} = 0.0$ MeV, $V_{33} = 21.96$ MeV



Dimension

I^π	0^+	1^+	2^+	3^+	4^+	5^+	6^+
SM	2343	6466	9884	11768	12424	11628	10073
DBM	4	2	7	3	4	1	1

Q-moment [e × fm^2] and B(E2) [e^2 × fm^4]

	Exp.	DBM	SM
$Q_2(2_1)$	-21±6	-15.37	-29.65
$2_1 \to 0_1$	217±17	73.85	211.92
$4_1 \to 2_1$	177±20	143.80	285.70

4. Program package

Program for jj-coupling shell-model calculation
<<jjSMQ>>
(Ver. 2.10; 2004)
K. Takada, M. Sato and S. Yasumoto
Department of Physics, Kyushu University

This set of programs (named "jjSMQ"; the last "Q" implies "Kyushu Univ") is a general-use program package to perform the jj-coupling shell-model calculation.

You can download the program package from
ftp://kutl.kyushu-u.ac.jp/pub/takada/jjSMQ/
through anonymous ftp.

4. Program package

Program for calculation of nuclear structure of even-even nuclei
by using the Dyson Boson Mapping
<<DBM3>>
(Ver. 1.02; 2005)

K. Takada, S. Tazaki* and S. Yasumoto**

Department of Physics, Kyushu University
*Department of Applied Physics, Fukuoka University
**International Health and Welfare University

This set of programs (named "DBM3") is a general-use program package to perform the calculation of nuclear structure of even-even nuclei by using the Dyson Boson Mapping theory.

You can download the program package from

ftp://kutl.kyushu-u.ac.jp/pub/takada/DBM3/

through anonymous ftp.

5. Concluding remarks

The important results obtained through our numerical analyses.

(1) By diagonalizing the boson Hamiltonian

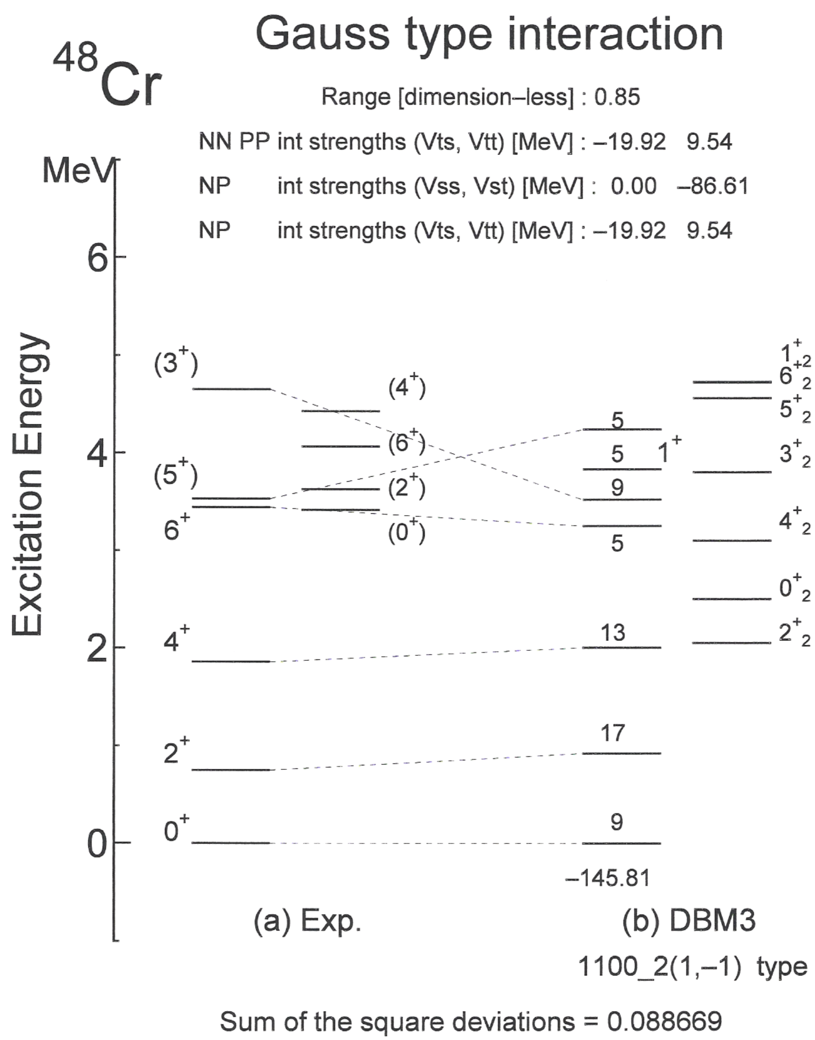
$$\boldsymbol{H} = \boldsymbol{H}_0 + \boldsymbol{H}^{(c)}_{\rm int} + \boldsymbol{H}^{(n)}_{\rm int}$$

within the simplest *s*- and *d*-boson space, we can well reproduce the energy levels of the exact shell-model calculations and the experimental levels as well.

(2) The term $\boldsymbol{H}^{(n)}_{\rm int}$ representing the normalization of the coupling effects between the collective and non-collective degrees of freedom is indispensable for the DBM method.

(3) Some of the Tamm-Dancoff phonons with the lowest energy eigenvalues are selected as the ``collective" phonons, and then the corresponding bosons constitute our ``collective " model space.

6. Some additions



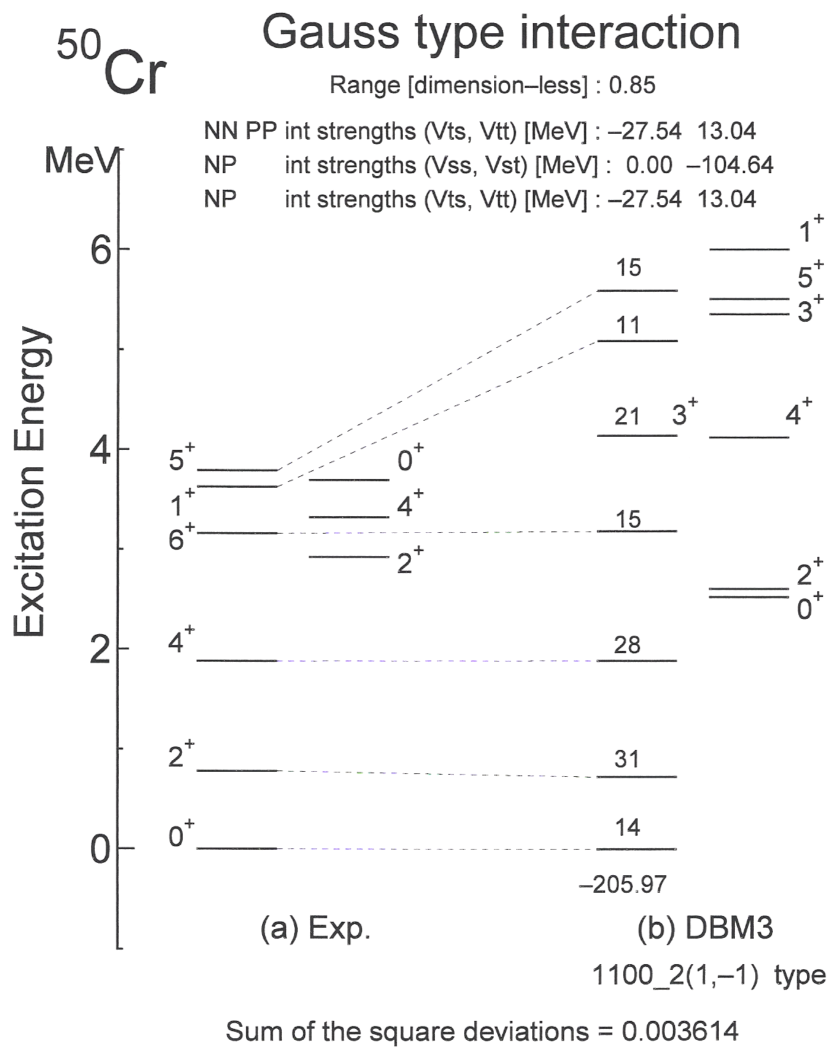
Dimension

I^{π}	Shell Model	DBM
0^+	41335	9
1^+	118269	5
2^+	182421	17
3^+	225725	9
4^+	246979	13
5^+	245387	5
6^+	226259	5

Q-moment [$e \times fm^2$] and B(E2) [$e^2 \times fm^4$]

	Exp.	DBM
$Q_2(2_1)$	unknown	8.93
$2_1 \to 0_1$	272.42	160.60
$4_1 \to 2_1$	unknown	209.54

6. Some additions



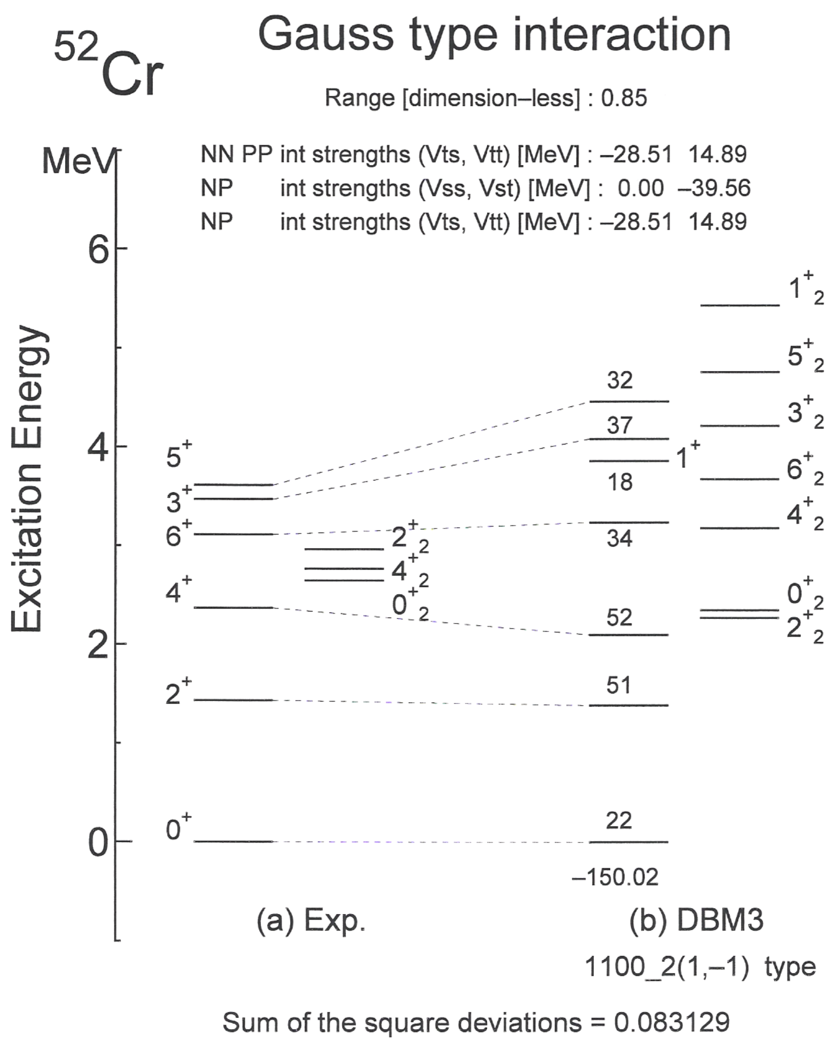
Dimension

I^π	Shell Model	DBM
0^+	267054	14
1^+	771409	11
2^+	1200017	31
3^+	1510328	21
4^+	1686373	28
5^+	1723299	15
6^+	1641492	15

Q-moment [e × fm^2] and B(E2) [e^2 × fm^4]

	Exp.	DBM
$Q_2(2_1)$	-36.7	-25.09
$2_1 \rightarrow 0_1$	217.2	180.54
$4_1 \rightarrow 2_1$	unknown	301.77

6. Some additions



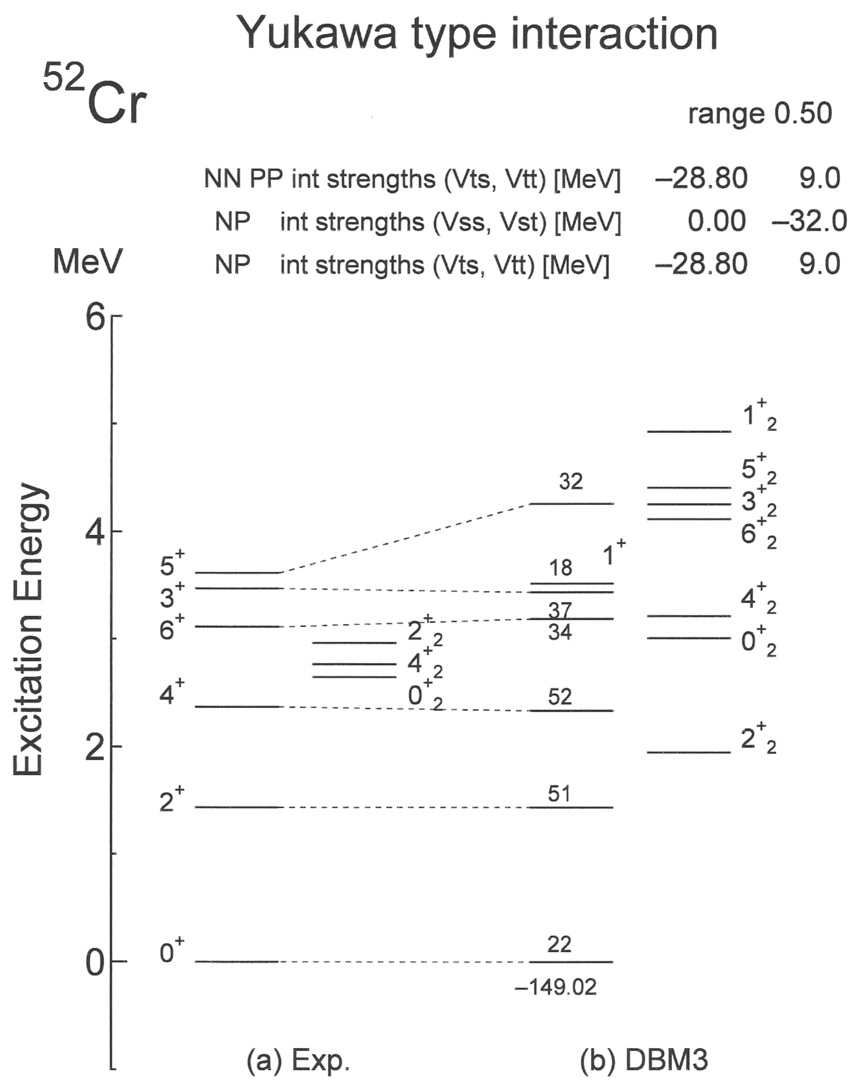
Dimension

I^π	Shell Model	DBM
0^+	773549	22
1^+	2242811	18
2^+	3505079	51
3^+	4447300	37
4^+	5016447	52
5^+	5195894	32
6^+	5029085	34

Q-moment [e × fm^2] and B(E2) [e^2 × fm^4]

	Exp.	DBM
$Q_2(2_1)$	-8.216	-25.40
$2_1 \rightarrow 0_1$	132.06	77.65
$4_1 \rightarrow 2_1$	unknown	184.62

6. Some additions



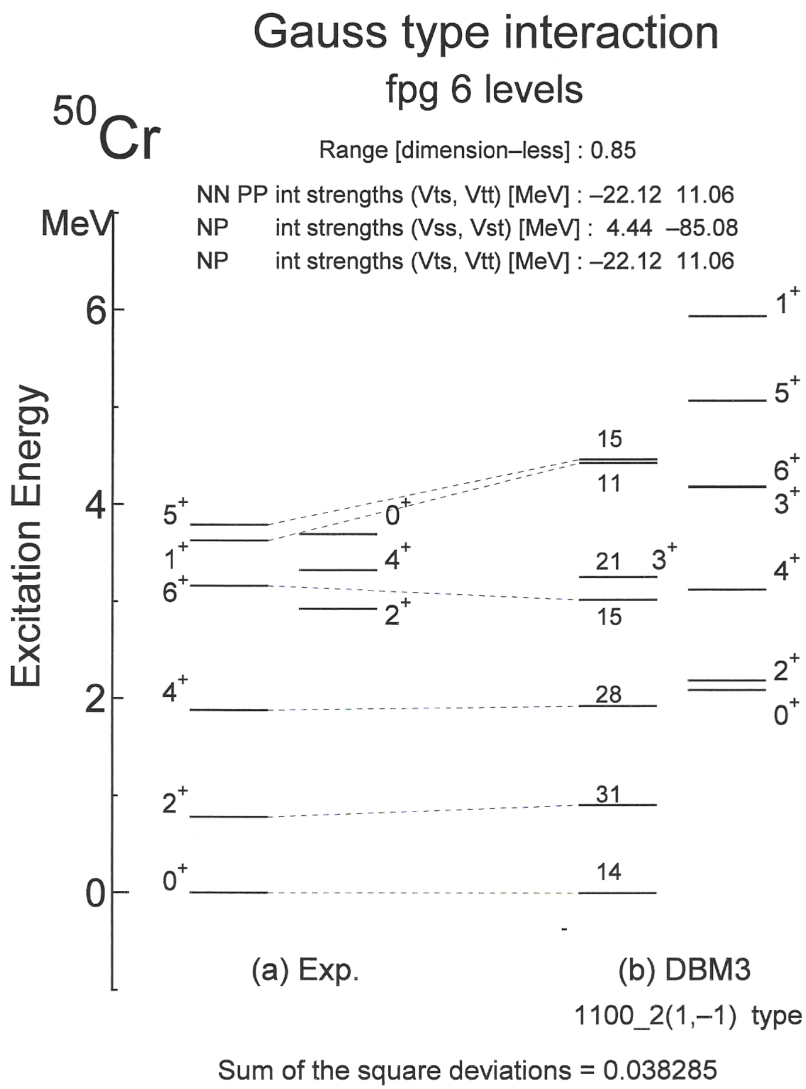
Dimension

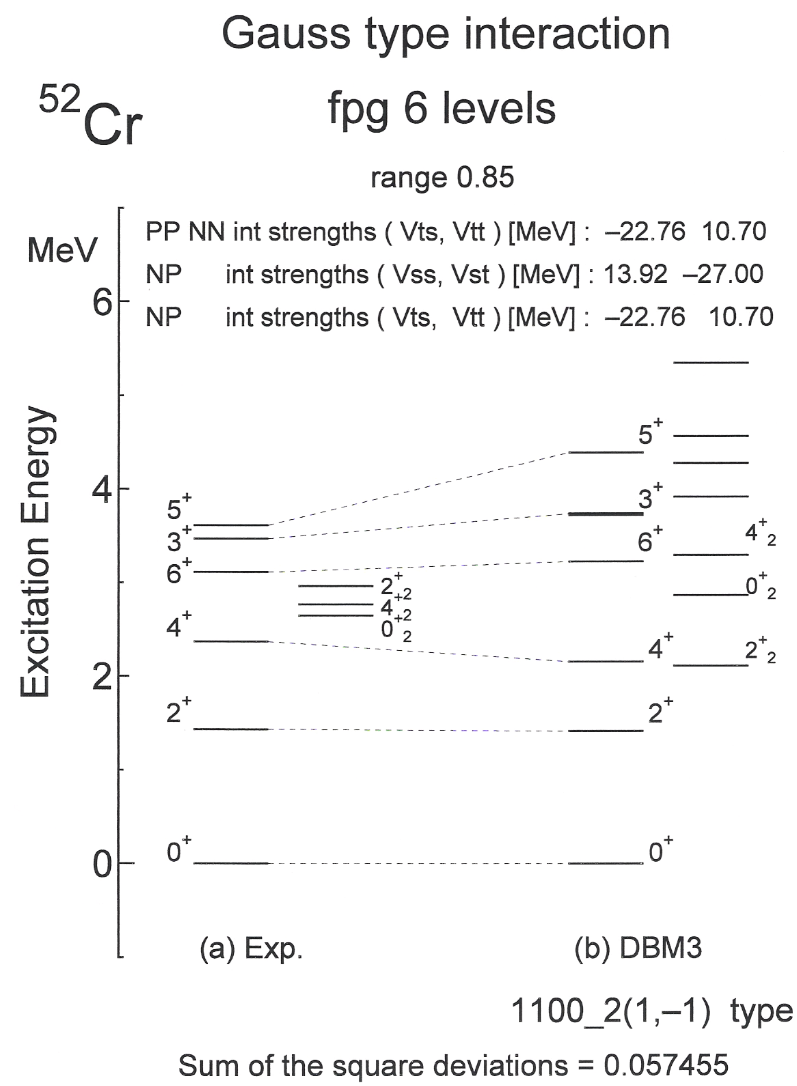
I^π	Shell Model	DBM
0^+	773549	22
1^+	2242811	18
2^+	3505079	51
3^+	4447300	37
4^+	5016447	52
5^+	5195894	32
6^+	5029085	34

Q-moment [e × fm^2] and B(E2) [e^2 × fm^4]

	Exp.	DBM
$Q_2(2_1)$	-8.216	-15.67
$2_1 \rightarrow 0_1$	132.06	72.56
$4_1 \rightarrow 2_1$	unknown	84.18

6. Some additions





Dyson Boson Mapping and Shell-Model Calculations for Even-Even Nuclei

E N D

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