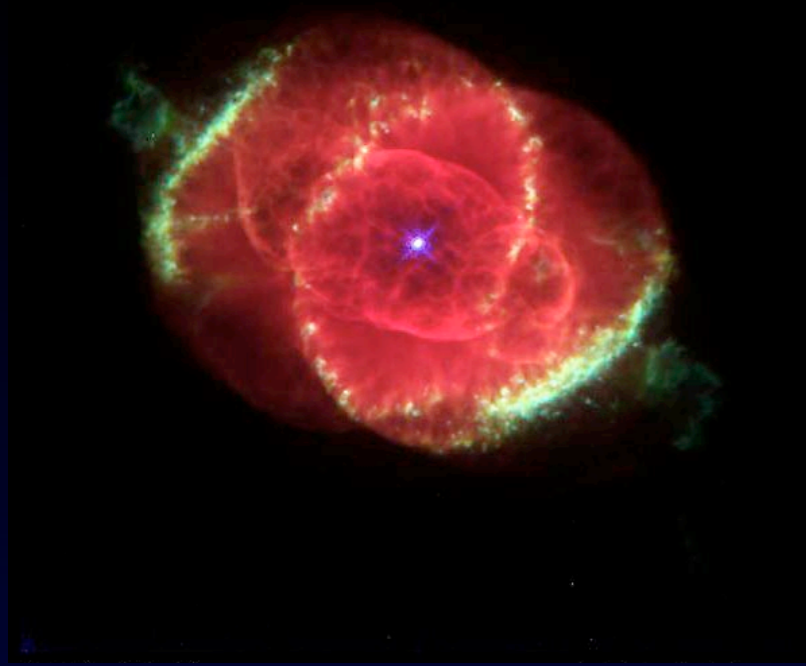
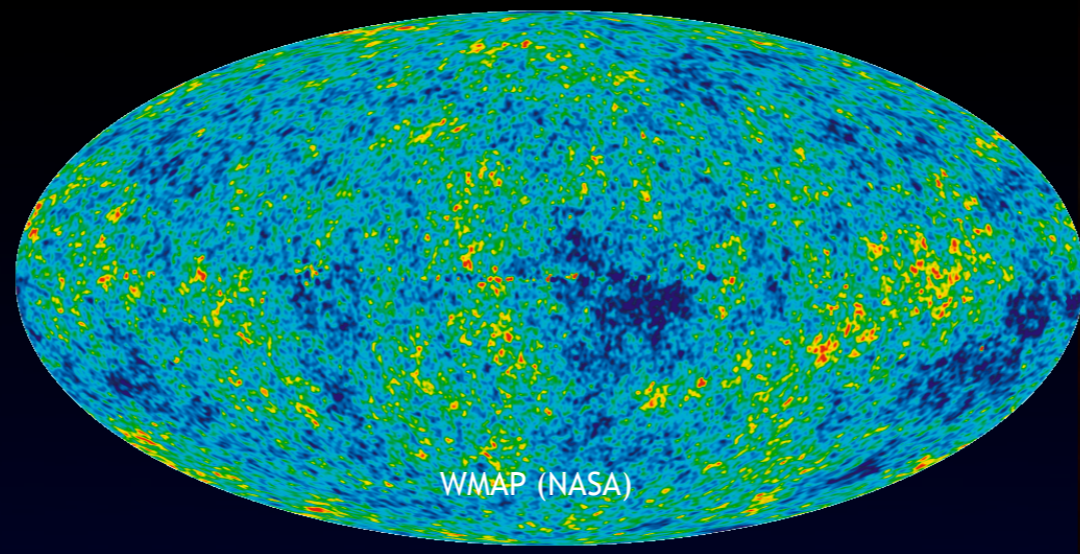
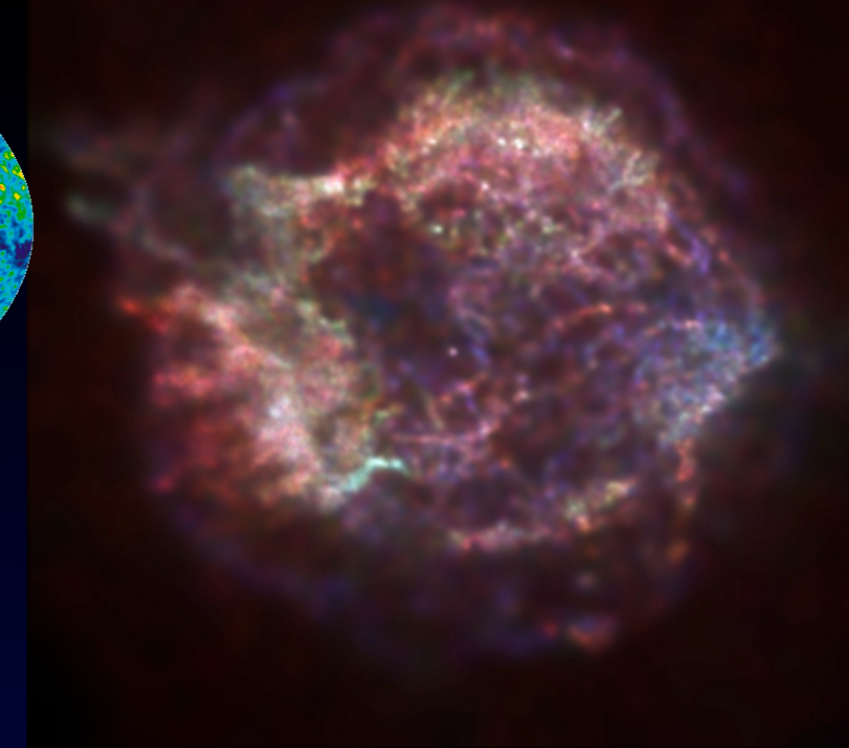


NGC6543 (Harrington & Borkowski /NASA)



SNR Cassiopeia A (Hughes et al/Chandra/NASA)



WMAP (NASA)

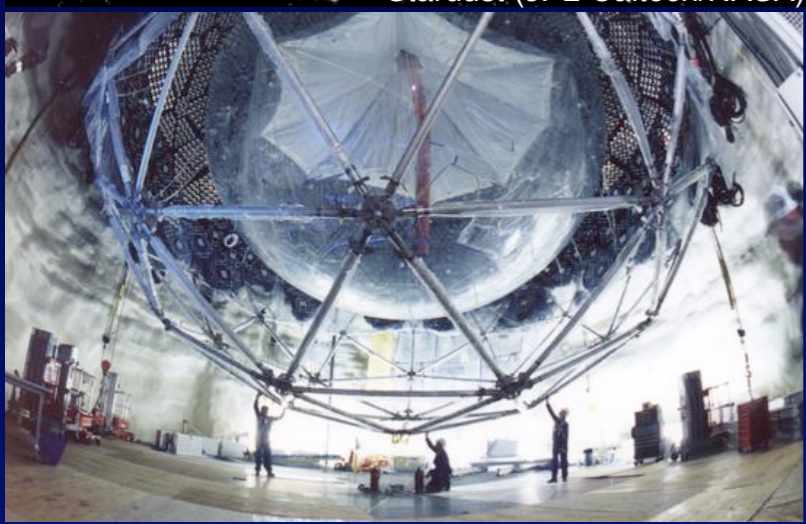
Nuclear Physics for Astrophysics



Stardust (JPL-Caltech/NASA)



Integral (ESA)



VLT (ESO)

W. Raphael Hix
ORNL Physics Division and
UTK Department of Physics
& Astronomy



Lecture Schedule

1. Nuclear Physics for Astrophysics

How do we learn about cosmic nuclear evolution

What nuclear data are needed

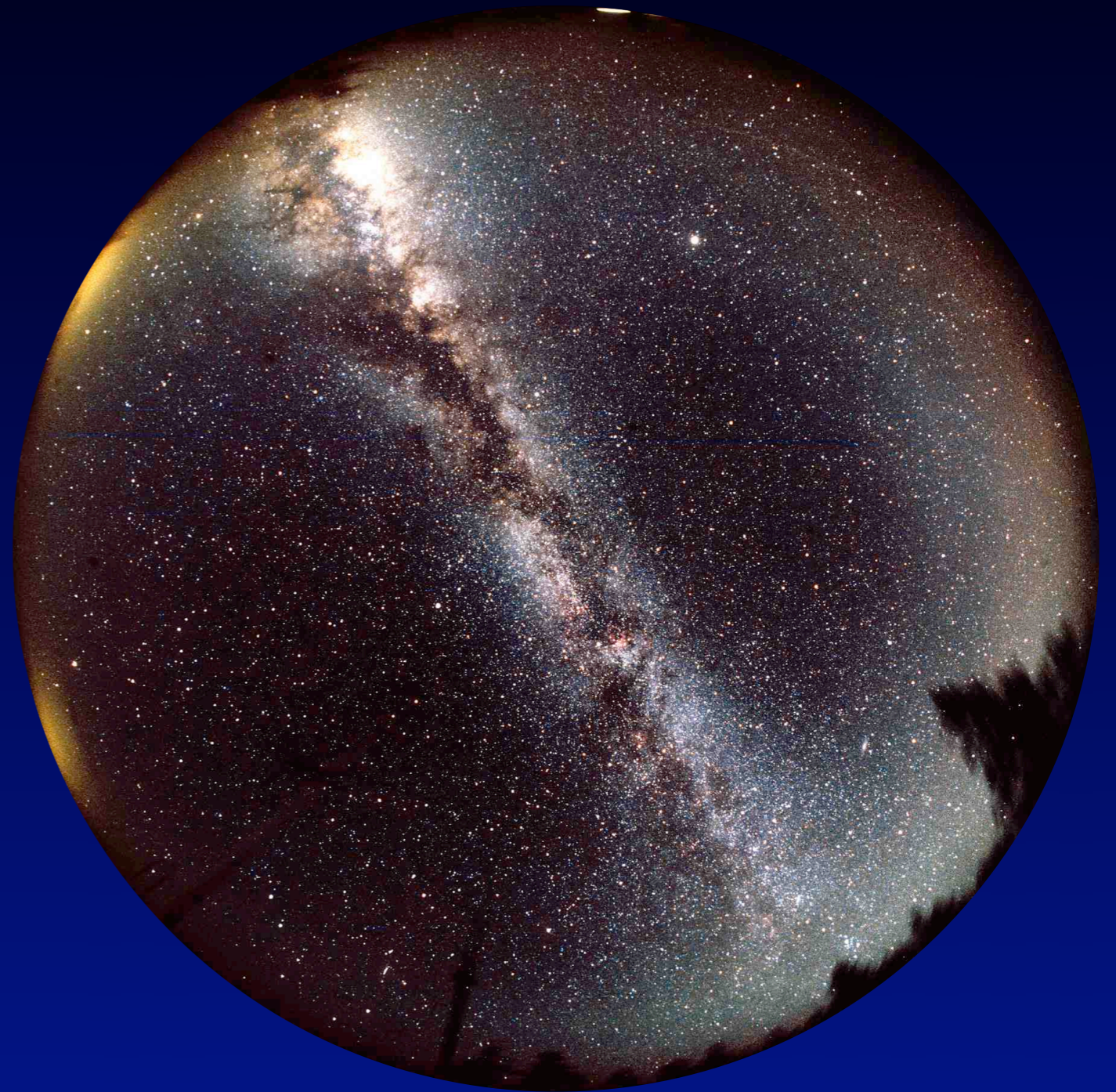
2. Lives of Stars

3. Supernovae

4. Stellar Afterlife

Why Study Astrophysics?

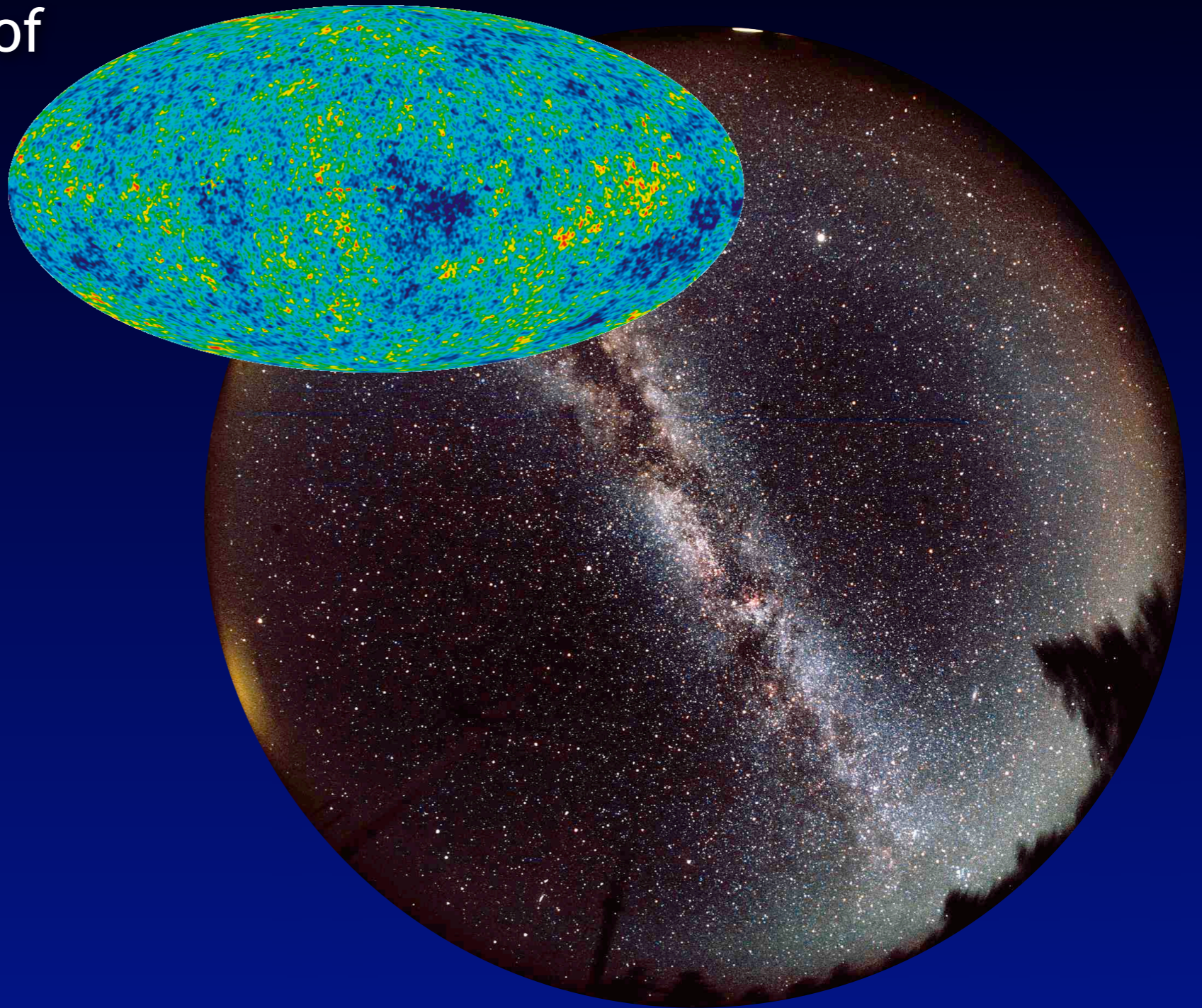
Explore the beauty of
the night sky



Why Study Astrophysics?

Explore the beauty of
the night sky

Understand our
place in the Cosmos

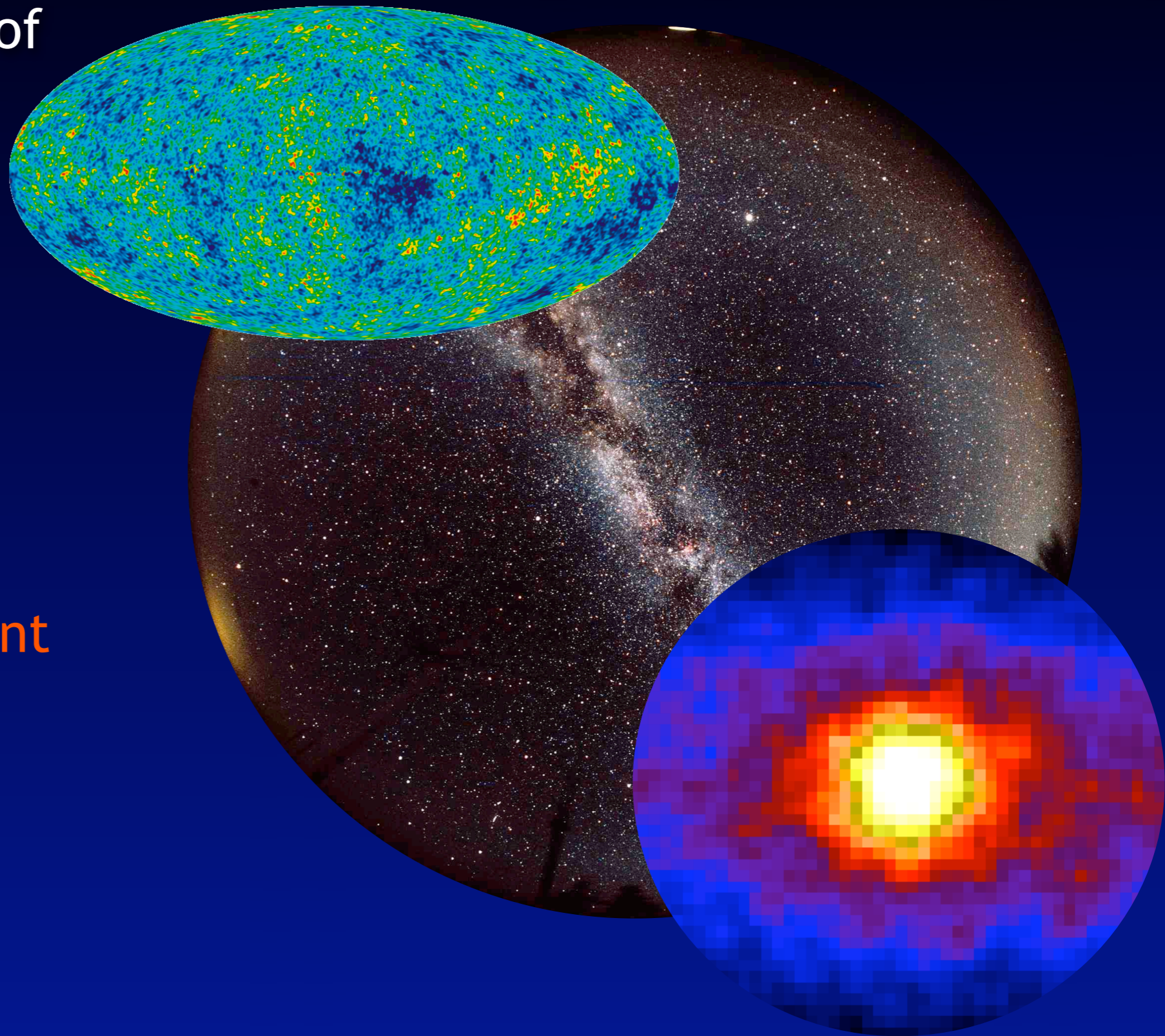


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Investigate physics
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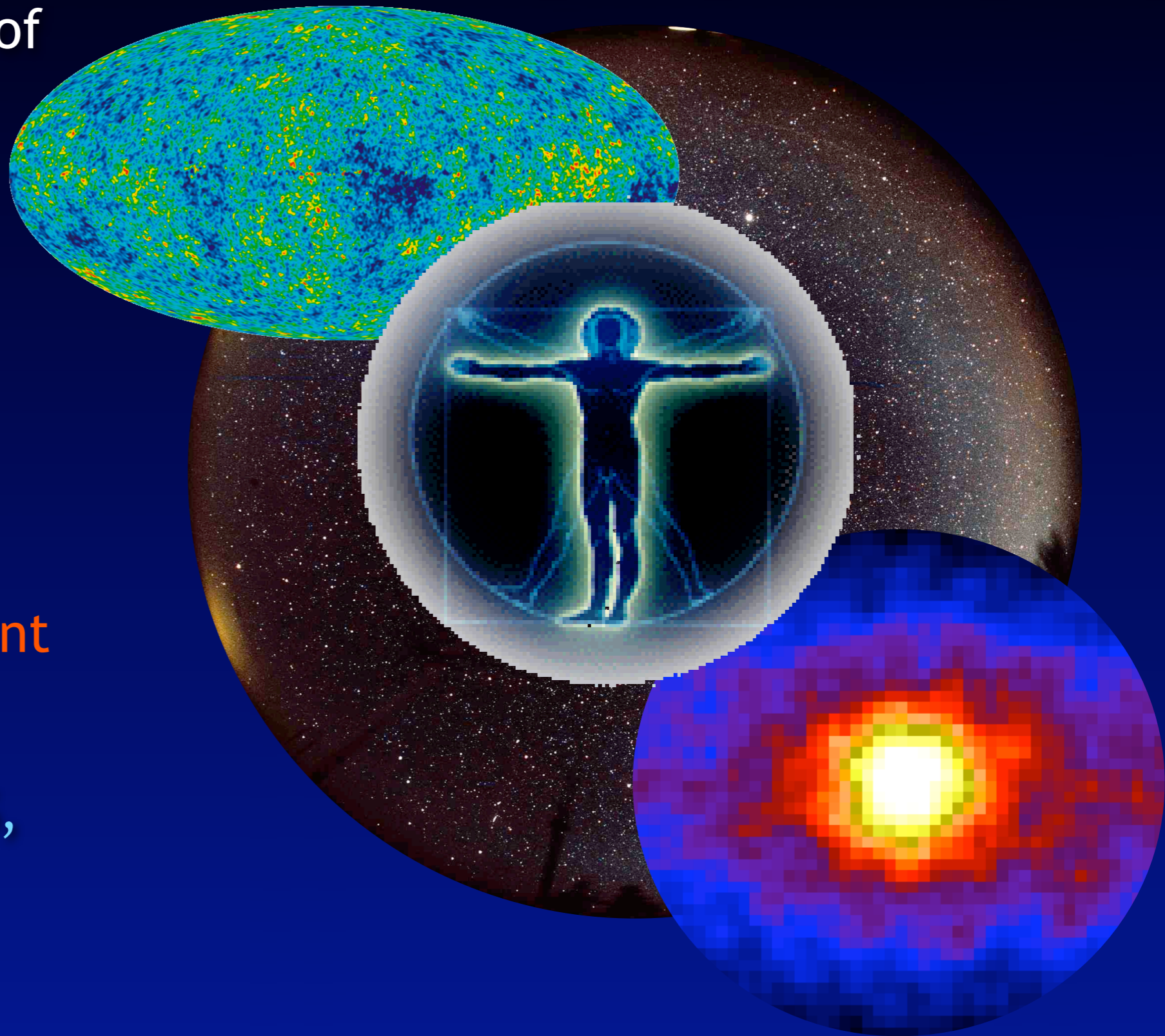
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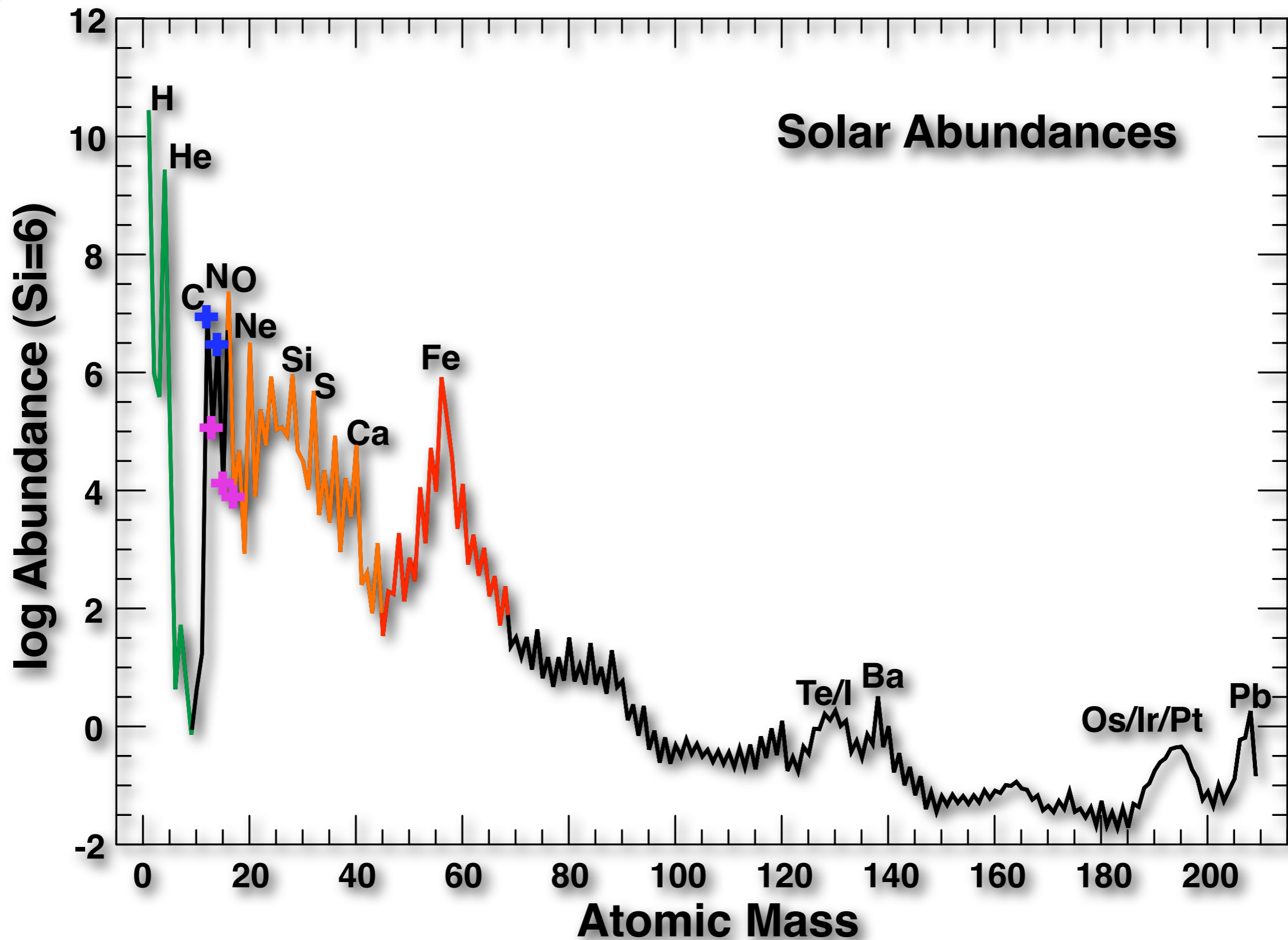
Understand our place in the Cosmos

Investigate physics inaccessible to terrestrial experiment

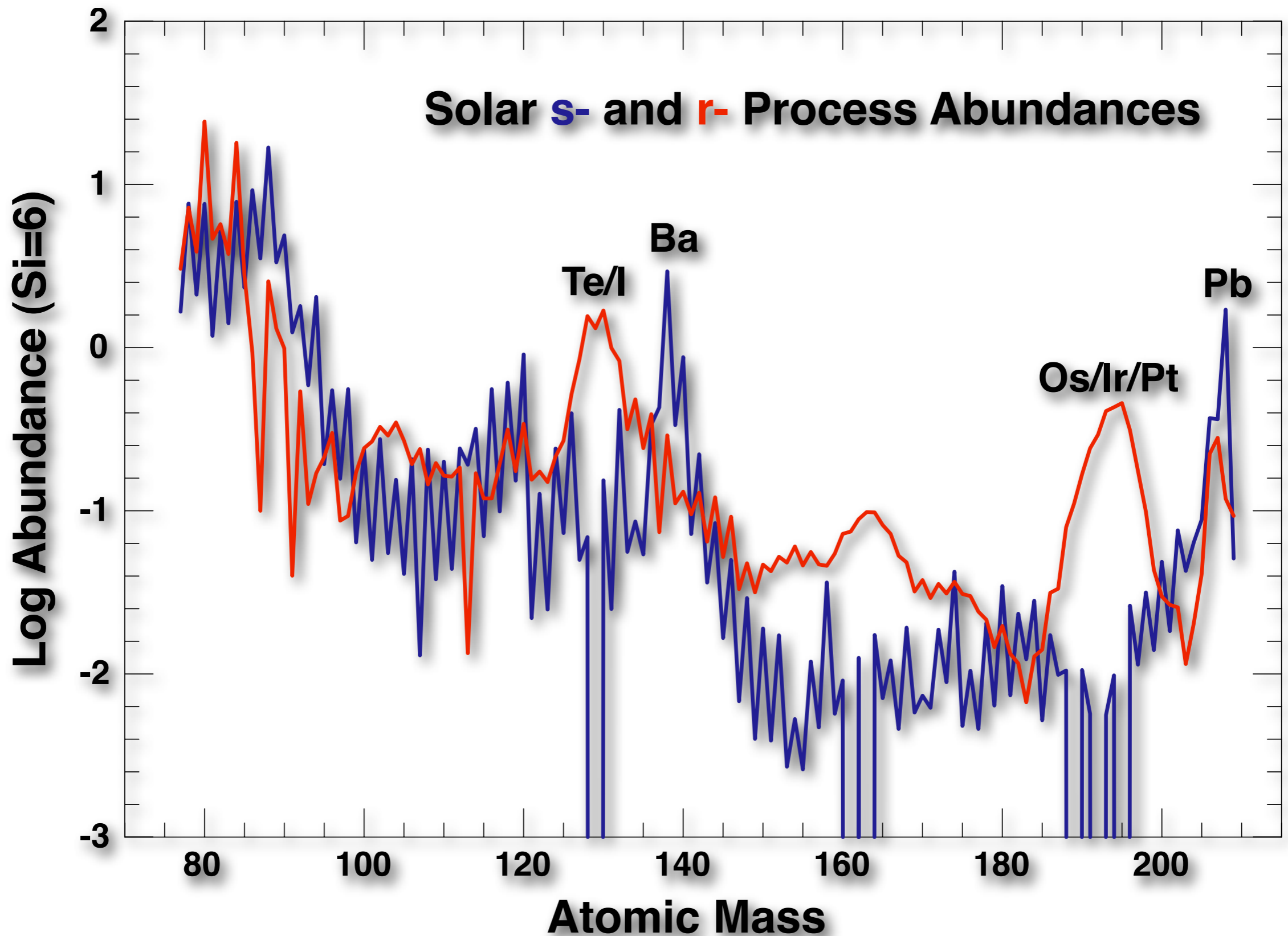
Explain our ORIGINS, how we came to be from stardust.



Of what are we made?



Of what are we made?



2 Essential Questions of Nuclear Astrophysics

How do nuclei get made?

When? Where? Is it an ongoing process?

How does making nuclei affect the stellar environment?

Quiescent or Explosive? Exothermic or Endothermic?



Alchemy

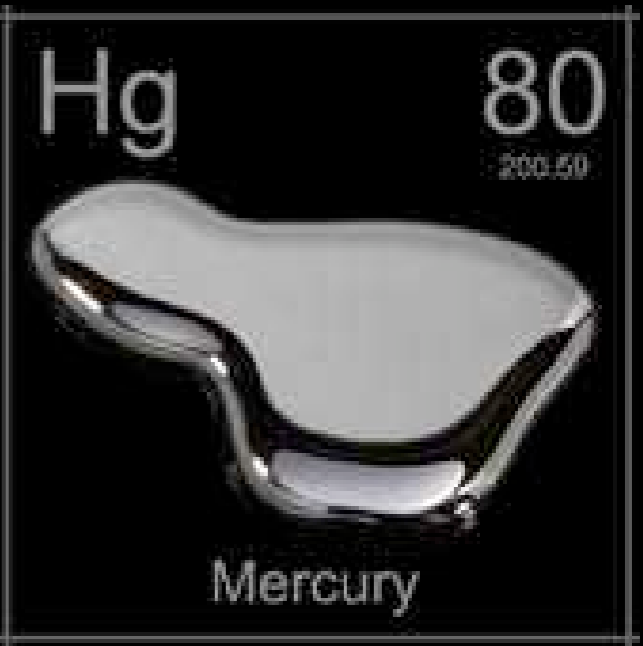
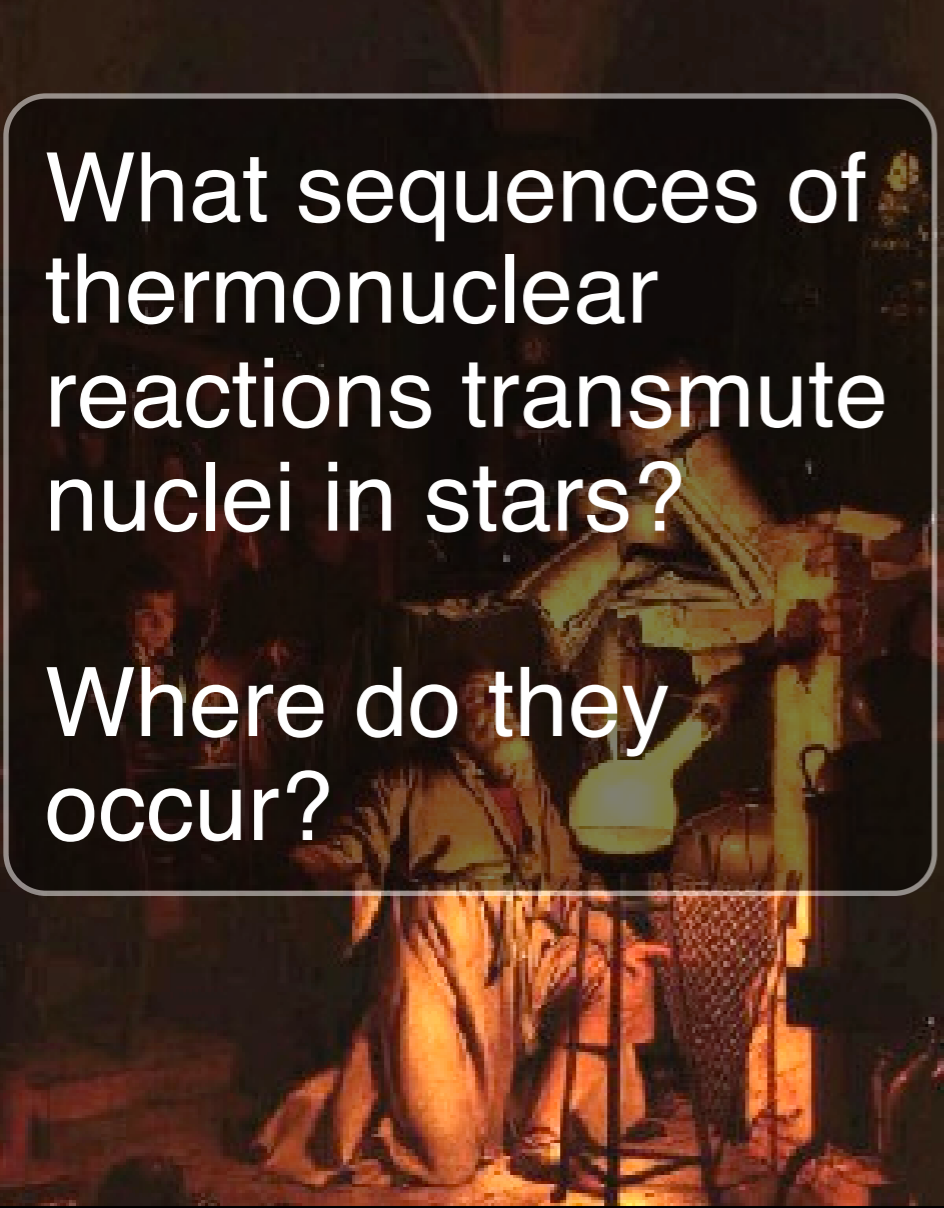
sought to transmute
common elements into
rare elements (to get
rich) ...



Alchemy



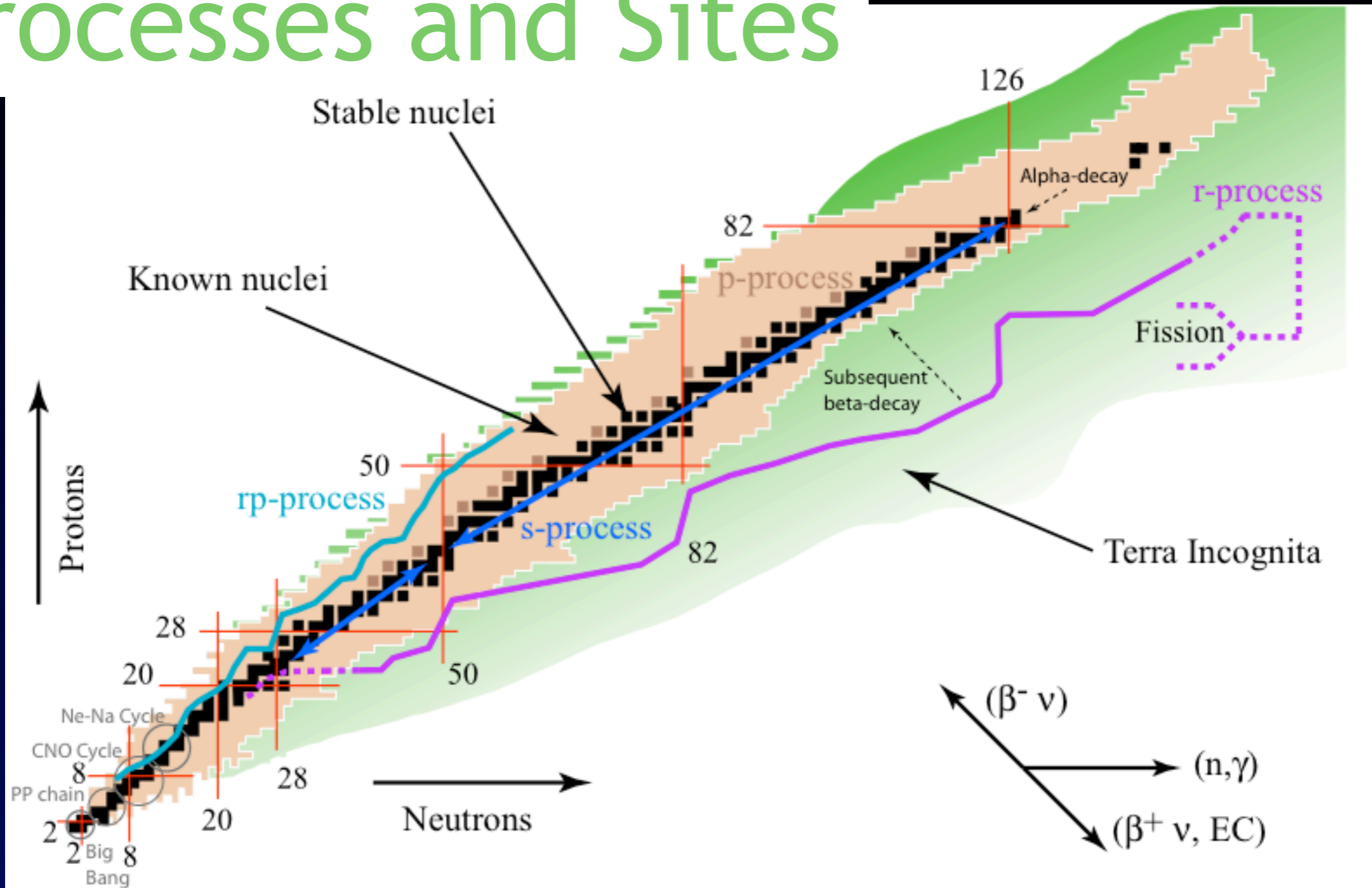
What sequences of thermonuclear reactions transmute nuclei in stars?
Where do they occur?



Stellar Alchemy

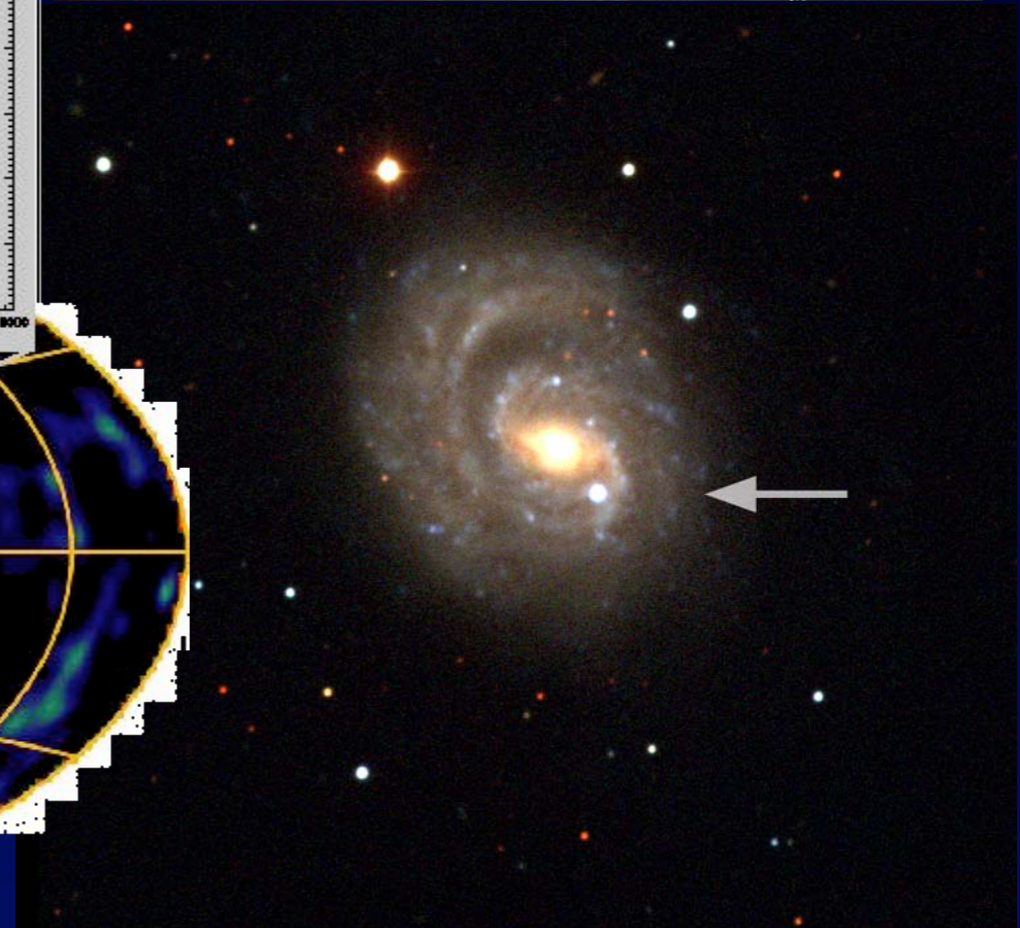
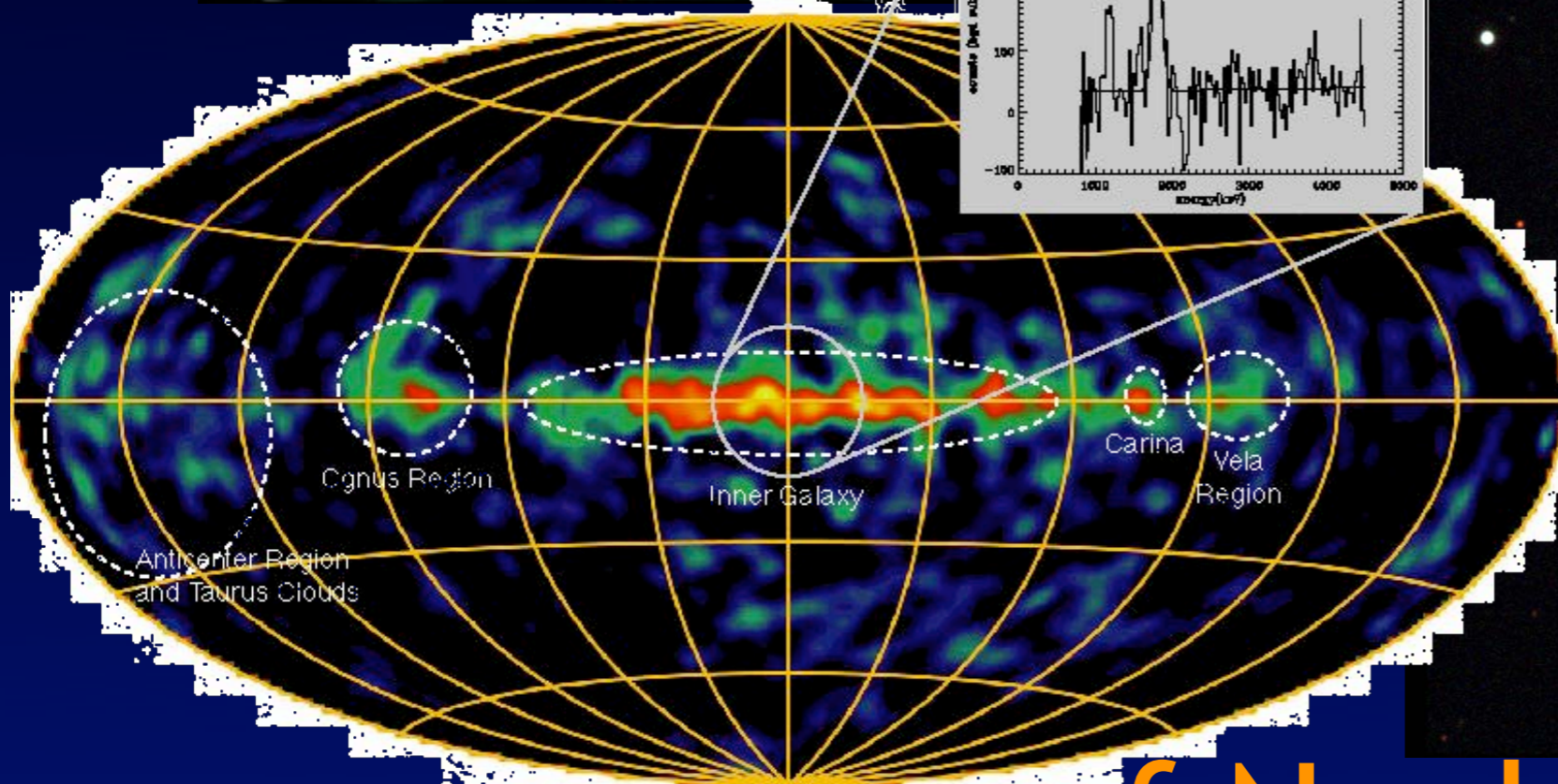
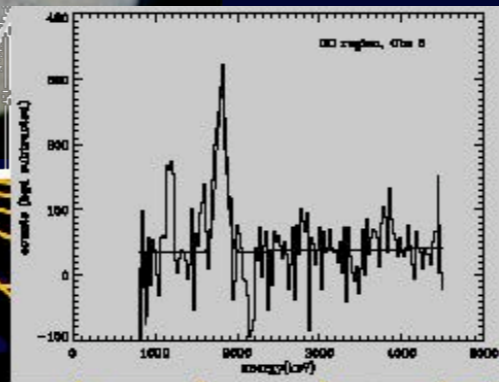


Processes and Sites



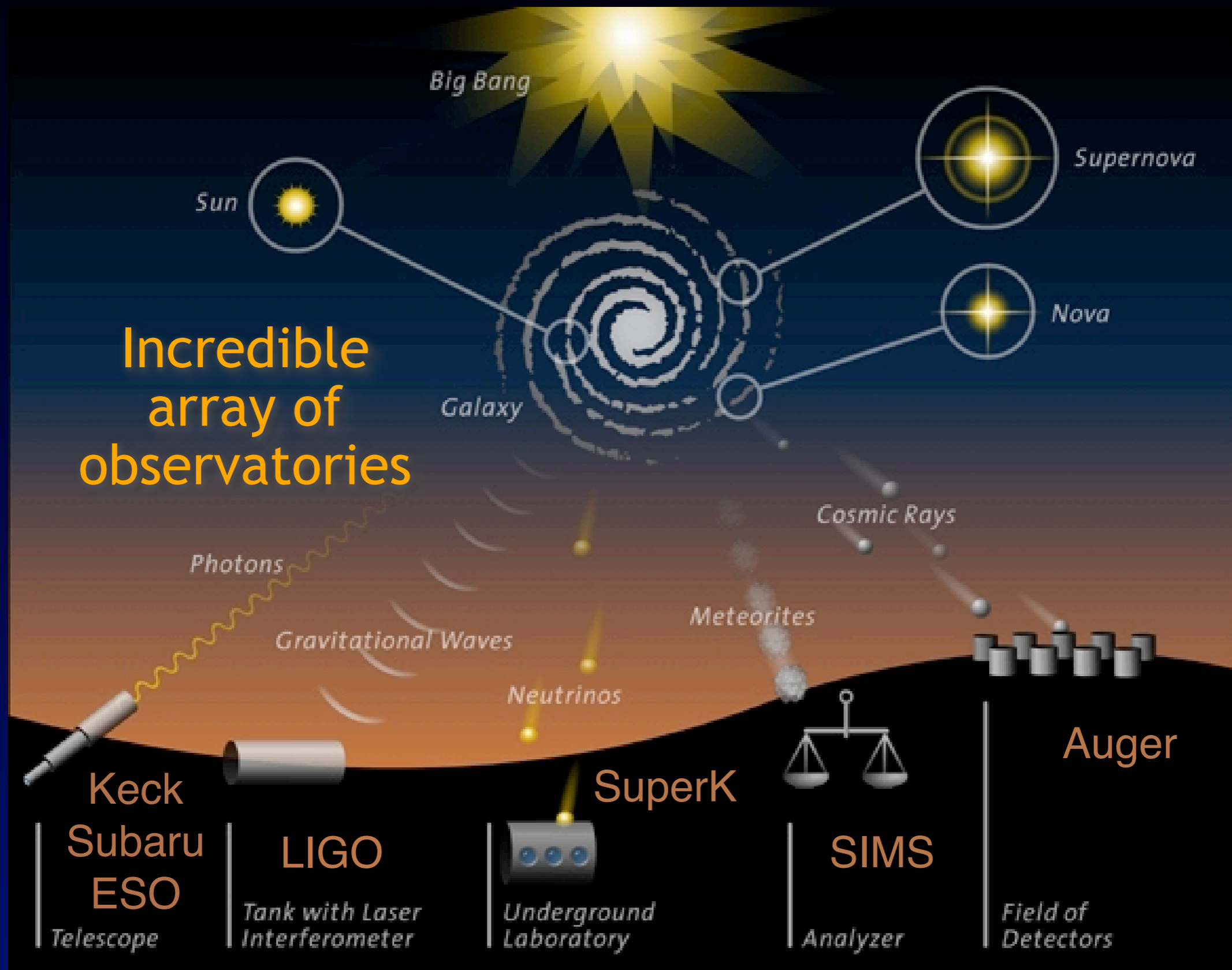
Understanding Origins means understanding **processes** that transmute nuclei and the **sites** where these processes occur.

Astrophysical Observations...

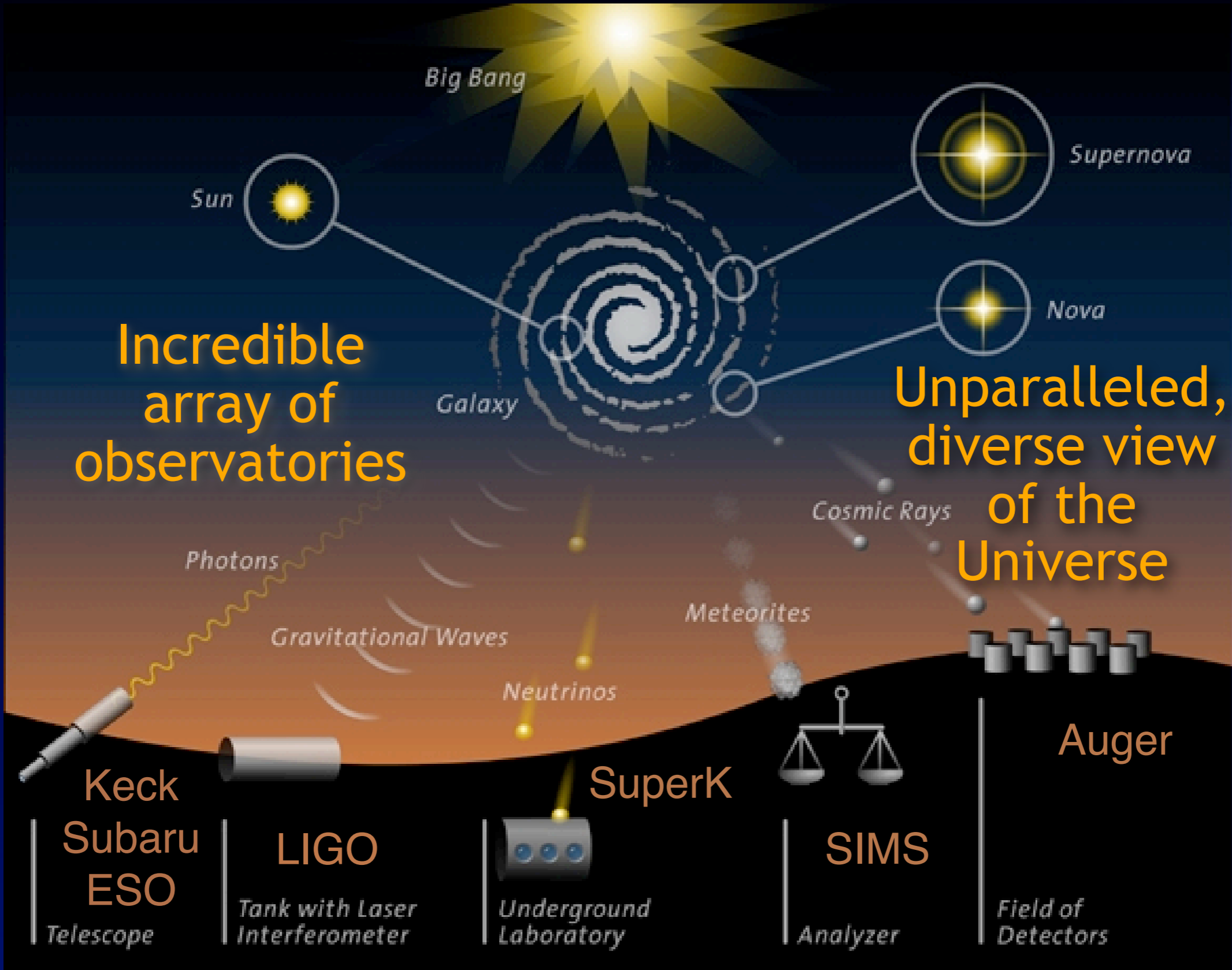


... of Nuclear Evolution

Golden Age of Observation



Golden Age of Observation



Photons of all sorts!



Photons of all sorts!



Chandra (NASA)

X-rays



Integral (ESA)

γ -rays



HST (NASA)

visible

VLT (ESO)



Spitzer (NASA)

infrared



WMAP (NASA)

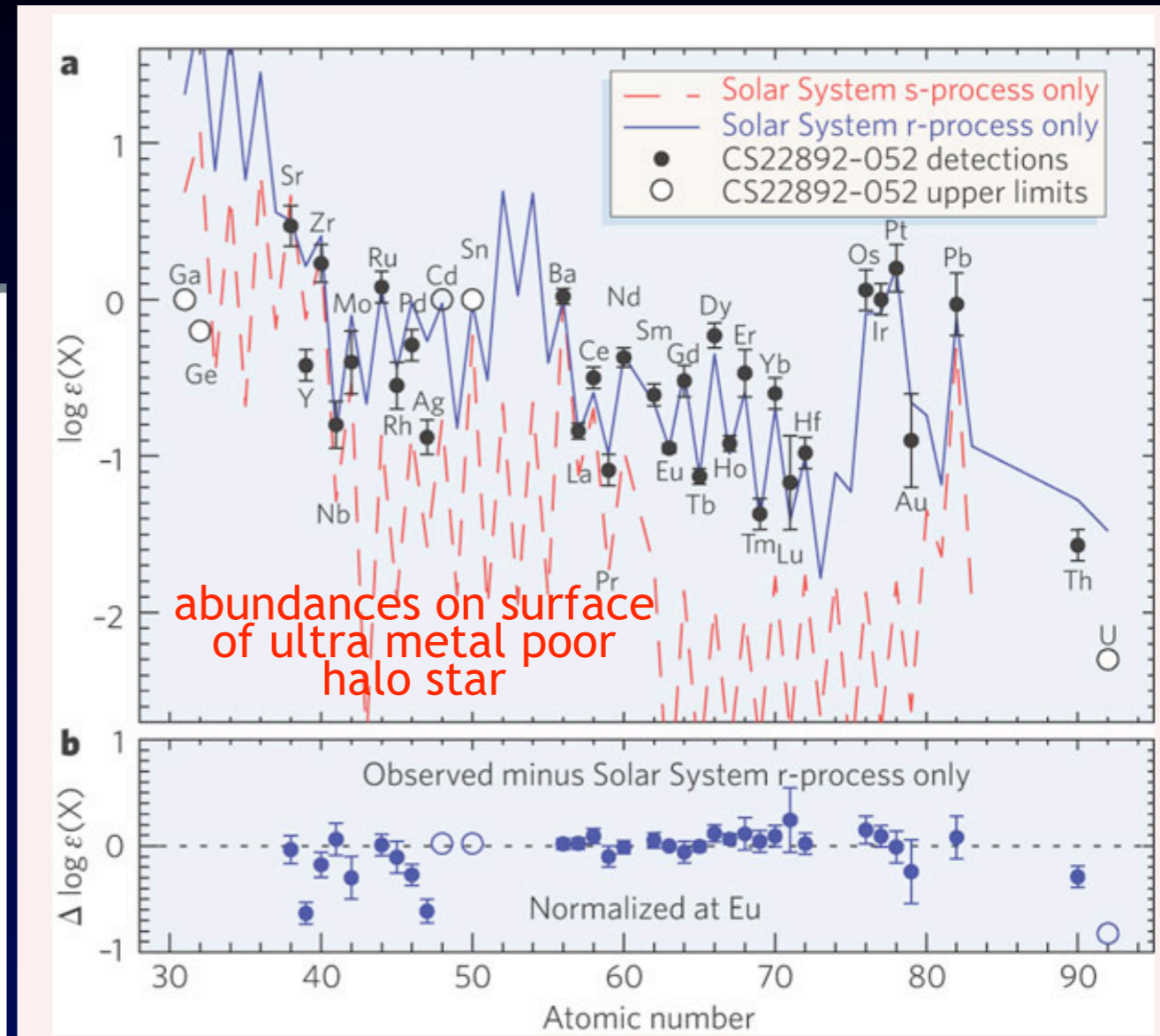
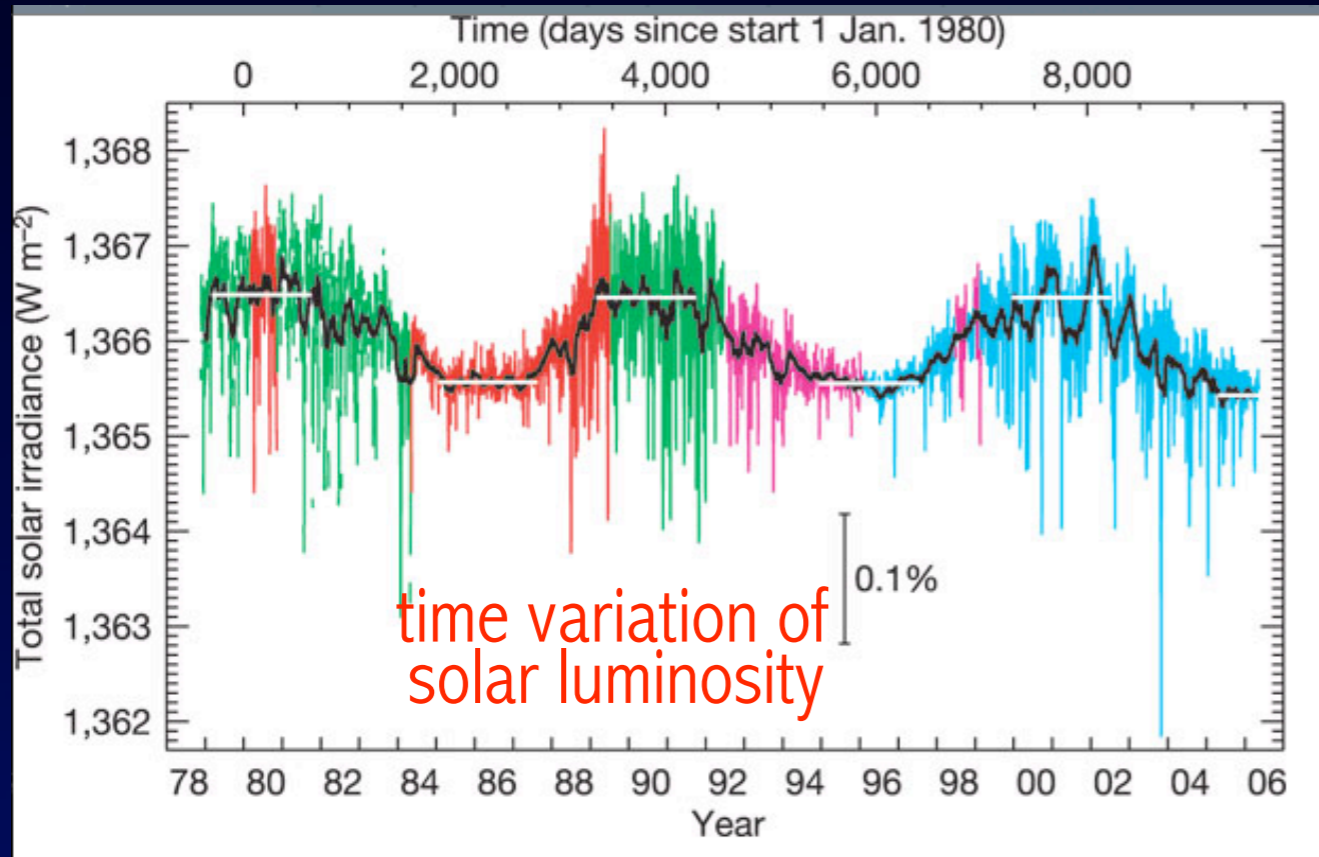
Radio



VLA (NRAO)

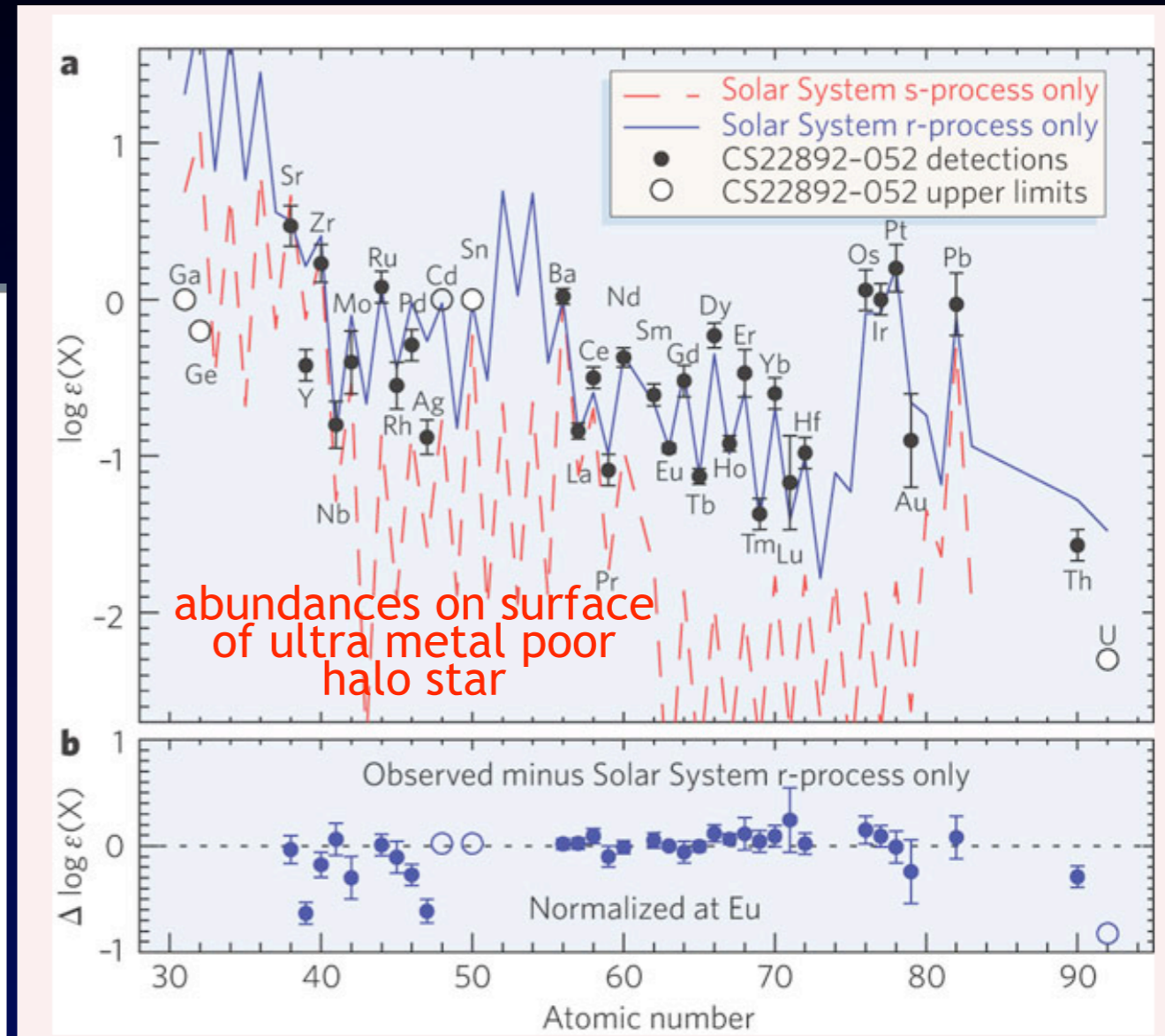
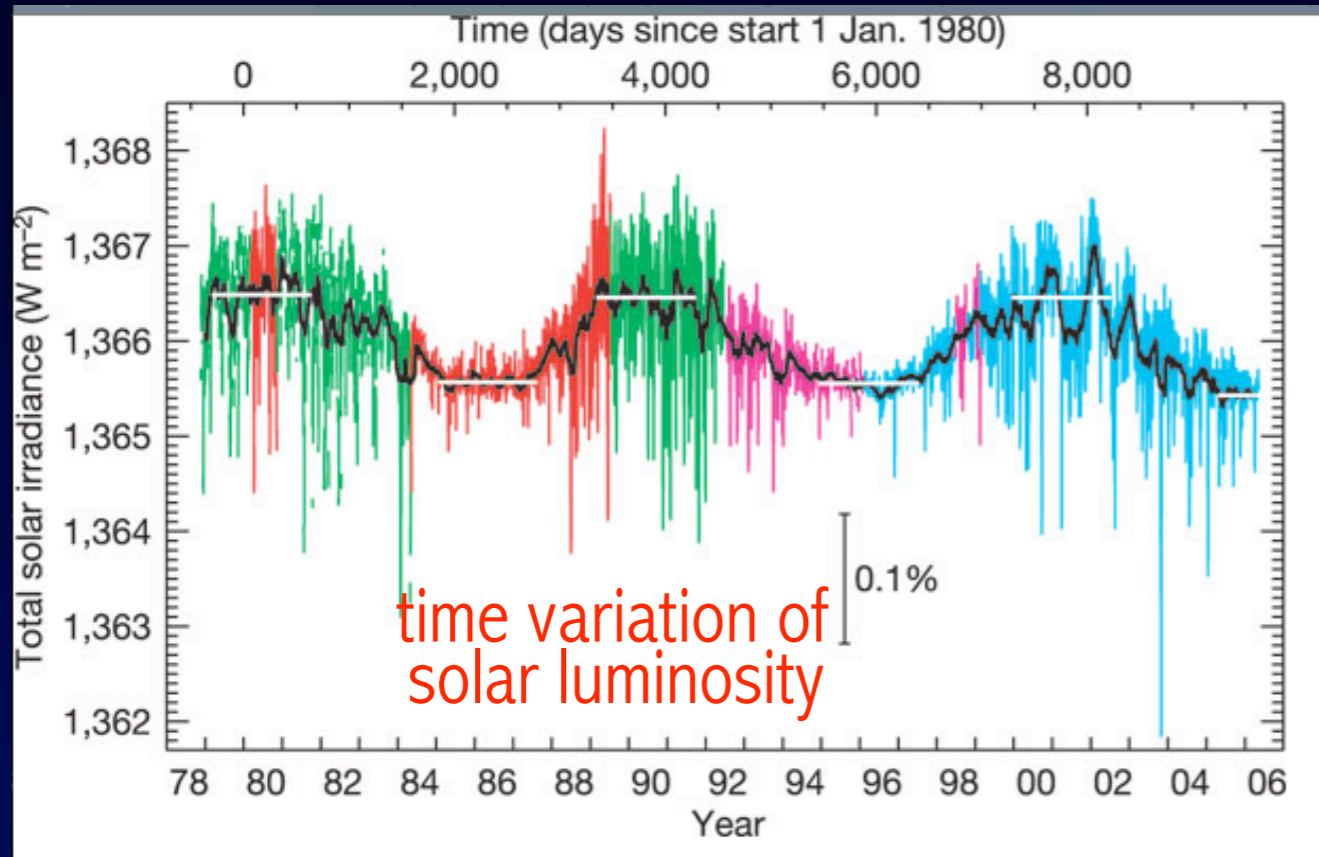
What do observations tell us?

1) Surface properties of stars



What do observations tell us?

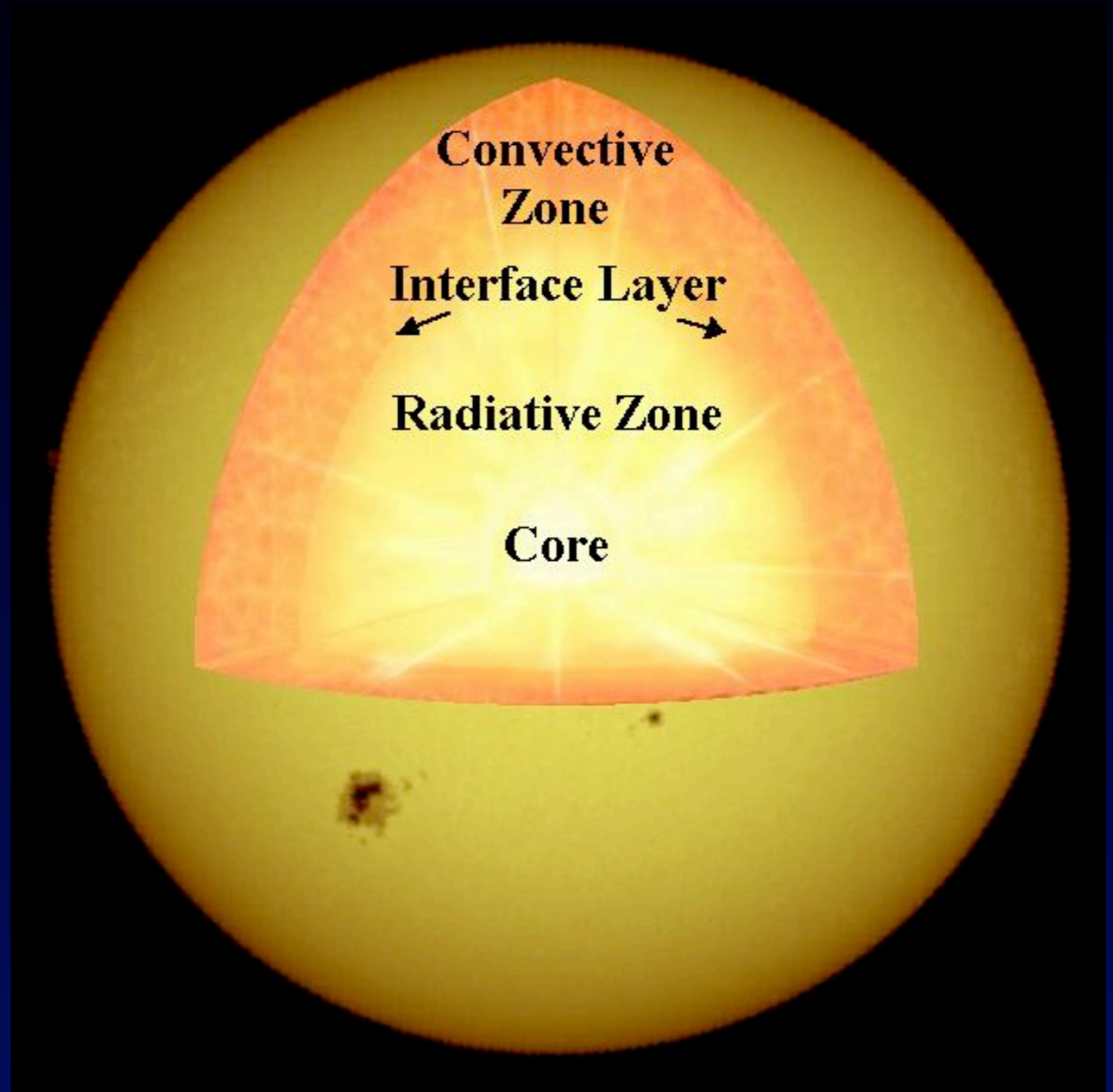
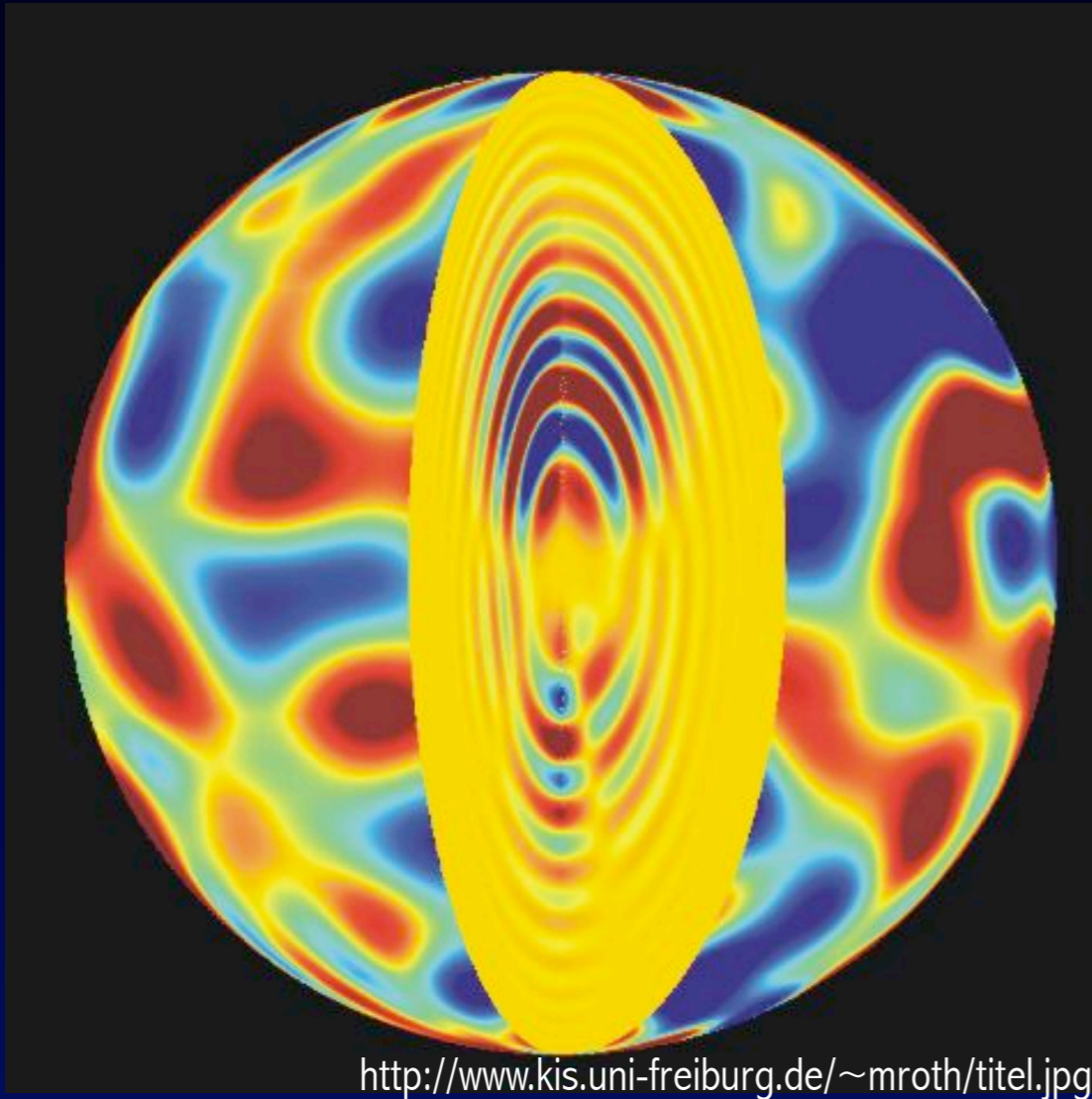
1) Surface properties of stars



luminosity, temperature, radius, surface abundances, variations (e.g., sunspots, coronal mass ejections)

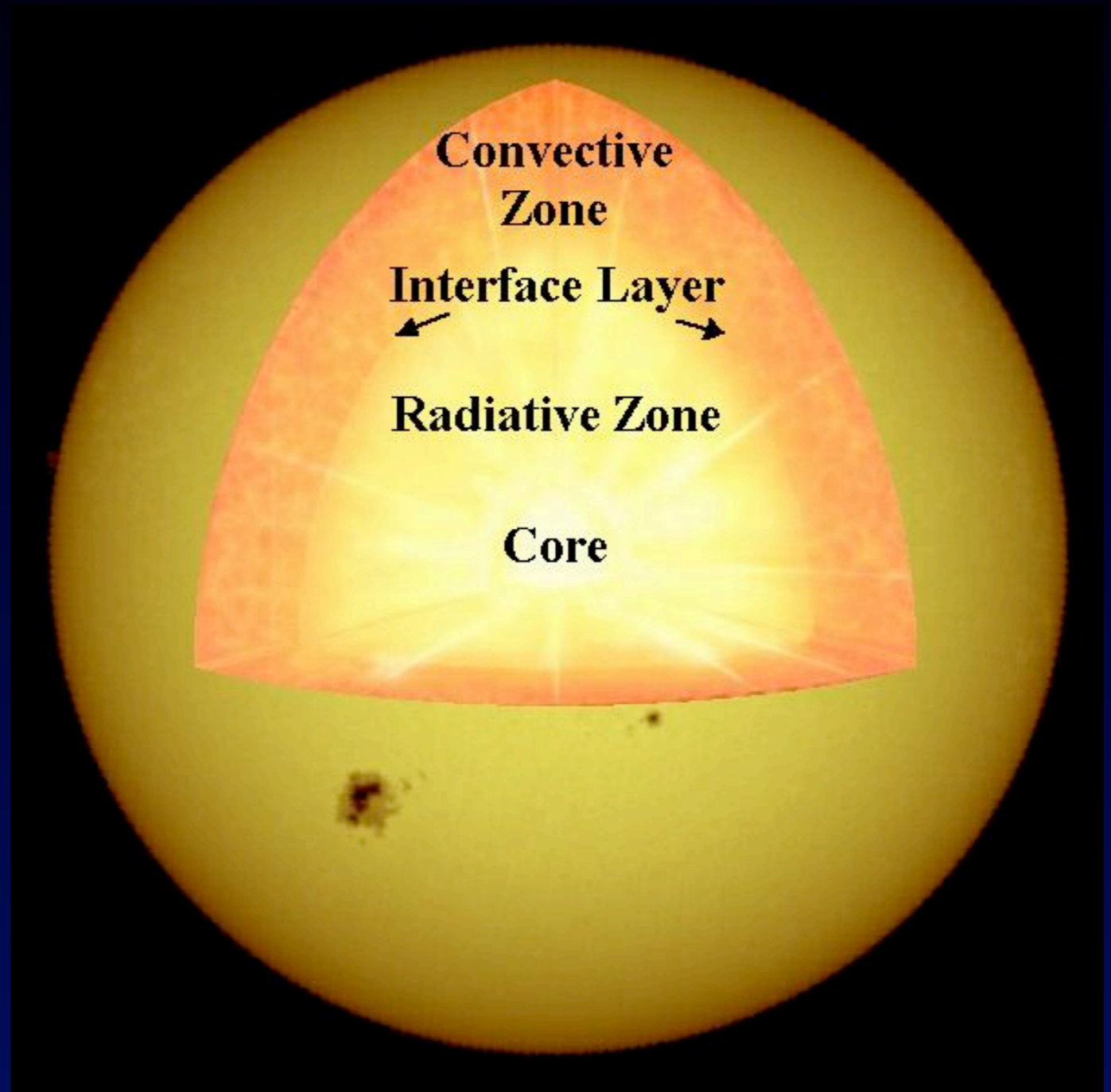
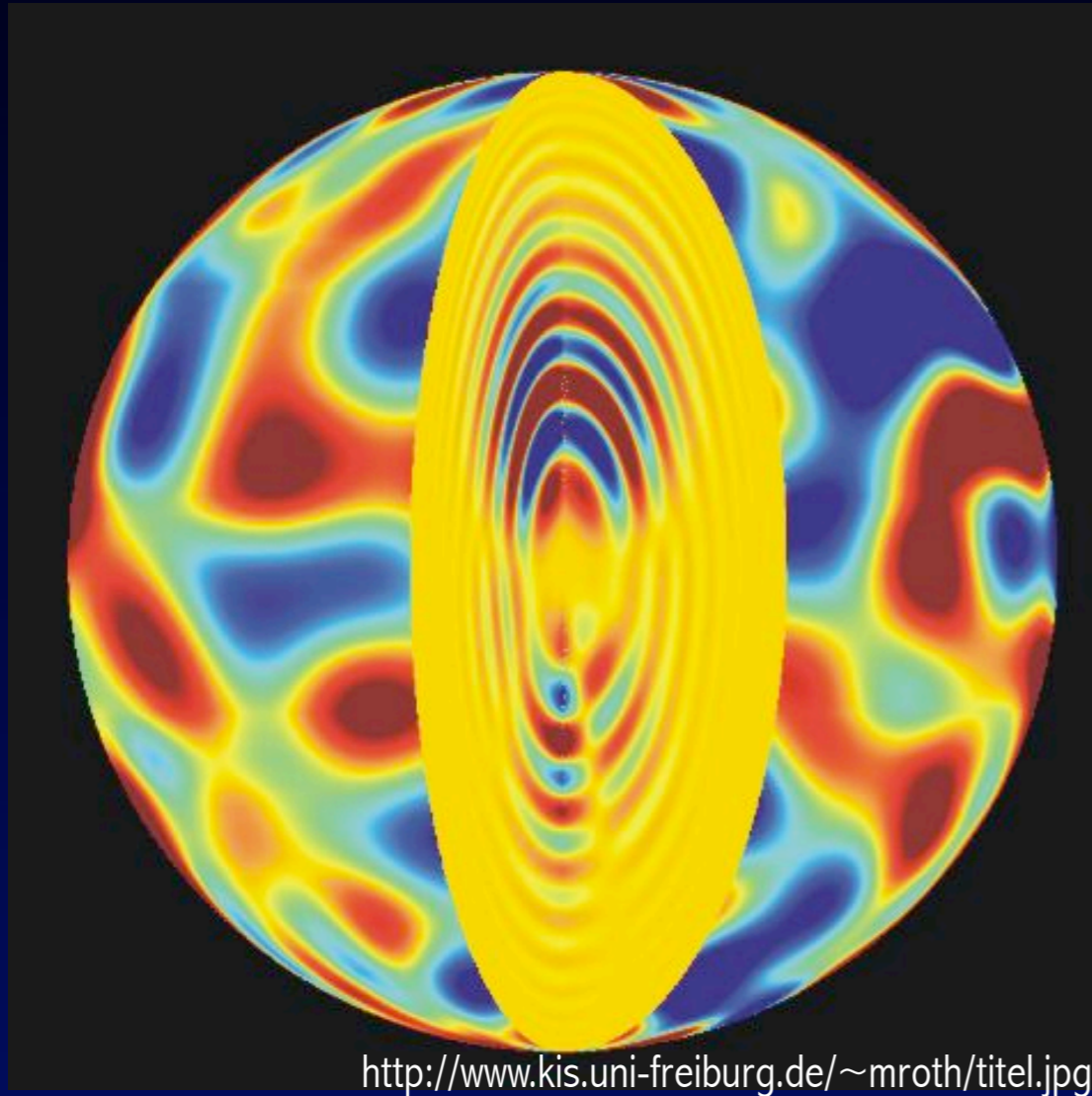
What do observations tell us?

2) Clues to the interiors of stars



What do observations tell us?

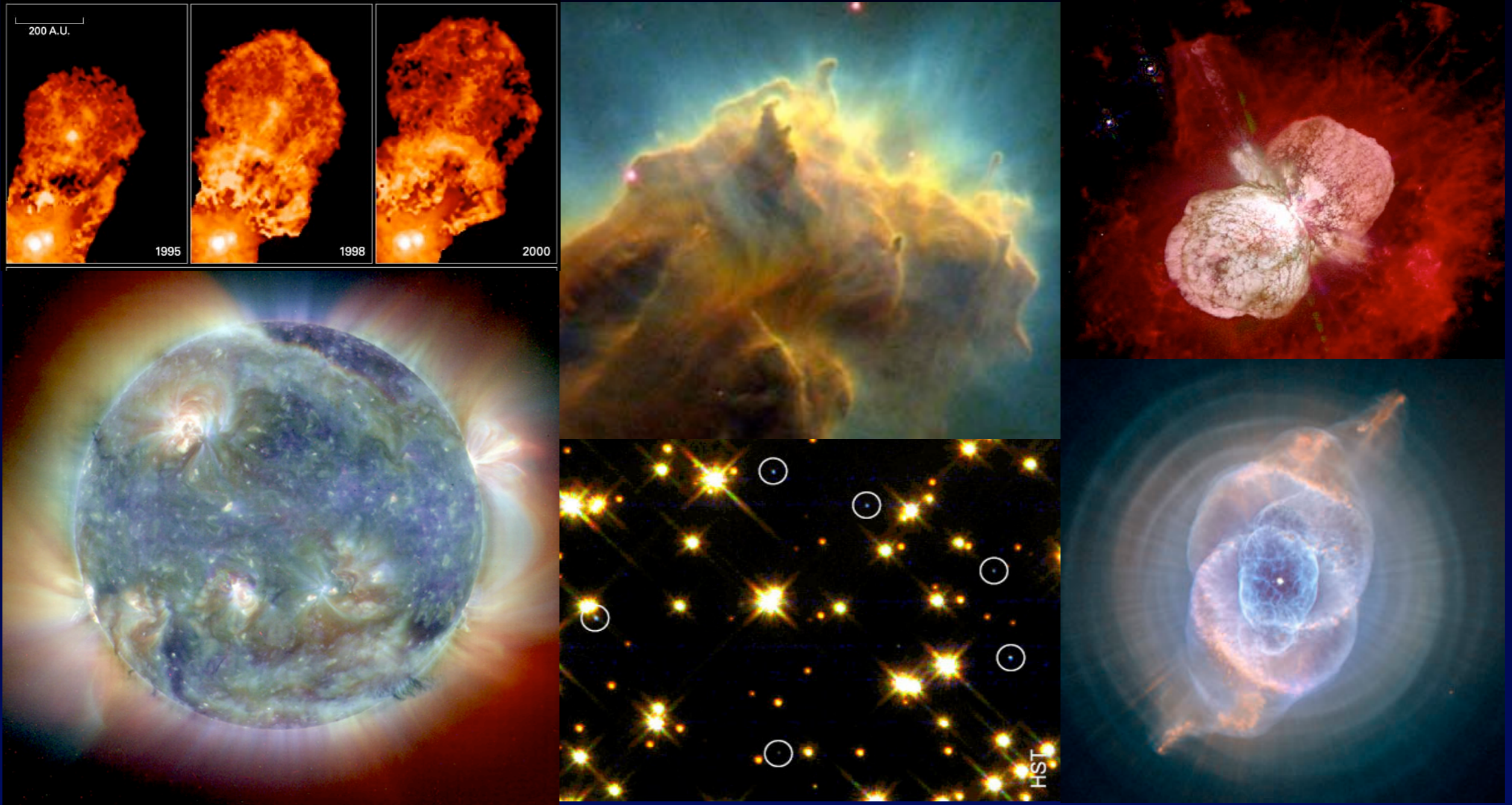
2) Clues to the interiors of stars



helioseismology - vibrations of solar surface probes interior
neutrinos - emitted in the core & (almost) free stream out

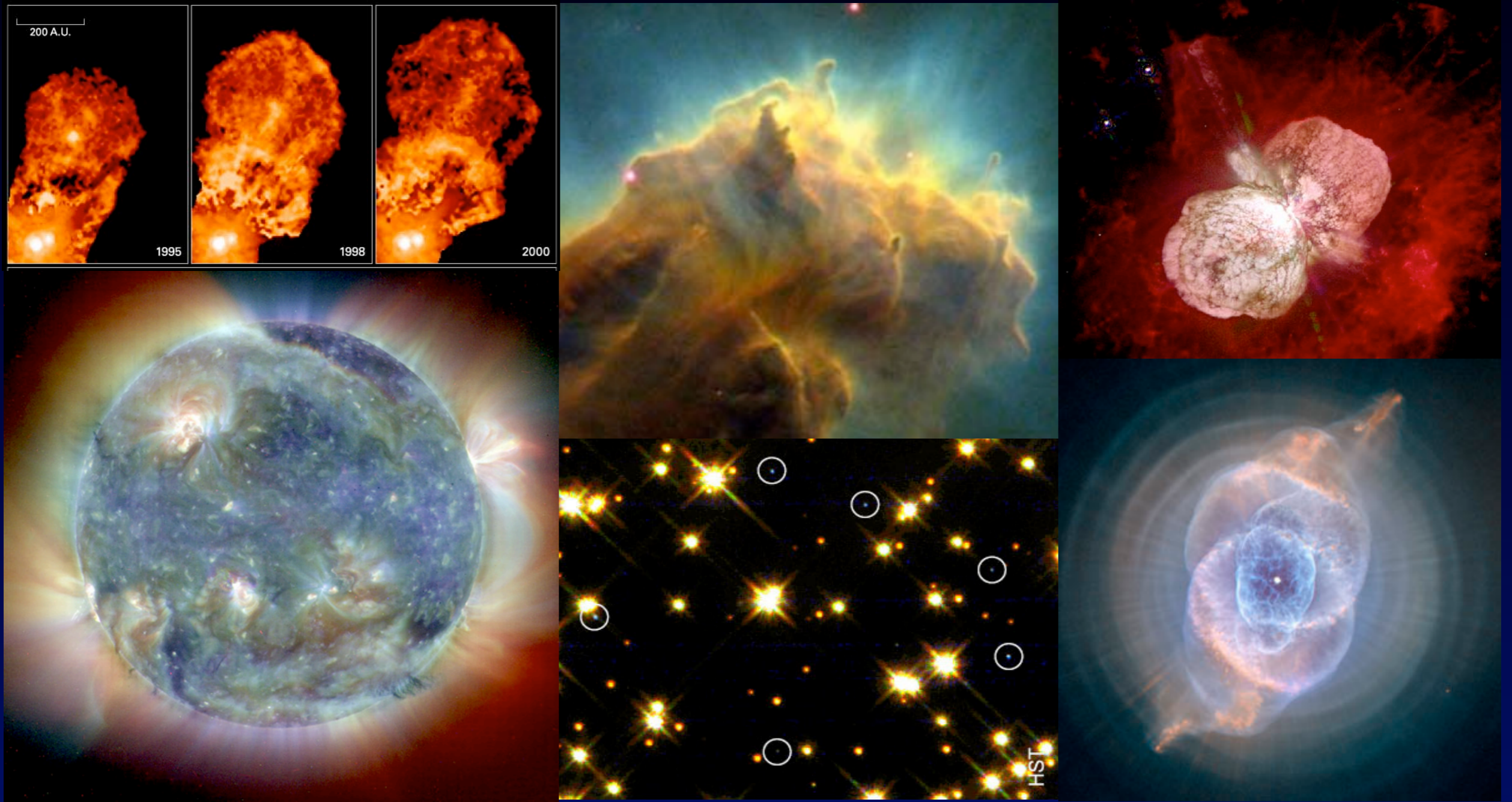
What do observations tell us?

3) Stages of stars lives



What do observations tell us?

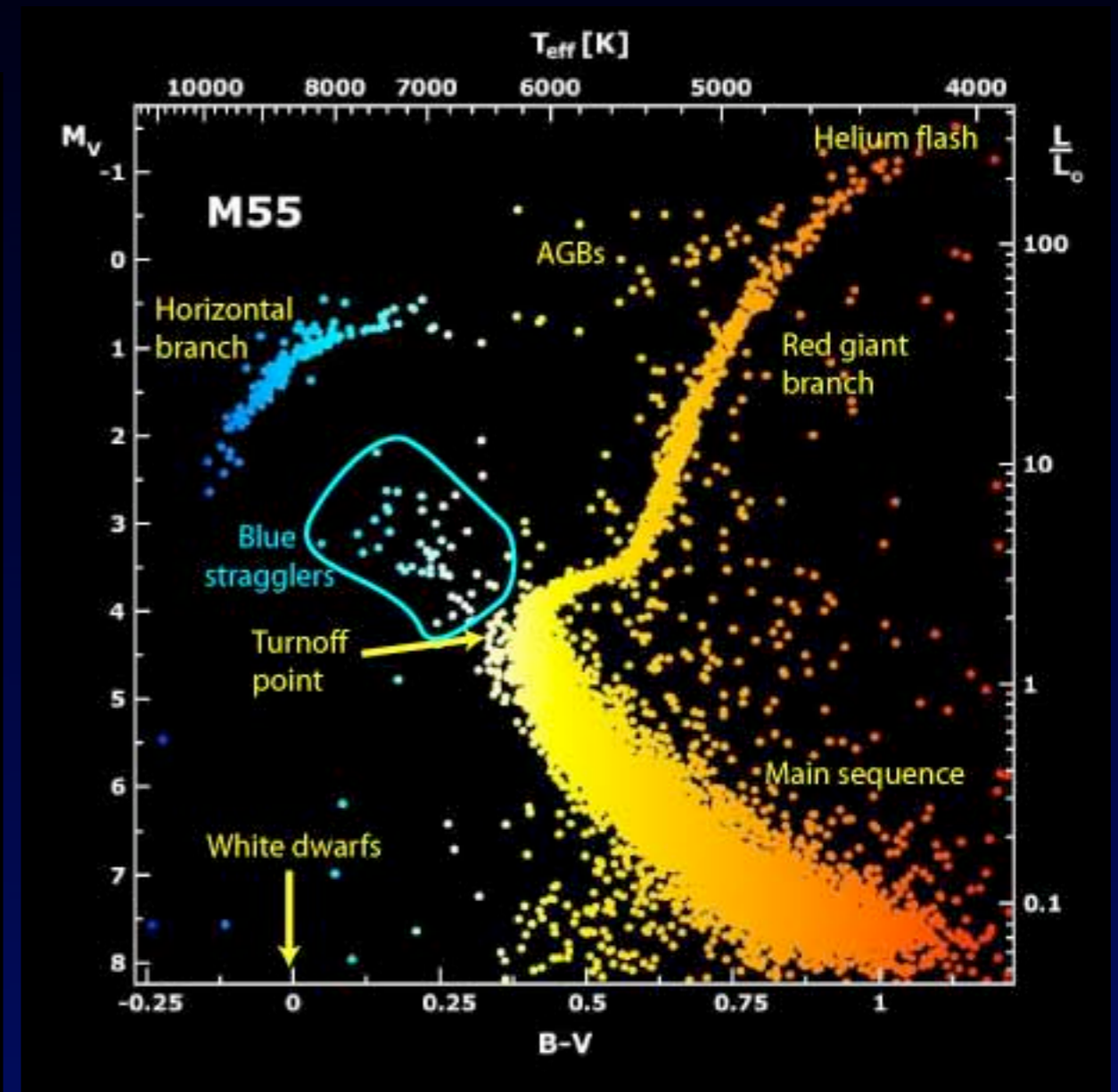
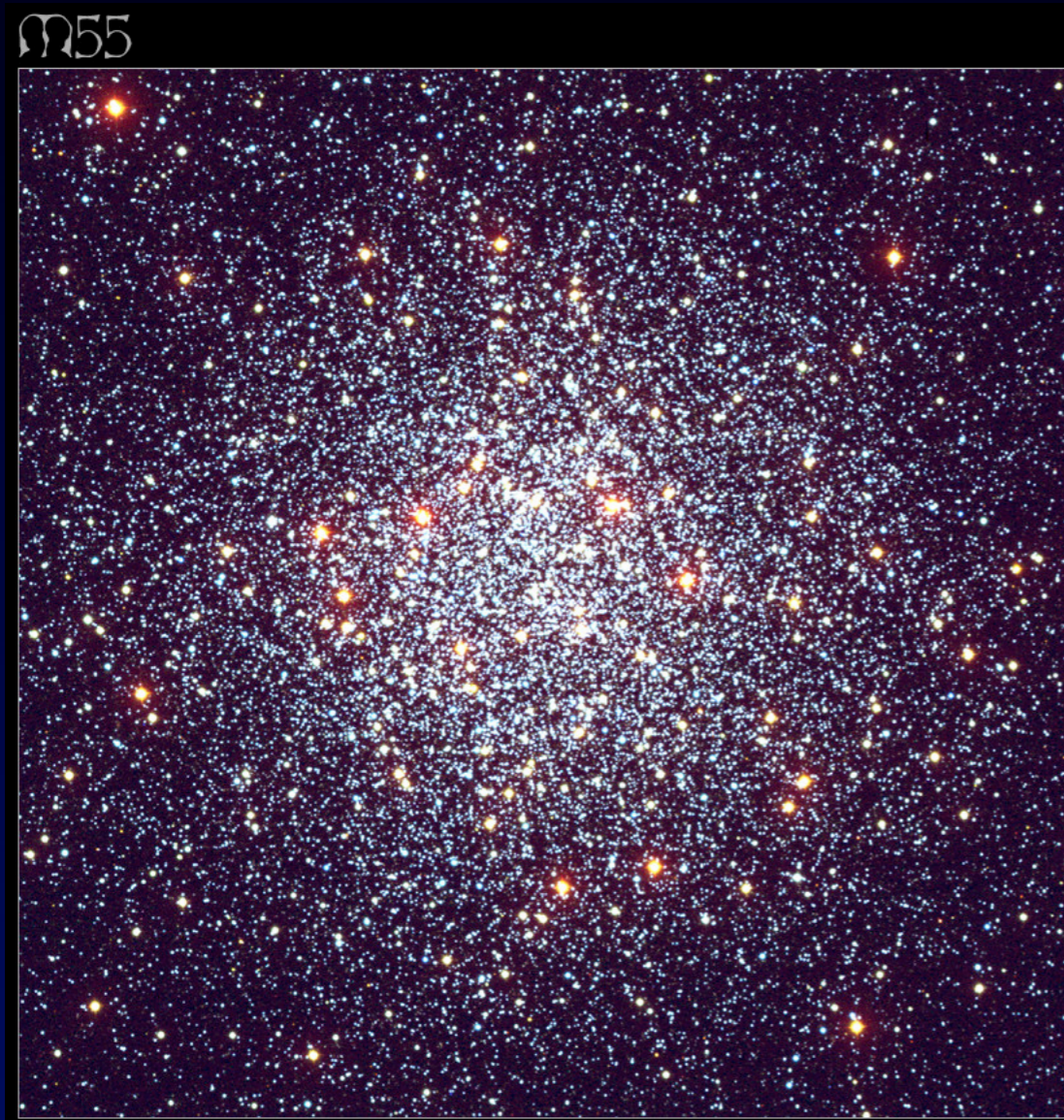
3) Stages of stars lives



Birth from clouds of gas and dust, normal burning, death in explosions or by fading out...

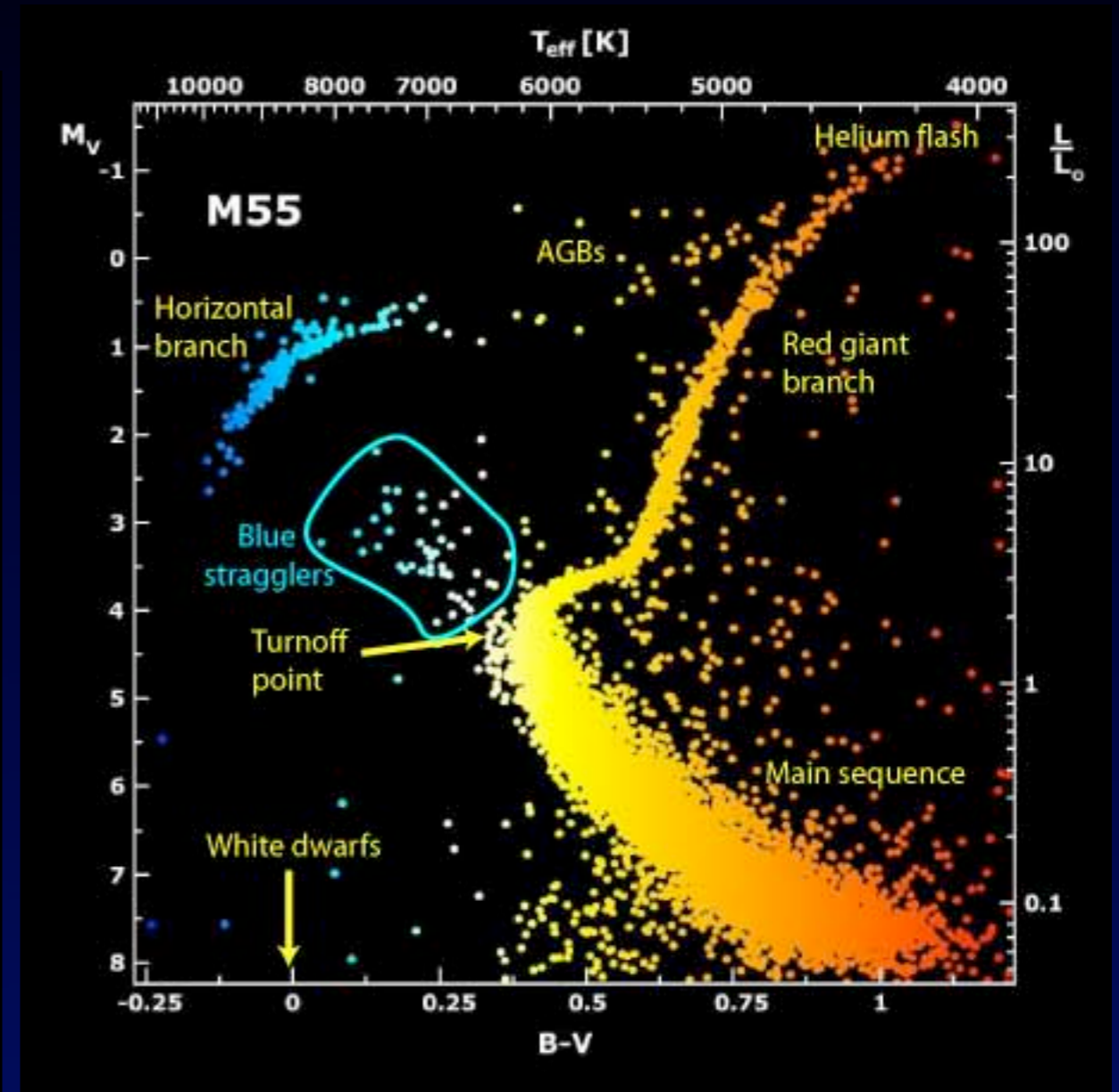
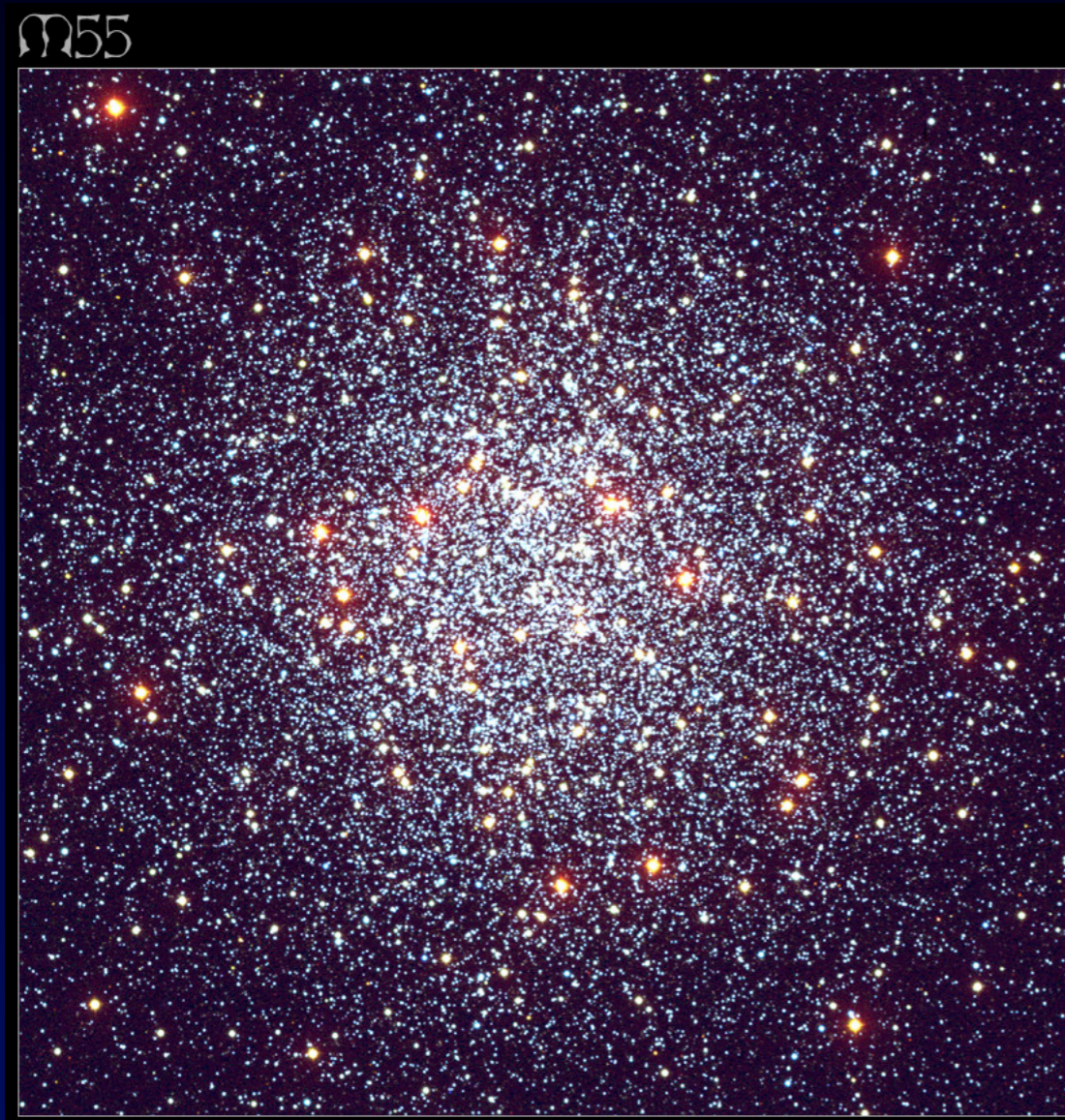
What do observations tell us?

4) Lifecycle of stars



What do observations tell us?

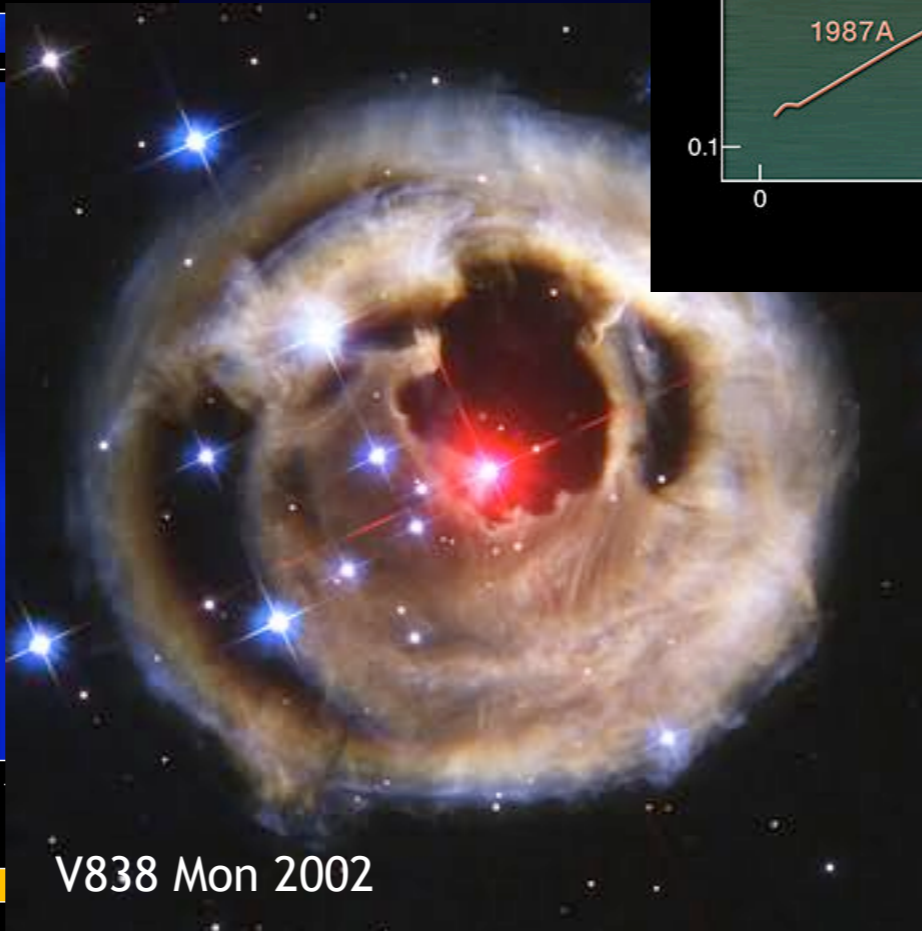
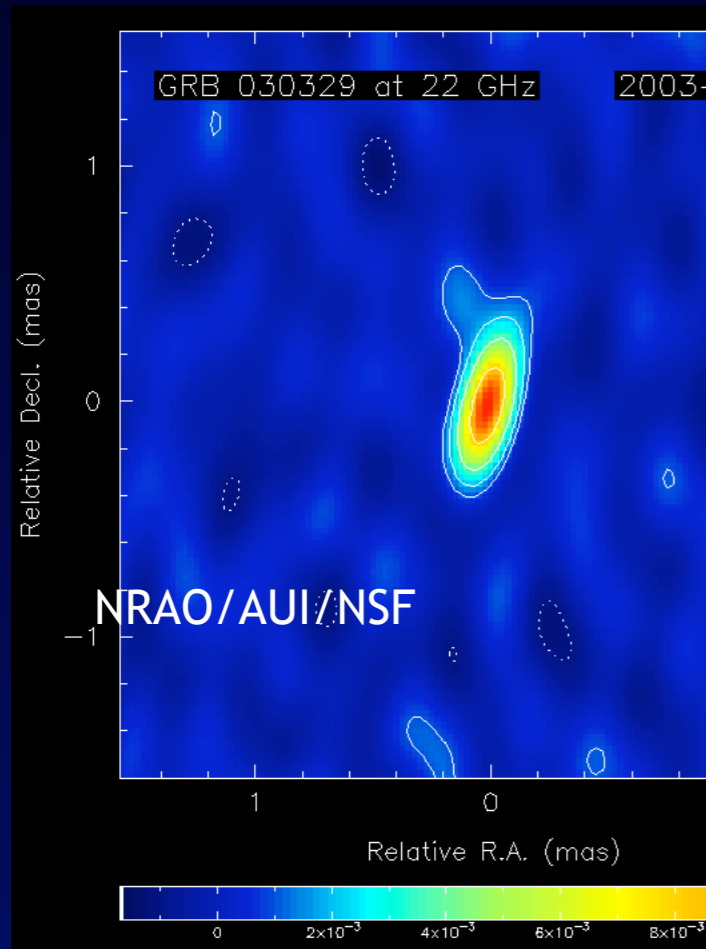
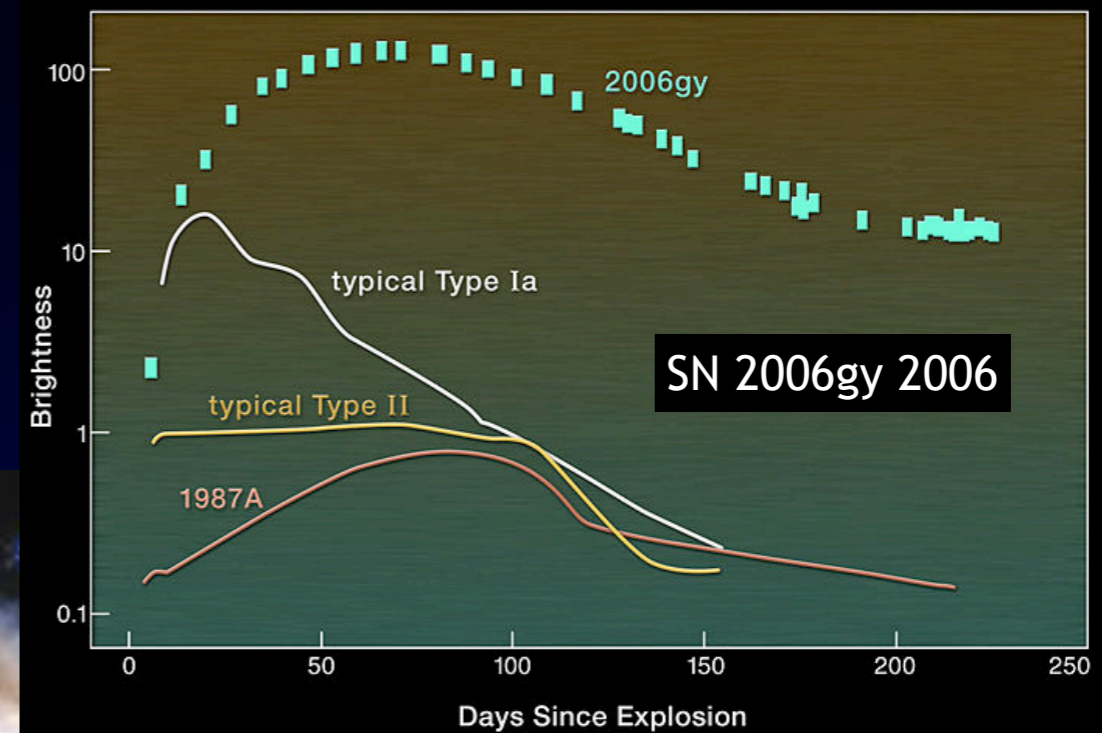
4) Lifecycle of stars



Census of these stages tells us the lifecycle.

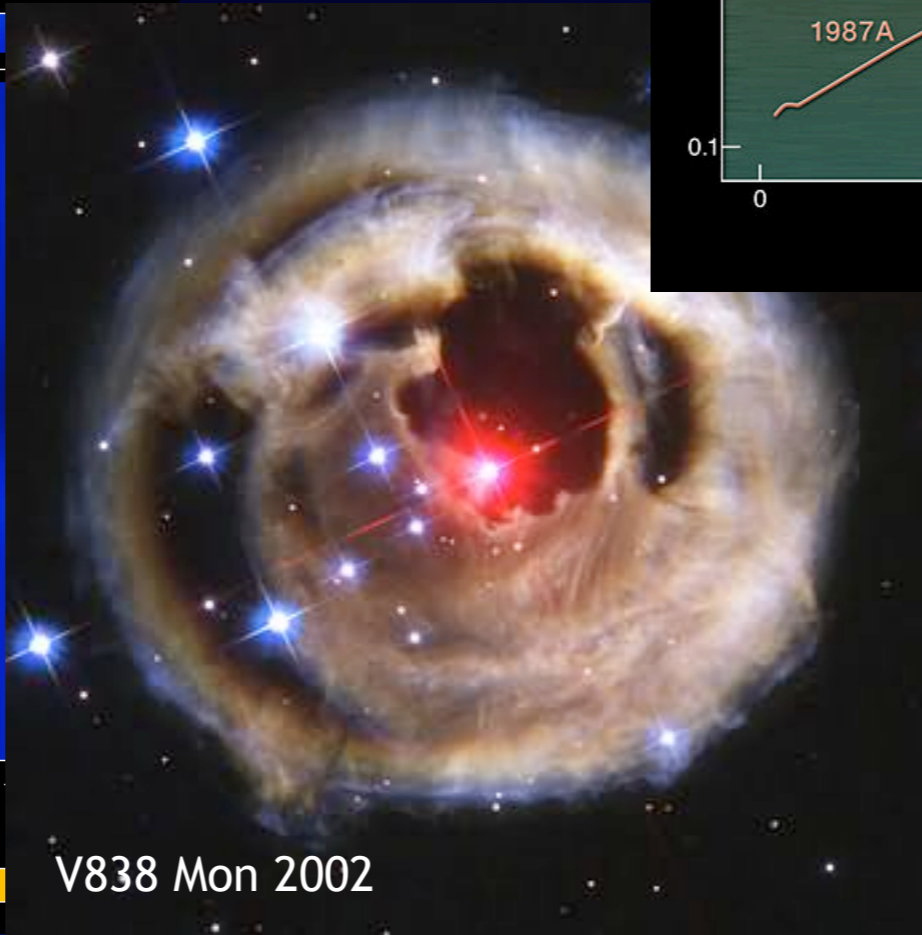
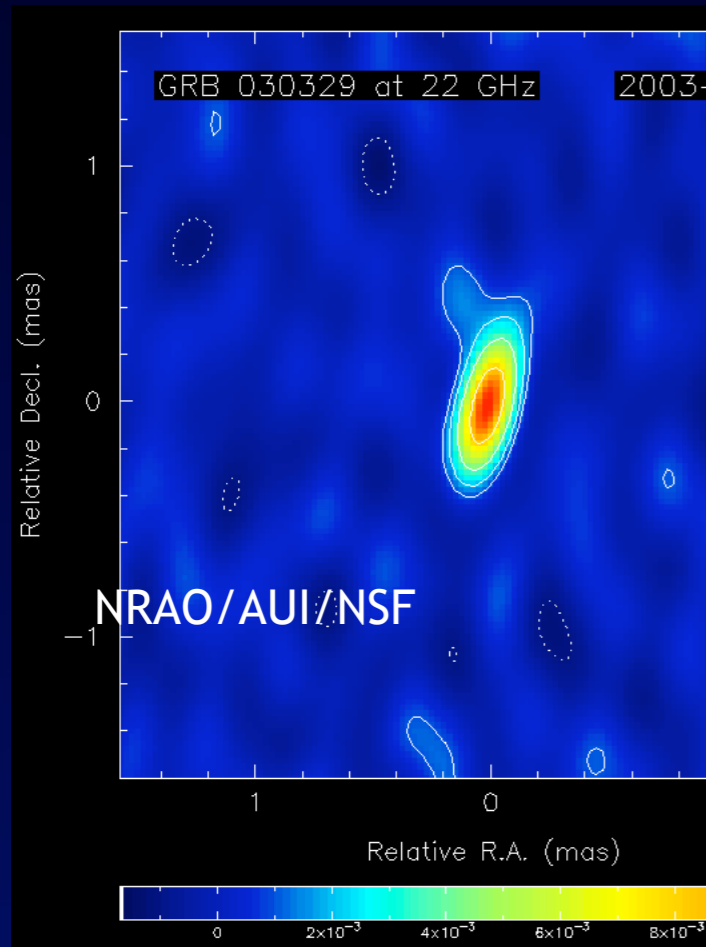
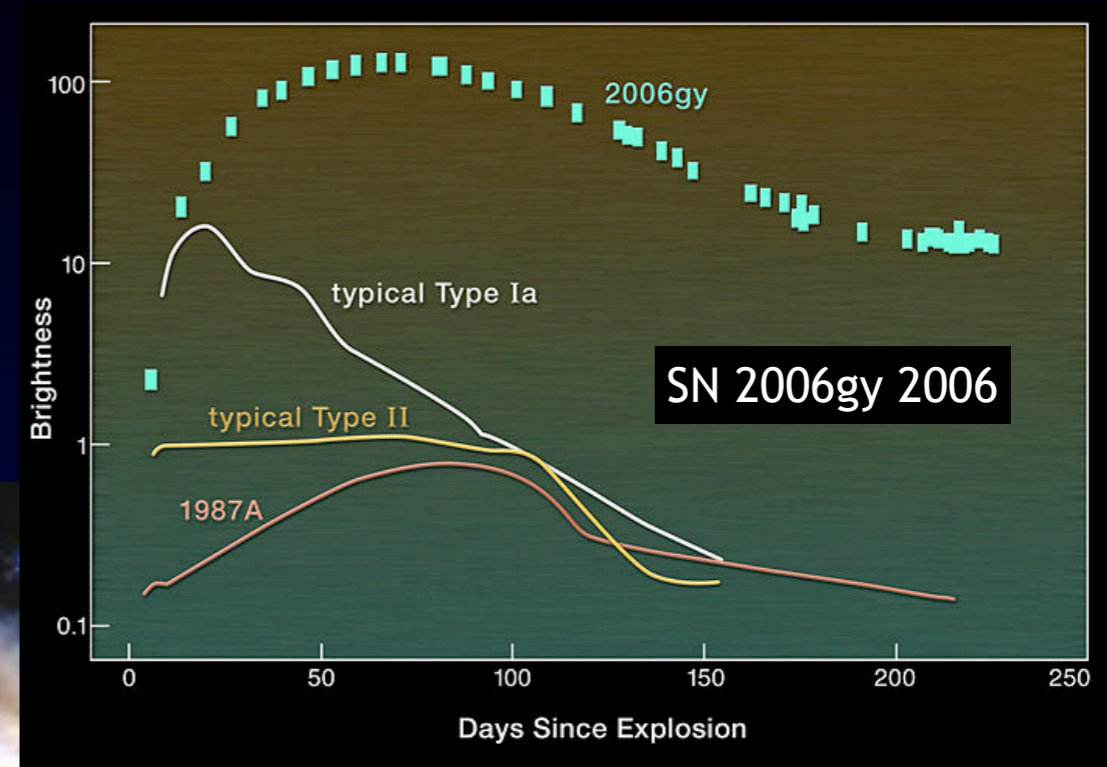
What do observations tell us?

5) New surprises and unexpected connections



What do observations tell us?

5) New surprises and unexpected connections



Hypernovae, GRB - supernova - collapsar connection, dwarf-classical nova connection

Nuclear Input

1) Reaction Rates

- For most astrophysical processes, a wide variety of **reaction rates** are needed.

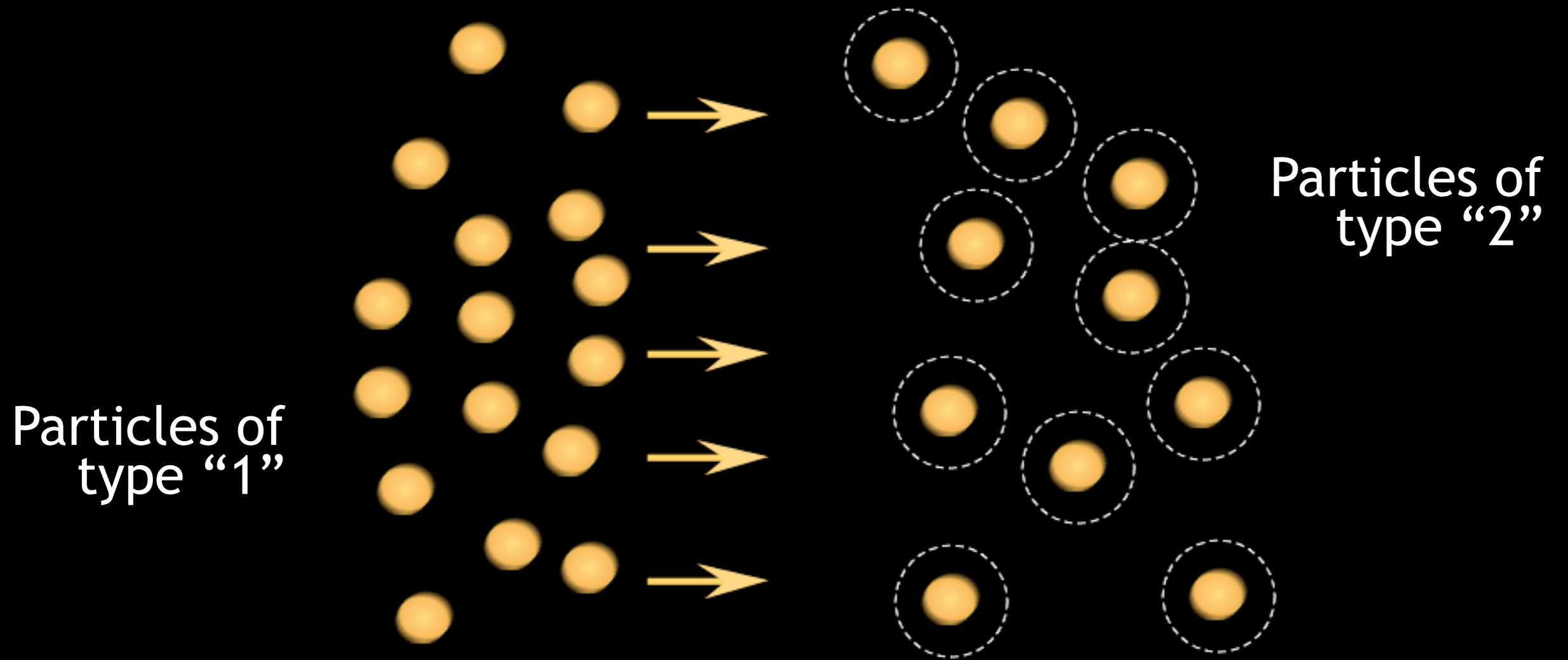
2) Masses and Partition Function

- In equilibrium, rates are unimportant because of detailed balance.
- Equilibrium populations depend on relative **masses** and **partition functions**.
- Partition Function is sum over **thermally populated levels**.

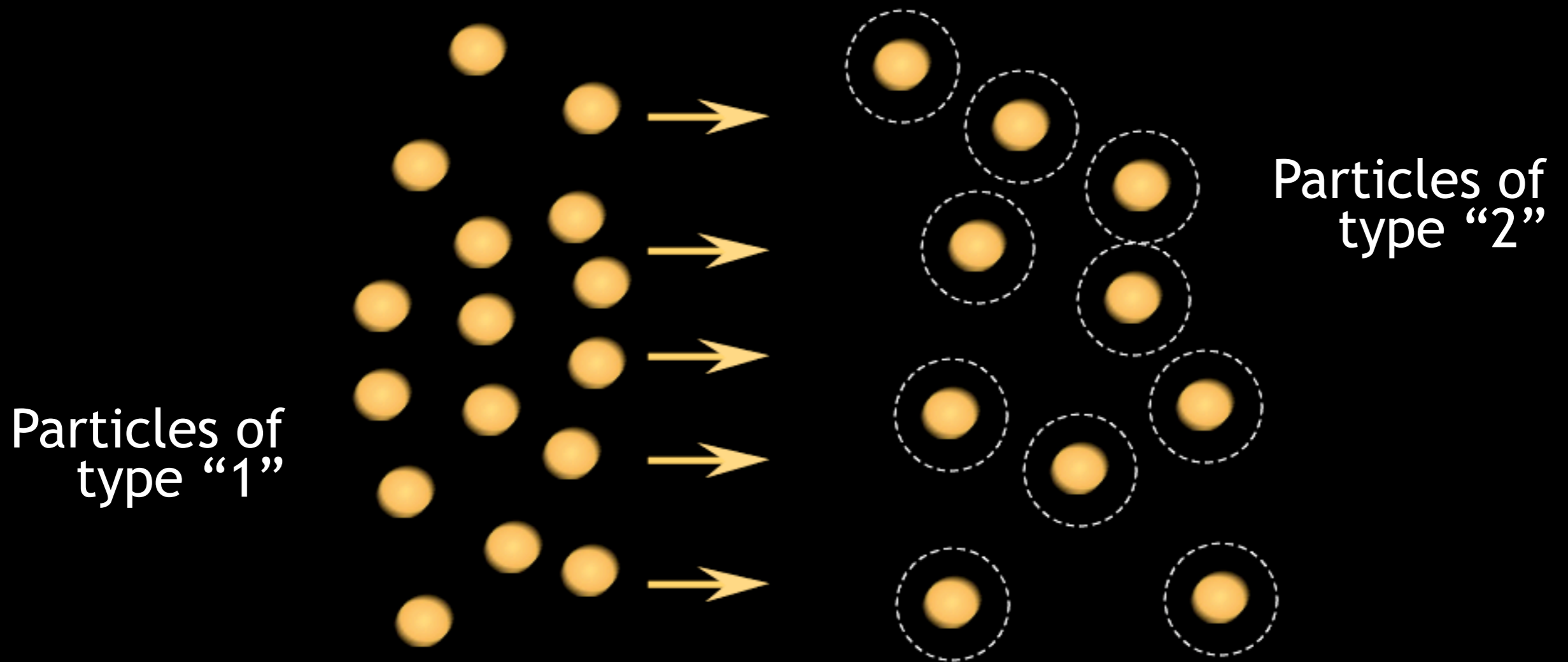
3) Nuclear Matter

- For Neutron Stars and their natal supernovae, significant fractions of a solar mass are at densities **similar to nucleons in a nucleus**.
- Think of nucleus with $A \sim 10^{57}$

What are Reaction Rates?



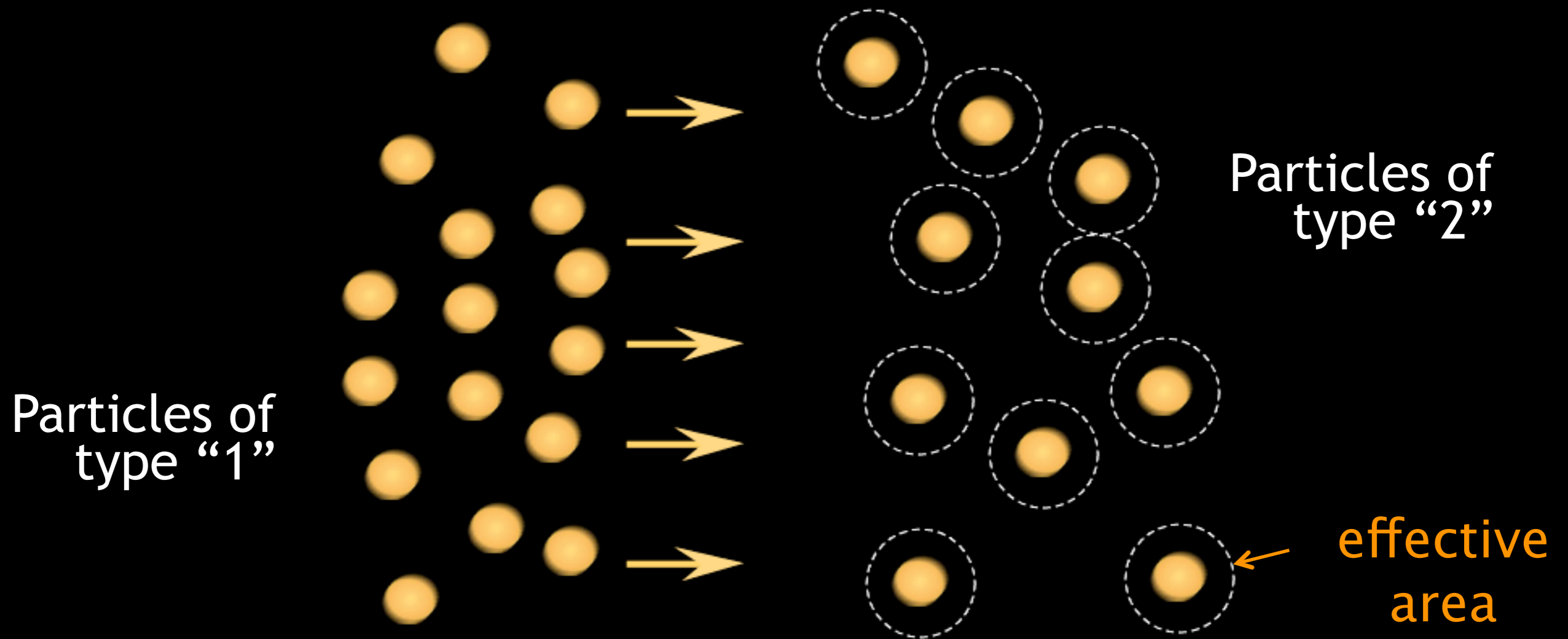
What are Reaction Rates?



Reactions / cm^3 / s =

- relative flux particles "1" ($\text{cm}^{-2} \text{s}^{-1}$)
- * number of target particles "2" (cm^{-3})
- * effective area of particle "2" a reaction (cm^2)

What are Reaction Rates?



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Reaction Rates as Differential Eqn.

Define

n_1 = number of particles of type “1” per volume (cm^{-3})

n_2 = number of particles of type “2” per volume (cm^{-3})

v = relative velocity (cm s^{-1})

$\sigma(v)$ = effective cross sectional area for a reaction (cm^2)

Then

$n_1 * v$ = relative flux of “1” relative to “2” ($\text{particles cm}^{-2} \text{ s}^{-1}$)

Reactions / cm^3 / s =

relative flux particles “1” ($\text{cm}^{-2} \text{ s}^{-1}$)

* (number of particles “2” cm^{-3})

* effective area of each particle “2” for a reaction (cm^2)

$$\text{Reactions / cm}^3 \text{ / s} = (n_1 * v) * n_2 * \sigma(v) = n_1 n_2 \sigma(v) v$$

$$\boxed{dn_1/dt = - n_1 n_2 \sigma(v) v}$$

Abundances and Mass Fractions

Number density, n_i , naturally depends on the mass density, ρ (g cm^{-3}).

It is possible to separate this dependence by defining the **abundance**

$$Y_i = n_i / \rho N_A, \text{ where } N_A \text{ is Avagadro's Number.}$$

Abundance has units of mole g^{-1} and is the fraction of a mole of species i per gram of matter, so it also called **Molar Fraction**.

Multiplying the abundance by the molecular weight of species i , A_i , which has units of g mole^{-1} gives the **mass fraction**

$$X_i = A_i Y_i$$

Mass fraction is convenient for presentation as $\sum_i X_i = 1$

Localizing Nuclear Effects

By splitting the hydrodynamic changes in density from the local nuclear changes we can derive an expression for dY/dt from dn/dt .

$$n_i = \rho N_A Y_i \longrightarrow dn_i/dt |_{\rho} = \rho N_A dY_i/dt$$

Combining with $dn_1/dt = -n_1 n_2 \sigma(v) v$

$$\rho N_A dY_1/dt = -n_1 n_2 \sigma(v) v$$

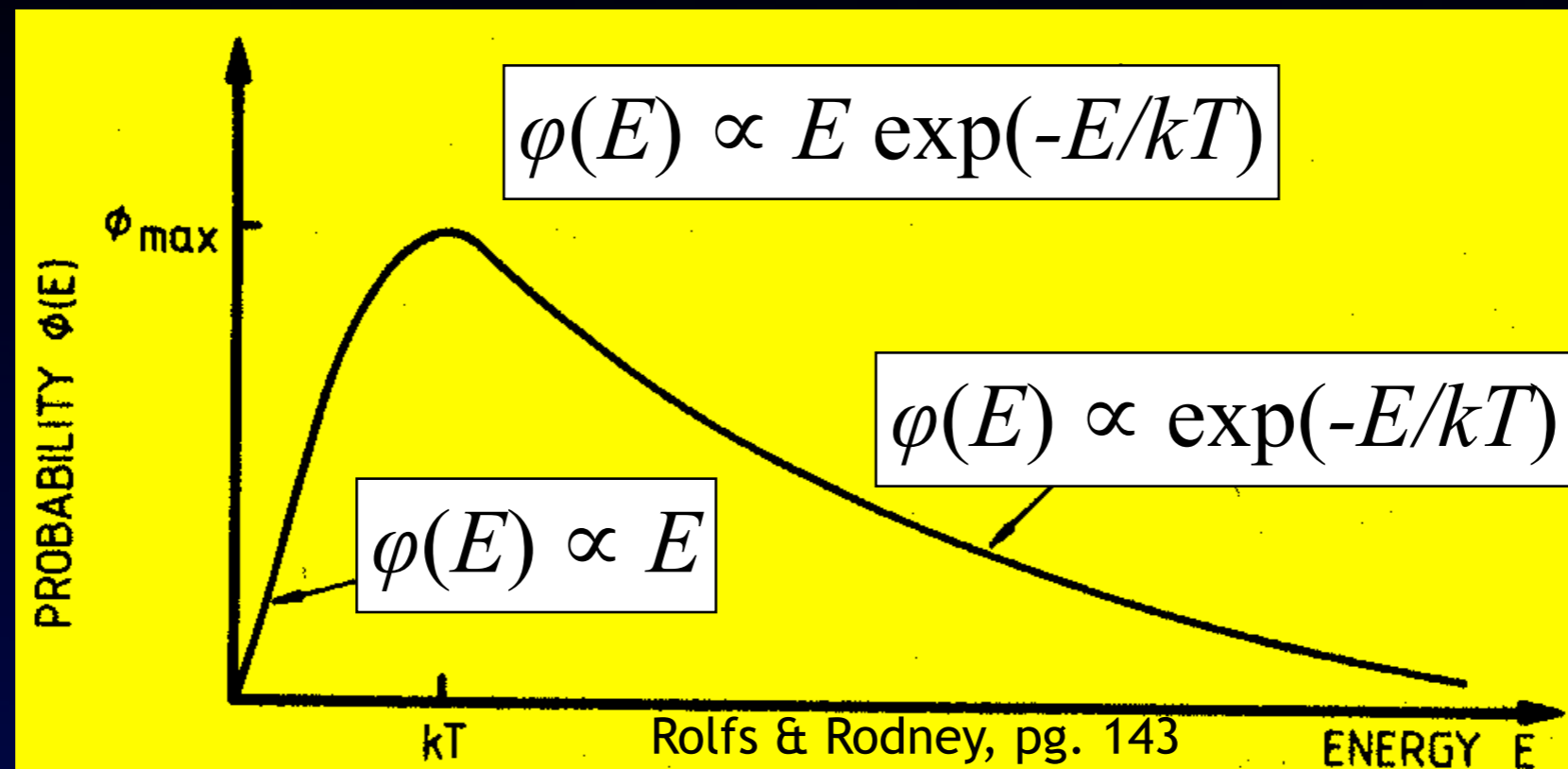
Replacing n_i with Y_i

$$\rho N_A dY_1/dt = -\rho N_A Y_1 * \rho N_A Y_2 * \sigma(v) v$$

Yields

$$dY_1/dt = -Y_1 Y_2 \rho N_A \sigma(v)$$

Maxwell-Boltzmann Distribution



Nuclei in the stellar plasma are far from monoenergetic.

For a given temperature, there is a distribution of relative velocities (relative energies) between any pair of particles in the star.

For most circumstances, a **Maxwell-Boltzmann Distribution** is sufficient.

Thermonuclear Reaction Rates

Integrating over the MB velocity distribution gives the **thermal average** cross section, denoted $\langle\sigma v\rangle$.

$$dY_1/dt = - Y_1 Y_2 \rho N_A \langle\sigma v\rangle$$

Define $N_A \langle\sigma v\rangle$ ($\text{cm}^3 \text{ s}^{-1} \text{ mole}^{-1}$) as the “Reaction Rate”.

Thermonuclear Reaction Rates

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Define $N_A \langle\sigma v\rangle$ ($\text{cm}^3 \text{ s}^{-1} \text{ mole}^{-1}$) as the “Reaction Rate”.

Important Distinction:

Cross Section: function of relative **velocity** (or energy)

Reaction Rate: function of **temperature** (not energy)

Thermonuclear Reaction Rates

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Important Distinction:

Cross Section: function of relative **velocity** (or energy)

Reaction Rate: function of **temperature** (not energy)

Thermal average of cross sections needed for rates!

But rate at any temperature depends on $\sigma(E)$ over **range of energies**.

MB Details

The Maxwell–Boltzmann distribution, for temperature, T , velocity, v , and mass, m_i , is

$$\varphi_i(v, T) = 4\pi v^2 (m_i/2\pi kT)^{3/2} \exp(-m_i v^2/2kT)$$

With $\int_0^{\infty} \varphi_i(v, T) dv = 1$

Reaction Rate is

$$N_A \langle \sigma v \rangle (T) = \int_0^{\infty} \int_0^{\infty} N_A \varphi_1(v_1, T) \varphi_2(v_2, T) \sigma(v_1 - v_2) (v_1 - v_2) dv_1 dv_2$$

Change to center of mass coordinates

v = Relative velocity = $(v_1 - v_2)$ & μ = reduced mass

V = Center of mass velocity & M = total mass

$$\varphi(v, T) = 4\pi v^2 (\mu/2\pi kT)^{3/2} \exp(-\mu v^2/2kT)$$

$$\varphi(V, T) = 4\pi V^2 (M/2\pi kT)^{3/2} \exp(-MV^2/2kT)$$

Reaction Rate in Detail

Using center of mass coordinates

$$N_A \langle \sigma v \rangle (T) = \int_0^\infty \int_0^\infty N_A \varphi(\mathbf{v}, T) \varphi(V, T) \sigma(\mathbf{v}) v dv dV$$

Since $\int_0^\infty \varphi(V, T) dV = 1$ we can integrate over V to get

$$N_A \langle \sigma v \rangle (T) = N_A 4\pi (\mu/2\pi kT)^{3/2} \int_0^\infty v^3 \sigma(\mathbf{v}) \exp(-\mu v^2/2kT) dv$$

Change to center of mass energy $E = 0.5 \mu v^2$

$$N_A \langle \sigma v \rangle (T) = N_A (8/\pi\mu)^{1/2} (kT)^{-3/2} \int \sigma(E) E \exp(-E/kT) dE$$

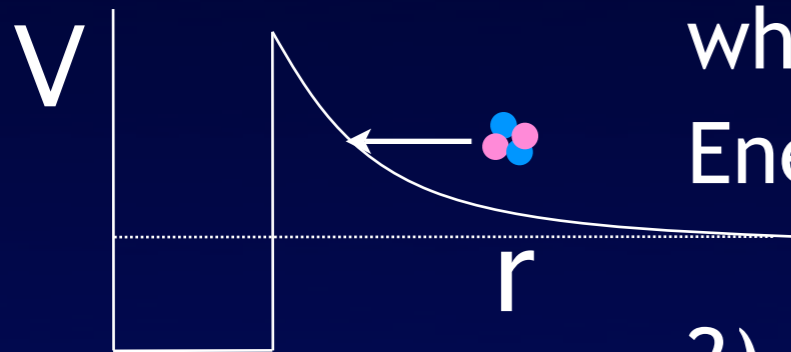
Cross section & Astrophysical S-factor

Helpful to Simplify the rate expression for charged particle induced reactions by exploiting two well known energy dependences for $\sigma(E)$

Charged particle Cross Sections $\sigma(E)$

1) are proportional to probability for coulomb barrier penetration $\exp[-(E_G/E)^{1/2}]$, where $E_G = 2\mu (\pi e^2 Z_1 Z_2 / h)^2$ is the Gamow Energy.

2) are proportional to the nuclear size (de Broglie wavelength²), $\pi \lambda^2 \sim 1/E$.

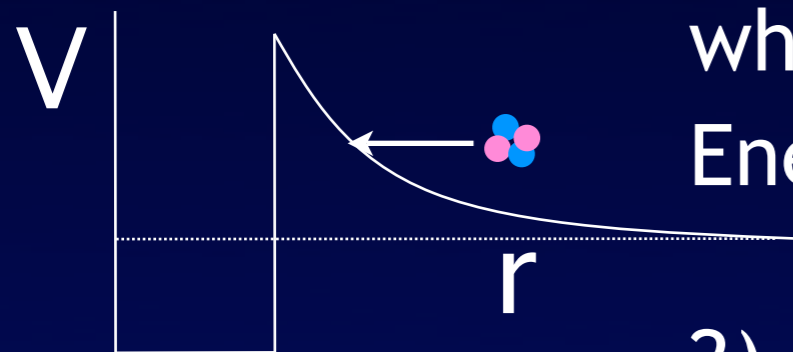


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Charged particle Cross Sections $\sigma(E)$

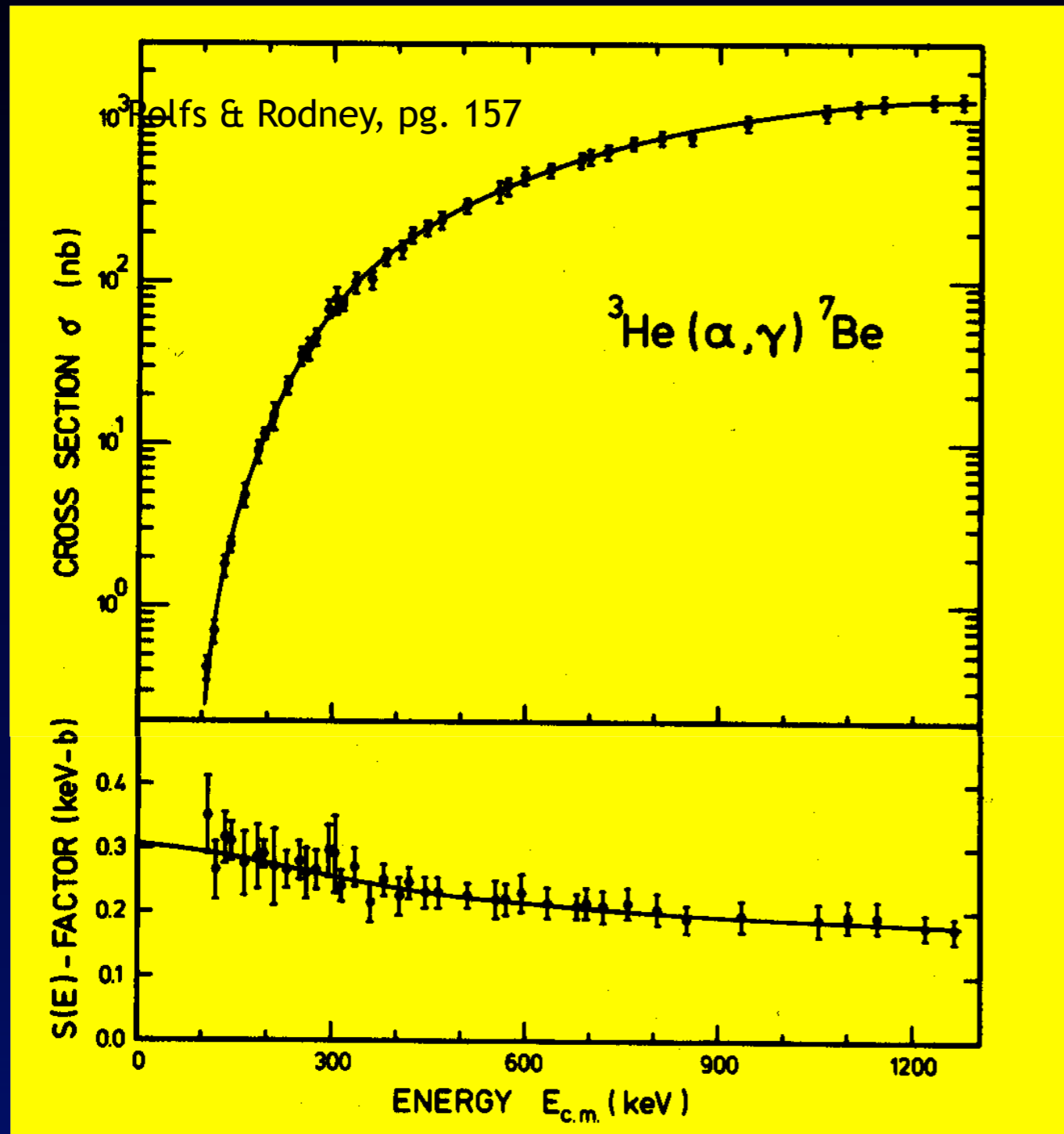
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2) are proportional to the nuclear size (de Broglie wavelength²), $\pi \lambda^2 \sim 1/E$.

The other energy dependencies are lumped together into $S(E)$ - the Astrophysical S-factor

Non-Resonant Reactions

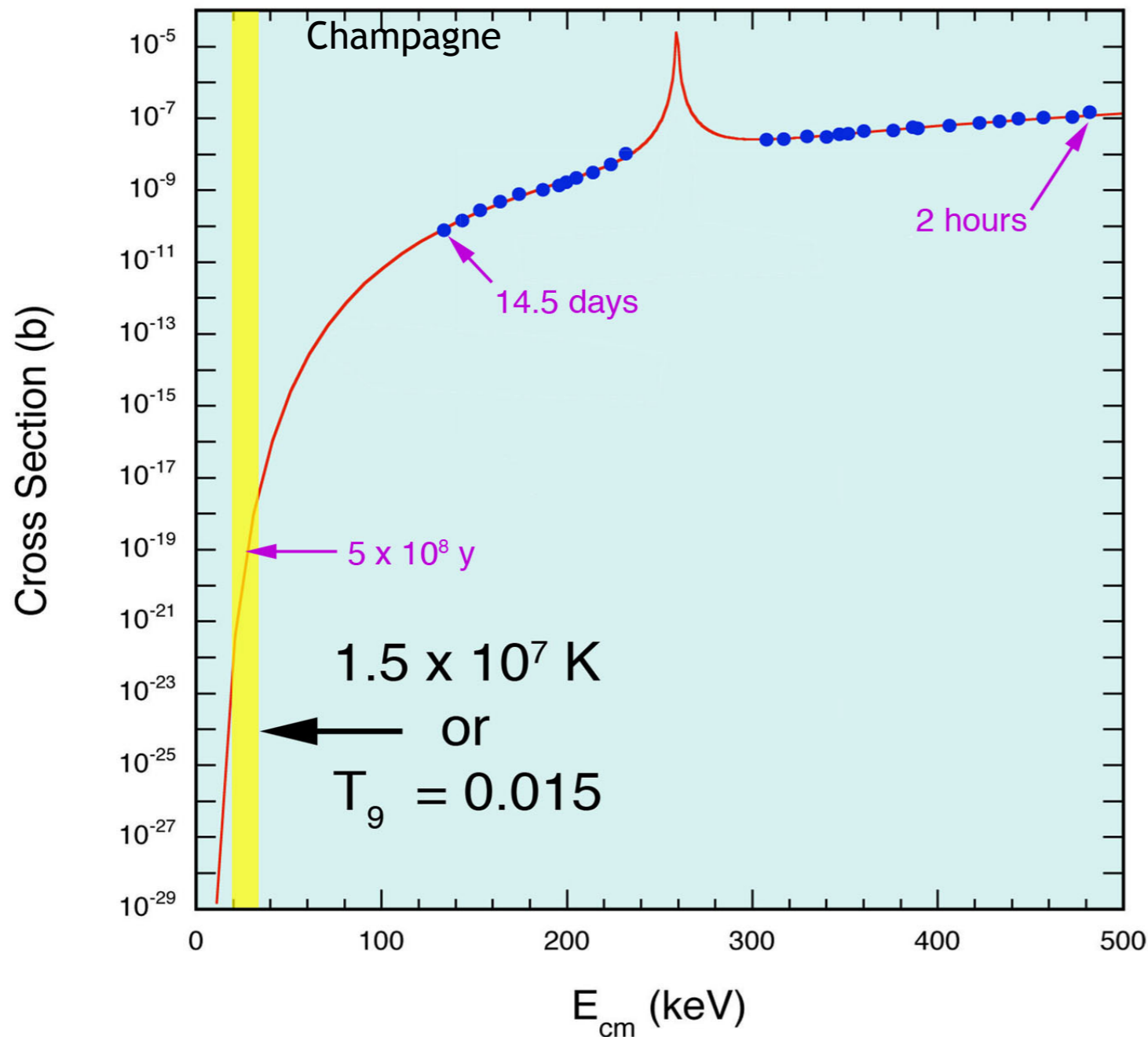


For many reactions, **MOST** of the energy dependence in $\sigma(E)$ is described by the penetrability & nuclear size terms.

$S(E)$ is very slowly varying

Advantageous to work with $S(E)$ rather than $\sigma(E)$

Extrapolating the S factor



For many reactions of astrophysical interest, the energies at which measurements are feasible are much larger than the energies at which the reaction occur in stars.

Extrapolating the S factor is much more reliable.

Reaction Rate with S factor

By writing the cross section as

$$\sigma(E) = (1/E) \cdot \exp[-(E_G/E)^{1/2}] \cdot S(E)$$

the Reaction Rate

$$N_A \langle \sigma v \rangle (T) = N_A (8/\pi\mu)^{1/2} (kT)^{-3/2} \int_0^{\infty} \sigma(E) E \exp(-E/kT) dE$$

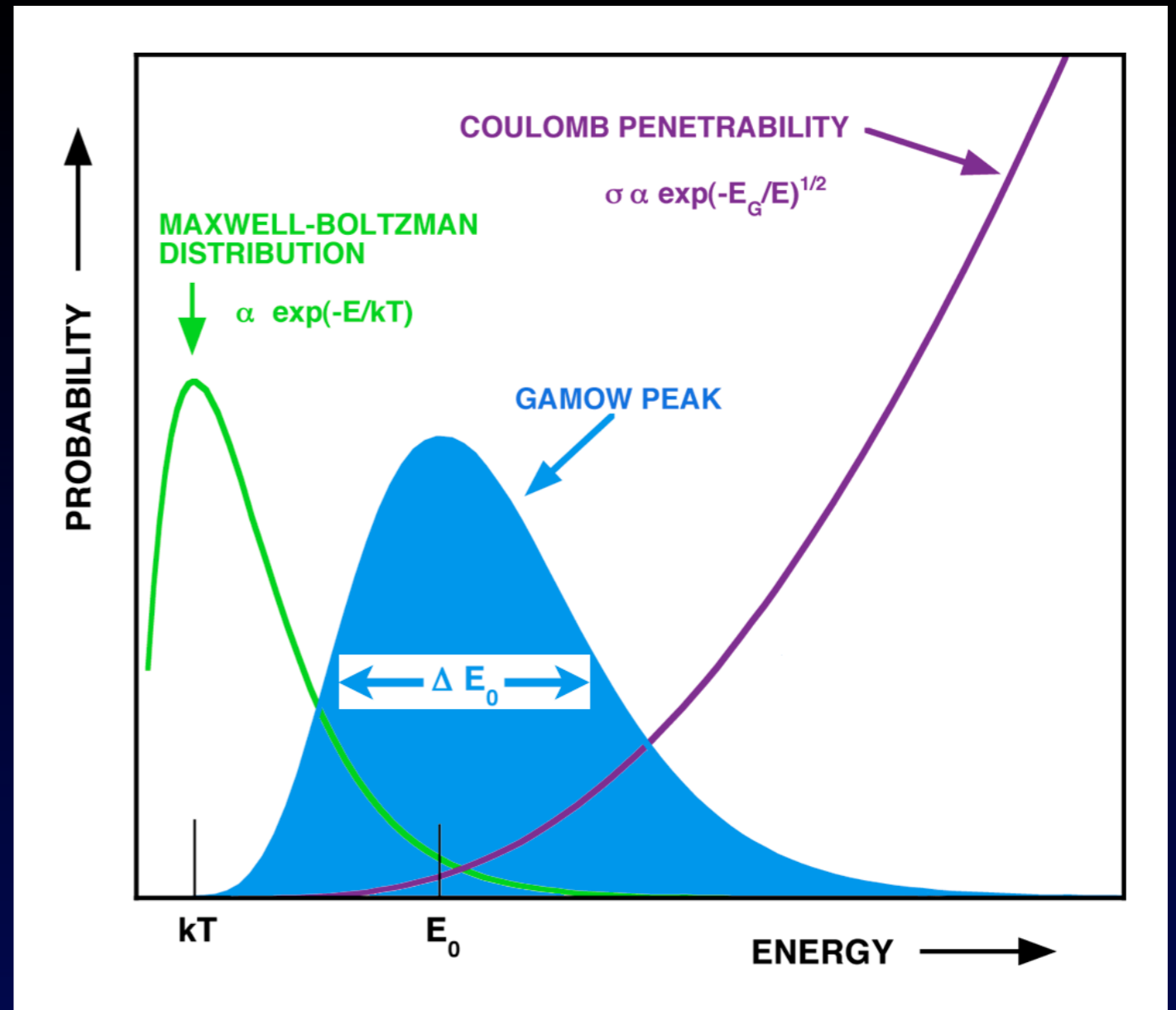
becomes

$$N_A \langle \sigma v \rangle (T) = N_A (8/\pi\mu)^{1/2} (kT)^{-3/2} \int_0^{\infty} S(E) \exp[-E/kT - (E_G/E)^{1/2}] dE$$

S-factor has units of Energy • Area ... typically keV * barn,
MeV * barn where 1 barn = 10⁻²⁴ cm²

Gamow Peak

If $S(E)$ is slowly varying, Integral is dominated by $\exp[-E/kT - (E_G/E)^{1/2}]$, which decreases at **low energy** because of the right hand term and decreases at **high energy** because of the left hand term.



Maximum at E_0 = “most effective stellar energy”

$$= E_G^{1/3} (kT/2)^{2/3}$$

“Width” of ΔE_0 = “Gamow Window”

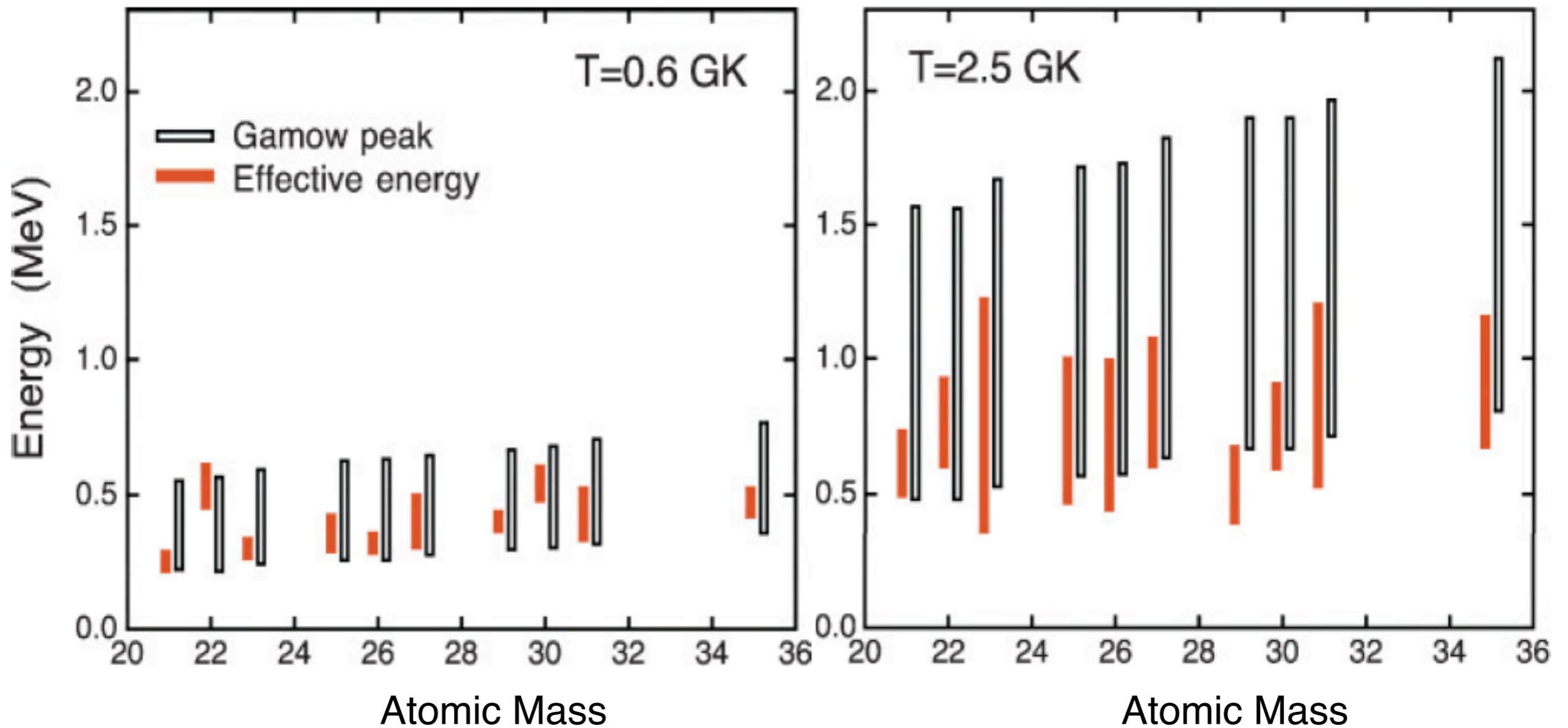
$$= [4/3^{1/2}] (E_0/kT)^{1/2}$$

Gamow Window Examples

Reaction	Site	MK	E_0
$p + p$	Sun	15	6
$p + {}^{14}\text{N}$	CNO	30	40
$\alpha + {}^{12}\text{C}$	Red Giant	200	300
$p + {}^{17}\text{F}$	Nova	300	232
$\alpha + {}^{30}\text{S}$	XRB	1000	1800

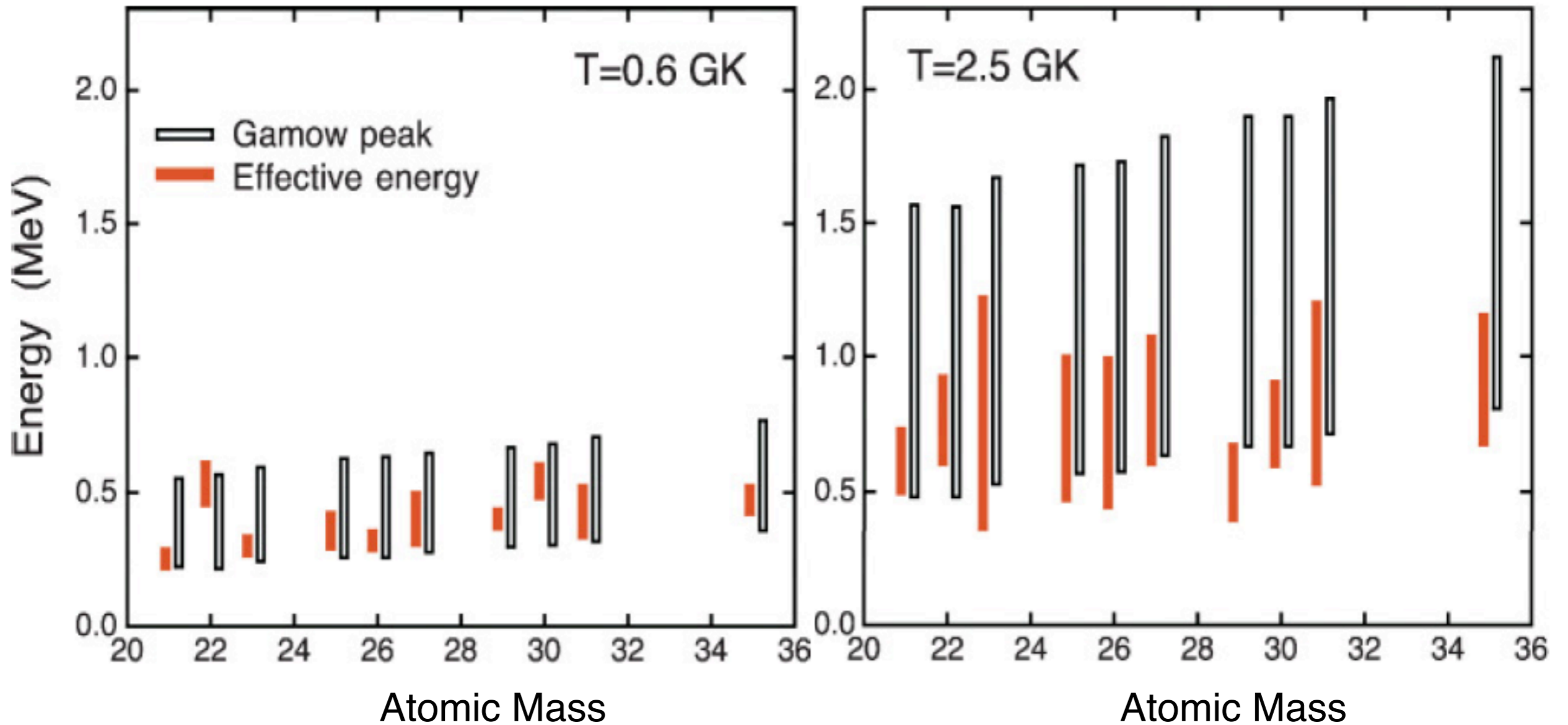
Gamow Window Examples

Newton, Iliadis, Champagne, et al. (2007)



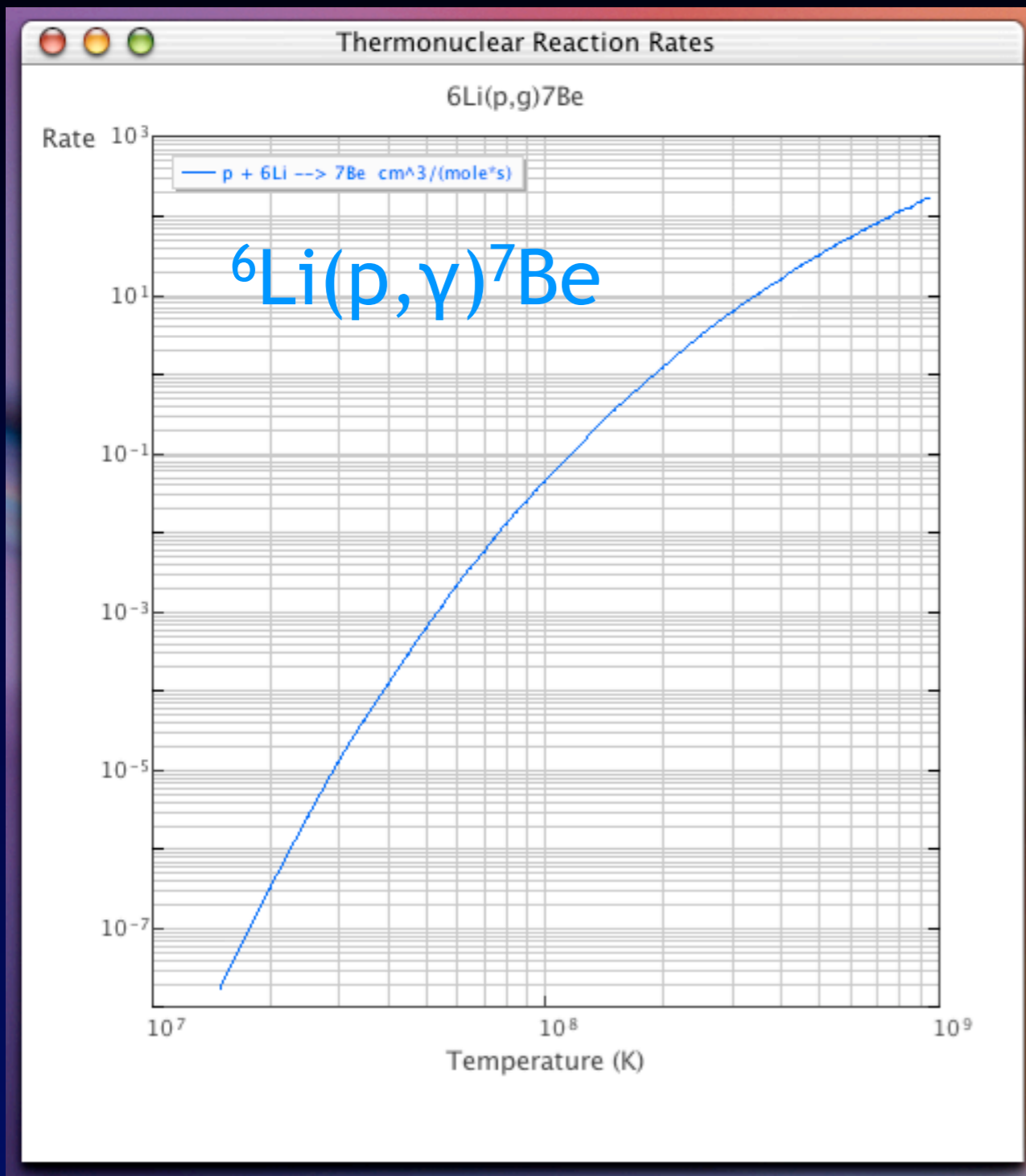
Gamow Window Examples

Newton, Iliadis, Champagne, et al. (2007)



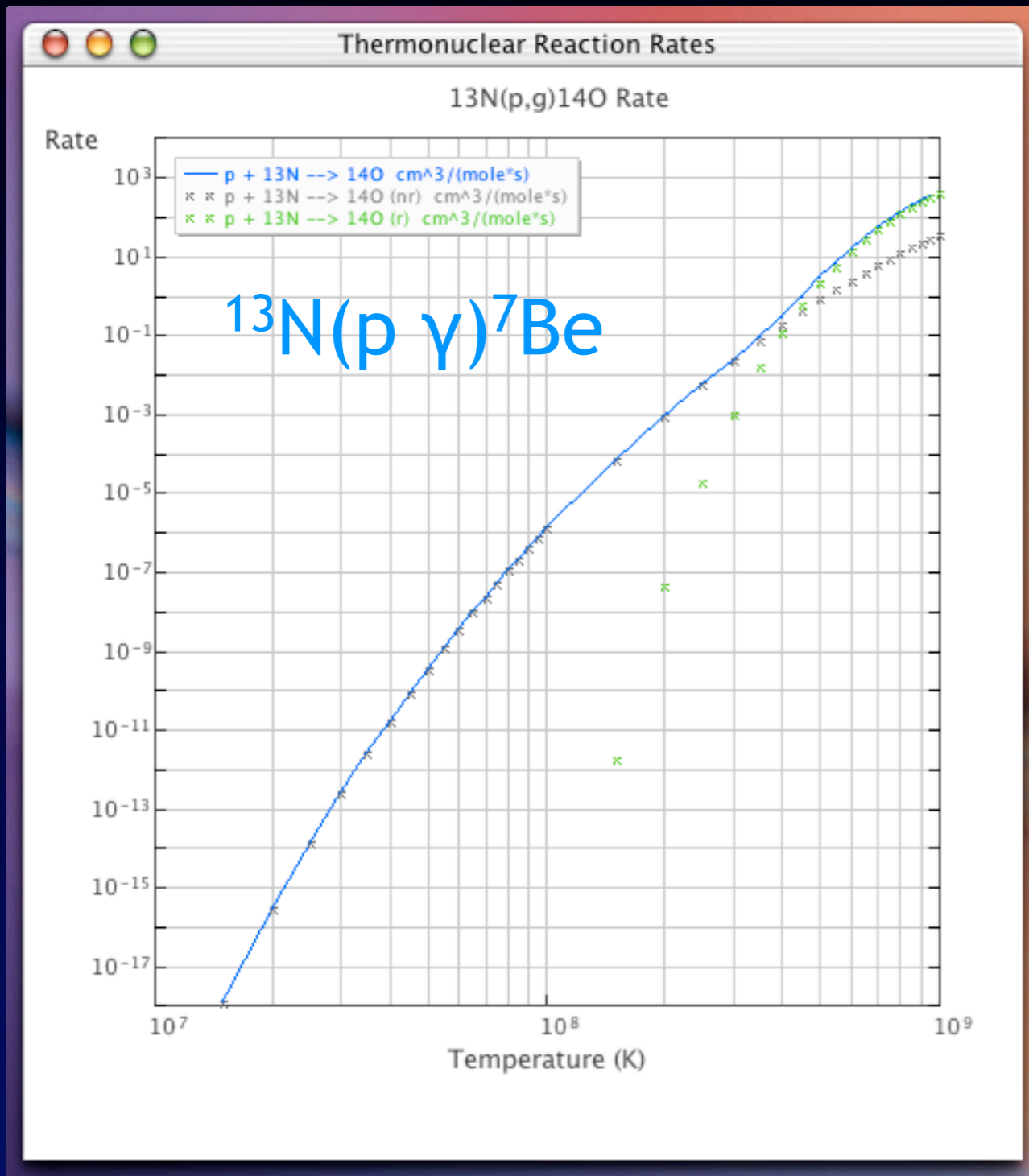
Gamow Peak applies to direct capture, but only 'BoE' estimate for resonances.

Reaction Rate Examples



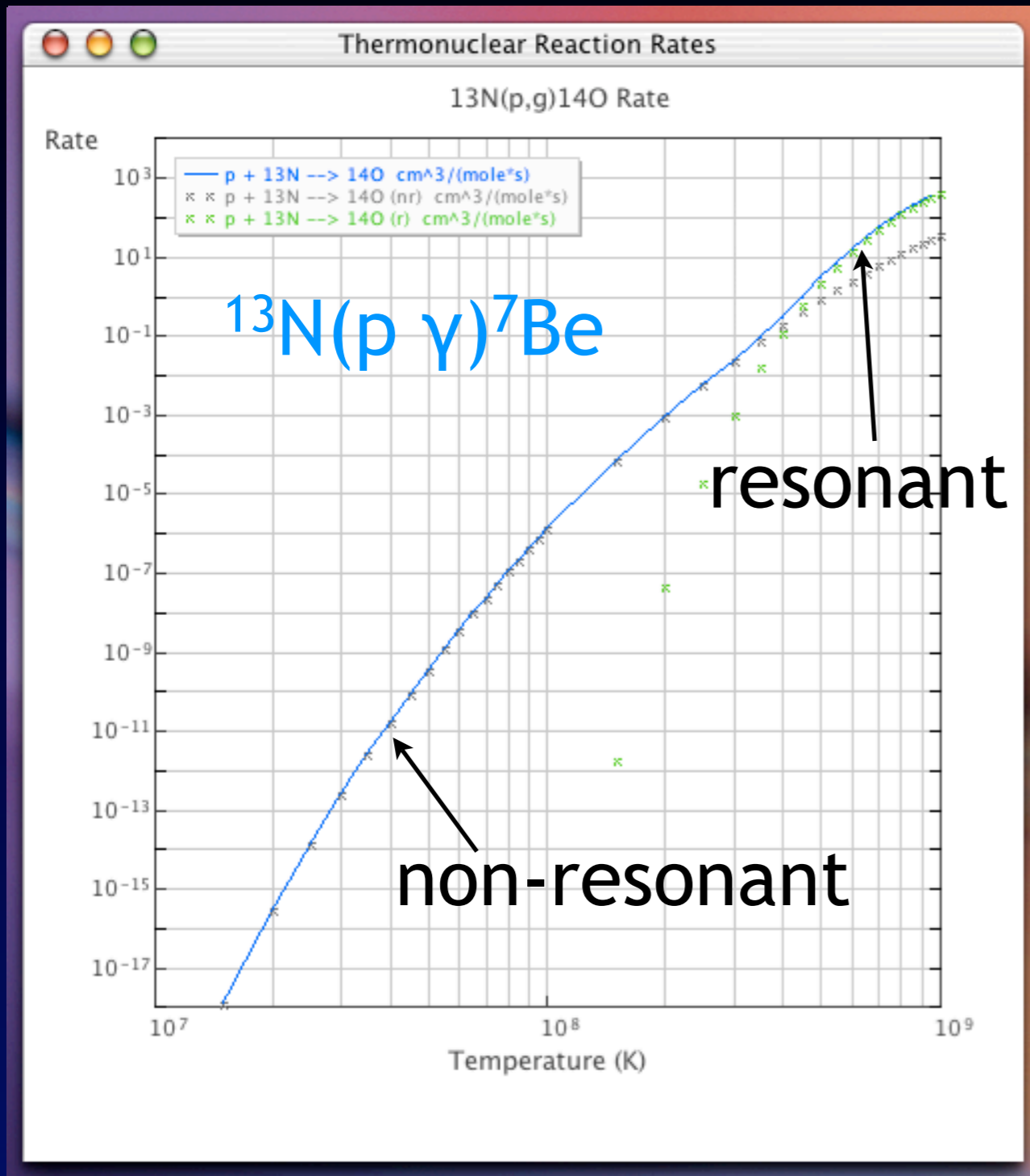
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Reaction Rate Examples



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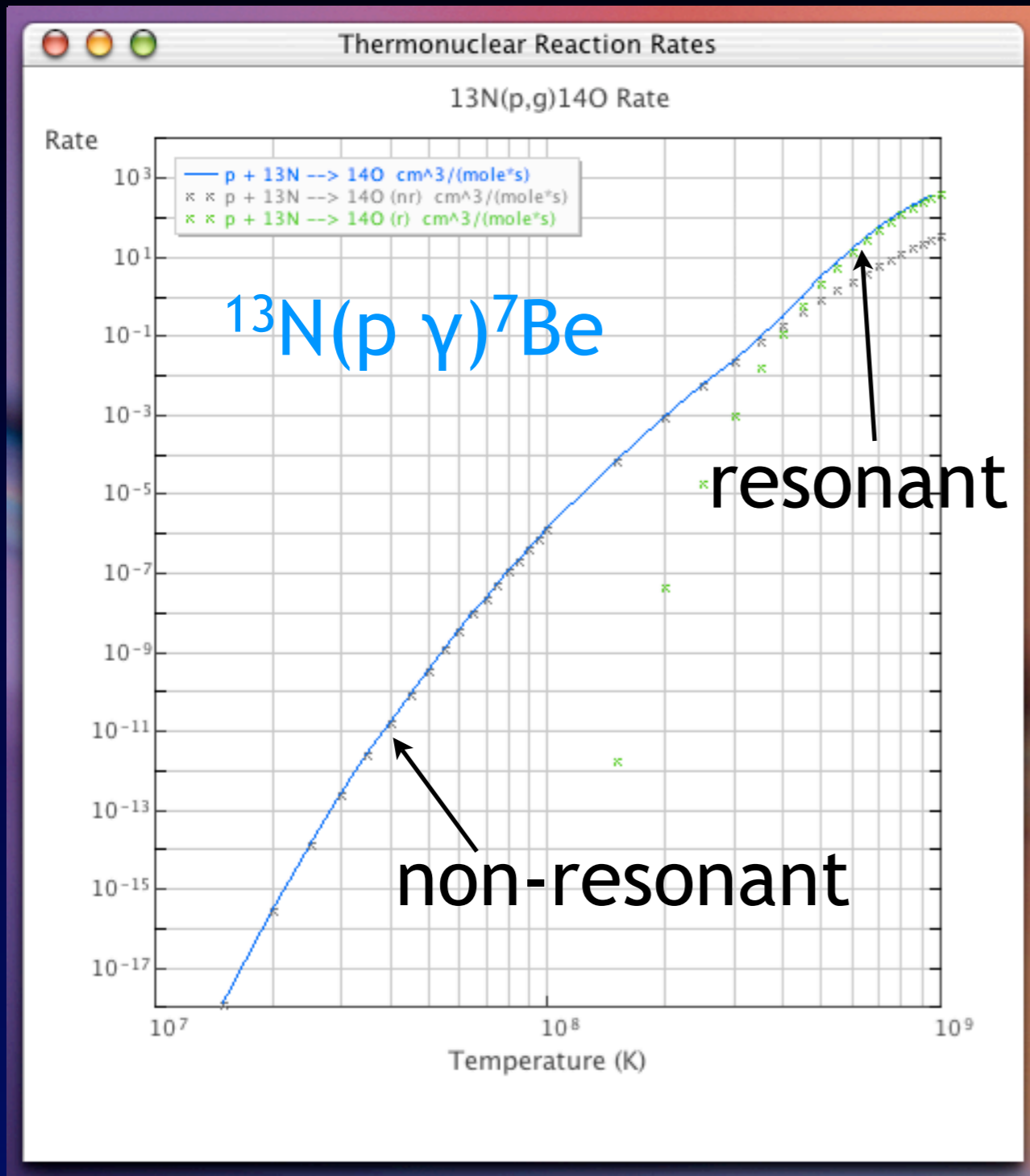
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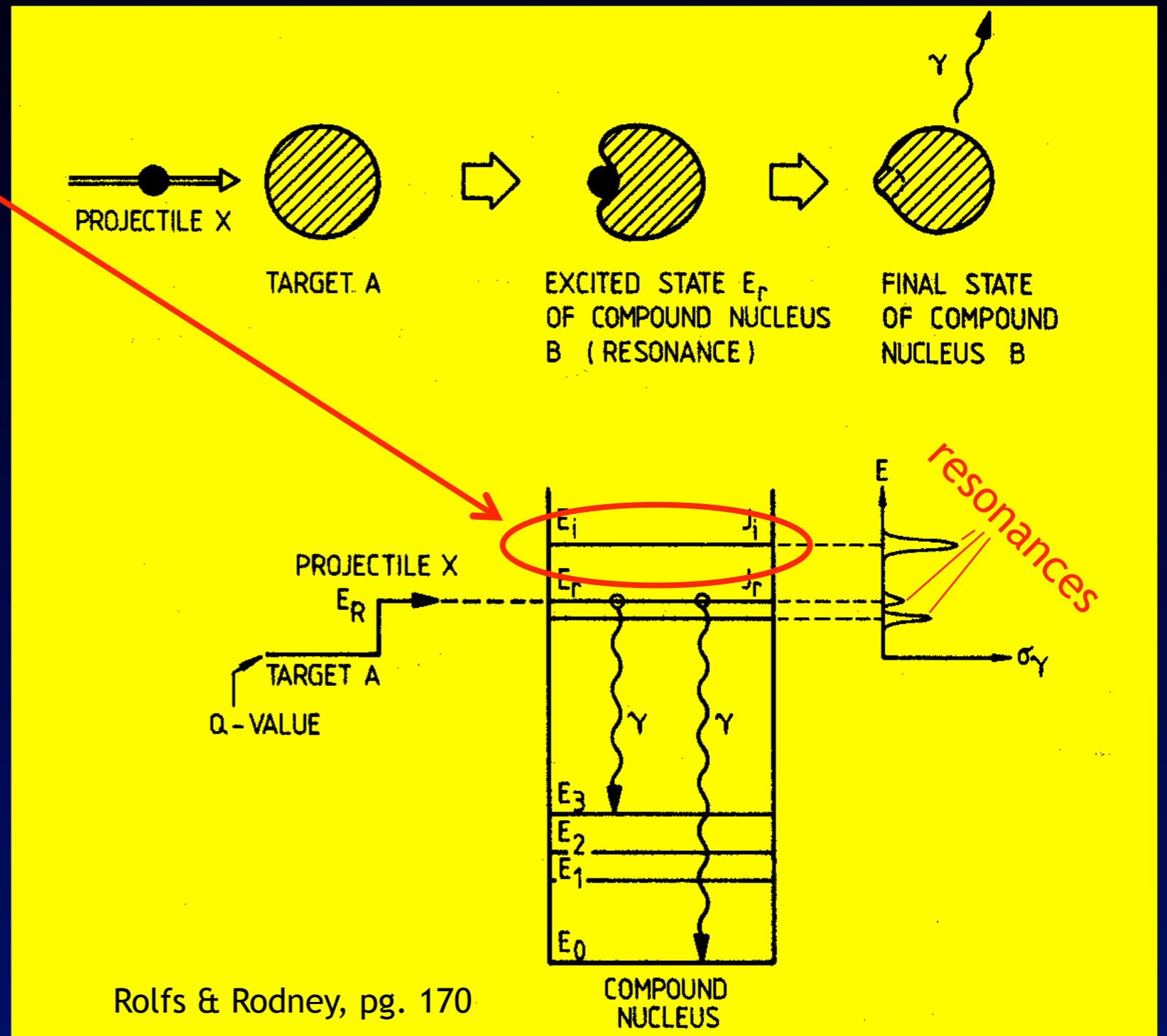
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Rate changes by many orders of magnitude as temperature changes by 100.

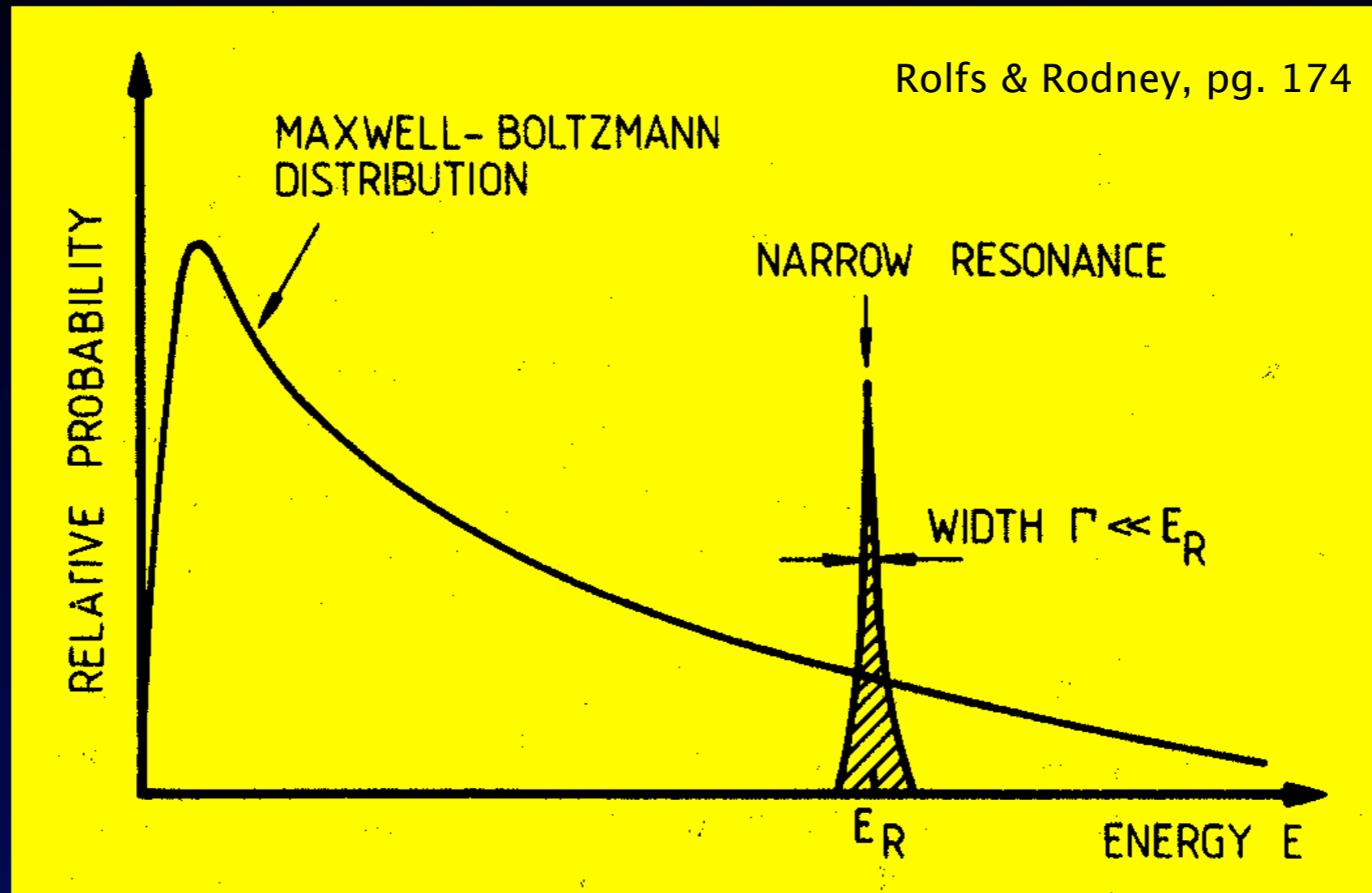
Resonant Reaction Rates

The presence of a **nuclear resonance** with an energy in the Gamow Window can dramatically increase the reaction rate - by factors of 10 to 10^7 in some cases

The search for these resonances and measurement of their properties is **extremely important.**



Narrow Resonances



For single resonances with a narrow total width, the resonance energy region effectively becomes the Gamow Peak

Breit-Wigner Formula

For a Narrow (Lorentzian) resonance the cross sections $\sigma(\mathbf{E})$

$$\sigma_{BW}(\mathbf{E}) = \frac{\pi (\hbar c)^2}{2\mu\mathbf{E}} * \frac{(2J_r+1)}{(2J_1+1)(2J_2+1)} * \frac{\Gamma_{in} \Gamma_{out}}{[(\mathbf{E}-\mathbf{E}_r)^2 + (\Gamma_{tot}/2)^2]}$$

where

J_r = spin of resonant state

J_1, J_2 = spin of reactant nuclei

\mathbf{E}_r = energy of resonant state

Γ_{in} = energy width of reaction channel to form state (entrance channel) = \hbar / τ_{in}

Γ_{out} = energy width of exit channel = \hbar / τ_{out}

Γ_{tot} = total energy width of resonance = \hbar / τ_{tot}

Resonant Reaction Rates

For **narrow** resonances

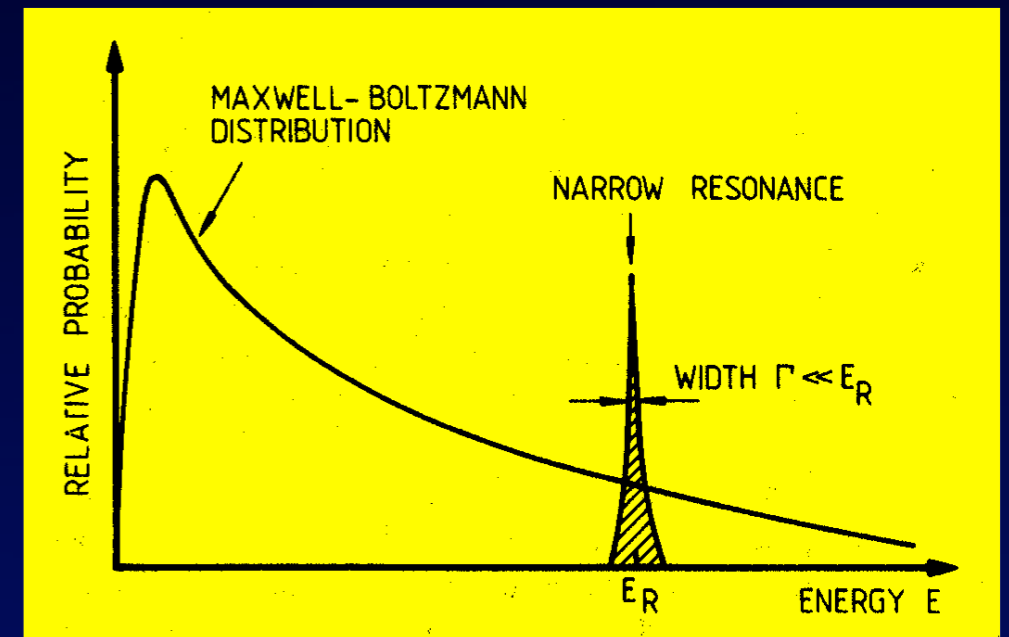
$$\begin{aligned}
 N_A \langle \sigma v \rangle (T) &= N_A (8/\pi\mu)^{1/2} (kT)^{-3/2} \int \sigma_{BW}(E) E \exp(-E/kT) dE \\
 &= N_A (8/\pi\mu)^{1/2} (kT)^{-3/2} E_r \exp(-E_r/kT) \int \sigma_{BW}(E) dE
 \end{aligned}$$

Integrating the cross section

$$\int \sigma_{BW}(E) dE = (\pi/2) \Gamma_{tot} \sigma_{BW}(E_r)$$

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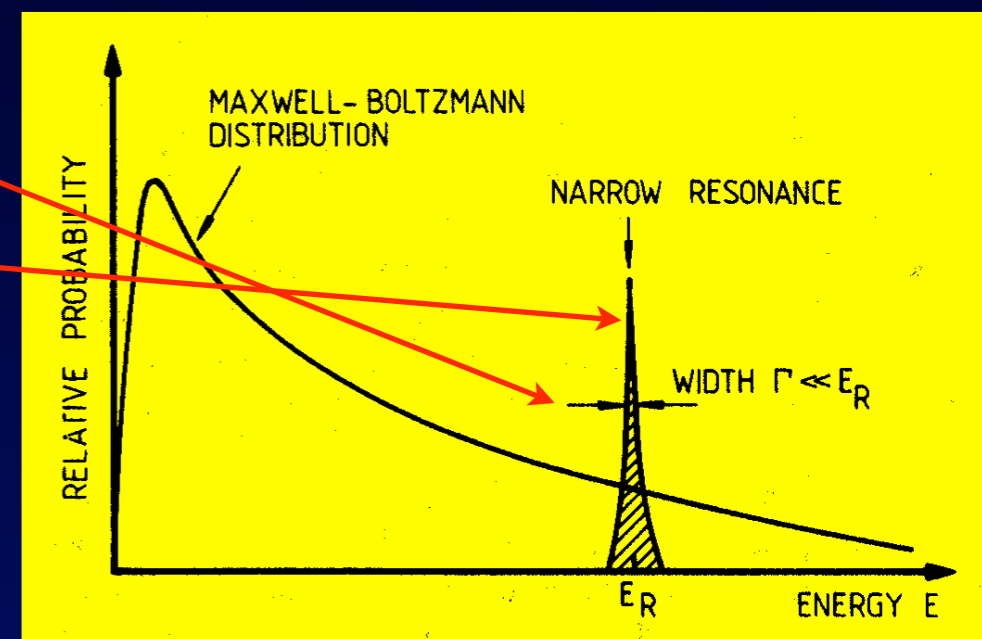
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Resonance Strength

Define

$$\omega = \frac{(2J_r+1)}{(2J_1+1)(2J_2+1)} = \text{Statistical factor}$$

$$\gamma = \frac{\Gamma_{in} \Gamma_{out}}{\Gamma_{tot}}$$

$\omega\gamma$ is the “resonance strength” (units are in energy)

In terms of $\omega\gamma$

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For multiple narrow resonances

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Linear Exponential
↓ ↓

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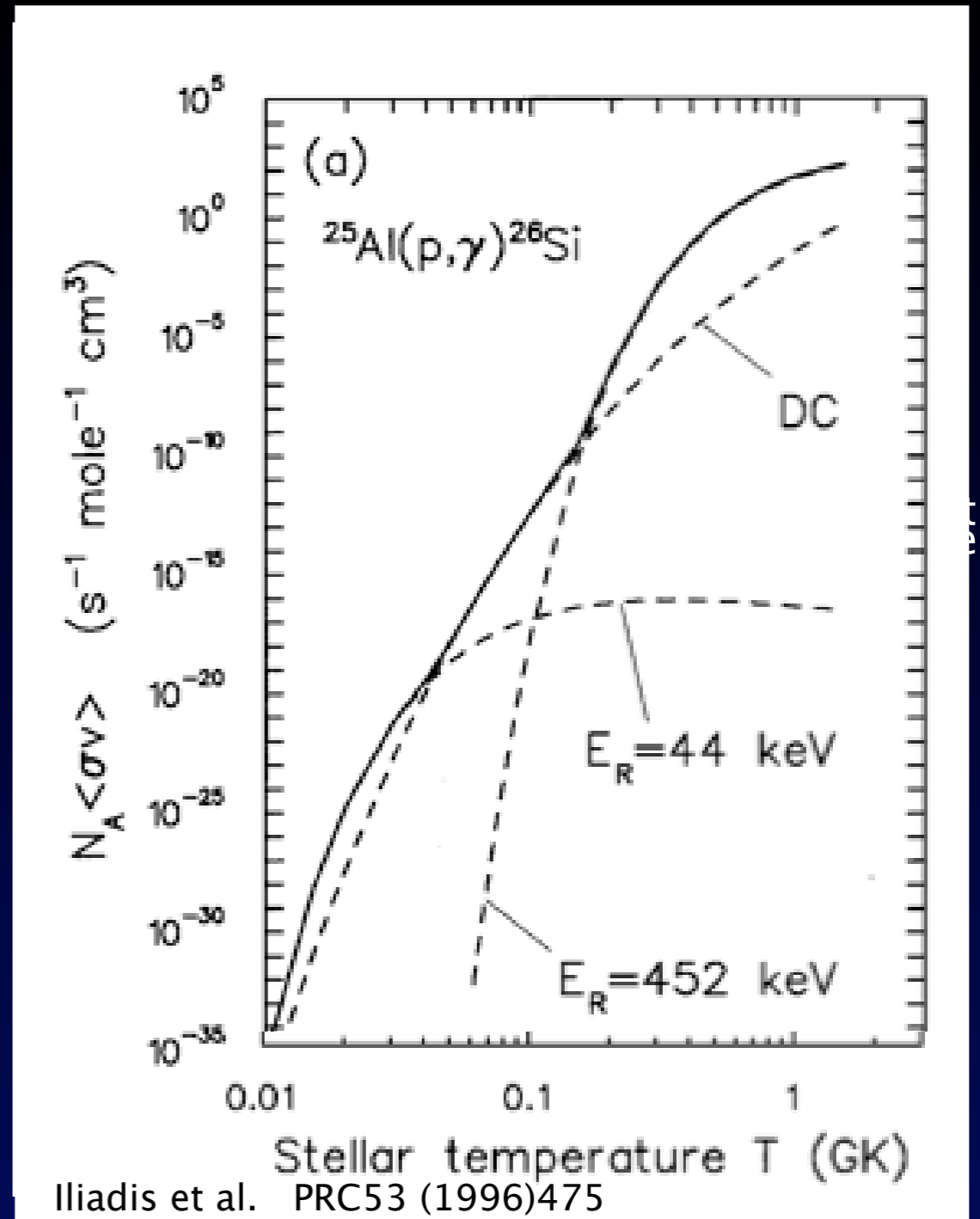
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Which Resonances?

The presence of resonances can make a **dramatic** difference (orders of magnitude) in the reaction rate

Resonances with **zero orbital angular momentum transfer** are important (strengths are often large)

Especially important are resonances with a **large strength** and a **low resonance energy**



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Summary

1. Nuclear Physics for Astrophysics

How do we learn about cosmic nuclear evolution

Myriad of observations from the ground, satellite and underground detectors provide us information about the changing composition of the Cosmos.

What nuclear data are needed

Reaction Rates, Masses and Partition functions

Data on Nuclear Matter

2. Lives of Stars

3. Supernovae

4. Stellar Afterlife