Recent Progress on the 0^+ Dominance

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Outline of my talk

- Introduction
- Formulation in single j-shells
- Estimation of the ground state energy
- Application to the *P(I)*
- Summary

In collaboration with A. Arima and Y.M. Zhao

Introduction

Johnson, Bertsch and Dean (1998)

0^+ predominance

$$\langle V^2_{\alpha,\alpha'} \rangle = c_J(1 + \delta_{\alpha\alpha'}),$$
$$\langle V_{\alpha,\alpha'} V_{\beta,\beta'} \rangle = 0, \qquad (\alpha, \alpha') \neq (\beta, \beta').$$

TABLE I. Percentage of ground states (g.s.) of the RQE that have $J = 0, T = T_z$ for our target nuclides, as compared to the percentage of all states in the model spaces that have these quantum numbers.

N	Ω	Nucleus	$J = 0, T = T_z$ g.s.	$J = 0, T = T_z$ Total space
6	12	^{22}O	76%	9.8%
6	20	^{46}Ca	75%	3.5%
$N = 4, Z = 4$	12	^{24}Mg	66%	1.1%

C. W. Johnson, G. F. Bertsch, D. J. Dean and I. Talmi
Second paper

Calculation on O, Ca, Mg

RQE : random quasiparticle ensemble

TBRE: two-body random ensemble

RQE-NP: random quasiparticle ensemble-no pairing

RQE-SPE: random quasiparticle ensemble with single-particle energies

TABLE I. Percentage of ground states for selected random ensembles that have $J=0$ for our target nuclides, as compared to the percentage of all states in the model spaces that have these quantum numbers. (Statistical error is approximately $1-3\%$.) Entries with dashes were not computed.

Nucleus	RQE	RQE-NP	TBRE	RQE-SPE	$J=0$ (total space)	$J=2$ (total space)
^{20}O	68%	50%	50%	49%	11.1%	14.8%
^{22}O	72%	68%	71%	77%	9.8%	13.4%
^{24}O	66%	51%	55%	78%	11.1%	14.8%
^{44}Ca	70%	46%	41%	70%	5.0%	9.6%
^{46}Ca	76%	59%	56%	74%	3.5%	8.1%
^{48}Ca	72%	53%	58%	71%	2.9%	7.6%
^{50}Ca	65%	45%	51%	61%	2.7%	7.1%
^{24}Mg	66%	–	44%	54%	4%	16%
^{26}Mg	62%	52%	48%	56%	4%	15%
^{28}Mg	59%	46%	44%	54%	4%	16%

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Formulation in single-j shells

In order to simplify argument, we take j^n -configuration.

Hamiltonian
$$\hat{H} = \sum_{J=0}^{2j-1} \sqrt{2J+1}\, G_J \left[A^{\dagger(J)} \tilde{A}^{(J)} \right]^{(0)}$$

$$A^{\dagger(J)} = \frac{1}{\sqrt{2}} \left[a_j^{\dagger} a_j^{\dagger} \right]^{(J)}, \quad \tilde{A}^{(J)} = -\frac{1}{\sqrt{2}} \left[\tilde{a}_j \tilde{a}_j \right]^{(J)}$$

Two-body random ensemble (TBRE) :

$$\rho(G_J) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{G_J^2}{2} \right]$$

The ensemble average : $\langle \ \rangle$

$$\langle G_J \rangle = 0, \qquad \langle G_J G_K \rangle = \delta_{JK}$$

Matrix elements of $\hat{H}$ for spin-*I* states

with dimensiton d_I

$$H_{I\beta\gamma} = < j^n I\beta \,|\, \hat{H} \,|\, j^n I\gamma > = \sum_{J=0}^{2j-1} \alpha^J_{I\beta\gamma} G_J$$

$$\alpha^J_{I\beta\gamma} = \frac{n(n-1)}{2} \sum_{K\delta} < j^{n-2} K\delta, j^2 J \,|\}\, j^n I\beta >< j^{n-2} K\delta, j^2 J \,|\}\, j^n I\gamma >$$

c.f.p. c.f.p.

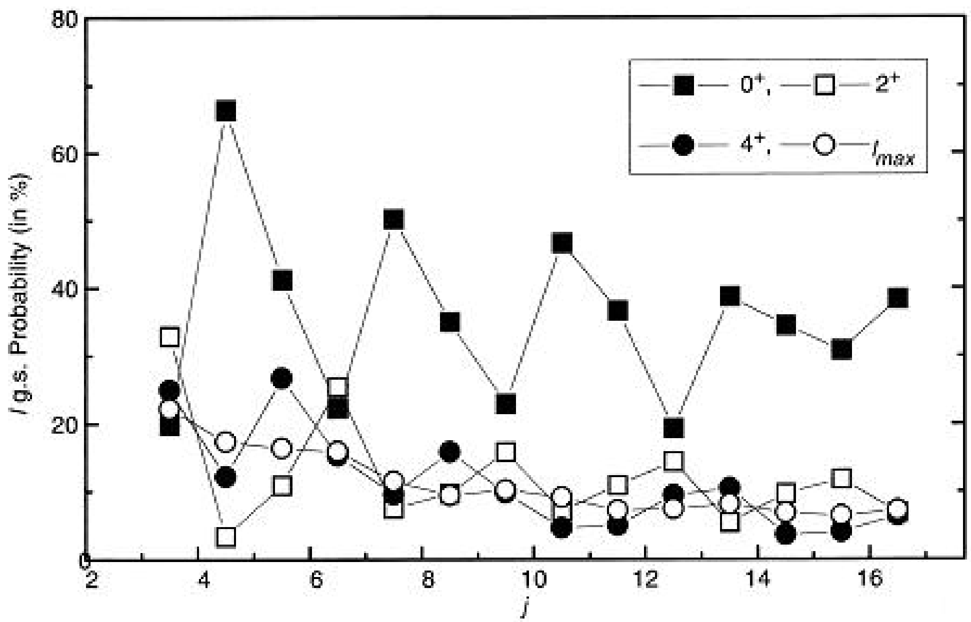
Definition of the matrix $\boldsymbol{\alpha}^J_I$ with dimensiton d_I

$$\left(\boldsymbol{\alpha}^J_I\right)_{\beta\gamma} \equiv \alpha^J_{I\beta\gamma} \qquad (\beta, \gamma = 1, \ldots, d_I)$$

0^+ dominance examples

Probabilities of Spin=*I* ground states : $P(I)$

All probabilities are obtained by 1000 runs of the TBRE Hamiltonian in j^4



One sees clearly the Spin = 0 dominance in this figure.

Empirical approach to predict P(I)

We set one of the two-body matrix elements $G_J = -1$

and all others 0. We find which angular momentum I

gives the lowest eigenvalue among all the eigenvalues

of the shell model diagonalization.

How many times does a certain angular momentum I gives

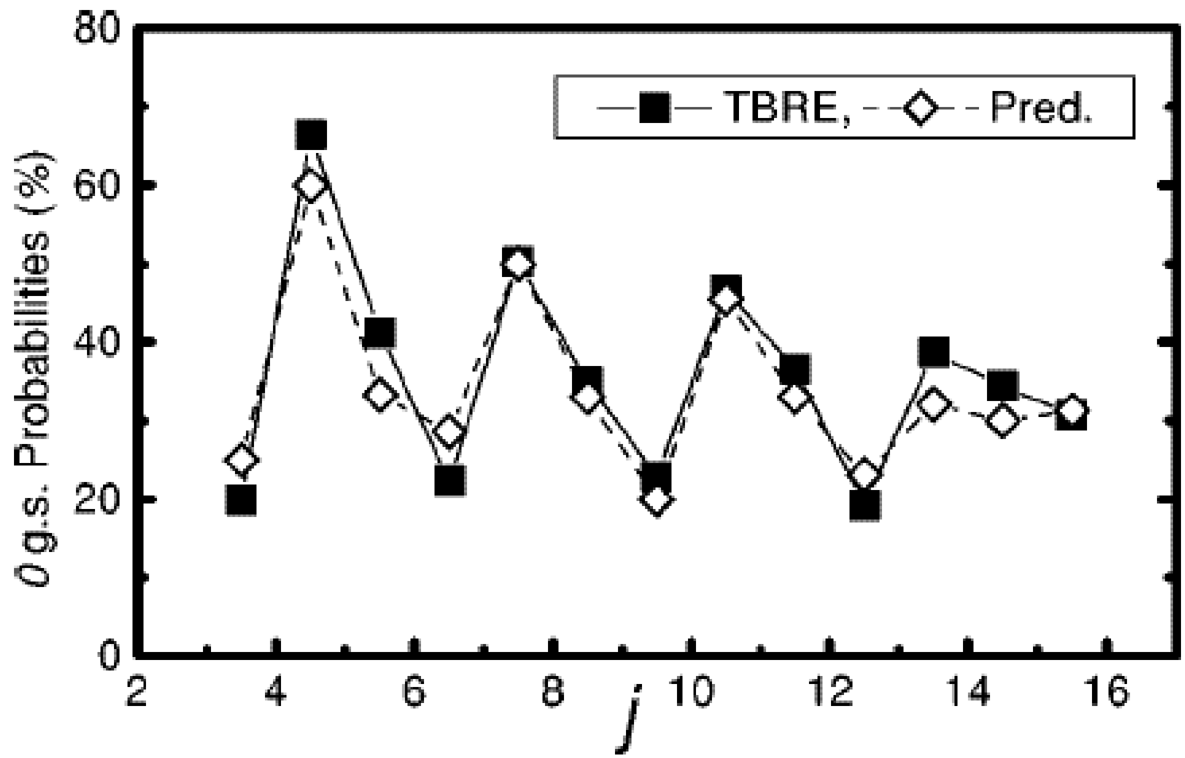
the lowest eigenvalues among all the possible eigenvalues ? N_I

We predict that the probability of I g.s. is given by

$P(I) = N_I / N$ where N is the number of G_J $(= (2j+1)/2)$

TABLE I. The angular momenta that give the lowest eigenvalues when $G_J = -1$ and all other parameters are 0 for four fermions in single-j shells.

$2j$	$G0$	$G2$	$G4$	$G6$	$G8$	$G10$	$G12$	$G14$	$G16$	$G18$	$G20$	$G22$	$G24$	$G26$	$G28$	$G30$
7	0	4	2	8												
9	0	4	0	0	12											
11	0	4	0	4	8	16										
13	0	4	0	2	2	12	20									
15	0	4	0	2	0	0	16	24								
17	0	4	6	0	4	2	0	20	28							
19	0	4	8	0	2	8	2	16	24	32						
21	0	4	8	0	2	0	0	0	20	28	36					
23	0	4	8	0	2	0	10	2	0	24	32	40				
25	0	4	8	0	2	4	8	10	6	0	28	36	44			
27	0	4	8	0	2	4	2	0	0	4	20	32	40	48		
29	0	4	8	0	0	2	6	8	12	8	0	24	36	44	52	
31	0	4	8	0	0	2	0	8	14	16	6	0	32	40	48	56


80
60
40
20
0
2
4
6
8
10
12
14
16
j
0 g.s. Probabilities (%)
TBRE, Pred.

Two main problems

- What is the origin of spin=0 dominance?

- What quantities characterize the ground state energy for each angular momentum?

  -- How to estimate the ground state energy in many-body problems?

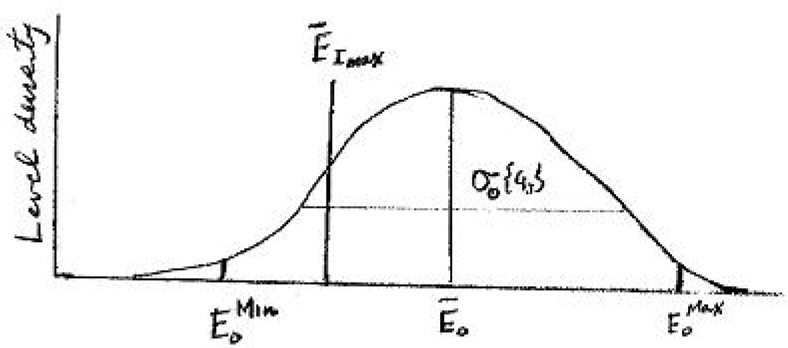
Estimation of the ground state energy

We assume the ground state energy of spin-I states as follows

$$E_I^{(\min)} = \bar{E}_I - \Phi(d_I)\ \sigma_I\{G_J\}$$

Average Width

The distribution of enegies are assumed to be Gaussian



Estimation of the factor $\Phi(d_I)$

Let us take the following guess to the lowest eigen-value $E_I^{(\min)}$ of the Hamiltonian $\hat{H}$ by assuming that eigen-energies $E_{I\beta}$ $(\beta = 1, 2, \cdots d_I)$ follow a gaussian distribution

$$\rho(E_I) = \frac{d_I}{\sqrt{2\pi}\sigma_I} \exp\left[-\frac{\left(E_I - \bar{E}_I\right)^2}{2\left(\sigma_I\right)^2} \right]$$

To estimate $E_I^{(\min)}$, we need to solve the following equation ;

$$\int_{E_I^{(\min)}}^{\bar{E}_I} \rho(E_I)\, dE_I = \frac{d_I}{2} - 1$$

This is converted to the following equation by the change of variable

$$\mathrm{Erfc}(t^M) = \frac{\sqrt{\pi}}{d_I} \qquad\qquad t^M = \frac{E_I^{(\min)} - \bar{E}_I}{\sqrt{2}\sigma_I}$$

where the error function is defined as $\mathrm{Erfc}(x) \equiv \int_x^{\infty} \exp\left[-t^2\right] dt$

We cannot solve this equation analytically, but for large d_I we get

$$t^M \approx -\sqrt{\ln d_I - \tfrac{1}{2}\ln(4\pi \ln d_I)}$$

by using the asymptotic expansion of the error function for its large argument.

Thus, we have

$\Phi(d_I)$

$$E_I^{(\min)} = \bar{E}_I - \sqrt{2\ln d_I - \ln(4\pi \ln d_I)}\ \sigma_I$$

Accordingly we have the estimate of the minimum energy $E_I^{(\min)}$

for $\{G_J\}$

$$E_I^{(\min)} = \bar{E}_I - \Phi(d_I)\ \sigma_I\{G_J\}$$

Average Width

Here $\Phi(d_I) = \sqrt{\ln d_I - \frac{1}{2}\ln(4\pi \ln d_I)}$ and the width

$$\sigma_I\{G_J\} = \sqrt{\frac{1}{d_I}\mathrm{Tr}\left[\left(\hat{H} - \bar{E}_I\right)^2\right]}$$

$$= \sqrt{\frac{1}{d_I}\sum_{J,K}\mathrm{Tr}\left[\left(\boldsymbol{\alpha}_I^J - \bar{\alpha}_I^J\mathbf{I}\right)\left(\boldsymbol{\alpha}_I^K - \bar{\alpha}_I^K\mathbf{I}\right)\right]G_J G_K}$$

Note that this guess is only valid for $d_I >> 1$

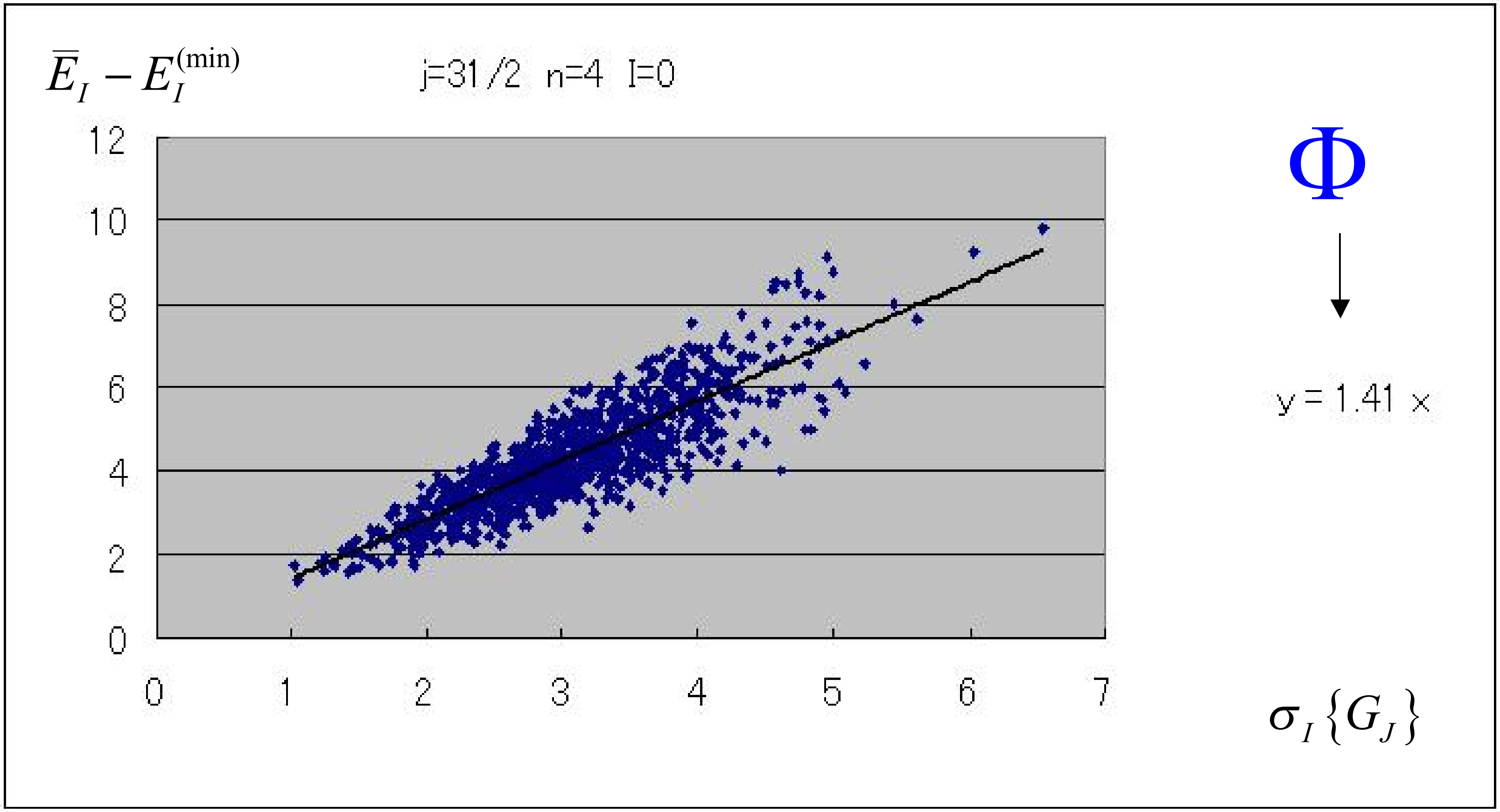
Numerical Check of our formula

Let us check our formula numerically.

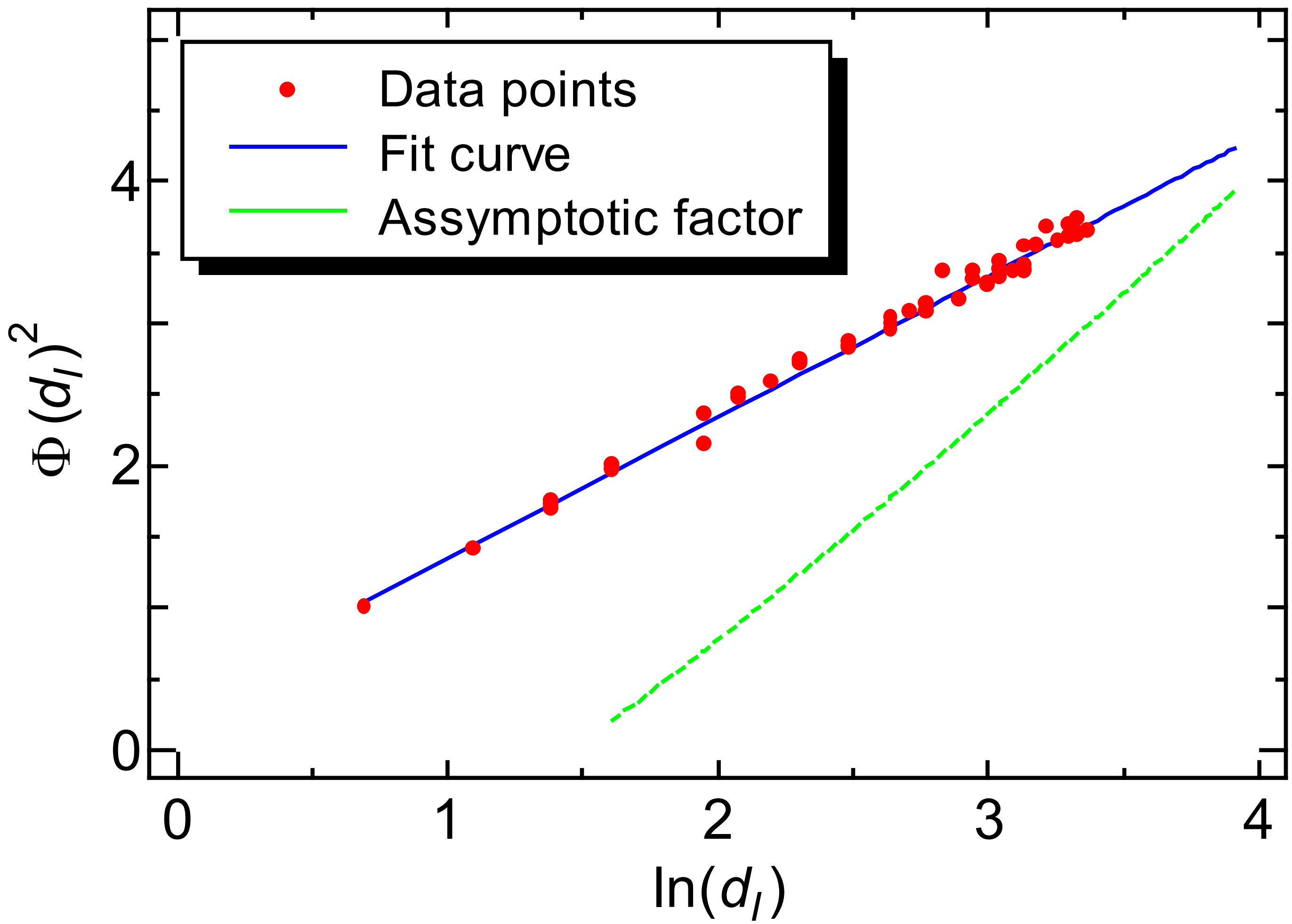
$$E_I^{(\min)} = \bar{E}_I - \Phi(d_I)\ \sigma_I\left\{G_J\right\}$$

We calculate $\bar{E}_I - E_I^{(\min)}$ as a function of $\sigma_I\left\{G_J\right\}$

$j = 31/2 \quad n = 4 \quad I = 0$



Factors Φ^2 for j=31/2, n=4



Application of our new formula

$$E_I^{(\min)} = \bar{E}_I - \Phi(d_I)\ \sigma_I\{G_J\}$$

$$\Phi(d_I) = \sqrt{a\ \ln(d_I)\ +\ b} \qquad a = 0.99,\quad b = 0.36$$

Surprisingly factors a, b are insensitive to the shell-size

particle number as far as single-j shells are concerned.

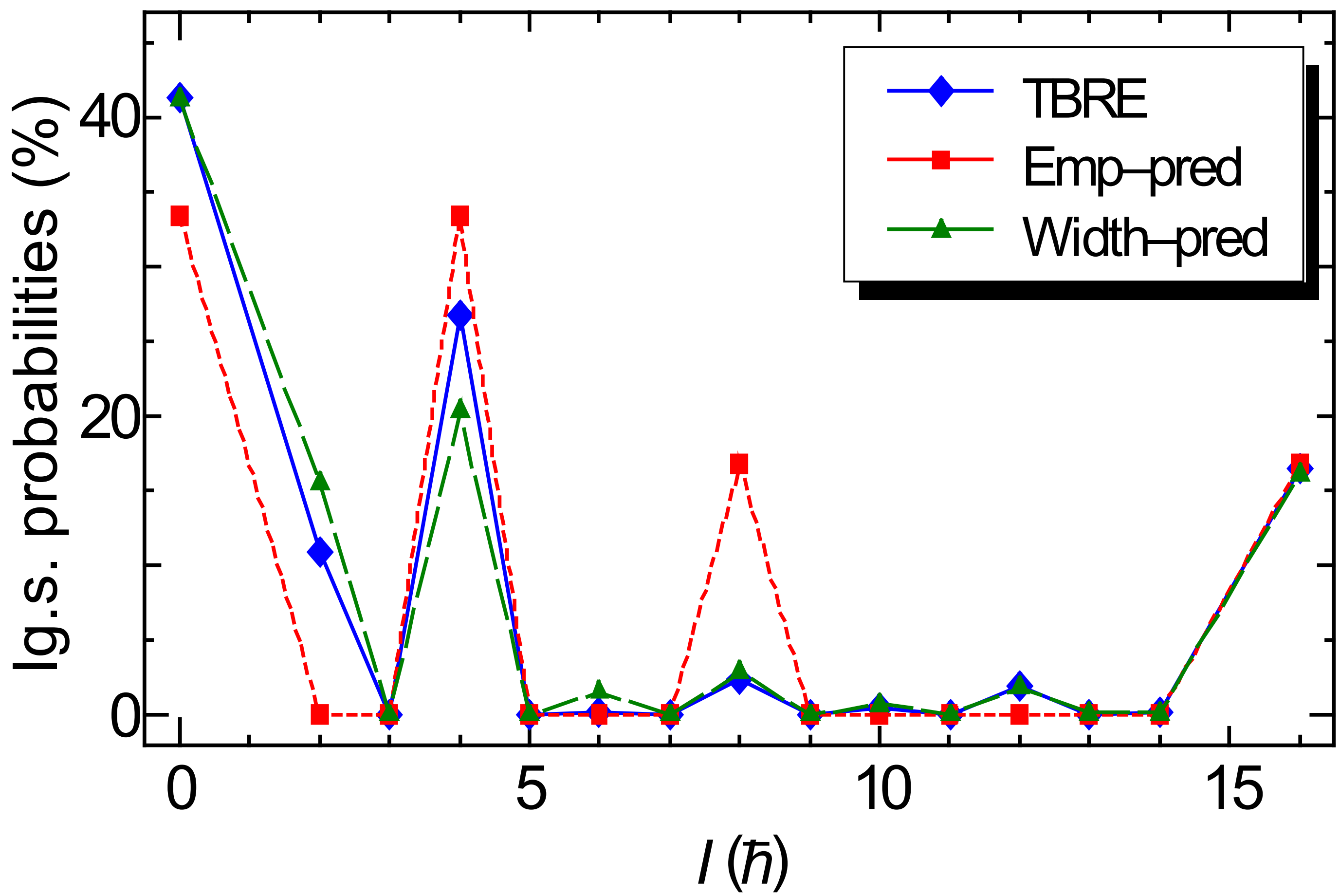
We calculate *P*(*I*) using this formula

“Width-prediction” method

Application to single-*j* shell

$j = \frac{11}{2}$ shell

P(*I*)

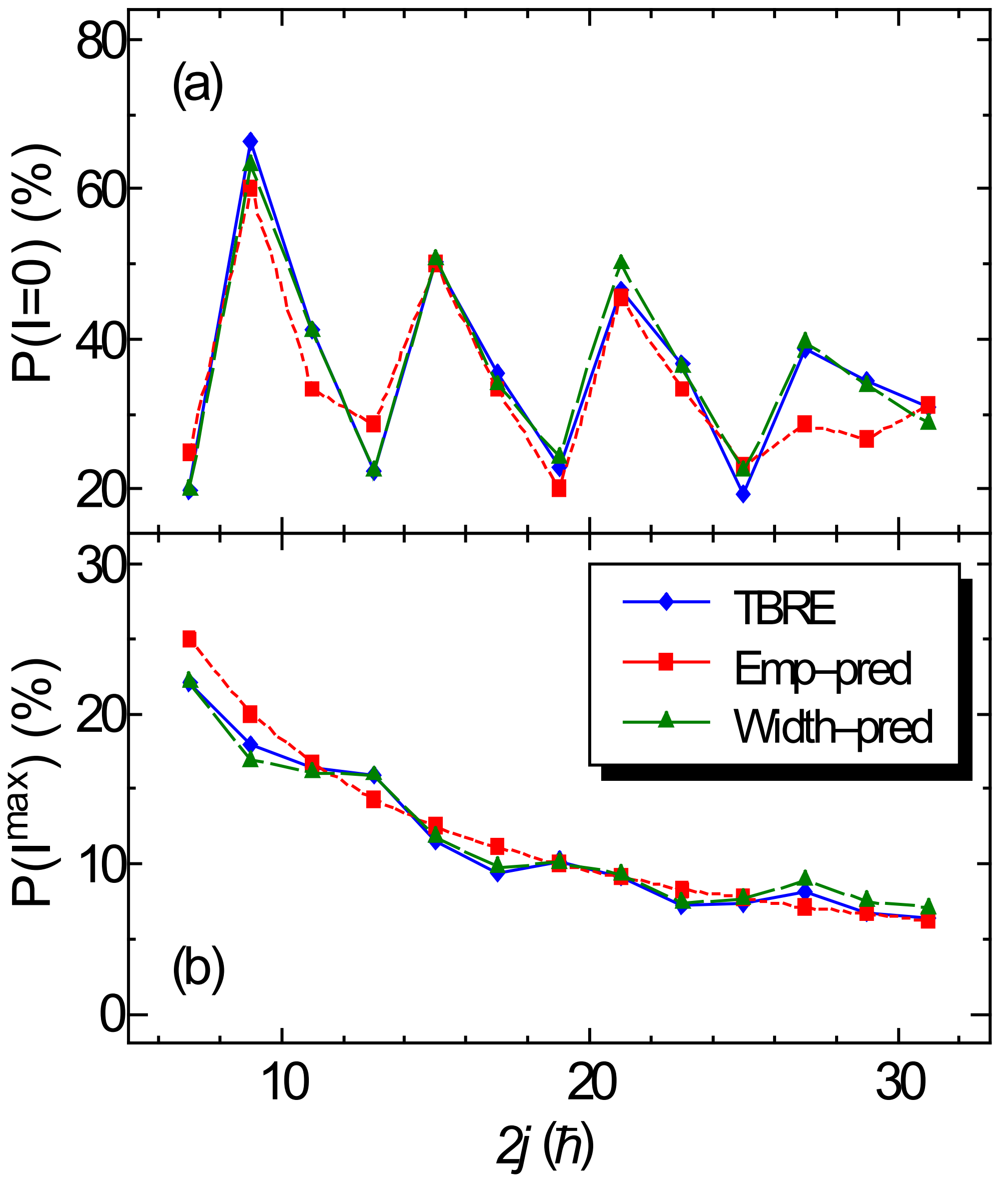


Fermion systems : Single-j

$j=7/2$ to $j=31/2$ $n=4$

P(I = 0)

P(I^{max})

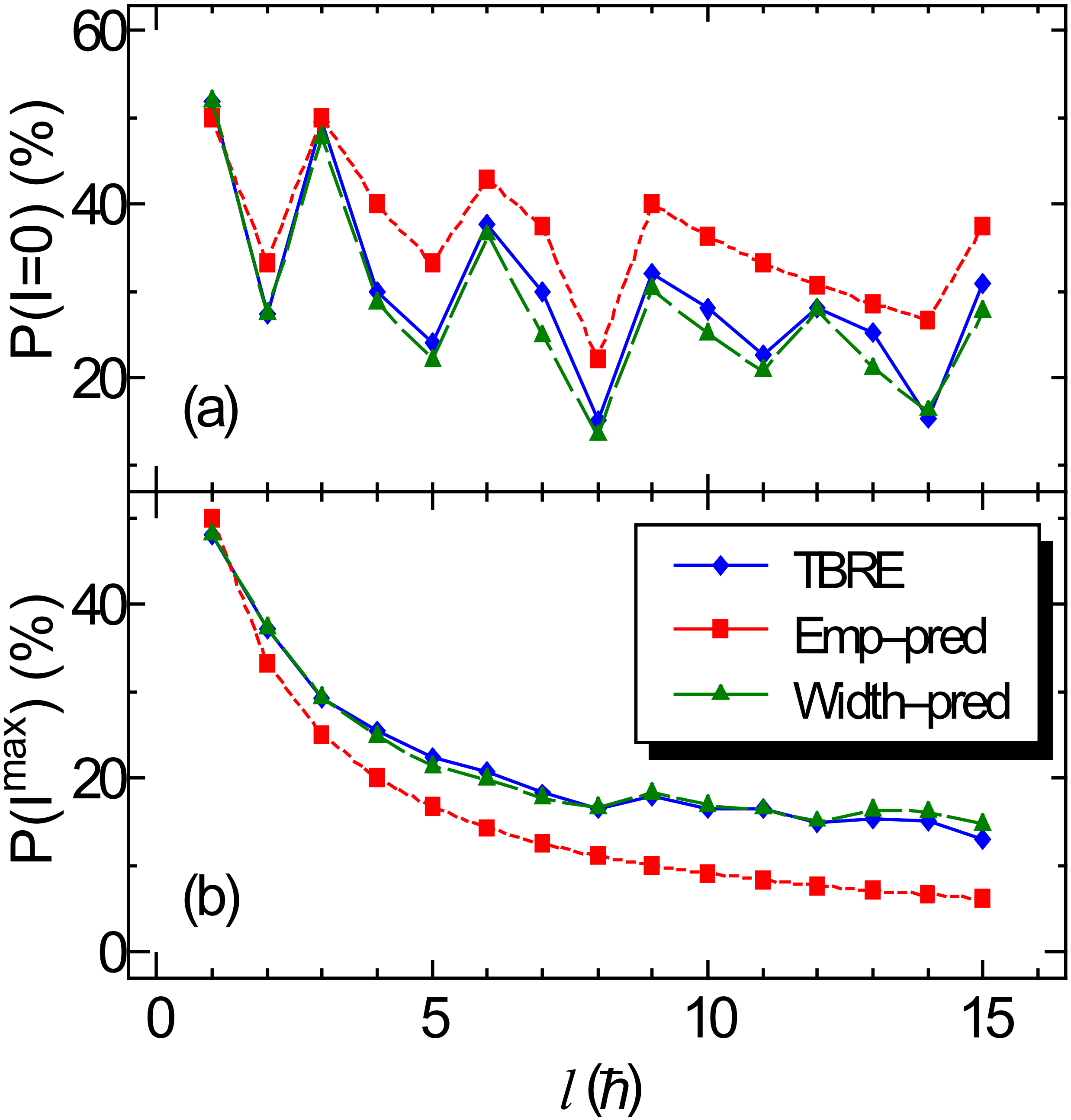


Boson systems : Single-l

l=1 to l=15 n=4

$P(I=0)$

$P(I^{max})$



Papenbrock and Weidenmuller

Phys. Rev. Lett. 93(2004)132503

$$E_I^{(\min)} = - r_I \ \sigma_I^P \left\{ G_J \right\}$$

$$\sigma_I^P \left\{ G_J \right\} = \sqrt{\frac{1}{d_I} \mathrm{Tr}\left[\left(\hat{H} \right)^2 \right]}$$

Their $P(I)$ for $j = 19/2$ and $n = 6$

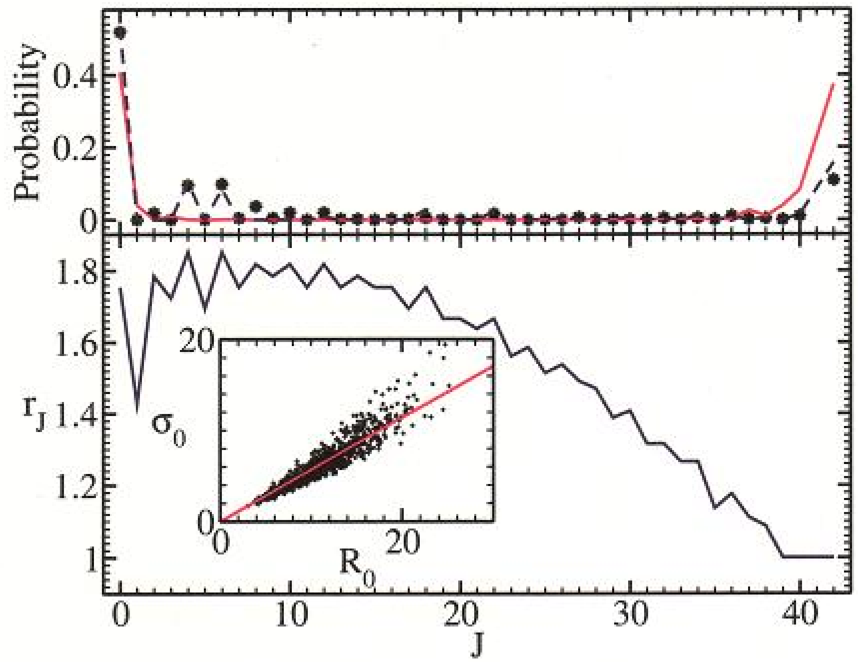


FIG. 1 (color online). Six fermions in a shell with spin $j = 19/2$. Top: Probability that the ground-state has spin J (data points); probability that spin J has the largest spectral width (solid line); probability that the product $r_J \sigma_J$ is maximal (dashed line). Bottom: Scaling factor r_J between the widths and spectral radii. Inset: Spectral radius R_0 versus width σ_0 (data points) and the linear fit (line) for total spin $J = 0$. (Results from 900 random realizations).

Microscopic origin of the 0+ dominance

$$H_{I\beta\gamma} = < j^n I\beta | \hat{H} | j^n I\gamma > = \sum_{J=0}^{2j-1} \alpha^J_{I\beta\gamma} G_J$$

$$\alpha^J_{I\beta\gamma} = \frac{n(n-1)}{2} \sum_{K\delta} < j^{n-2}K\delta, j^2 J |\} j^n I\beta > < j^{n-2}K\delta, j^2 J |\} j^n I\gamma >$$

c.f.p.   c.f.p.

$$\bar{\alpha}^J_I \equiv \frac{1}{d_I} \sum_{\beta}^{d_I} \alpha^J_{I\beta\beta} = \frac{1}{d_I} \mathrm{Tr}\left(\boldsymbol{\alpha}^J_I\right)$$

$$\left(\sigma^J_I\right)^2 = \frac{1}{d_I} \mathrm{Tr}\left(\left(\boldsymbol{\alpha}^J_I - \bar{\alpha}^J_I \mathbf{I}\right)^2\right) = \frac{1}{d_I} \sum_{\beta,\gamma} \alpha^J_{I\beta\gamma} \alpha^J_{I\gamma\beta} - \left(\bar{\alpha}^J_I\right)^2$$

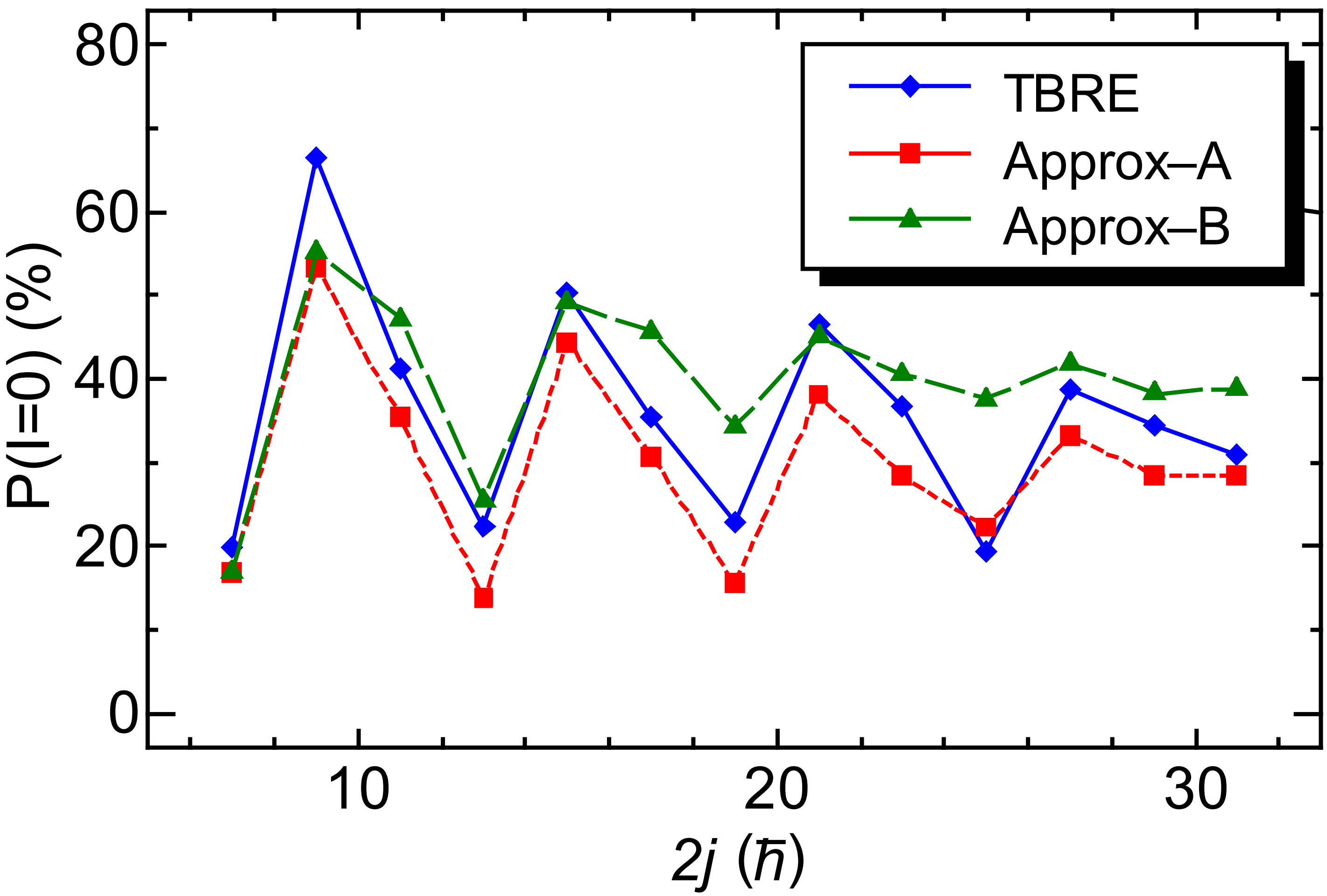
"Random phase approximation"

$$\frac{1}{d_I}\sum_{J,K}\mathrm{Tr}\left[\left(\boldsymbol{\alpha}_I^J-\bar{\alpha}_I^J\mathbf{I}\right)\left(\boldsymbol{\alpha}_I^K-\bar{\alpha}_I^K\mathbf{I}\right)\right]G_J G_K \approx \sum_{J,K}\sigma_I^J\sigma_I^K G_J G_K$$

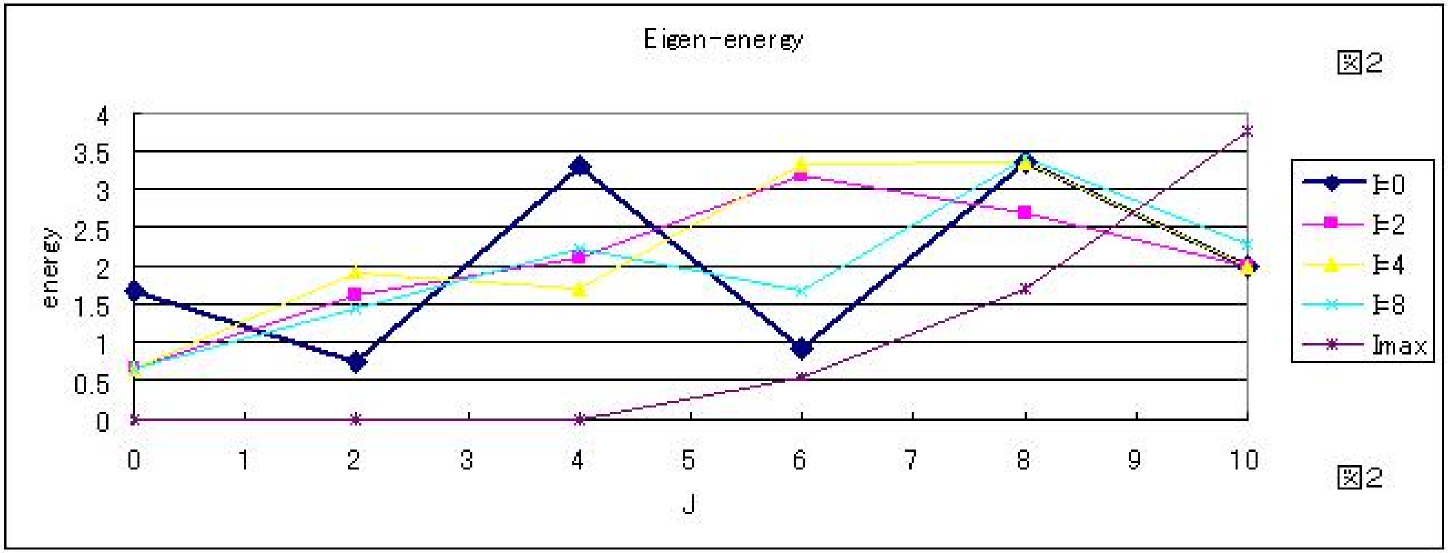
$$E_I^{Min} \approx \bar{E}_I - \Phi(d_I)\left|\sum_J \sigma_I^J G_J\right|$$

$$= \sum_J \bar{\alpha}_I^J G_J - \Phi(d_I)\left|\sum_J \sigma_I^J G_J\right|$$ ***Approx-A***

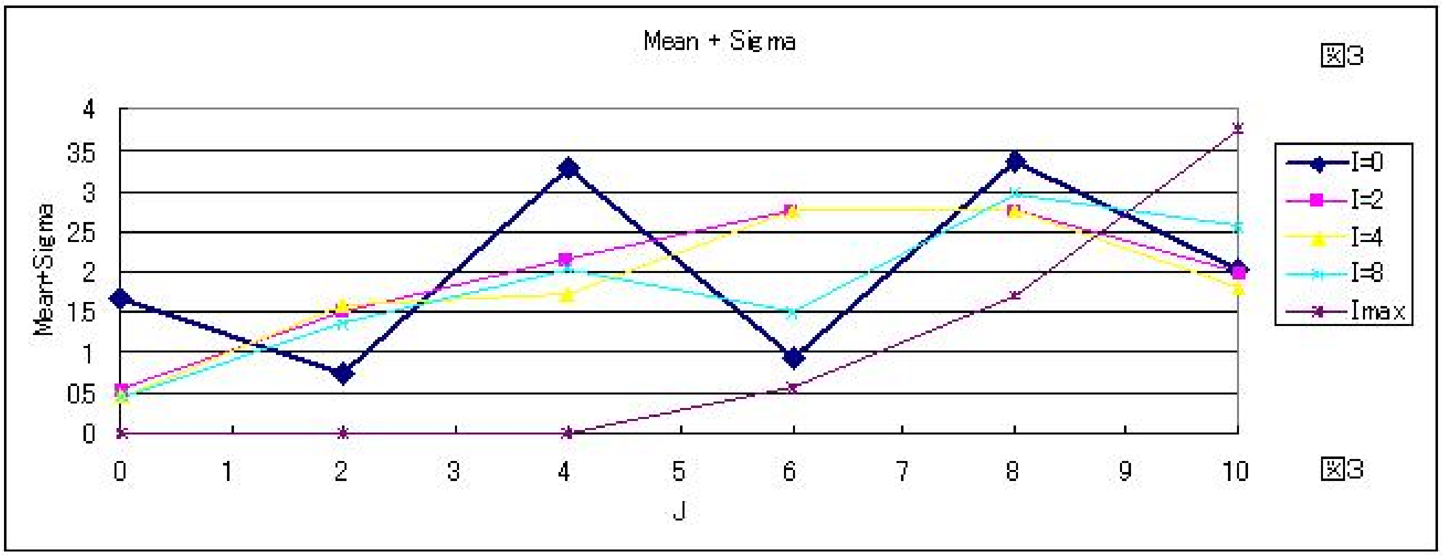
$$= \sum_J \bar{\alpha}_I^J G_J - \left|\sum_J \sigma_I^J G_J\right|$$ ***Approx-B***



80
60
40
20
0
P(I=0) (%)
10
20
30
2j (ħ)
TBRE
Approx–A
Approx–B



Eigen-energy
図2
energy
4
3.5
3
2.5
2
1.5
1
0.5
0
0
1
2
3
4
5
6
7
8
9
10
J
I=0
I=2
I=4
I=8
Imax
図2



Mean + Sigma
図3
Mean+Sigma
4
35
3
25
2
15
1
05
0
0
1
2
3
4
5
6
7
8
9
10
J
I=0
I=2
I=4
I=8
Imax
図3

Large fluctuation of alpha and sigma

$$H_{I\beta\gamma} = < j^n I\beta \,|\, \hat{H} \,|\, j^n I\gamma > = \sum_{J=0}^{2j-1} \alpha_{I\beta\gamma}^{J} G_J$$

$$\alpha_{I\beta\gamma}^{J} = \frac{n(n-1)}{2} \sum_{K\delta} < j^{n-2} K\delta, j^2 J \,|\} j^n I\beta >< j^{n-2} K\delta, j^2 J \,|\} j^n I\gamma >$$

c.f.p. c.f.p.

$$\bar{\alpha}_I^J \equiv \frac{1}{d_I} \sum_{\beta}^{d_I} \alpha_{I\beta\beta}^{J} = \frac{1}{d_I} \mathrm{Tr}\left(\boldsymbol{\alpha}_I^J\right)$$

$$\sum_J \bar{\alpha}_I^J = \frac{n(n-1)}{2}$$

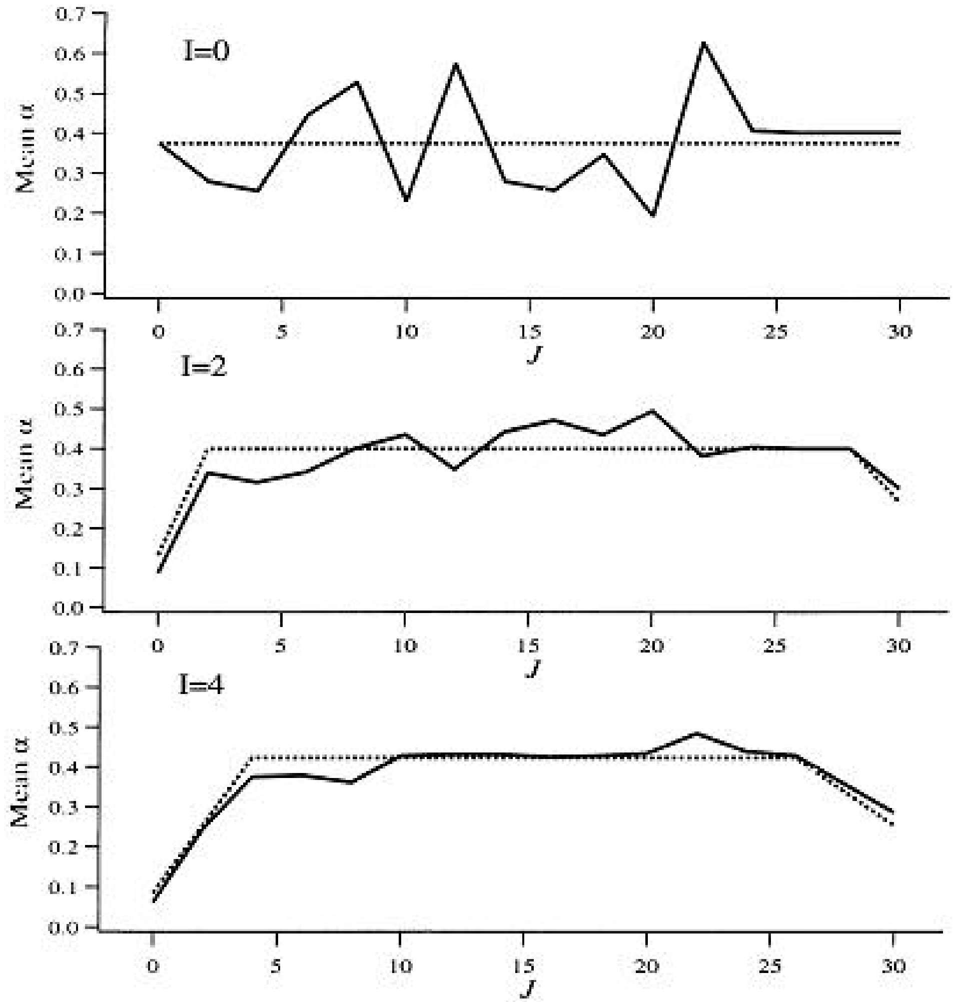


Figure 3. Predicted $\bar{\alpha}_I^J$ (dotted line) and actual values (solid line) for $I = 0, 2, 4$ as a function of J with $j = 31/2$ and $n = 4$.

Conclusion

- A new formula is proposed to estimate the ground state of spin-I states of a TBRE hamiltonian.
- The probability $P(I)$ using our new formula gives a good agreement with $P(I)$ of TBRE for both fermions and bosons in single orbitals
- The microscopic origin of the spin-0 dominance is much easier to access by using our new formula.