Lecture at 2007 CNS Summer School

Di-neutron correlation and collective excitation in neutron-rich nuclei

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Outline

1. Introduction: What is the di-neutron correlation

2. Origin of the di-neutron correlation: dilute neutron matter

Strong coupling pairing and relation to Bose-Einstein condensation

3. di-neutron correlation in medium and heavy mass n-rich nuclei

Density functional theory and Hartree-Fock-Bogoliubov method

4. Exotic modes of excitation and surface di-neutron mode

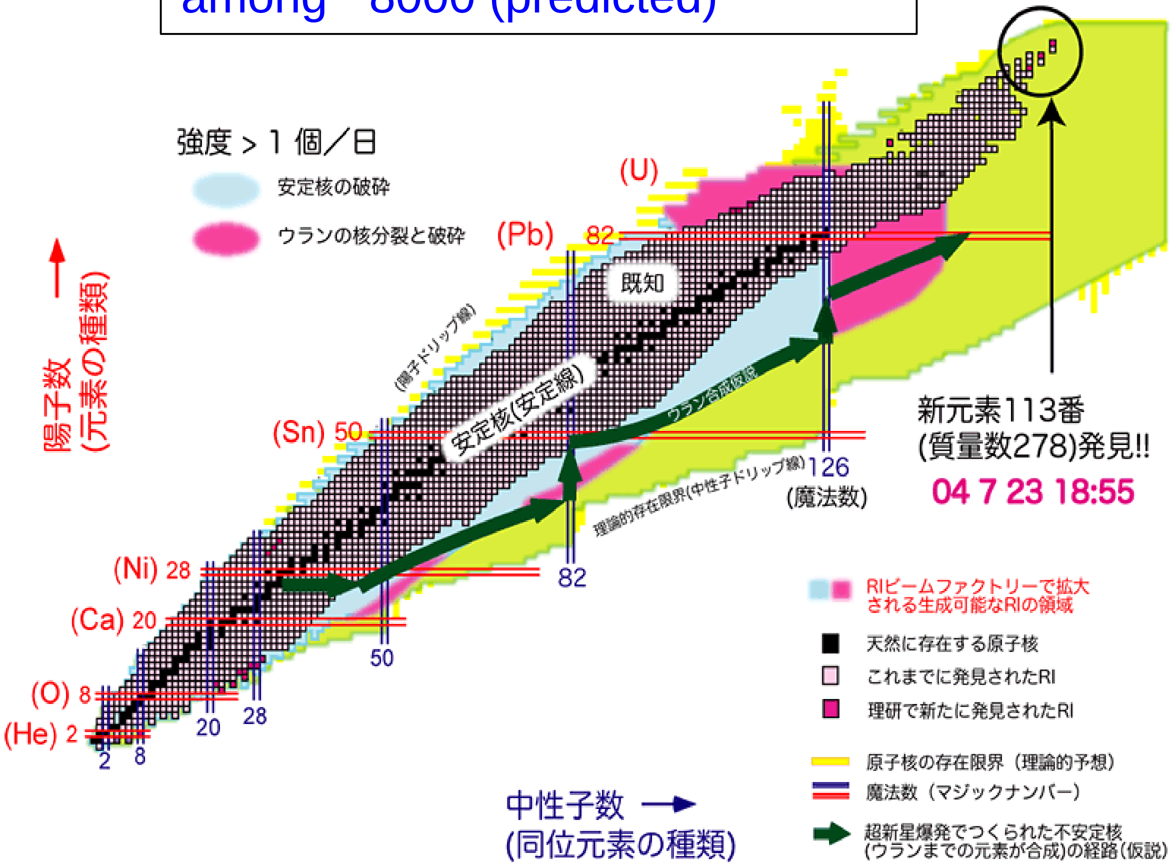
Linear response in the density functional theory and continuum QRPA method

New region of nuclear physics

RI beam facilities in the third generation

RIBF@RIKEN, FAIR@GSI, RIA ....

~4000 isotopes may be produced among ~8000 (predicted)



- **Unstable nuclei in the whole mass regions**
- **Nuclei around the r-process S_n~2MeV , N/Z ~2)**

  ex ^{78}Ni region
- **Extension of near-drip-line region S_n~0MeV)**

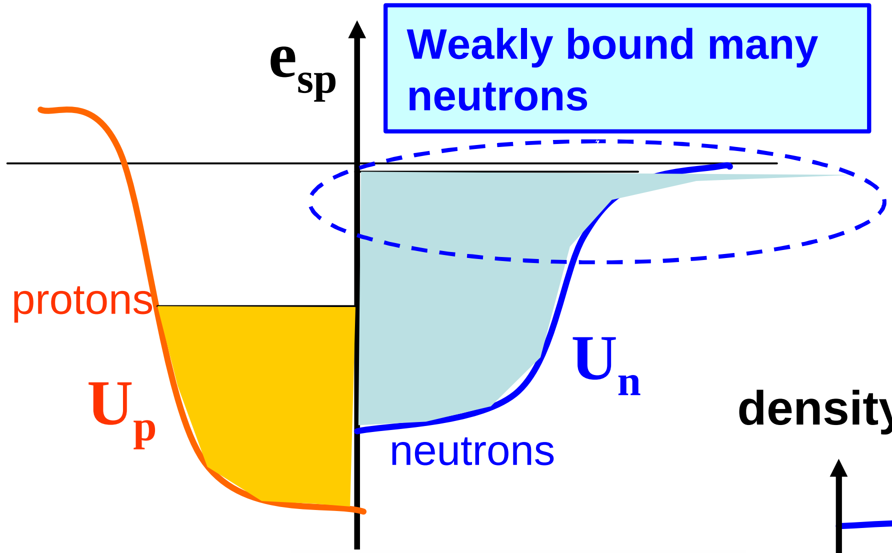
  From O to medium-mass nuclei ~Ca

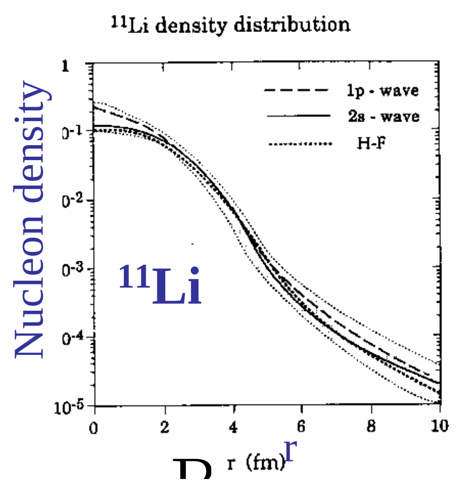
RIBF website

1. Introduction

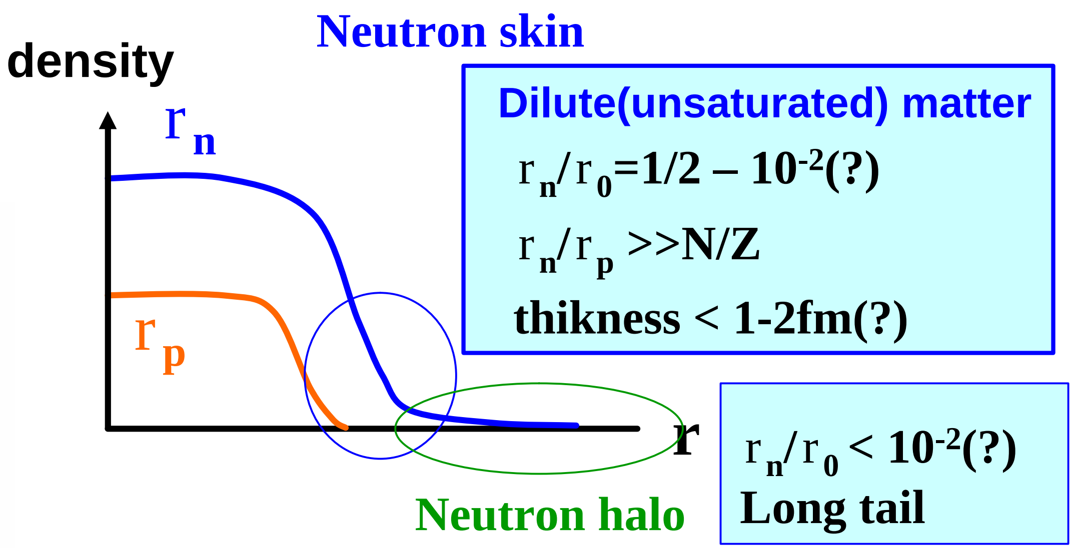
What is the di-neutron correlation?

Exotic features in n-rich nuclei





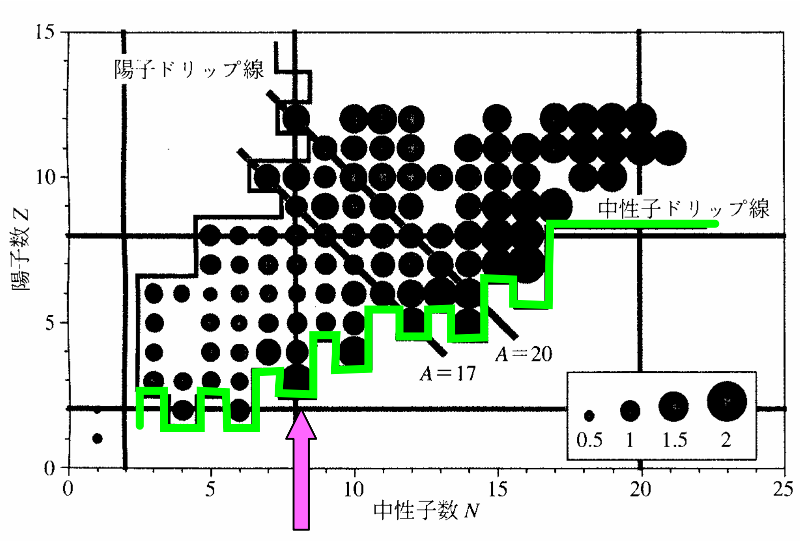
Tanihata et al.
PLB287 (1992)



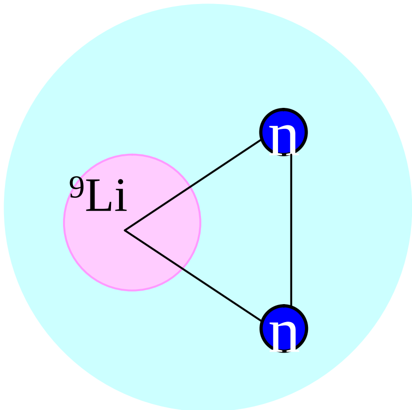
Exotic features in n-rich nuclei (2)

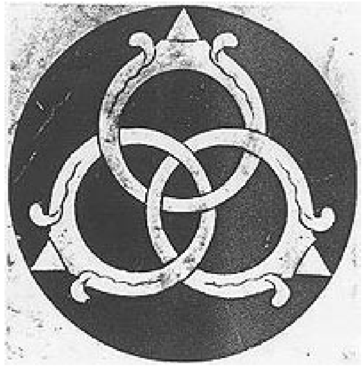
^{11}Li

A. Ozawa et. Al.



A Borromean ring nucleus



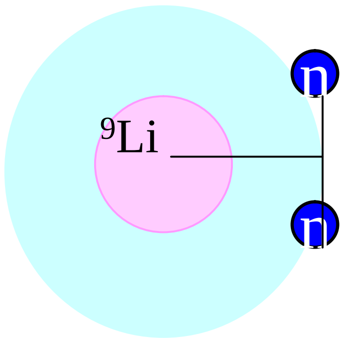


n-n pair correlation is essential !

Exotic features in n-rich nuclei (3)
Di-neutron correlation

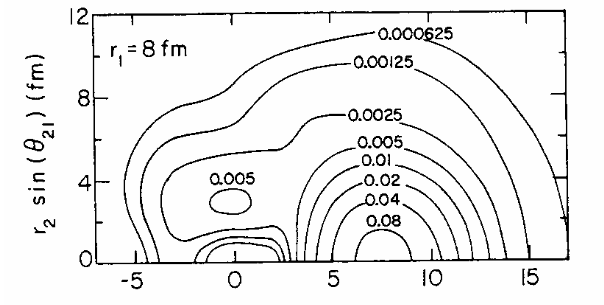
Many predictions since ~late 1980' for 2n-halo nuclei ^{11}Li and ^{6}He

Di-neutron config.

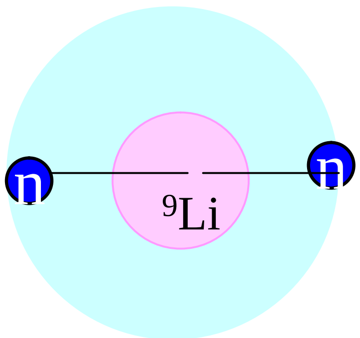


G.F.Bertsch, H.Esbensen, Ann. Phys. 209(1991) 327

Also K. Ikeda. 1992, P.G. Hansen et.al.1987, M.V.Zhukov et al., 1993

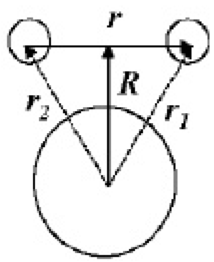


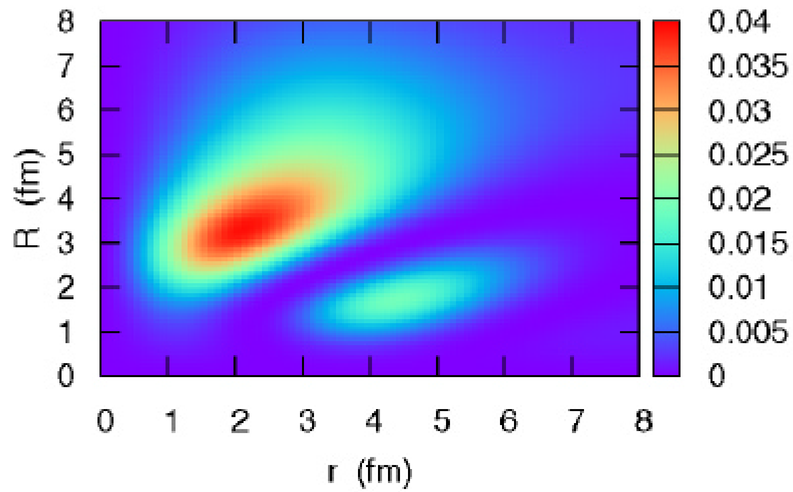
Cigar config.



K.Hagino et al., Phys.Rev. Lett.99, 022506(2007)

Also K. Kato

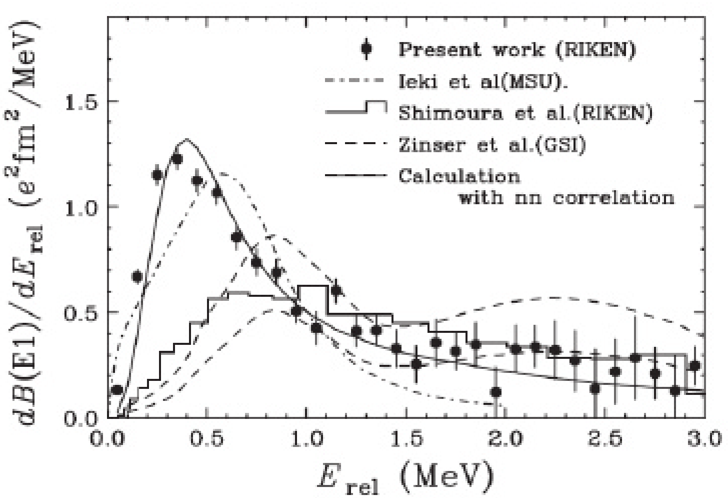


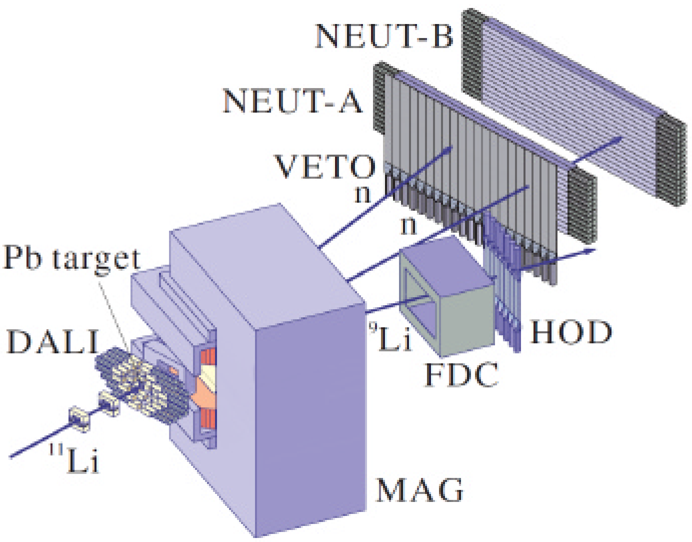


Coulomb breakup and soft dipole excitation

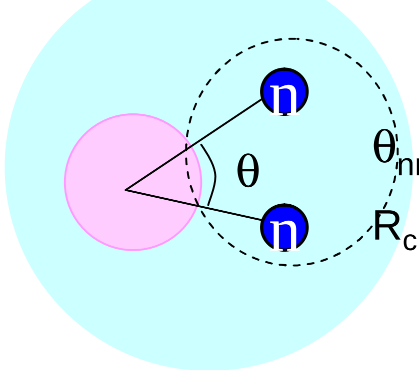
2n halo nuclei, eg. 11Li

New Coulomb break-up exp. on 11Li at RIKEN
Nakamura et al.
PRL30,252502 (2006)





Suggesting a spatial di-neutron correlation

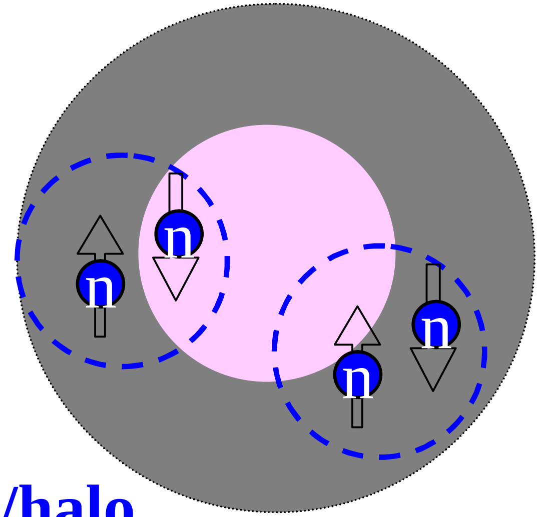


$\theta_{nn}=48^{+14}_{-18}$ deg

$R_{c,2n}$=5.01+-0.32 fm

Condensation of di-neutrons ?
Possible new feature of the pair correlation

Di-neutrons = spatially compact pairs
= boson like objects in nuclei

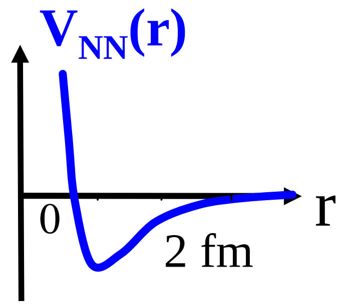


skin /halo

Several weakly bound neutrons in medium mass n-rich nuclei

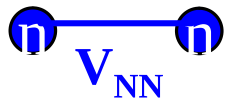
Many-body correlation like boson condensation ???
(BEC like)

ABC's of nuclear pair correlation

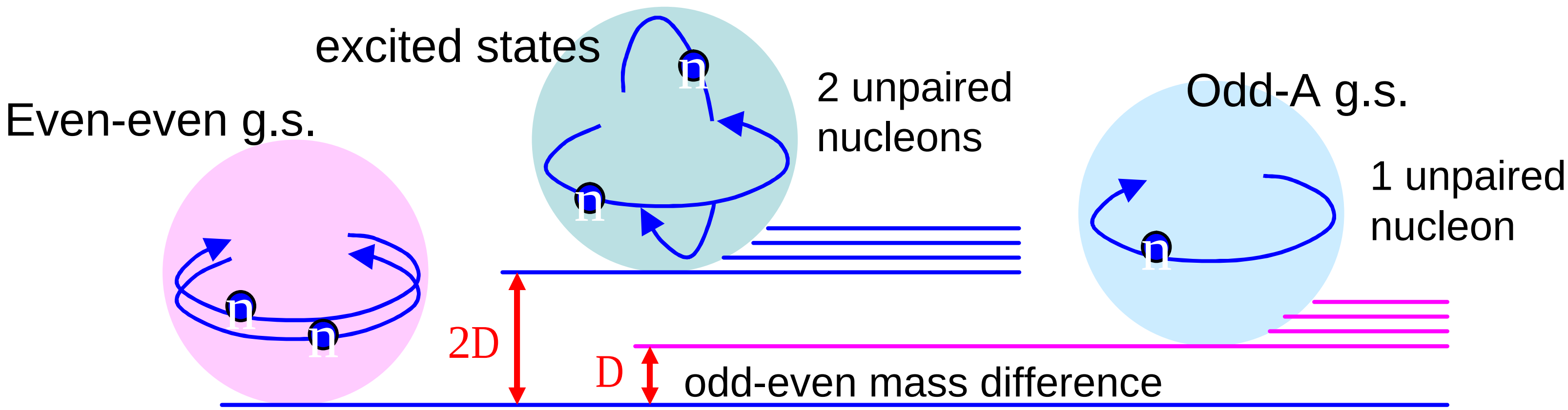


The origin: strong attraction of the nuclear force + "something"

Two neutrons in a nucleus form a pair coupled to $J^p=0^+$



Standard pair correlation in stable nuclei

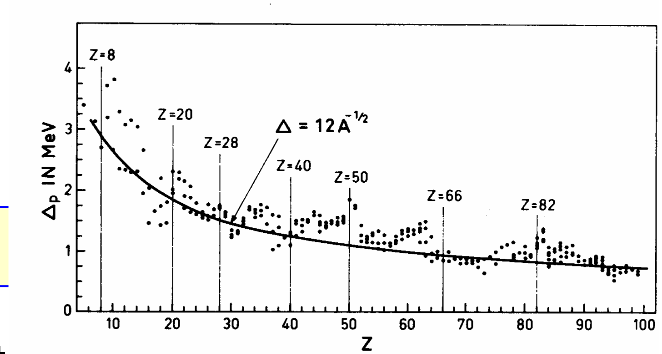


Pairing gap D = binding energy of the coupled pairs

Typical size of the pairing gap is 1-3MeV

D = 12/sqrt{A} MeV

D << e_F much smaller than Fermi energy ~40MeV



Weak coupling pairing => hard to expect the di-neutron correlation

2. Origin of the di-neutron correlation

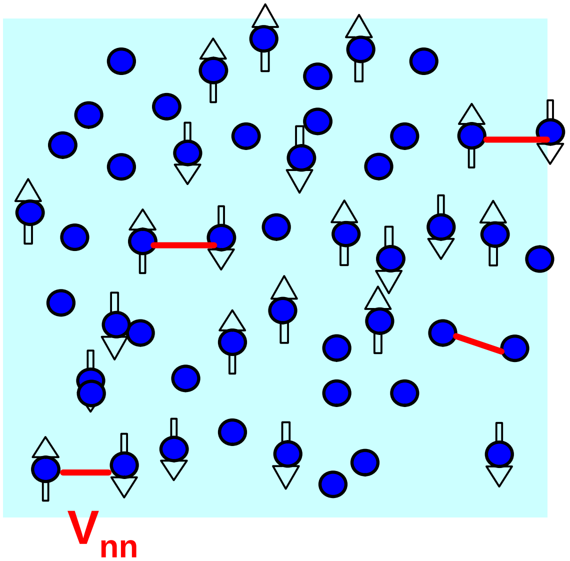
Strong coupling pairing and
Bose-Einstein condensation

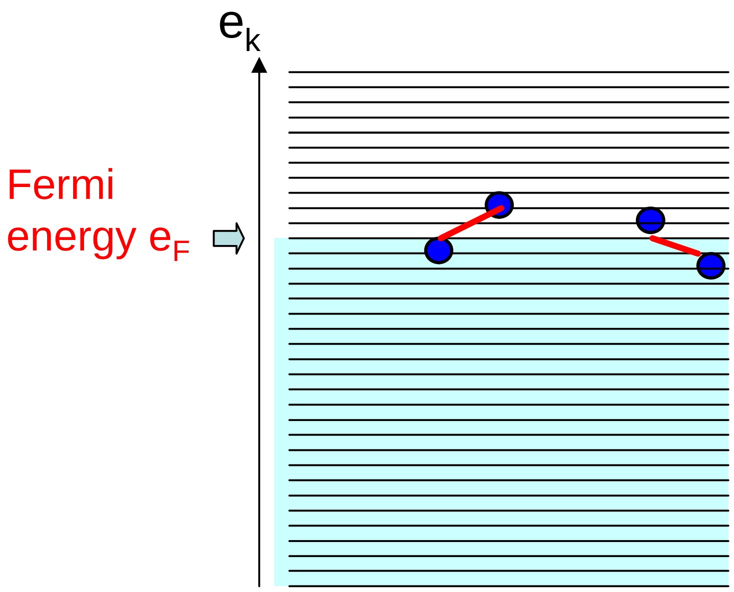
Dilute neutron matter

Pairing correlation in uniform matter

Electrons in metal

Neutrons in a neutron star Etc.





Single-particle states

$$e_k = \frac{\hbar^2 k^2}{2m}$$

$$\varphi_{k\uparrow,\downarrow} = e^{ikr}$$

momentum **k**

Fermi momentum k_F

$$e_F = \frac{\hbar^2 k_F^{\ 2}}{2m}$$

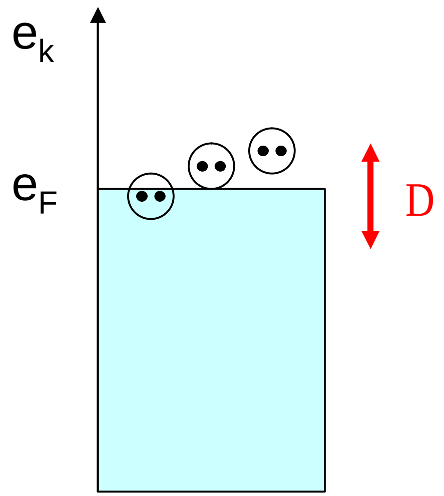
Density

$$\rho = 2\int_0^{k_F} d\vec{k} = \frac{k_F^{\ 3}}{3\pi^2}$$

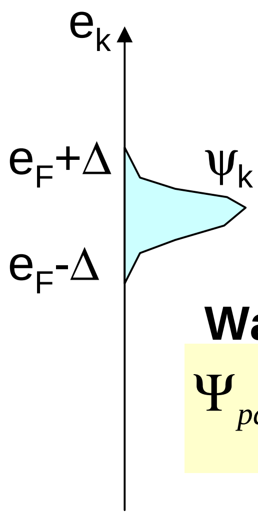
Pairing correlation in uniform matter

Bardeen-Cooper-Schrieffer 1957 Theory of superconducting metal

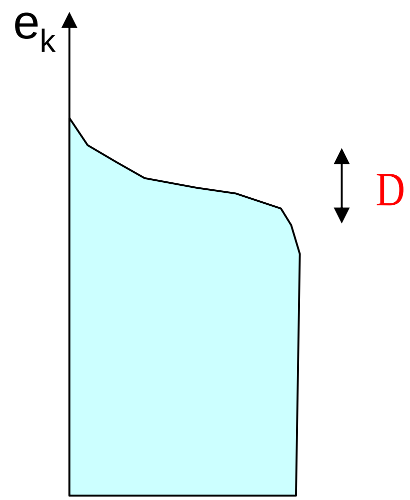
Fermi sea + correlated pairs (Cooper pairs)



pairing gap Δ ~ binding energy of the correlated pair



Occupation probability of s.p. orbits



Wave function of pairs

$$\Psi_{pair}(\vec{r}_1,\vec{r}_2)=\sum_k e^{i\vec{k}(\vec{r}_1-\vec{r}_2)}\psi(k)$$

Pairs are formed by employing the states **k** and **-k** within an interval **$e_F-\Delta < e_k < e_F+\Delta$**

Weak coupling pairing

Small pairing gap D<<e$_F$, small binding energy of the correlated pair

a narrow interval energy interval e$_F$ – D < e$_k$ < e$_F$ + D
S.p. states in a momentum interval kF-dk < k < kF +dk

$$\Delta = \frac{de_k}{dk} dk = \frac{\hbar^2 k_F}{m} dk$$

Size of Cooper pair (coherence length)

$$x \approx \frac{1}{dk} = \frac{\hbar^2 k_F}{m\Delta}$$

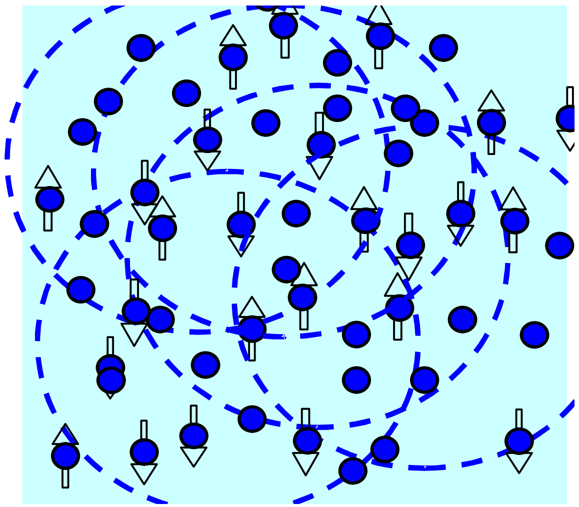
Cf. average interparticle distance

$$d = r^{-1/3} \approx 3k_F$$

Pair size / interparticle distance

$$\frac{x}{d} \approx \frac{\hbar^2 k_F^2}{2m} \approx \frac{e_F}{\Delta} >> 1$$

Pair size is much larger than the average interparticle distance



Strong coupling pairing & Bose-Einstein condensation

If the interaction is sufficiently strong, then the Fermion pairs form tightly bound states

Size of pairs

$$x = \frac{\hbar}{\sqrt{2mE_{bind}}}$$

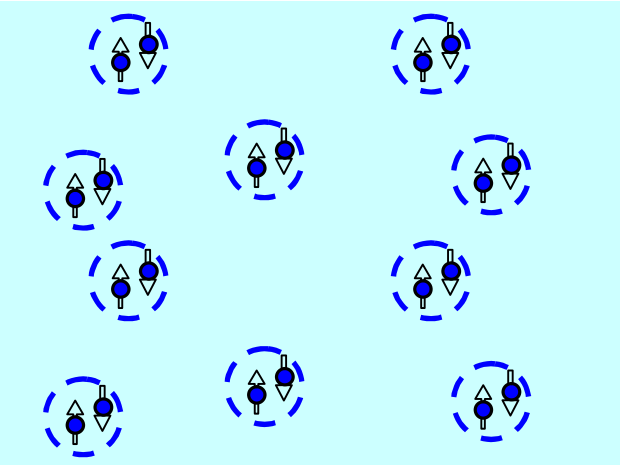
E_{bind}= binding energy of a pair

Pair size / interparticle distance

$$\frac{x}{d} \approx \sqrt{\frac{e_F}{E_{bind}}} < 1$$

For $E_{bind} > e_F$

If the pair size x is smaller than interparticle distance d, the pairs are almost like "boson"s.



The g.s. is a Bose-Einstein condensation of "bosons"

BCS-BEC crossover

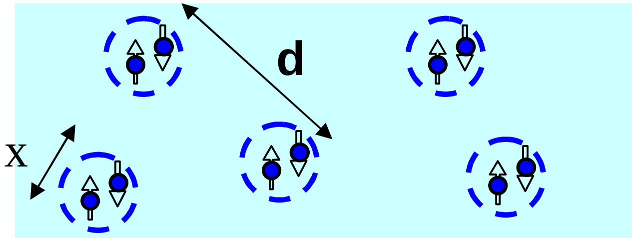
Leggett 1980, Nozieres & Schmitt-Rink 1985

BEC

Interaction: **Strong**

In free space: **Bound pair (boson) is already formed**

In medium: **Bound pairs (bosons) condensate**



$x/d<<1$, $D/e_F>1$

Crossover

D/e_F

1.3 ~ 0.2

x/d

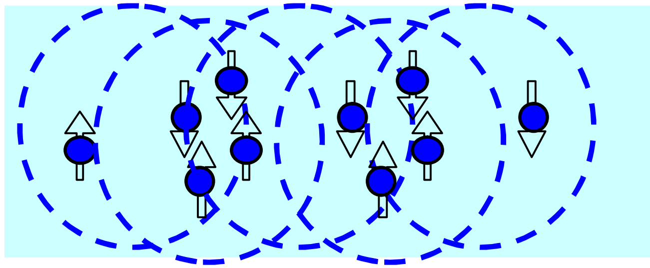
0.2~ 1.10

BCS

Interaction: **Weak**

In free space: **No bound pair is formed**

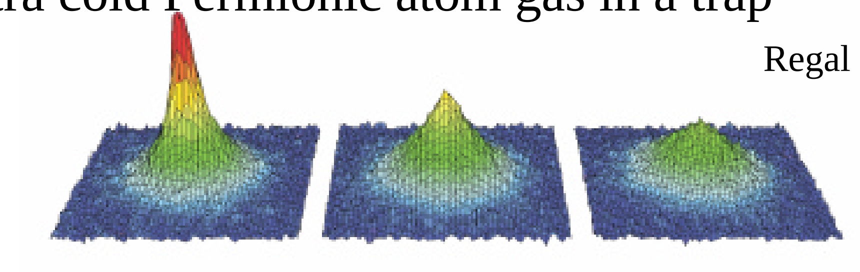
In medium: **Bound pairs (Cooper pairs) are formed, then condensate**



$x/d>1$, $D/e_F<<1$

Observed in ultra cold Fermionic atom gas in a trap

Regal et at. PRL 92(2004)040403



Demonstration of BCS-BEC crossover

A solvable BCS-BEC crossover model Engelbrecht 1997, Marini et al 1998

Fermions in uniform matter

An attractive contact interaction **$v(r-r')=-V_0\, d(r-r')$**

V_0 is related to the scattering length **a** of NN interaction

Dimensionless parameter **$1/k_F a$**

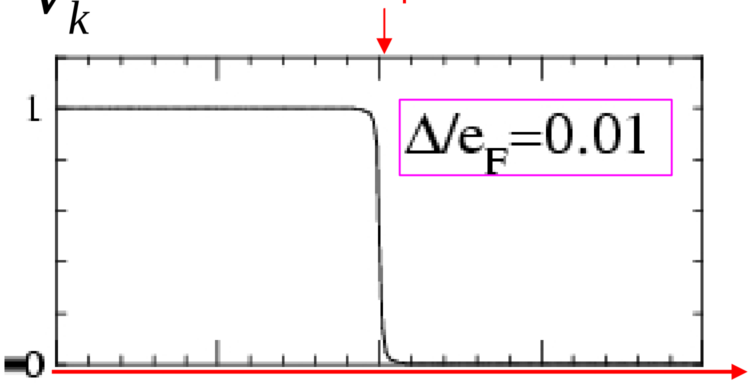
The force strength **V_0** is varied

From weak coupling to strong coupling

Variation of the pair structure

Occupation probability

$v_k^{\;2}$



Single-particle energy e_k

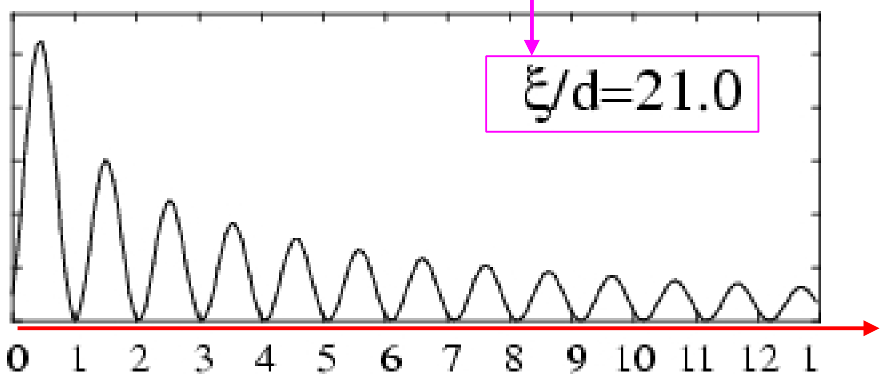
0

Pair wave function

$r^2|\Psi_{pair}(r_{12})|^2$

r.m.s. radius of pair

$x = \int_{r<r_d} |\Psi_{pair}(\vec{r})|^2 \, r^2 d\vec{r}$

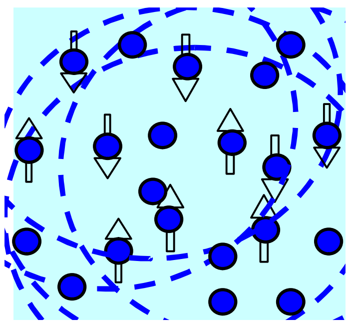


Relative distance r_{12}/d

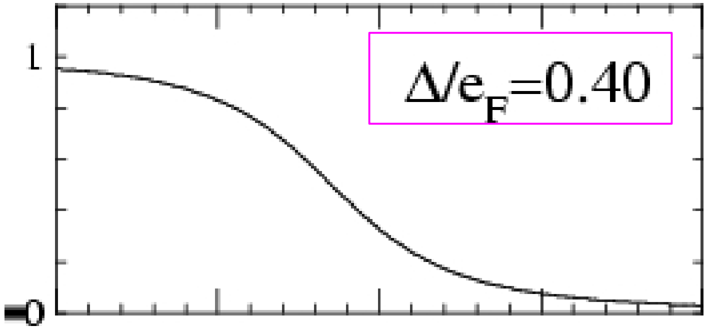
in unit of the average interparticle distance d

Weak coupling BCS pairing

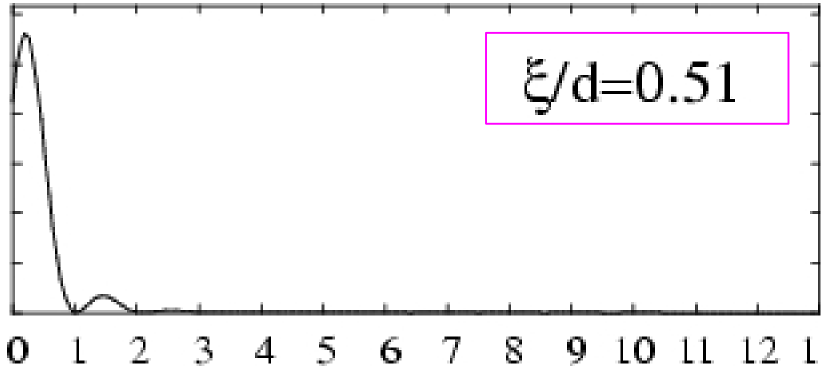
x/d >>1



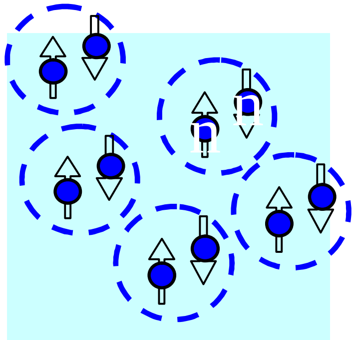
Occupation probability



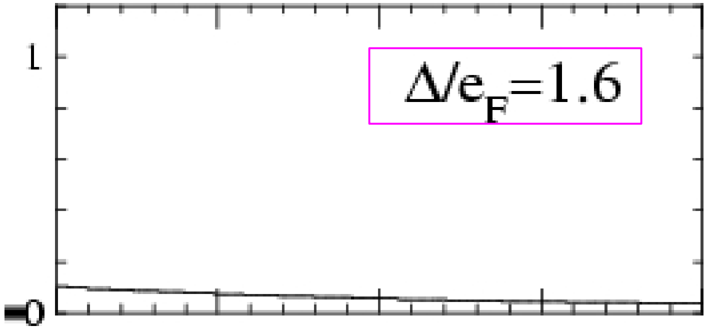
Pair wave function



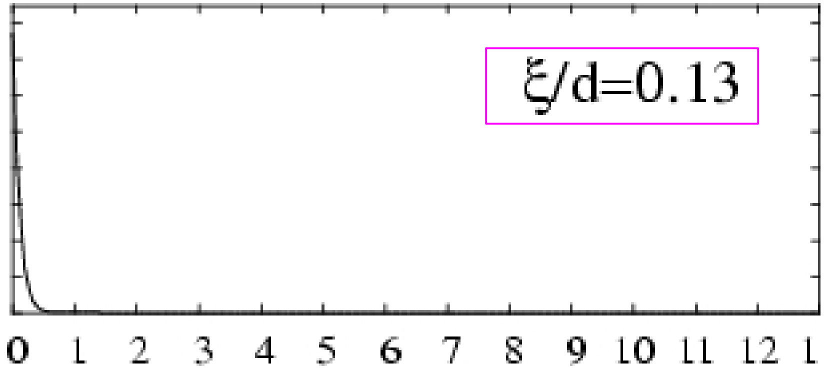
Crossover region of weak BCS and BEC



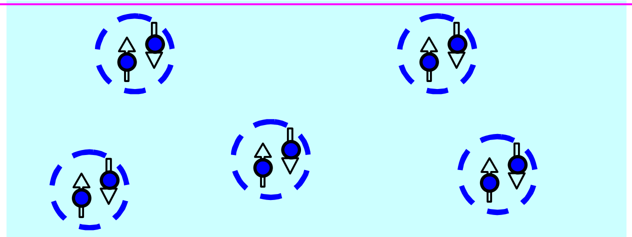
Occupation probability



Pair wave function



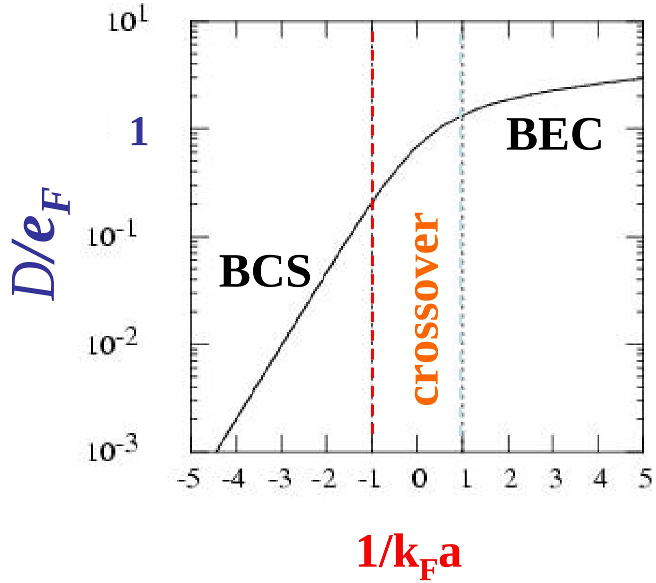
Bose-Einstein Condensation



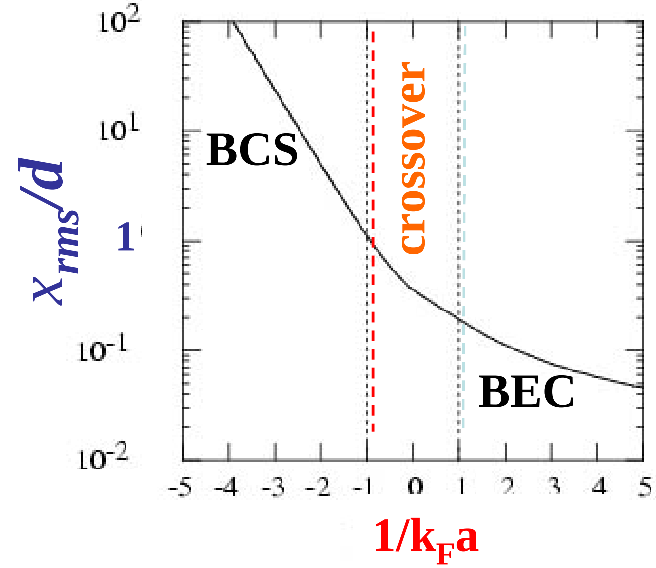
Signatures of BCS-BEC crossover

	Weak BCS	crossover region	BEC
D/e_F	< 0.2	0.2 ~ 1.3	> 1.3
x/d	> 1.1	1.1 ~ 0.2	< 0.2
$1/k_Fa$	< -1	-1 ~ 1	> 1

Pair gap vs Fermi energy

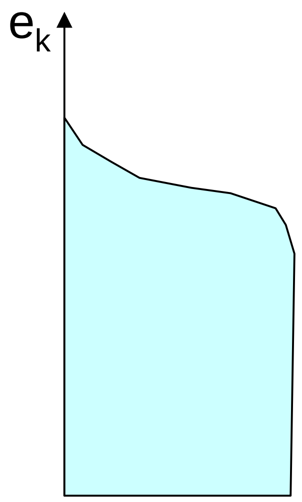


r.m.s. radius vs av. inter-particle distance



Variational BCS theory for the pair correlated Fermi system

Bardeen, Cooper, Schrieffer 1957 ~



pair correlated variational state

$$\left|\Psi\right\rangle = \prod_k (u_k + v_k c^+_{k\uparrow} c^+_{-k\downarrow}) \left|0\right\rangle$$

Time reversal orbits **k** and **-k** are pairwise occupied with amplitudes $\mathbf{v_k}$ (& unoccupied amplitudes $\mathbf{u_k}$)

Hamiltonian

$$H = \sum_i (e_i - \lambda) c_i^+ c_i + \tfrac{1}{4} \sum_{ij} \left\langle i\bar{i} \right| v \left| j\bar{j} \right\rangle c_i^+ c_{\bar{i}}^+ c_{\bar{j}} c_j$$

$$i = k\uparrow \qquad \bar{i} = -k\downarrow$$

Energy functional

$$\left\langle H \right\rangle = \sum_i (e_i - \lambda) \left\langle c_i^+ c_i \right\rangle + \tfrac{1}{4} \sum_{ij} \left\langle i\bar{i} \right| v \left| j\bar{j} \right\rangle \left\langle c_i^+ c_{\bar{i}}^+ \right\rangle \left\langle c_{\bar{j}} c_j \right\rangle$$

Functional of density matrix $\left\langle c_i^+ c_i \right\rangle$

pair density matrix $\left\langle c_i^+ c_{\bar{i}}^+ \right\rangle, \ \left\langle c_{\bar{j}} c_j \right\rangle$

Condensation of Fermi pairs

Variational BCS theory for the pair correlated Fermi system

Energy variation and minimum

$$0 = d\langle H \rangle = d\langle h \rangle$$

$$\langle H \rangle = \sum_i (e_i - l)\langle c_i^+ c_i \rangle + \tfrac{1}{4}\sum_{ij} \langle i\bar{i}|v|j\bar{j}\rangle \langle c_i^+ c_{\bar{i}}^+ \rangle \langle c_{\bar{j}} c_j \rangle$$

Generalized mean-field Hamiltonian

$$h = \sum_i (e_i - l) c_i^+ c_i + \tfrac{1}{2}\sum_{ij} \langle i\bar{i}|v|j\bar{j}\rangle \left(\langle c_i^+ c_{\bar{i}}^+ \rangle c_j c_{\bar{j}} + c_i^+ c_{\bar{i}}^+ \langle c_j c_{\bar{j}} \rangle \right)$$

$$= \sum_i (e_i - l) c_i^+ c_i + \tfrac{1}{2}\sum_i \Delta_i^* c_i c_{\bar{i}} + \Delta_i c_i^+ c_{\bar{i}}^+$$

Pair potential & pair gap

$$\Delta_i = \frac{\partial \langle H \rangle_{pair}}{\partial \langle c_i^+ c_{\bar{i}}^+ \rangle}$$

Equivalent to solve single-particle states for h

$$\begin{pmatrix} e_i - l & \Delta_i \\ \Delta_i & -(e_i - l) \end{pmatrix} \begin{pmatrix} u_i \\ v_i \end{pmatrix} = E_i \begin{pmatrix} u_i \\ v_i \end{pmatrix}$$

Bogoliubov equation

⇨

Density $\langle c_i^+ c_i \rangle = v_i^2$

Pair density $\langle c_i c_{\bar{i}} \rangle = u_i v_i$

Pair wave fn $\Psi_{pair}(r_{12}) = \sum_k e^{ikr_{12}} u_k v_k$

Dilute neutron matter

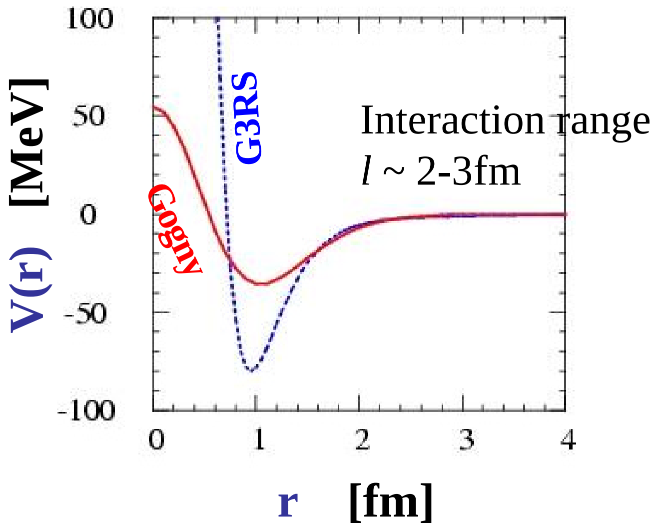
Cf.   inner crust of neutron star

Pairing property in dilute nucleon matter

1. **n-n interaction in ^1S channel**
   Strong attraction at low k: scat. length a=-18.5 fm (exp)
   Less attractive with increasing $k > l^{-1} \sim 0.5$ fm

   Bare force G3RS with core (Tamagaki 1968)

   Gogny force (finite range effective int.)



2. **Solve the variational BCS eq.**

Pairing gap

$$\Delta(k) = \sum_p V(\vec{k} - \vec{p}) \frac{\Delta(p)}{\sqrt{(e_p - m)^2 + \Delta(p)^2}}$$

Cooper pair wave function

$$\Psi_{pair}(r_{12}) = \sum_k e^{ikr_{12}} u_k v_k$$

r.m.s radius of Cooper pair)

$$x_{rms} = \int_{r<r_d} |\Psi_{pair}(\vec{r})|^2 r^2 d\vec{r}$$

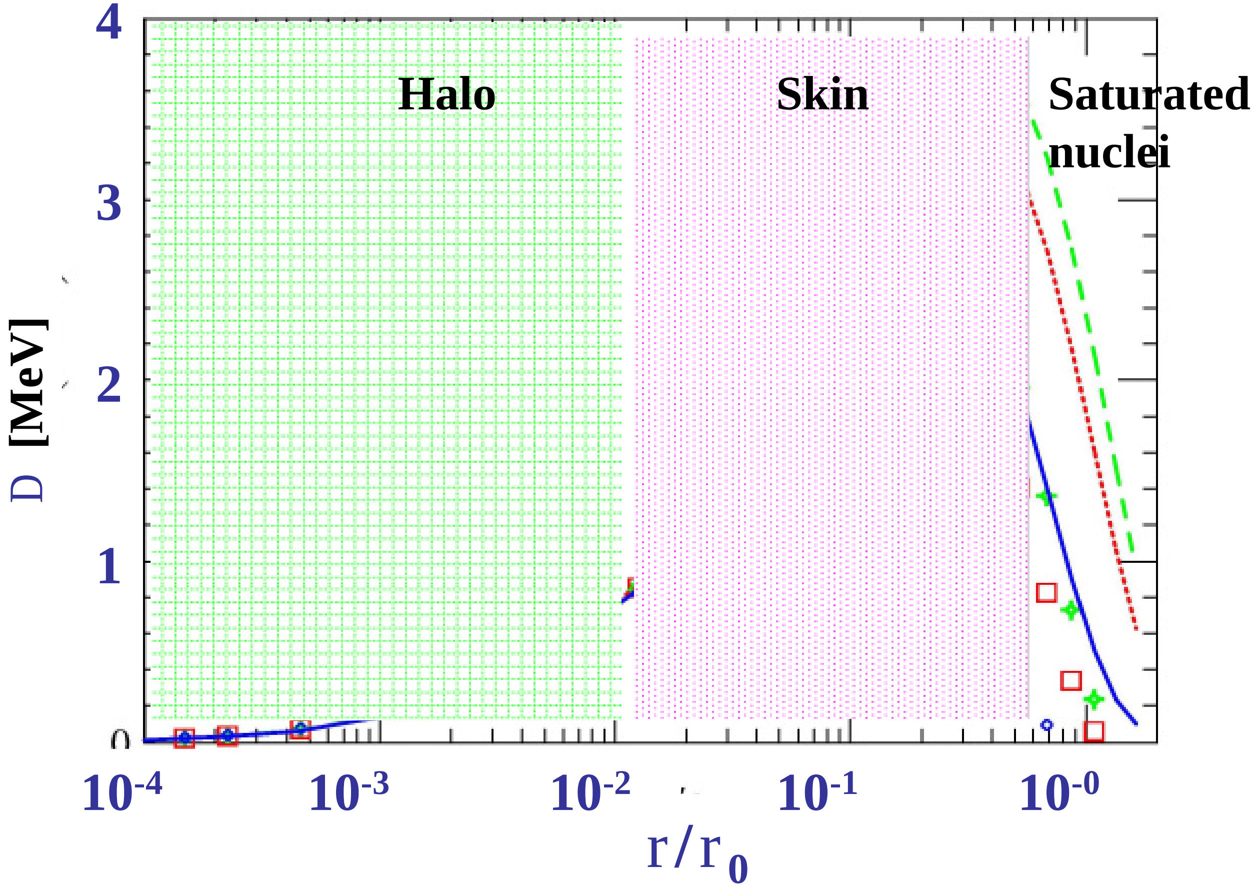
NB. The results at the BCS level is well established

But large ambiguity in the higher order effects: polarization, induced interaction etc..)

For a review, see Dean, Hjorth-Jensen, Rev. Mod. Phys. 75 (2003),
Lombardo and Schulze, Lecture Notes in Physics 578

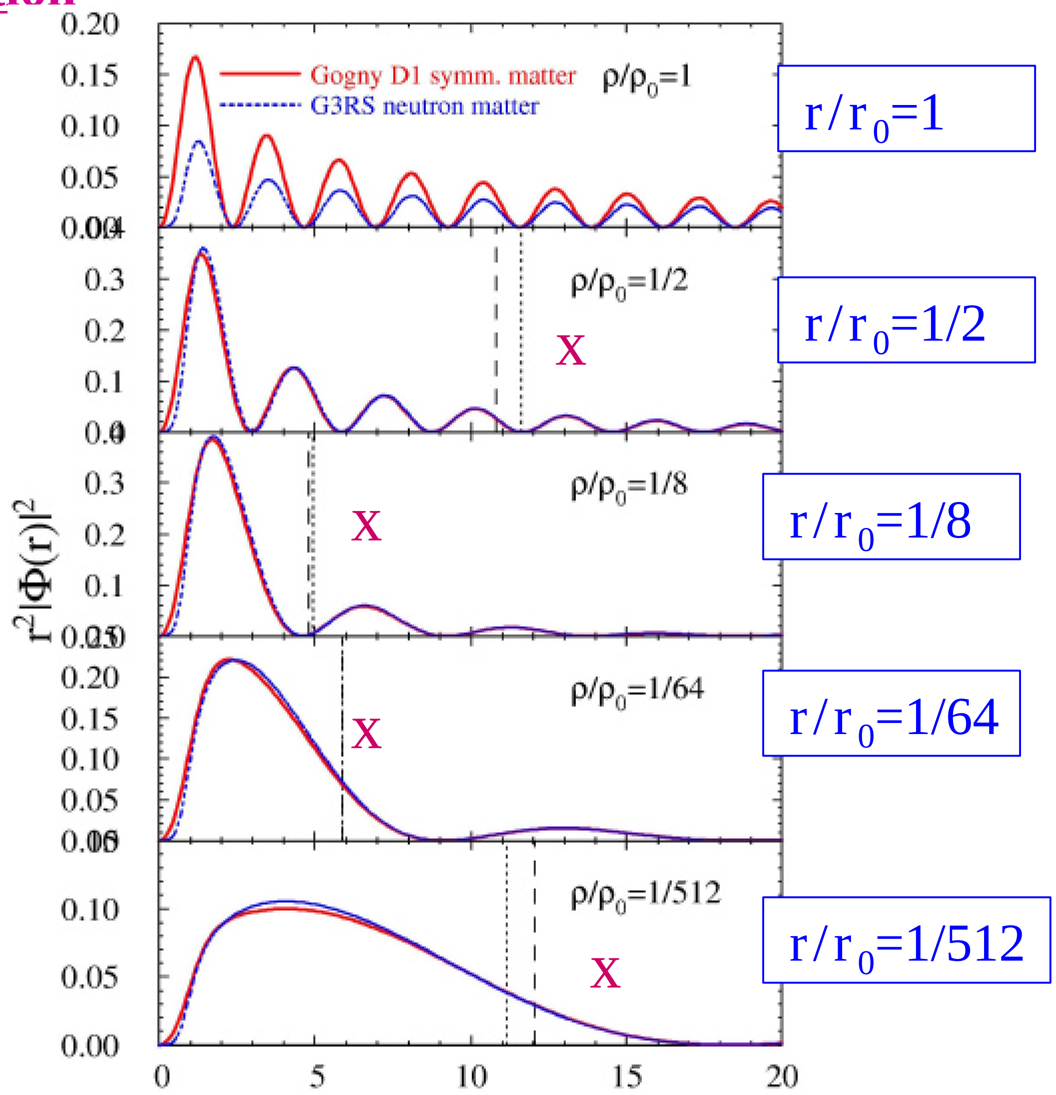
Pairing gap in dilute nucleon matter

Maximum pairing gap at around k_F=0.8 fm^{-1} (ρ/ρ_0=0.1)



Neutron Cooper pair wave function

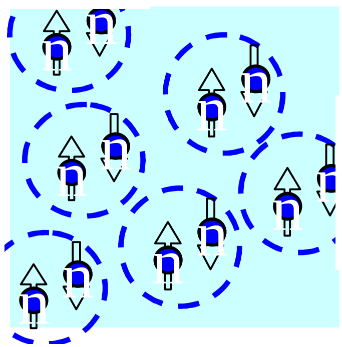
Cooper pair wave function



r (fm) **Relative distance between paired neutrons**

Strong di-neutron correlation at skin & halo density region

r/r_0 < **1/10**

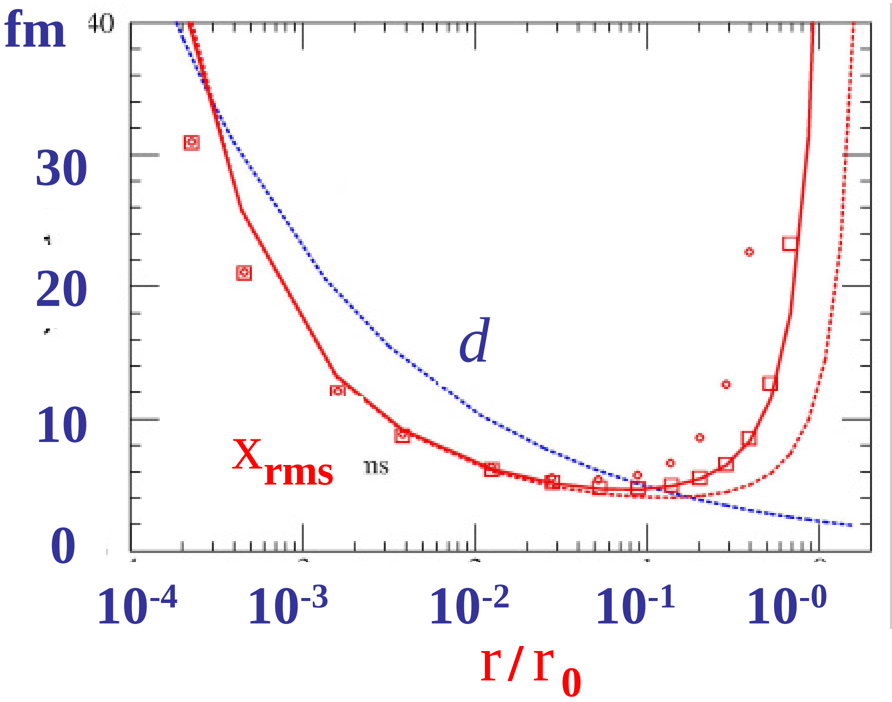


Size of Cooper pair

Size of neutron Cooper pair

r.m.s. radius $x_{rms} = \int_{r<r_d} |\Psi_{pair}(\vec{r})|^2 r^2 d\vec{r}$

inter-neutron distance $d = \rho^{-1/3}$



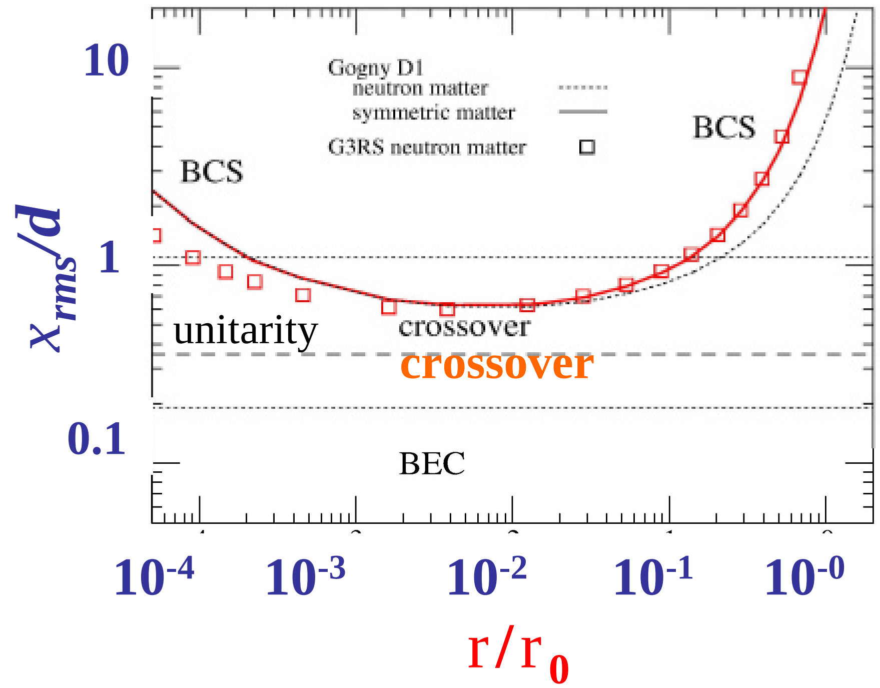
Size smaller than average distance $x<d$ at r/r_0=1/5-10^{-4} (skin and halo)

Small pair size **x~5 fm** at r/r_0=1/5-1/20 (skin)

Strong di-neutron correlation at these densities

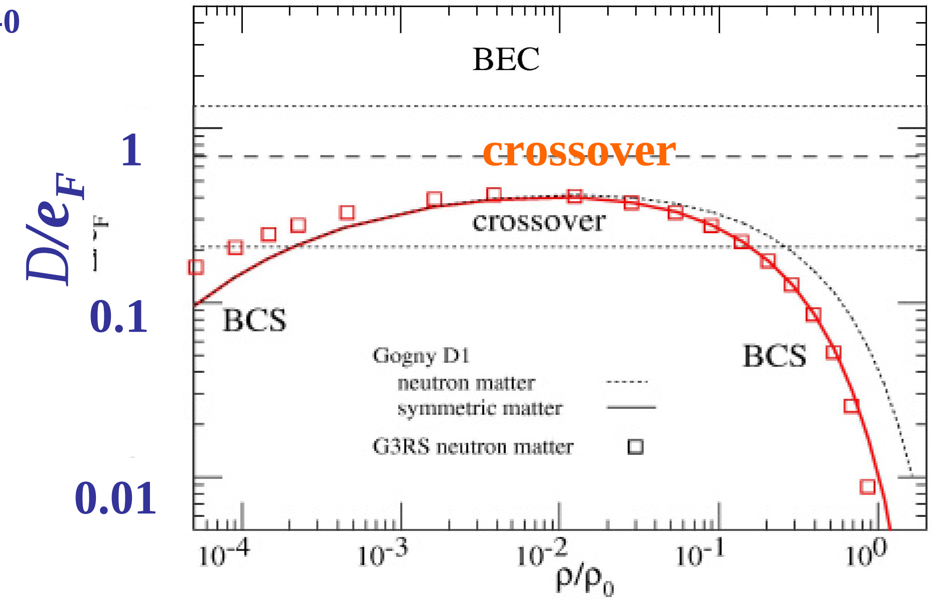
Di-neutron correlation is an BCS-BEC crossover phenomenon

r.m.s. radius vs. inter-neutron distance x/d

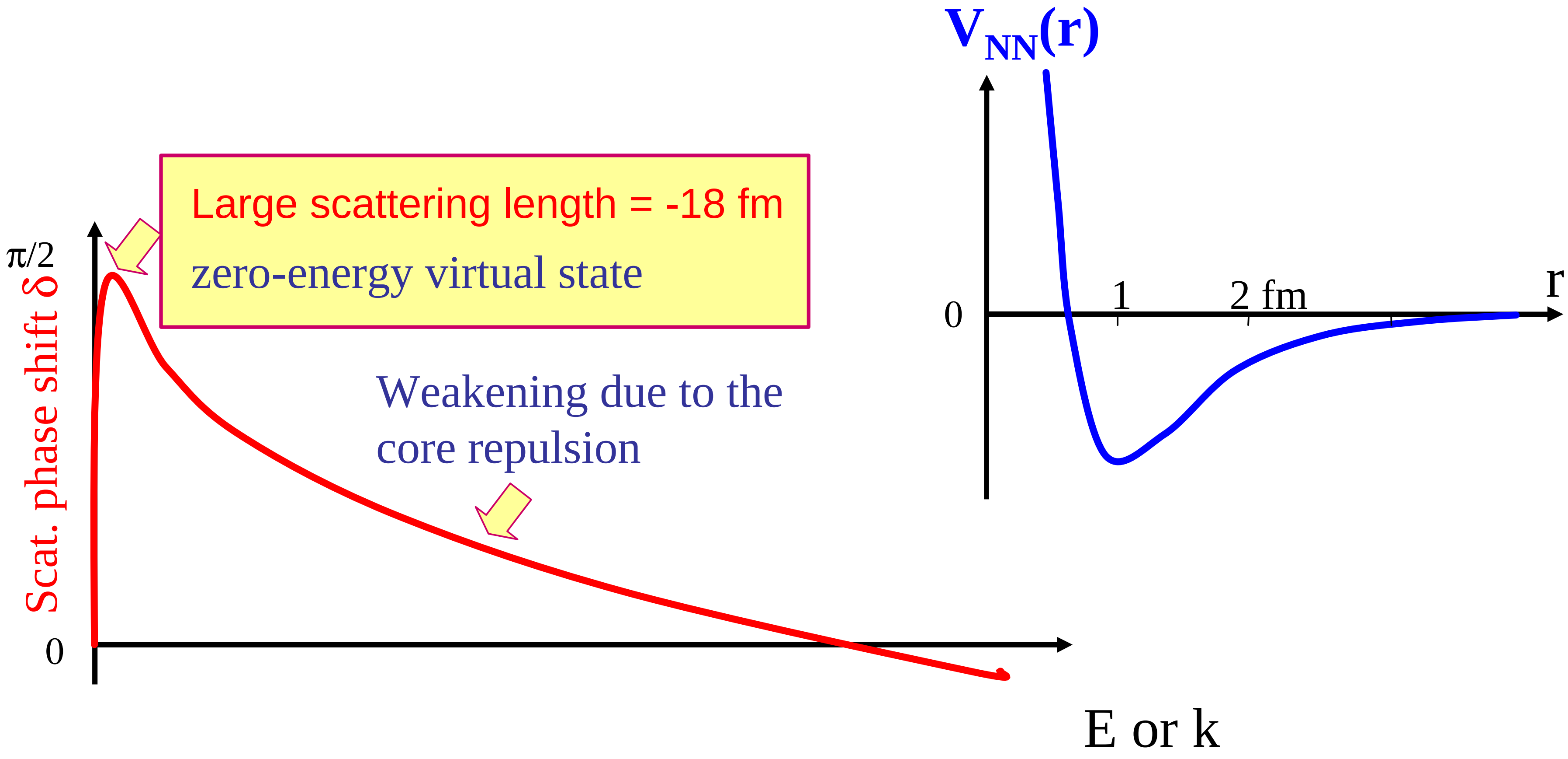


BCS-BEC crossover in a wide interval of densities
r/r_0**=1/5-10**$^{-4}$
At neutron densities in skin and halo

Pair gap vs. Fermi energy D/e$_F$



Stronger interaction at low density



Low density → Low Fermi momentum

→ **Stronger $V_{NN}(k)$**

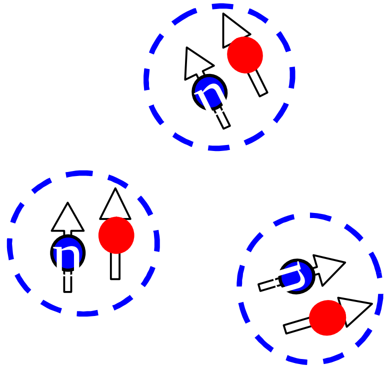
→ **Stronger correlation**

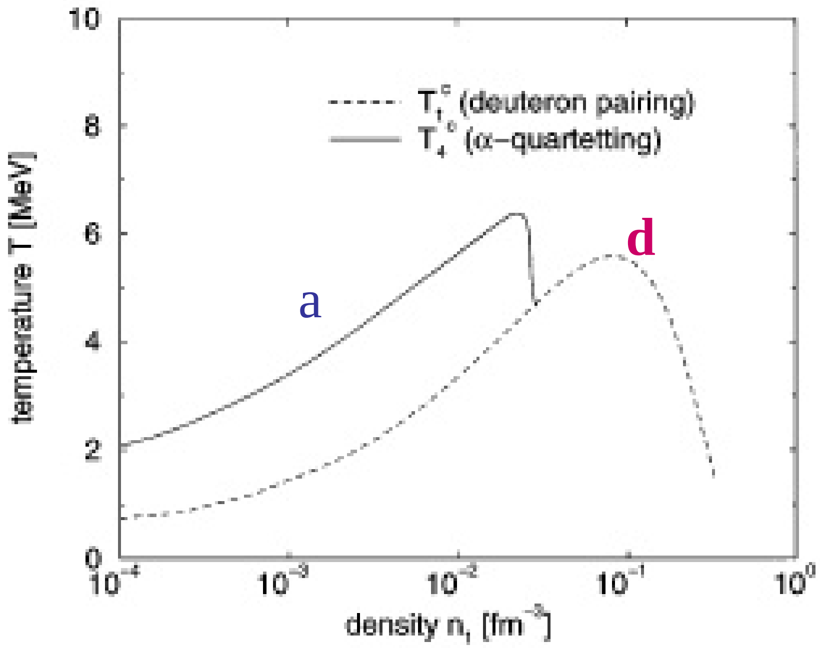
Dilute symmetric matter is more complex: deuteron (pn pair) & alpha condensations

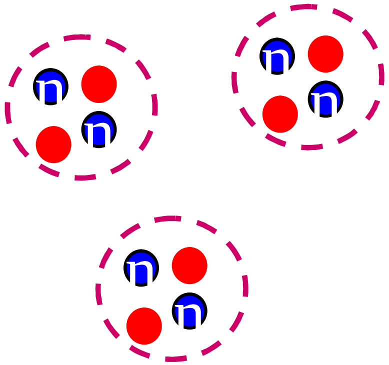
BEC's at low density

$r/r_0 \sim 1\text{-}10^{-1}$ **Deuteron condensation (pn-pairing)**

$r/r_0 < 10^{-1}$ **Alpha condensation (pnpn-quartetting)**

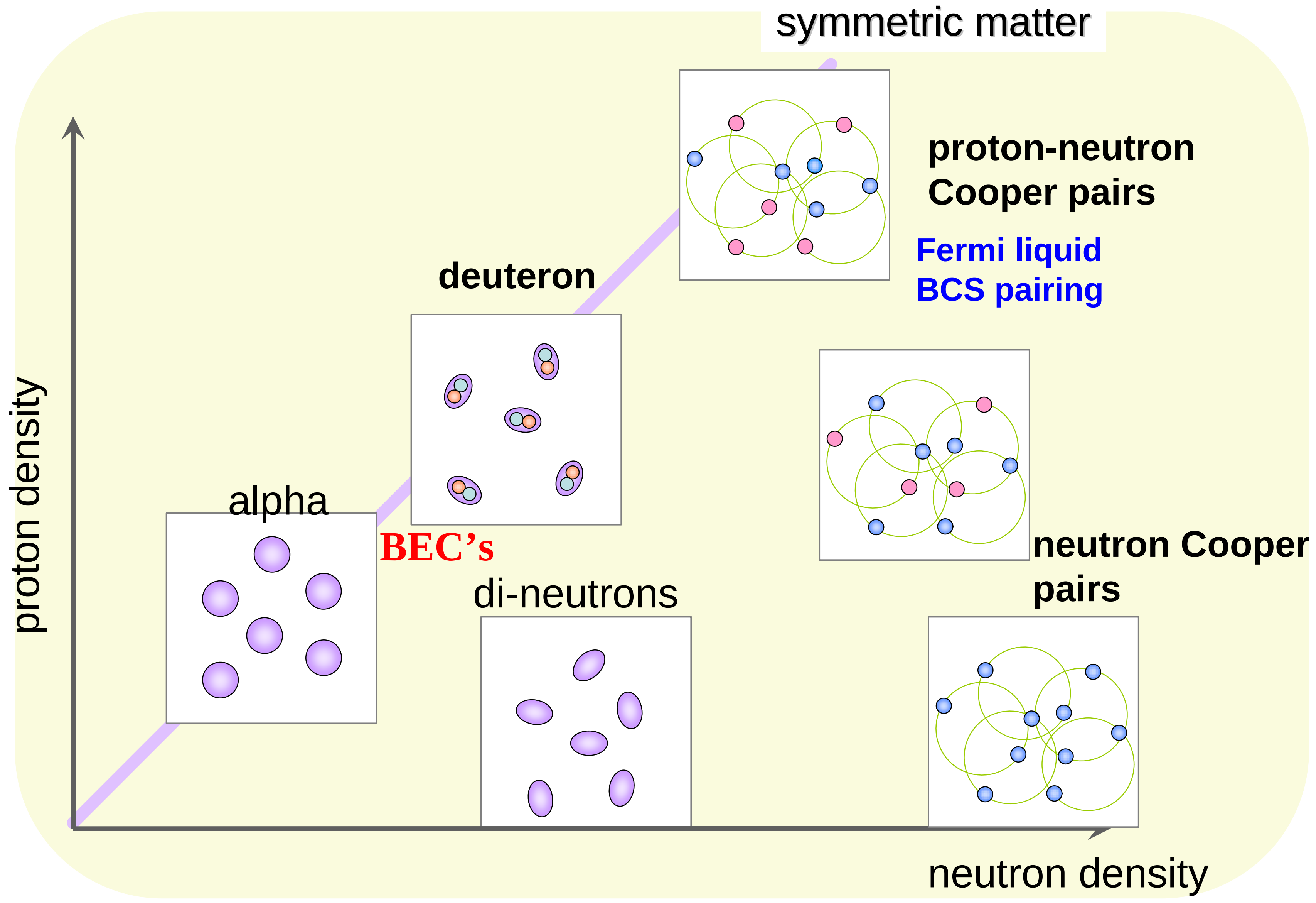






Roepke et al. PRL 1998

Correlations at dilute matter

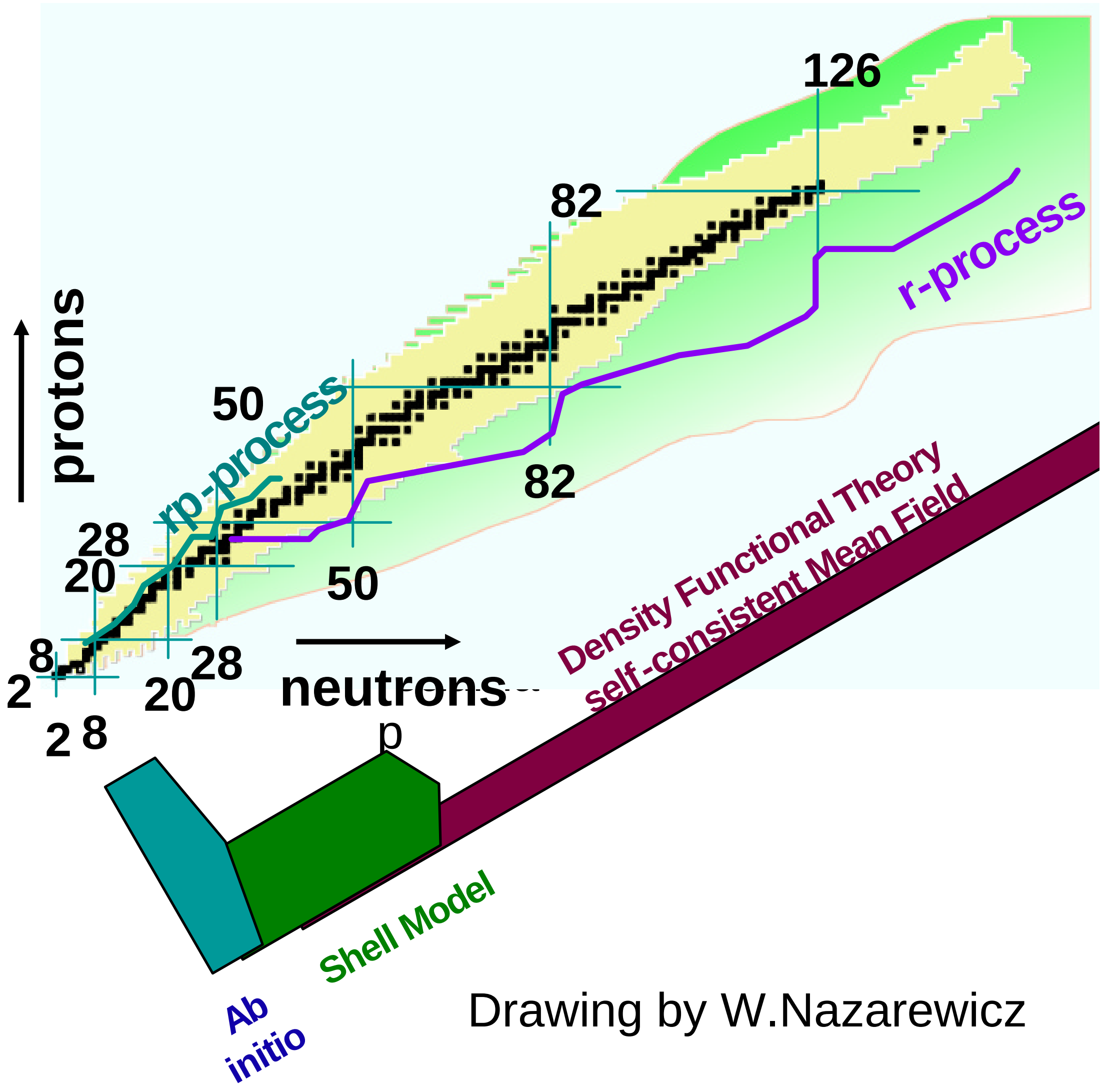


Drawing by Y. Kanada-En'yo

asymmetric matter

3. Di-neutron correlation in medium & heavy mass n-rich nuclei

Selfconsistent mean-field methods: A promising theoretical framework



Drawing by W.Nazarewicz

Hartree-Fock theory

Hartree-Fock-Bogoliubov theory

Density functional theory

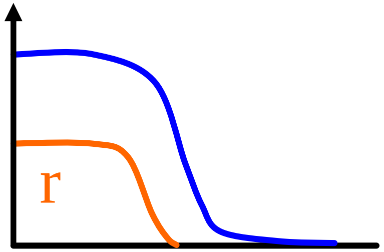
As a review, Bender,Heenen, Reinhard, RMP75 (2003)

Self-consistent mean-field theory & Density functional theory

1. **"Exact" ground state density if proper functional is given**
2. **Valid for both weak & strong coupling**

Hartree-Fock-Bogoliubov in coordinate space

Dobaczewski et al 1984,



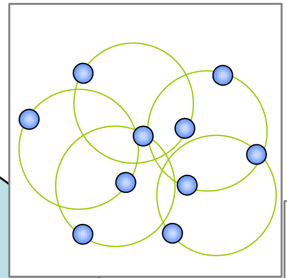
Density

$$r(r) = \sum y_i(r)^* y_i(r)$$

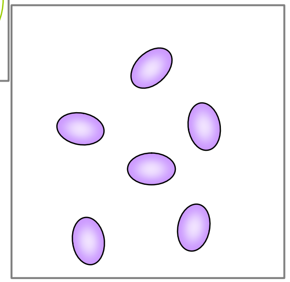
Pair density

$$\tilde{r}(r) = \sum f_i(r) y_i(r)$$

BCS pairing



BEC



Energy functional

$$E[r(r)] + E_{pair}[\tilde{r}(r)]$$

Single-particle states

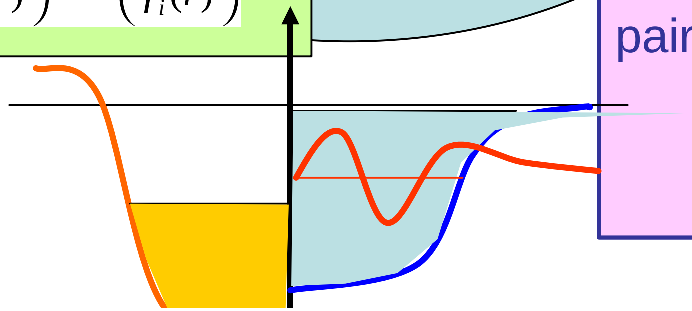
$$\begin{pmatrix} T + \Gamma(r) - l & \Delta(r) \\ -\Delta^*(r) & -T - \Gamma(r) + l \end{pmatrix} \begin{pmatrix} y_i(r) \\ f_i(r) \end{pmatrix} = E_i \begin{pmatrix} y_i(r) \\ f_i(r) \end{pmatrix}$$

mean-potential

$$\Gamma(r) = \frac{\partial E[r]}{\partial r(r)}$$

pair-potential

$$\Delta(r) = \frac{\partial E_{pair}[\tilde{r}]}{\partial \tilde{r}(r)}$$



Skyrme functional and pairing energy functional

The Skyrme functional | Pair correlation energy functional

$$E = E_{Skyrme}[\rho, \vec{\nabla}\rho, \Delta\rho, \tau, \vec{j}, \vec{s}, \overleftrightarrow{J}] + E_{pair}[\rho, \tilde{\rho}_+, \tilde{\rho}_-]$$

Density and derivatives

Kinetic energy density

Current density

Spin density

Spin orbit tensor

Pair density

Parameter sets

SIII

SkM*

SLy4, etc

As a review, Bender,Heenen, Reinhard, RMP75 (2003)

$$\begin{aligned}
E_{Skyrme} &= \int d\boldsymbol{r}\mathcal{H}(\boldsymbol{r}) \\
\mathcal{H}(\boldsymbol{r}) &= \frac{\hbar^2}{2m}\tau(\boldsymbol{r}) + B_1\rho^2(\boldsymbol{r}) + B_2\sum_q \rho_q^2(\boldsymbol{r}) \\
&+ B_3[\rho(\boldsymbol{r})\tau(\boldsymbol{r}) - \boldsymbol{j}^2(\boldsymbol{r})] + B_4\sum_q[\rho_q(\boldsymbol{r})\tau_q(\boldsymbol{r}) - \boldsymbol{j_q}^2(\boldsymbol{r})] \\
&+ B_5\rho(\boldsymbol{r})\Delta\rho(\boldsymbol{r}) + B_6\sum_q \rho_q(\boldsymbol{r})\Delta\rho_q(\boldsymbol{r}) \\
&+ B_7\rho^{\alpha+2}(\boldsymbol{r}) + B_8\rho^\alpha(\boldsymbol{r})\sum_q \rho_q^2(\boldsymbol{r}) \\
&+ B_9\{(\rho(\boldsymbol{r})\nabla\cdot\boldsymbol{J}(\boldsymbol{r}) + \boldsymbol{j}(\boldsymbol{r})\cdot\nabla\times\boldsymbol{\rho}(\boldsymbol{r})) + \sum_q(\rho_q(\boldsymbol{r})\nabla\cdot\boldsymbol{J}_q(\boldsymbol{r}) + \boldsymbol{j}_q(\boldsymbol{r})\cdot\nabla\times\boldsymbol{\rho}_q(\boldsymbol{r}))\} \\
&+ B_{10}\boldsymbol{\rho}^2(\boldsymbol{r}) + B_{11}\sum_q \boldsymbol{\rho}_q^2(\boldsymbol{r}) + B_{12}\rho^\alpha(\boldsymbol{r})\boldsymbol{\rho}^2(\boldsymbol{r}) + B_{13}\rho^\alpha(\boldsymbol{r})\sum_q \boldsymbol{\rho}_q^2(\boldsymbol{r})
\end{aligned}$$

Pair density functional,   less known

Pairing interaction / Pairing energy functional = Density Dependent Delta Interaction

$$E_{pair,n} = V_n[\rho_n]\left|\tilde{\rho}_n(r)\right|^2 \qquad V_{pair,n}(\vec{r},\vec{r}') = V_n[\rho_n]\delta(\vec{r}-\vec{r}')$$

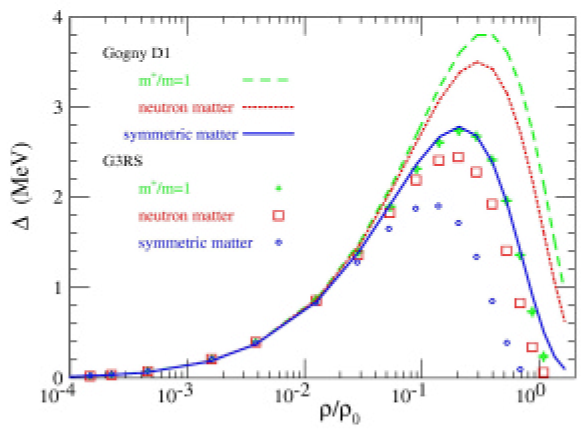
Density-dependent strength

$$V_n[\rho_n] = V_0\left(1 - h\left(\rho_n(\vec{r})/0.08\right)^{0.59}\right)$$

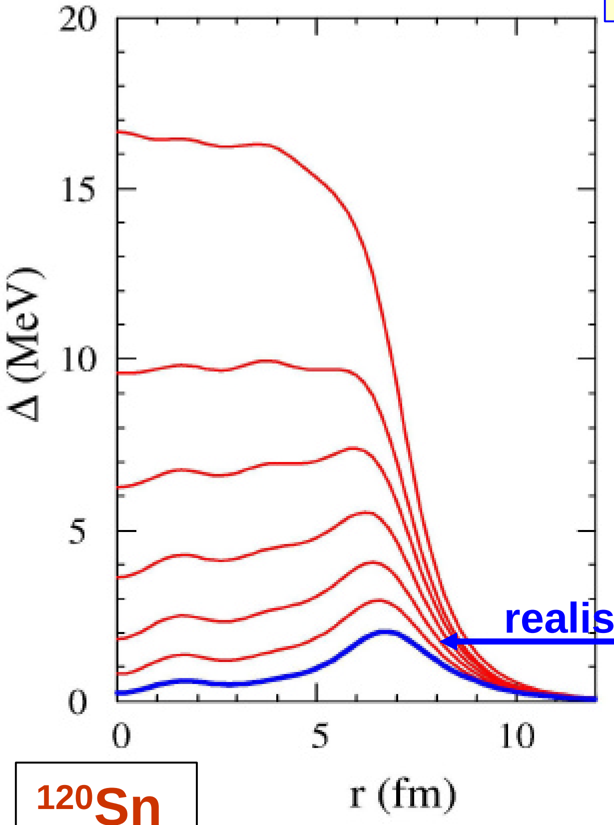
Esbensen-Bertsch 1991
Garrido etal 1999,
Matsuo 2006

$^1S_0$ scattering length
a=-18fm

Pairing gap in neutron matter (bare force BCS)



Neutron pair potential D(r)

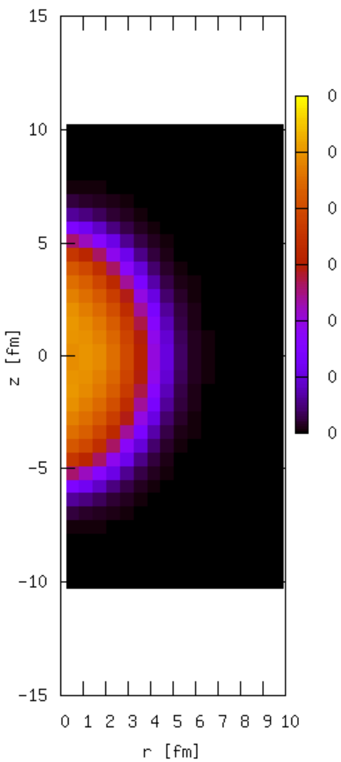


^{120}Sn

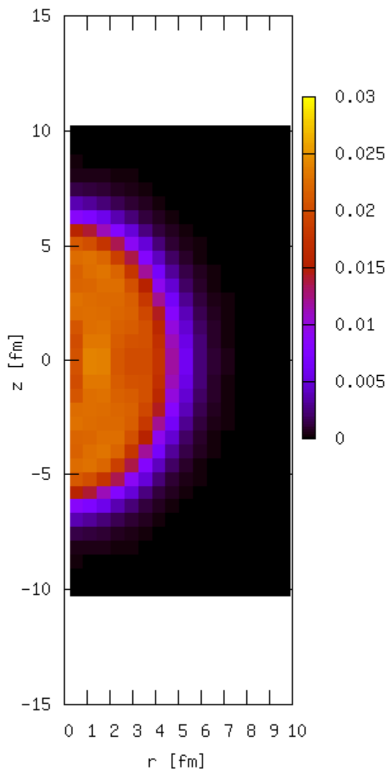
h=**0**
η=0.3
η=0.5
h=**0.71** Reproduce D_{Sn}~1.2M
η=0.84 Reproduce $D_{matter}^{bare\text{-}force}$

An example: ^{62}Cr

Neutron density



Neutron pair density



Pairs in ^{84}Ni
6 weakly bound neutrons above N=50

Two body correlation function

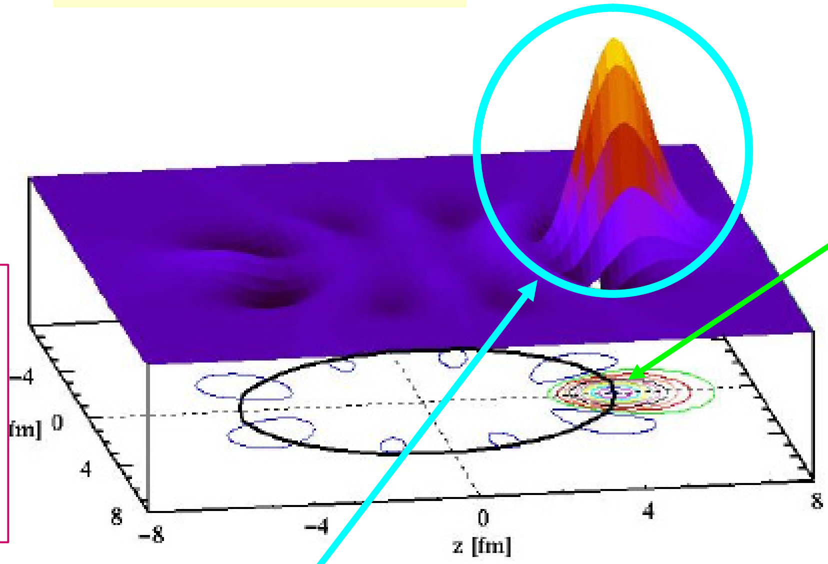
$$r_2^{corr}(\vec r'\uparrow;\vec r\downarrow)=\sum_{i\neq j} d(\vec r-\vec r_i)d_{s_i\uparrow}d(\vec r'-\vec r_j)d_{s_j\downarrow}-r_1(\vec r'\uparrow)r_1(\vec r\downarrow)$$

$$\approx|\Psi_{pair}(\vec r\uparrow,\vec r'\downarrow)|^2$$

Pair wave function

^{84}Ni

Skyrme-Hartree-Fock-Bogoliubov calc. with SLy4 & mix-type DDDI

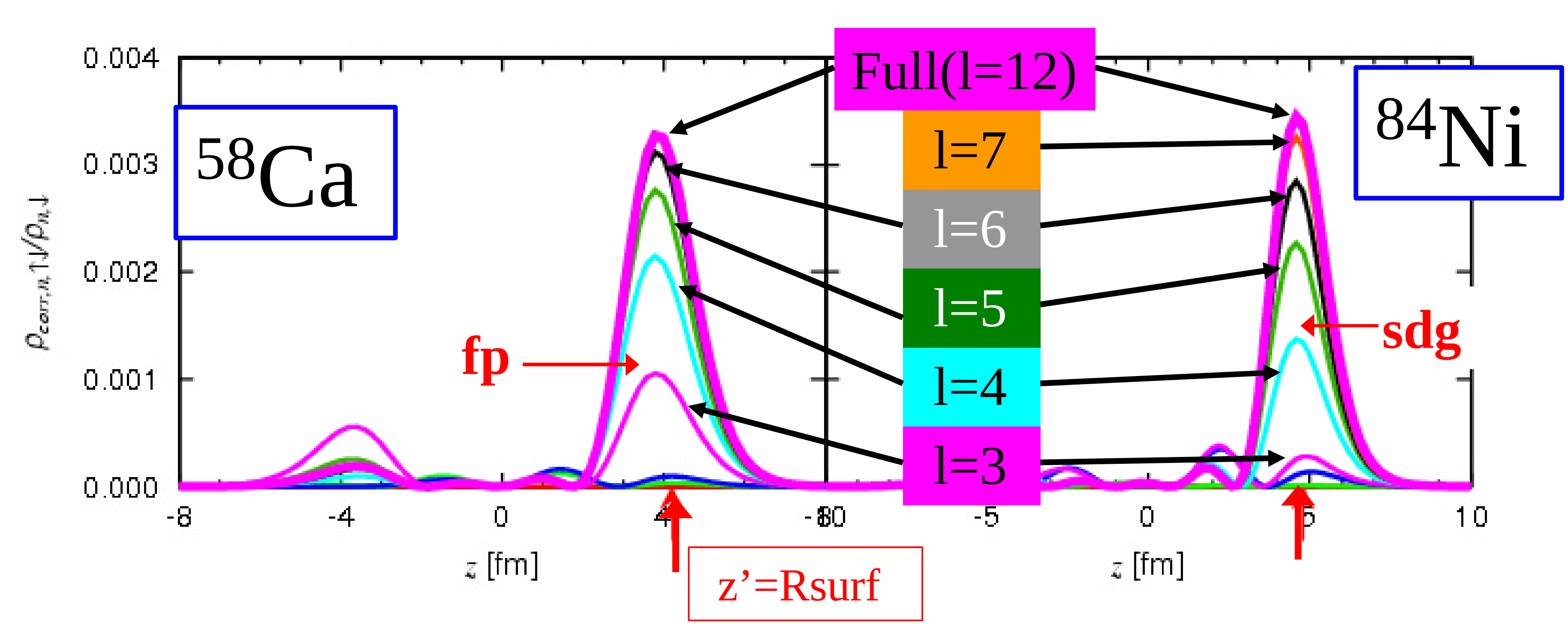


one neutron fixed at $\vec r'$

Strong di-neutron correlation

Strongly correlated at short relative distances |r-r’| < 2-3fm

Di-neutron correlation and high-L orbits

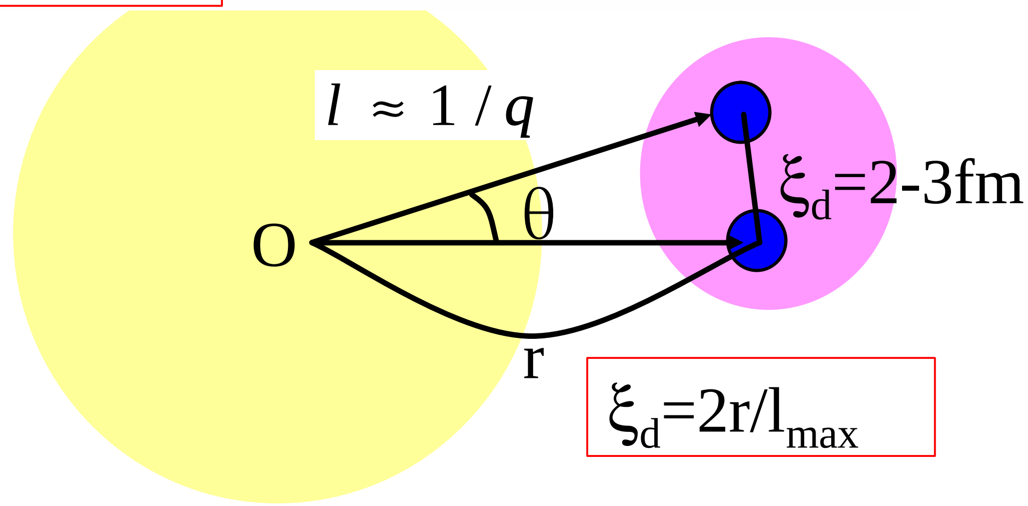


The coherent superposition of

high-L oribts *l*=3-8

in the continuum

forms the di-neutron correlation



Di-neutron correlation in medium-mass n-rich nuclei

HFB calculation using a finite range effective force (Gogny)

Pillet, Sandulescu, Schuck, PRC76, 024310 (2007)

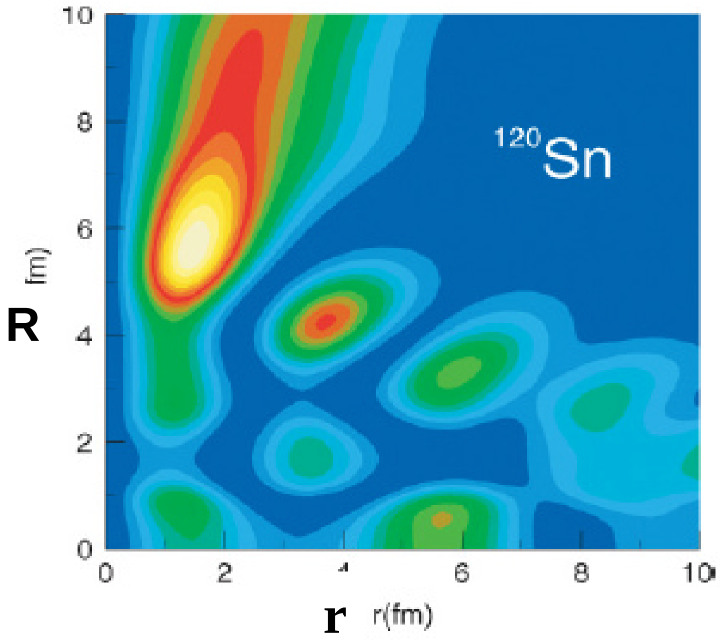
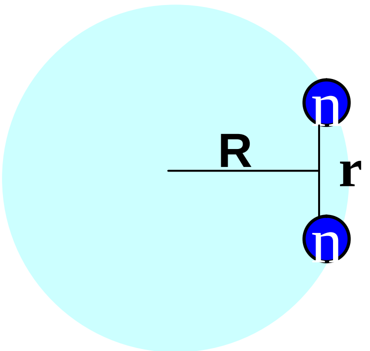
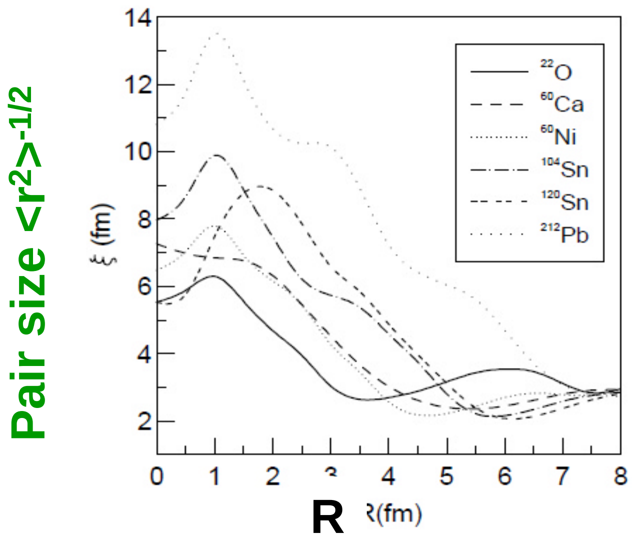


FIG. 7. (Color online) $W(R, r)$ for ^{120}Sn.



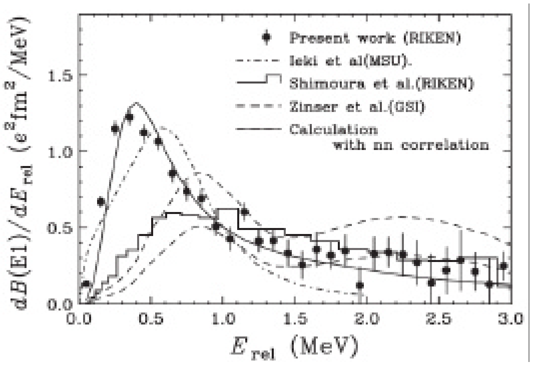


4. Exotic modes of excitation and surface di-neutron mode

Experimental facts: low-energy E1 strength

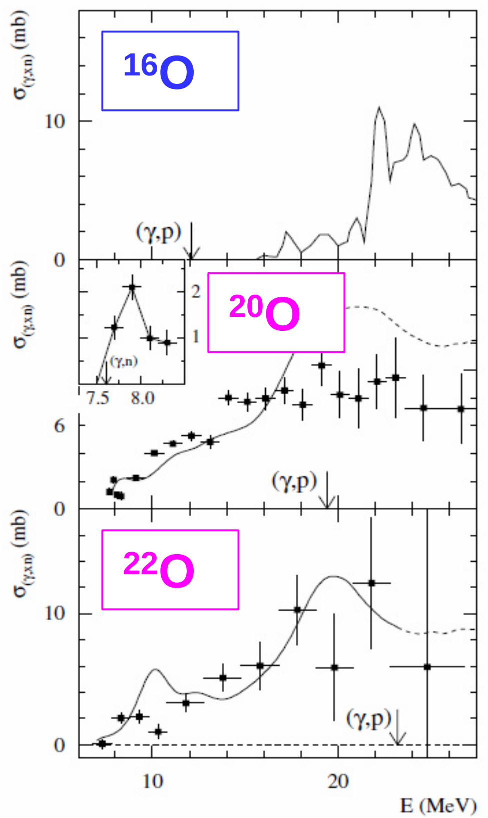
Light halo nuclei ^{11}Li

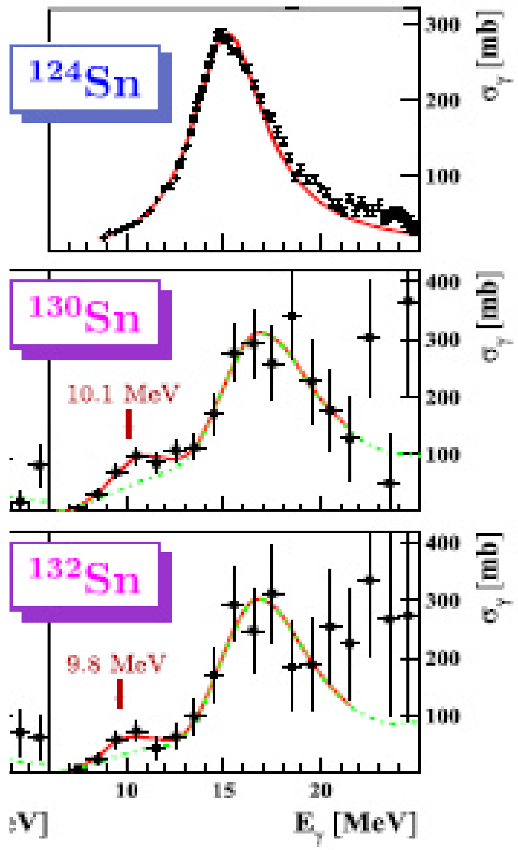
Nakamura et al. PRL 2006



Very large B(E1) strength at low-energy, just above threshold E_{th}=0.3 MeV

Heavy mass n-rich nuclei



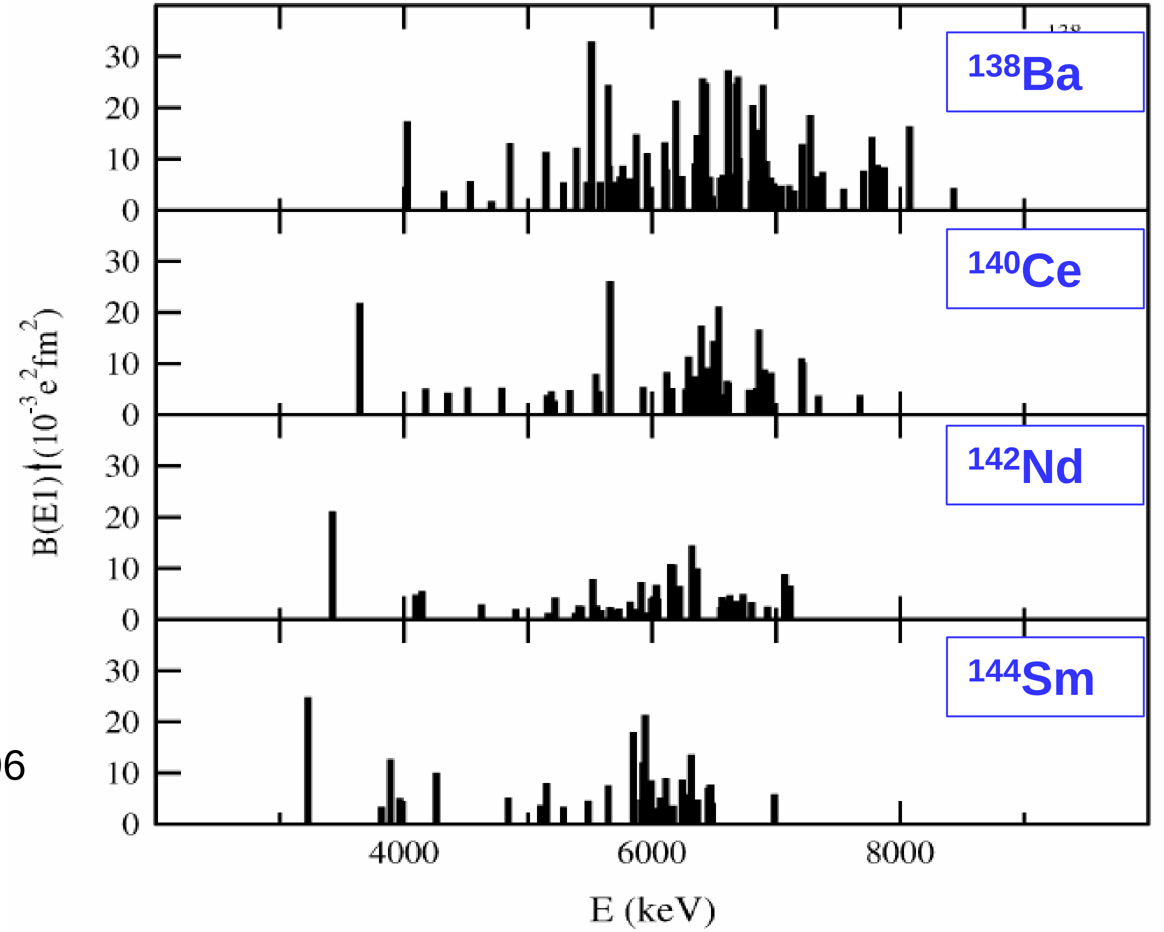


Leistenschneider et al. PRL 2001

Adrich et al. PRL 2006

Experimental facts: Low-energy E1 strength

Heavy stable nuclei Z=82 isotones

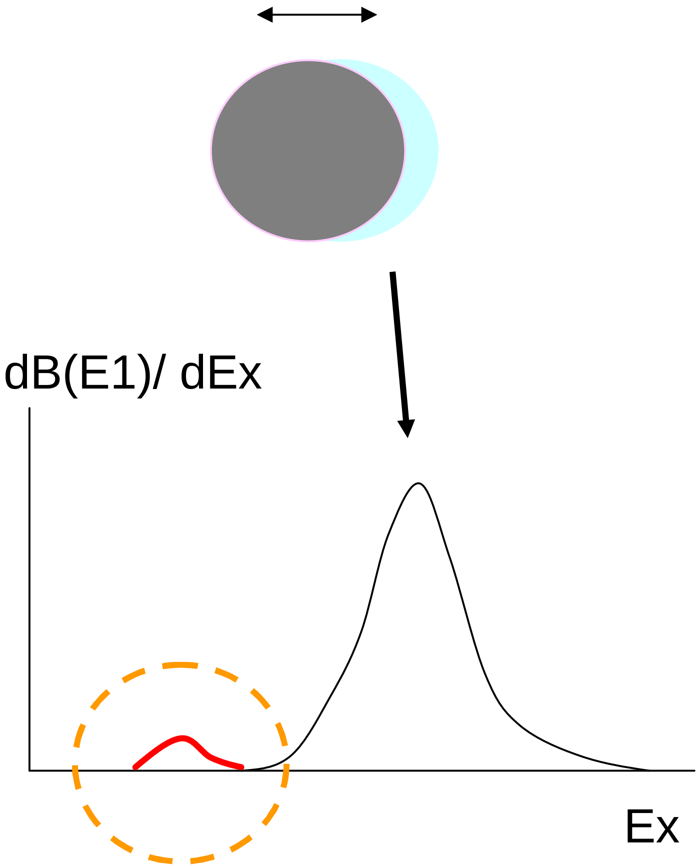


Small but significant E1 strength below threshold

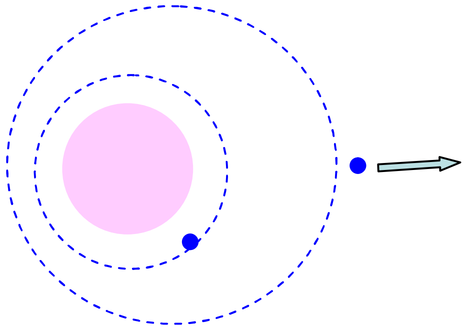
Volz et al. NPA 2006

Exotic modes of excitation ?

Giant dipole resonance

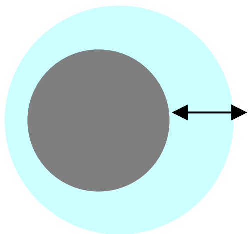


Candidates for the nature of the soft dipole



Break-up emission of a weakly bound neutron

Eg. Sagawa et al. 1995

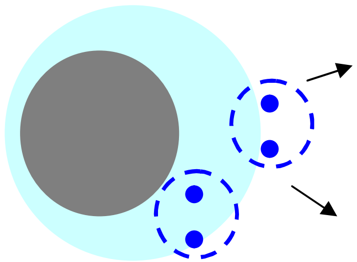


Pygmy dipole resonances

Collective excitation of neutron skin (excess neutrons)

Suzuki,Ikeda,Sato 1987

We explore 3-rd senario



Surface di-neutron mode

& Break-up emission of neutron pairs

Eg. Shimoura et al. PLB 1995

Microscopic theory to describe exotic modes of excitations in n-rich nuclei

"Microscopic" means that the model incorporates all the three aspects.

1. **Collective vibrations**

2. **Weakly bound orbits and particle emission (unbound continuum states)**

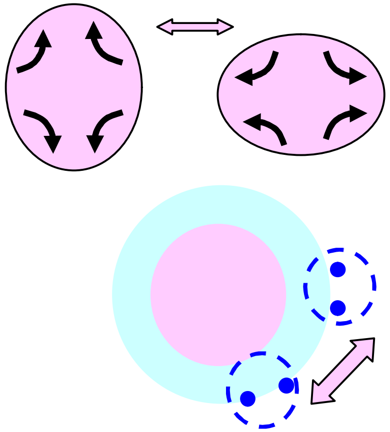
3. **Pair correlation/ di-neutron correlation**

⇨

The Quasiparticle Random Phase Approximation (QRPA) based on the **density functional theory / selfconsistent mean-field method**

As a review, Paar, Vretenar, Khan, Colo, Rep. Prog. Phys. 2007

Time-dependent density functional theory (TDHFB)

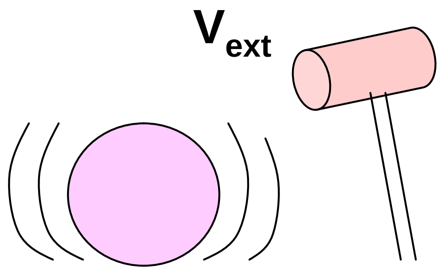


Time-evolving density

$$r(r,t)=\sum y_i(r,t)^* y_i(r,t)$$

Time-evolving pair density

$$\tilde{r}(r,t)=\sum f_i(r,t) y_i(r,t)$$



Nucleons can move collectively through the common mean-fields

Energy functional

$$E[r(r,t)]+E_{pair}[\tilde{r}(r,t)]$$

External perturbation

$$e^{-iwt}V_{ext}(r)$$

Time-dependent single-particle motion

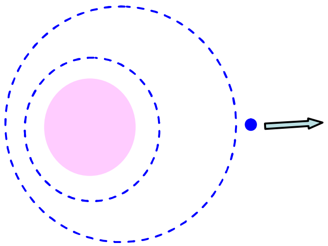
$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} y_i(r,t) \\ f_i(r,t)\end{pmatrix}=\begin{pmatrix} T+\Gamma(r,t)-l & \Delta(r,t) \\ -\Delta^*(r,t) & -T-\Gamma(r,t)+l\end{pmatrix}\begin{pmatrix} y_i(r,t) \\ f_i(r,t)\end{pmatrix}$$

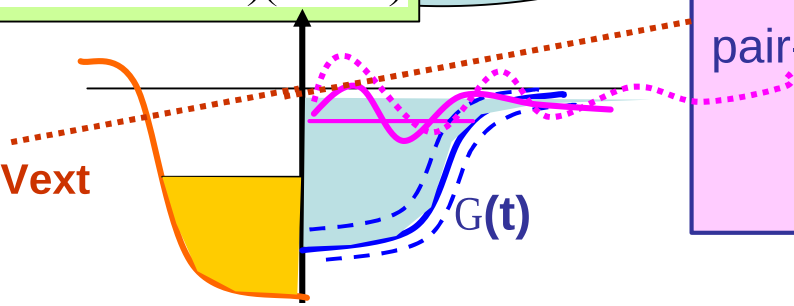
Time-dependent mean-potential

$$\Gamma(r,t)=\frac{\partial E[r]}{\partial r(r)}$$

pair-potential

$$\Delta(r,t)=\frac{\partial E_{pair}[\tilde{r}]}{\partial \tilde{r}(r)}$$





Small amplitude limit of TDHFB
Quasiparticle Random Phase Approximation (QRPA)

Density oscillation & pair density oscillation

$$\begin{pmatrix} dr(r) \\ d\tilde{r}(r) \\ d\tilde{r}^*(r) \end{pmatrix} = \int dr' \left(R^{ab}{}_0(r,r',w) \right) \begin{pmatrix} d\Gamma(r') + V_{ext}(r') \\ d\Delta(r') \\ d\Delta^*(r') \end{pmatrix}$$

Linear response equation

Response function (ph, pp, hh)

$$R_0(\vec{r},\vec{r}',w) = \int_C dE G(r,r',E) G(r',r,E+w)$$

$$dr(r,w) = \sum dy_i(r,w)^* y_i(r) + b.w.$$

$$d\tilde{r}(r,w) = \sum df_i(r,w) y_i(r) + b.w.$$

Energy functional

$$E[r] + E_{pair}[\tilde{r}]$$

External perturbation

$$e^{-iwt} V_{ext}(r)$$

Induced mean-fields

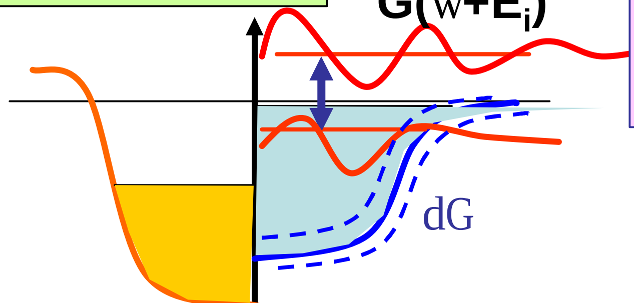
$$d\Gamma(r,w) = \frac{\partial^2 E[r]}{\partial r^2} dr(r,w)$$

$$d\Delta(r,w) = \frac{\partial^2 E_{pair}[\tilde{r}]}{\partial \tilde{r}^2} d\tilde{r}(r,w)$$

Response in single-particle motion

$$\begin{pmatrix} dy_i(r,w) \\ df_i(r,w) \end{pmatrix} = G(w+E_i) \begin{pmatrix} d\Gamma(r,w) & d\Delta(r,w) \\ d\Delta^*(r,w) & -d\Gamma(r,w) \end{pmatrix} \begin{pmatrix} y_i(r) \\ f_i(r) \end{pmatrix}$$

can be solved using the HFB Green function



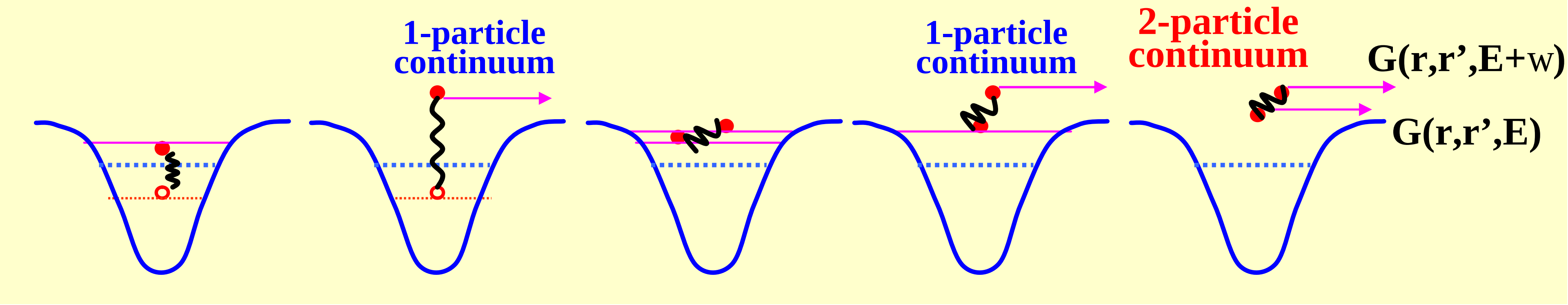
Continuum QRPA

M. Matsuo, Nucl. Phys. A696, 371 (2001)
also E. Khan et al. Phys. Rev. C66, 024309 (2002)

Nuclei near the neutron drip-line

⇒ Particle escaping in the continuum

⇒ Correlation among the continuum and weakly bound orbits



1. **Use exact single-particle Green function G(r,r',E) with proper r→∞ asymptotics and out-going wave boundary condition** Belyaev's construction 1987

$$G(r,r',E)=\sum_{st=1,2} c^{st} j^{out,s}(r_>,E)\, j^{reg,t}(r_<,E)$$

Regular and outgoing waves

2. **Summing up continuum states using a contour integral in the complex E-plane**

$$R_0(\vec{r},\vec{r}',w)=\frac{1}{2pi}\int_C dE\, G(r,r',E)G(r',r,E+w)+b.w.$$

Physical quantities obtained in QRPA

Linear response equation

$$\begin{pmatrix} dr(r) \\ d\tilde{r}(r) \\ d\tilde{r}^*(r) \end{pmatrix} = \int dr' \left(R^{ab}{}_0(r,r',w) \right) \begin{pmatrix} d\Gamma(r') + V_{ext}(r') \\ d\Delta(r') \\ d\Delta^*(r') \end{pmatrix}$$

Response function (ph, pp, hh)

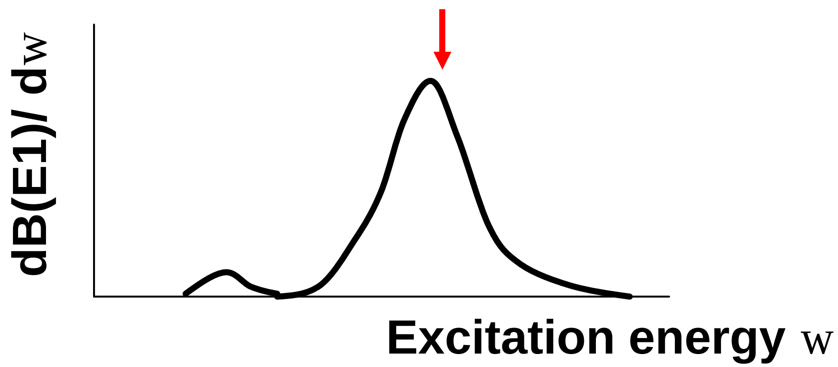
$$R_0(\vec{r},\vec{r}\,',w) = \int_C dE G(r,r',E) G(r',r,E+w)$$

Solve the linear response equation at each excitation energy ω

$$dr(r,w), d\tilde{r}(r,w), d\tilde{r}^*(r,w)$$

E1 Strength function as a function of w

$$\frac{dB(E1)}{dw} = -\frac{1}{p} \int dr\, r Y_{10} dr(r,w)$$



Transition density at a given w

Profiles of density oscillation

particle-hole transition density

$$\rho^{ph}(\mathbf{r}) = \langle i | \Sigma_\sigma \psi^\dagger(r\sigma)\psi(r\sigma) | 0 \rangle$$

particle-pair transition density

$$P^{pp}(\mathbf{r}) = \langle i | \psi^\dagger(r\downarrow)\psi^\dagger(r\uparrow) | 0 \rangle$$

hole-pair transition density

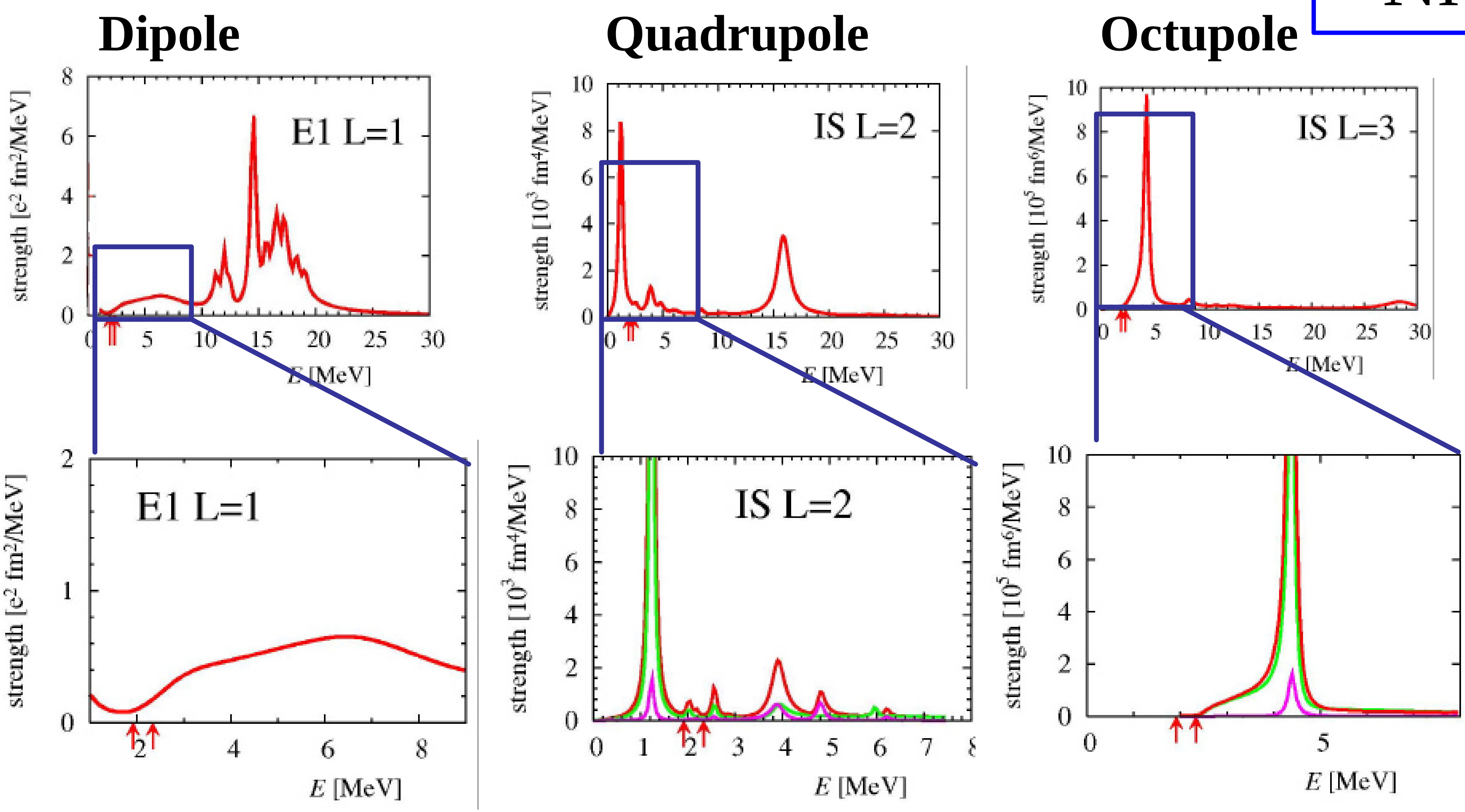
$$P^{hh}(\mathbf{r}) = \langle i | \psi(r\uparrow)\psi(r\downarrow) | 0 \rangle$$

Multipole responses in ^{84}Ni

Skyrme-HFB + Continuum QRPA calc.
with SLy4 (Landau-Migdal approx) & mix-type DDDI

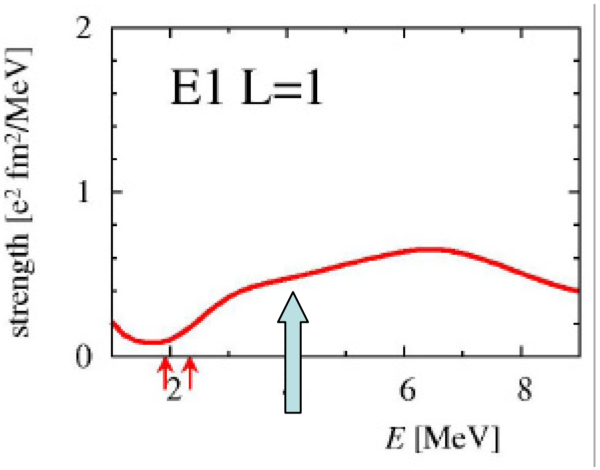
^{84}Ni

Multipole strength function



Soft dipole as Surface di-neutron mode

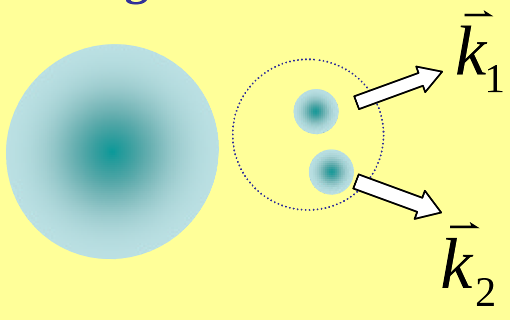
Strength function



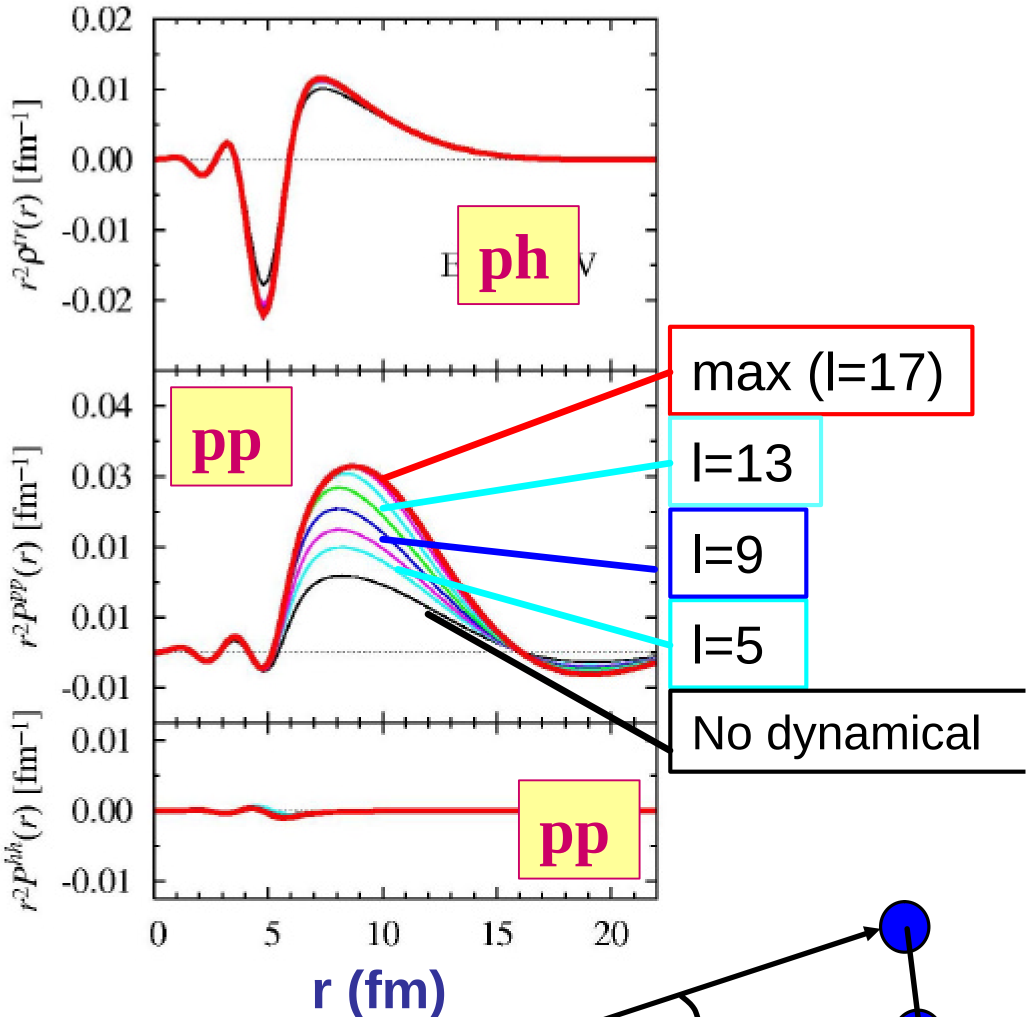
Emission of di-neutrons

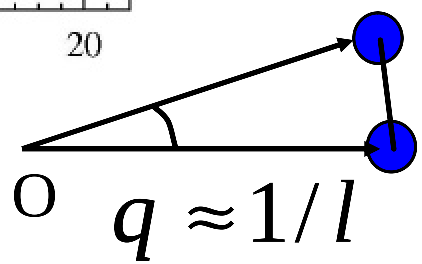
pp ≫ ph, hh

High-L orbits



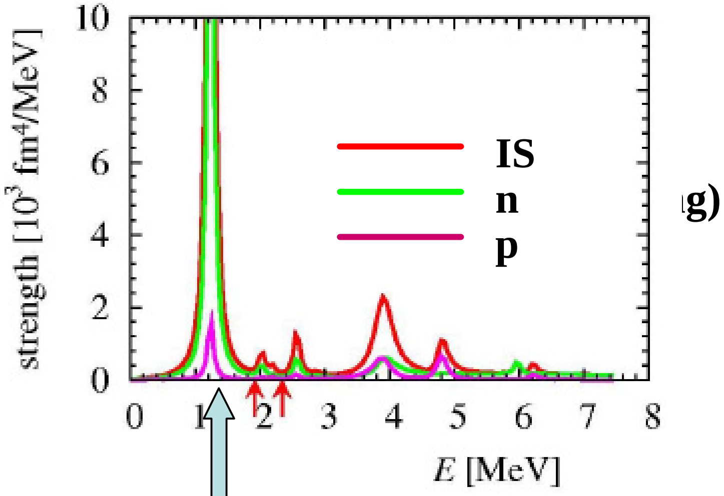
Neutron transition densities @ E=4.0MeV



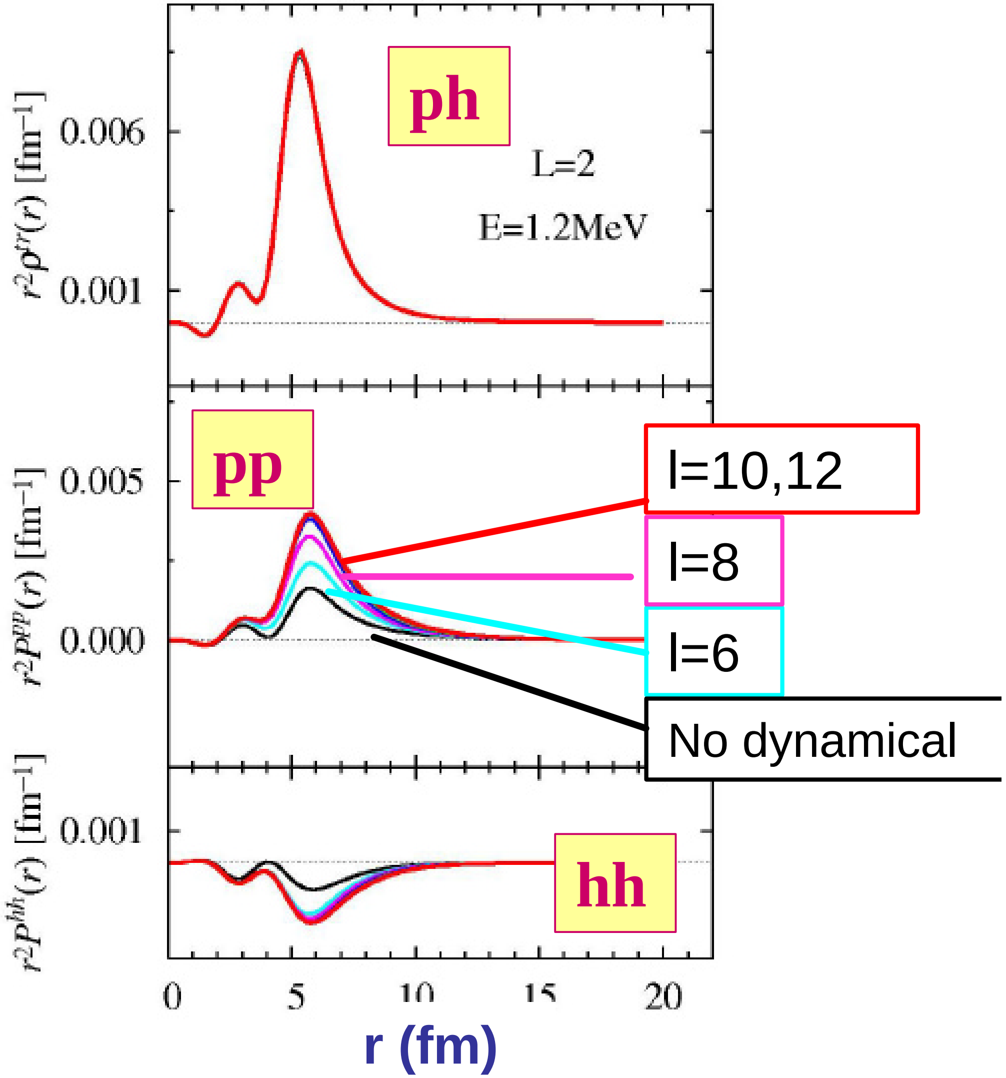


low-lying 2^+ : surface vibration

Strength function



Neutron transition densities @ E=1.2MeV



1. **Pronounced 2_1^+ state below E_{th}**

2. Surface vibration of ph-character

3. **No pair emission**

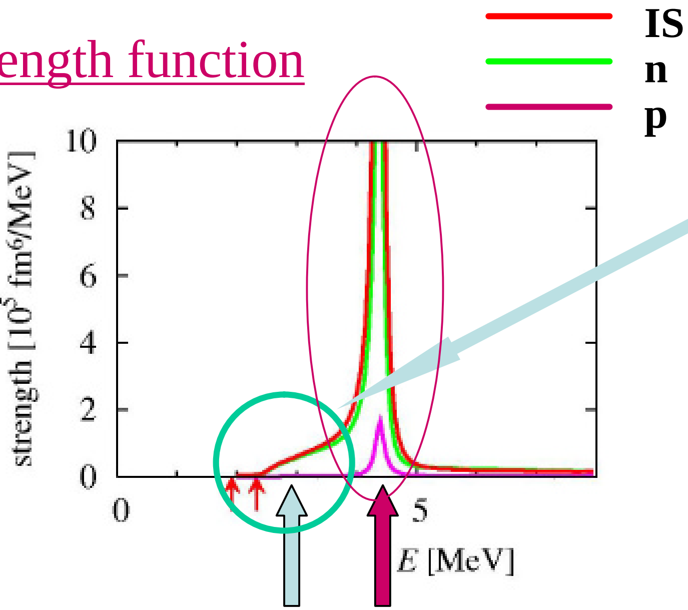
(few emission even in the resonance case)

4. Some pairing effect,

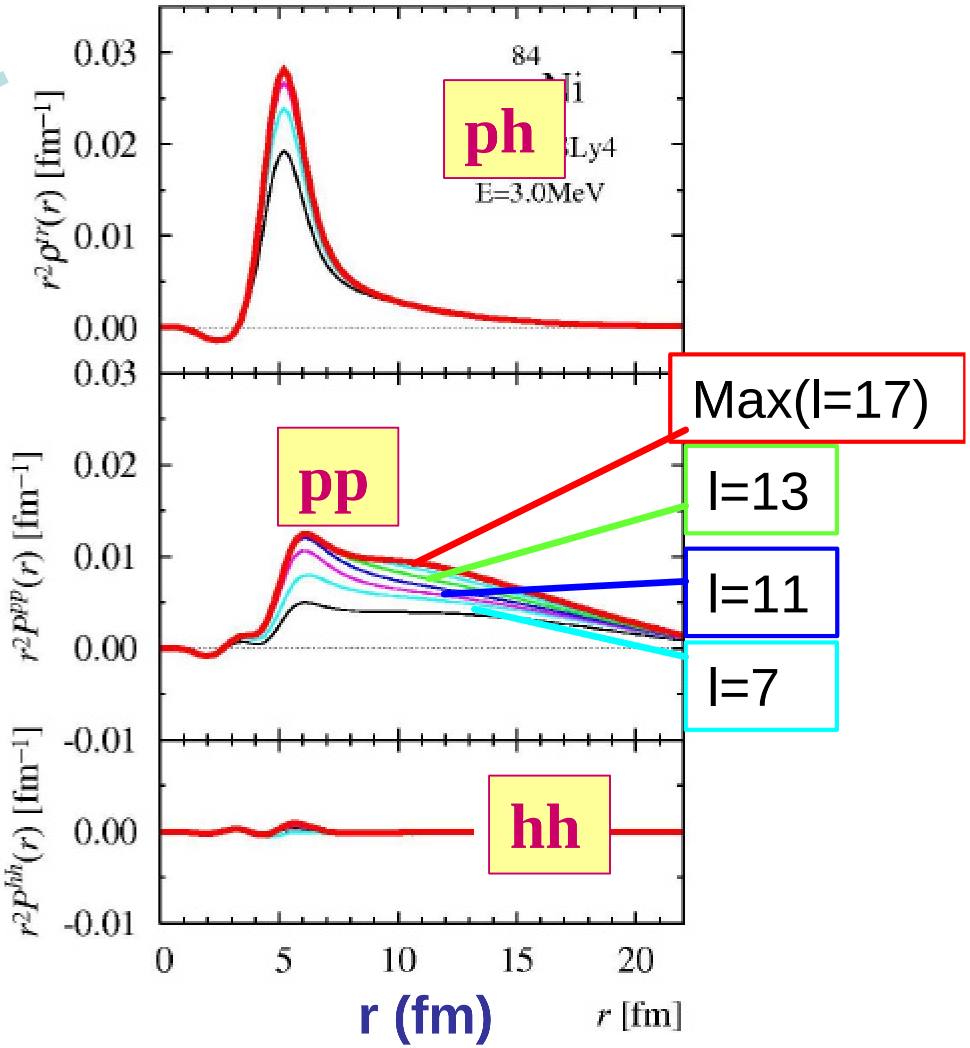
but di-neutron feature is weak.

Octupole: surface vibration + surface dineutron mode

Strength function



Neutron transition densities @ E=3.0MeV



There are two modes

A. Surface vibration of ph character
- **Similar to 2_1^+**
- **Sharp resonance even above E_{th}**

B. Continuum surface di-neutron mode
- **emission of di-neutron**
- **Similar to soft dipole,**

  but the di-neutron feature is slightly weaker