#### CISS07 8/29/2007

#### Comprehensive treatment of correlations at different energy scales in nuclei using Green's functions

Lecture 1: 8/28/07	Propagator description of single-particle motion and the link with experimental data
Lecture 2: 8/29/07	From Hartree-Fock to spectroscopic factors < 1: inclusion of long-range correlations
Lecture 3: 8/29/07	Role of short-range and tensor correlations associated with realistic interactions
Lecture 4: 8/30/07	Dispersive optical model and predictions for nuclei towards the dripline
Adv. Lecture 1: 8/30/07	Saturation problem of nuclear matter & pairing in nuclear and neutron matter
Adv. Lecture 2: 8/31/07	Quasi-particle density functional theory

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### Outline

- SRC for free particles
- Ladders in the medium
- Self-energy and Dyson equation
- Nuclear matter simplifications
- Self-consistent Green's functions in nuclear matter & results
- SRC in finite nuclei: where are the high-momentum nucleons
- Summary of sp strength in closed-shell nuclei
- Other nuclei
- N very different from Z
- Nuclear matter with isospin polarization

#### Short-range correlations for two free particles

Solve the Schrödinger equation or the equivalent "T"-matrix

$$\left\langle k\ell \left| \Gamma_{pp}^{JST}(k_0) \right| k'\ell' \right\rangle = \left\langle k\ell \left| V^{JST} \right| k'\ell' \right\rangle + \frac{m}{2\hbar^2} \sum_{\ell''} \int_0^\infty \frac{dq}{\left(2\pi\right)^3} q^2 \left\langle k\ell \left| V^{JST} \right| q\ell'' \right\rangle \frac{1}{k_0^2 - q^2 + i\eta} \left\langle q\ell'' \left| \Gamma_{pp}^{JST}(k_0) \right| k'\ell' \right\rangle$$

 $Effective\ interaction$ 



Sum of ladder diagrams

#### Sum of ladder diagram takes care of SRC Also in the medium!

#### Relative wave function and potential



Green's functions III 4

### Ladder diagrams in the medium (options)

Ladders in the medium  

$$\begin{split} \overbrace{\mathbf{\Gamma}_{pphh}}^{f} &= \bullet \dots \bullet + \frac{1}{2} \underbrace{\prod_{pphh}}^{r} = \bullet \dots \bullet + \frac{1}{4} \underbrace{\prod_{pphh}}^{g} = \bullet \dots \bullet + \frac{1}{4} \underbrace{\prod_{pphh}}^{g} \\ &\Rightarrow self - energy calculation \\ \langle k\ell | \Gamma_{pphh}^{JST}(K,E) | k'\ell' \rangle &= \langle k\ell | V^{JST} | k'\ell' \rangle + \frac{1}{2} \sum_{\epsilon^*} \int_{0}^{s} \frac{dq}{(2\pi)^3} q^2 \langle k\ell | V^{JST} | q\ell'' \rangle G_{pphh}^{f}(q;K,E) \langle q\ell'' | \Gamma_{pphh}^{JST}(K,E) | k'\ell' \rangle \\ G_{pphh}^{f} has different form depending on the level of sophistication \\ Nuclear matter: \\ G_{BG}^{f}(k_{1},k_{2};E) &= \frac{\theta(k_{1}-k_{F})\theta(k_{2}-k_{F})}{E-\epsilon(k_{1})-\epsilon(k_{2})+i\eta} \\ Bethe-Goldstone \\ G_{GF}^{f}(k_{1},k_{2};E) &= \frac{\theta(k_{1}-k_{F})\theta(k_{2}-k_{F})}{E-\epsilon(k_{1})-\epsilon(k_{2})-i\eta} \\ G_{GF}^{f}(k_{1},k_{2};E) &= \frac{\theta(k_{1}-k_{F})\theta(k_{2}-k_{F})}{E-\epsilon(k_{1})-\epsilon(k_{2})-i\eta} \\ G_{g}^{f}(k_{1},k_{2};E) &= \frac{\theta(k_{1}-k_{F})\theta(k_{2}-k_{F})}{E-\epsilon(k_{1})-\epsilon(k_{2})+i\eta} - \frac{\theta(k_{F}-k_{1})\theta(k_{F}-k_{2})}{E-\epsilon(k_{1})-\epsilon(k_{2})-i\eta} \\ G_{g}^{f}(k_{1},k_{2};E) &= \frac{\theta(k_{1}-k_{F})\theta(k_{2}-k_{F})}{E-\epsilon(k_{1})-\epsilon(k_{2})+i\eta} - \frac{\theta(k_{F}-k_{1})\theta(k_{F}-k_{2})}{E-\epsilon(k_{1})-\epsilon(k_{2})-i\eta} \\ G_{g}^{f}(k_{1},k_{2};E) &= \int_{\epsilon_{F}}^{\infty} dE_{1} \int_{\epsilon_{F}}^{\infty} dE_{2} \frac{S_{p}(k_{1};E_{1})S_{p}(k_{2};E_{2})}{E-E_{1}-E_{2}+i\eta} - \int_{-\infty}^{\epsilon_{F}} dE_{2} \frac{S_{h}(k_{1};E_{1})S_{h}(k_{2};E_{2})}{E-E_{1}-E_{2}-i\eta} \\ SCGF$$

#### Phase shifts for dressed nucleons



PRC60, 064319 (1999) also PRC58, 2807 (1998) Green's functions III 6

Dyson equation and spectral functions in nuclear matter (some simplifications)  $G(k;E) = G^{(0)}(k;E) + G^{(0)}(k;E)\Sigma(k;E)G(k;E)$ Dyson equation  $=\frac{1}{E-\varepsilon(k)-\Sigma(k;E)}$  $G^{(0)}(k;E) = \frac{\theta(k-k_F)}{E-\varepsilon(k)+in} + \frac{\theta(k_F-k)}{E-\varepsilon(k)-in}$  Noninteracting sp propagator  $S_{p}(k;E) = -\frac{1}{\pi} \frac{\operatorname{Im}\Sigma(k;E)}{\left(E - \varepsilon(k) - \operatorname{Re}\Sigma(k;E)\right)^{2} + \left(\operatorname{Im}\Sigma(k;E)\right)^{2}} \qquad \begin{array}{c} \text{particle} \\ \text{spectral function} \end{array}$  $S_{h}(k;E) = \frac{1}{\pi} \frac{\operatorname{Im}\Sigma(k;E)}{\left(E - \varepsilon(k) - \operatorname{Re}\Sigma(k;E)\right)^{2} + \left(\operatorname{Im}\Sigma(k;E)\right)^{2}}$ hole spectral function numerator sp strength  $G(k;E) = \int_{\alpha}^{\infty} dE' \frac{S_p(k;E')}{E - E' + i\eta} + \int_{\alpha}^{\varepsilon_F} dE' \frac{S_h(k;E')}{E - E' - i\eta}$ denominator where Green's functions III 7



### Recent developments

•	St. Louis	$\Rightarrow$ Complete self-consistency for spectral functions	
		(Ramos, Vonderfecht, Gearhart, Roth/Stoddard)	
•	Ghent	⇒ Discrete method (Van Neck, Dewulf)	
•	Cracow	⇒ Separable & soft interactions (Bozek, Czerski, Soma)	
•	Tübingen	⇒ Finite temperature & soft interactions (Müther, Frick)	
•	Barcelona	$\Rightarrow$ Finite temperature & soft interactions (Polls, Ramos, Rios)	
•	Giessen	$\Rightarrow$ Interaction related to cross section (Lenske et al)	

### Illustrative results for mean-field input



Strength distribution as in nuclei ...

peak strength +10% at lower energy + global depletion!!

#### Where does the strength go?



 $k < k_{\rm F}$ : 17% >  $\varepsilon_{\rm F}$  with 13% above 100 MeV (7% above 500 MeV) Without tensor force only 10.5% above  $\varepsilon_{\rm F}$  Green's functions III 11 Short-range correlations in nuclear matter and n(k)



B.E.Vonderfecht et al. Nucl. Phys. A555, 1 (1993) E.R.Stoddard, thesis (self-consistent ladders)

M. van Batenburg (thesis, 2001) & L. Lapikás from <sup>208</sup>Pb (e,e´p) <sup>207</sup>Tl

Occupation of deeply-bound proton levels from EXPERIMENT



Green's functions III 13

Two effects associated with short-range correlations

- Depletion of the Fermi sea
- Admixture of high-momentum components to replace depleted strength

### Location of high-momentum components

high momenta



 $require\ specific\ intermediate\ states$ 

External line k (large).

Intermediate holes  $\langle k_{\rm F}$ , say total momentum  $\sim 0$ . Momentum conservation: intermediate particle -k

 $\Rightarrow$  Energy intermediate state ~ <  $\varepsilon_{2h}$  > -  $\varepsilon(k)$ 

 $\Rightarrow$  the higher k the more negative the location of its strength

 $\Rightarrow$  no high-momentum components near  $\varepsilon_{\rm F}$ 

#### High-momenta near $\varepsilon_F$ ?



I. Bobeldijk et al., Phys. Rev. Lett. 73, 2684 (1994)

#### SRC (only) calculated in <sup>16</sup>O



### Quality of quasihole wave function



Green's functions III 18

#### Prediction of high-momentum components



p<sub>1/2</sub> spectral function at fixed energies in <sup>16</sup>O Phys. Rev. C49, R17 (1994)

#### Momentum distribution <sup>16</sup>O



Confirms expectation:

High momentum nucleons can be found at large negative energies

Green's functions III 20

### What are the rest of the protons doing?

Jlab E97-006 Phys. Rev. Lett. 93, 182501 (2004) D. Rohe et al.



- Location of high-momentum components
- Integrated strength agrees with theoretical prediction Phys. Rev. C49, R17 (1994)  $\Rightarrow$  0.6 protons for <sup>12</sup>C

# Integrated strength $\Rightarrow$ n(k)

momentum dependence



 $\rightarrow$  theory and experiment  $\pm$  agree

From: Sick, ECT\* workshop July 2007





#### enhancement for Au

not yet understood consequence of n-p correlations as N > Z??, rescattering ??

would like to get S(k, E) for  $N \neq Z$ 

#### Consequences of correlated sp strength



M12 and M14 transitions in (e,e') only 50% of ph estimate



Phys. Rep. 242 ( 94) 119

# We now essentially know what all the protons are doing in the ground state of a "closed-shell" nucleus !!!

- Unique for a correlated many-body system
- Information available for electrons in atoms (Hartree-Fock)
- Not for electrons in solids
- Not for atoms in quantum liquids
- Not for quarks in nucleons

⇒ Demonstrates the value of the study of the nucleus for its intrinsic interest as a quantum many-body problem! Location of single-particle strength in nuclei



SRC

### What about open-shell systems?

#### Semi-magic nuclei Green's function calculation



SRC the same GRs similar only difference near  $\varepsilon_F$ removal & addition probabilities

similar size for 2s<sub>1/2</sub> !!

#### $\Rightarrow$ pairing

## **Deformation?**

<sup>142</sup>Nd(e,e´p) Z=60; N=82 compare with <sup>146</sup>Nd(e,e´p) Z=60; N=86 Nucl. Phys. **A560**, 811 (1993)

$E_{x}$ <sup>141</sup> $Pr$ $J^{\pi}$	S	exp		
0.000	5/2⁺	0.23		
0.145	7/2+	0.39		
1.118	11/2-	0.05		
1.298	1/2+	0.03		
		$E_{x}$ <sup>145</sup> Pr $J^{\pi}$	S	exp
		0.000	7/2+	0.19
		0.063	5/2+	0.17
		0.189	5/2⁺	0.03
Wave functions in both		0.348	3/2+	0.02
nuclei are the same!		0.555	7/2+	0.03

## Systems

# with N very different from Z?

SRC still the same (tensor force disappears for n and "increases" for p for N>Z) (see PRC71,014313(2005)) Collectivity of excited states could be reduced So less fragmentation and removal of sp strength becomes more like mean-field (+ SRC+ whatever is left of tensor force for n but perhaps strong effect for p!) Continuum effects (soft dipoles ...)

#### SCGF for isospin-polarized nuclear matter

