Weak and Electromagnetic Interactions in Nuclei

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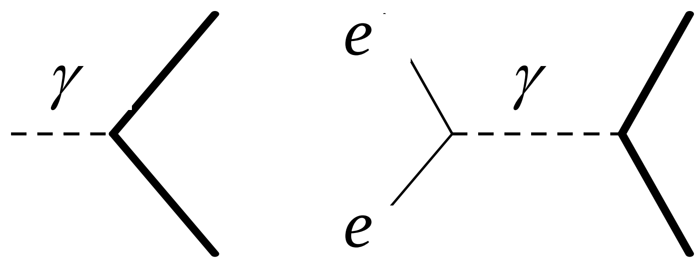
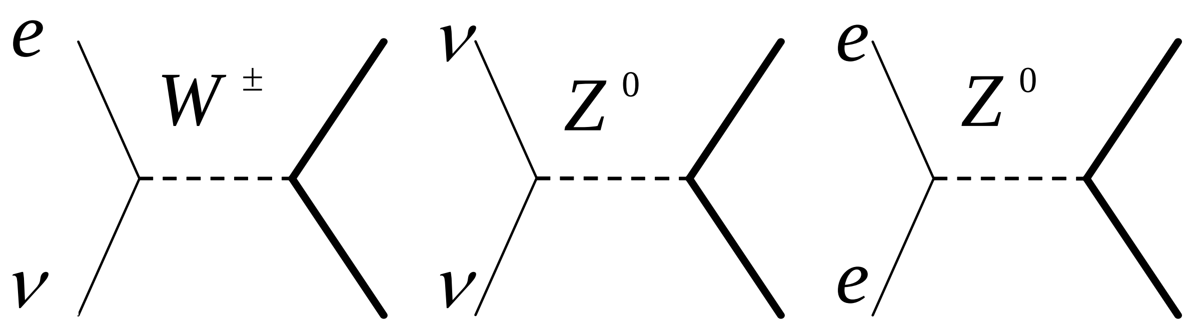
CSN Summer School
Aug. 28, 2007

Interactions	Situation	Strength (Cross sections)	Handling
EM (electro-magnetic)	Known	moderate (enough) $(10^{-36} - 10^{-26} cm^2/sr)$	clear
Weak (W)	Known within the standard model Not well determined part remains at hadron level	weak (small) $(10^{-42} - 10^{-38} cm^2)$	clear
Strong (S)	Not completely known at hadron level (need phenomenology)	strong (large) $(\square 10^{-(27-22)} cm^2)$	non-easy (distortion + absorption)

EM and Weak Interactions in Nuclei

- 1. Probe of nuclear structure



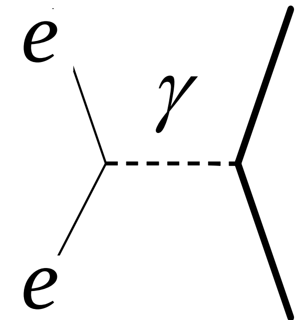


- 2. Probe of weak neutral current

e.g. $\bar{s}\gamma_\mu s, \quad \bar{s}\gamma_\mu\gamma_5 s$

EM

$$T_{fi}^{\gamma} = \frac{4\pi\alpha}{q_{\mu}^{2}} i\bar{e}\gamma_{\mu} e \cdot J_{\mu}^{\gamma}$$



$$J_{\mu}^{\gamma} = i < N \,|\, \frac{2}{3}\bar{u}\gamma_{\mu}u - \frac{1}{3}\bar{d}\gamma_{\mu}d - \frac{1}{3}\bar{s}\gamma_{\mu}s \,|\, N >$$

$$= i < N \,|\, \frac{1}{6}(\bar{u}\gamma_{\mu}u + \bar{d}\gamma_{\mu}d - 2\bar{s}\gamma_{\mu}s) + \frac{1}{2}(\bar{u}\gamma_{\mu}u - \bar{d}\gamma_{\mu}d) \,|\, N >$$

isoscalar (j^8) isovector (j^3)

$$= i\bar{u}(p')[F_1(q_{\mu}^2)\gamma_{\mu} + \frac{1}{2m}F_2(q_{\mu}^2)\sigma_{\mu\nu}q_{\nu}]u(p)$$

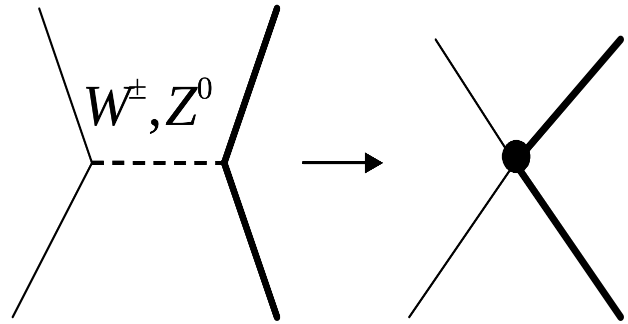
$$= i\bar{u}(p')\frac{1}{2}[(F_1^S + F_1^V\tau_3)\gamma_{\mu} + \frac{1}{2m}(F_2^S + F_2^V\tau_3)\sigma_{\mu\nu}q_{\nu}]u(p)$$

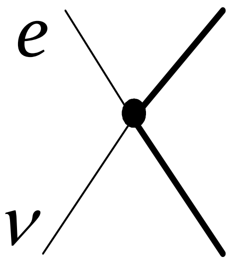
$$F_1^S(0) = F_1^V(0) = 1, \quad F_2^S(0) = -0.12 \quad F_2^V(0) = 3.706$$

$$\vec{J}^{\gamma} = F_1\frac{1}{2m}(\vec{p} + \vec{p}\,') + (F_1 + F_2)\frac{1}{2m}(-i\vec{\sigma} \times \vec{q})$$

$$J_0^{\gamma} = F_1 - (F_1 + F_2)(\frac{q^2}{8m^2} - \frac{i}{4m^2}\vec{q} \cdot (\vec{\sigma} \times \vec{q}))$$

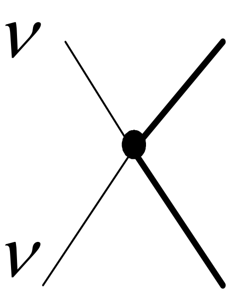
Weak



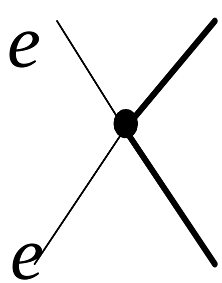


$$L_{eff}^{(\pm)} = i\frac{G}{\sqrt{2}}\{[\overline{e}\gamma_{\mu}(1+\gamma_5)\nu_e + (e \leftrightarrow \mu)]J_{\mu}^{(+)}$$

$$+[\overline{\nu}_e\gamma_{\mu}(1+\gamma_5)e + (e \leftrightarrow \mu)]J_{\mu}^{(-)}\}$$



$$L_{eff}^{(\nu)} = i\frac{G}{\sqrt{2}}[\overline{\nu}_e\gamma_{\mu}(1+\gamma_5)\nu_e + (e \rightarrow \mu) + ..]J_{\mu}^{(0)}$$



$$L_{eff}^{(e)} = -i\frac{G}{\sqrt{2}}[\overline{e}\gamma_{\mu}(1+\gamma_5)e - 4\sin^2\theta_W\overline{e}\gamma_{\mu}e]J_{\mu}^{(0)}$$

Charged Current

$$J_\mu^{(+)} = i < N \,|\, \bar{u}\gamma_\mu(1+\gamma_5)[d\cos\theta_C + s\sin\theta_C]$$

$$+\, \bar{c}\gamma_\mu(1+\gamma_5)[-d\sin\theta_C + s\cos\theta_C]\,|\, N >$$

$$J_\mu^{(+)} = i\cos\theta_C\{< N \,|\, \gamma_\mu\tau_\pm \,|\, N > + < N \,|\, \gamma_\mu\gamma_5\tau_\pm \,|\, N >\}$$

V(vector) AV(axial-vector)

$$< N \,|\, \gamma_\mu\tau_\pm \,|\, N > = \bar{u}(p')[F_1\gamma_\mu + \frac{1}{2m}F_2\sigma_{\mu\nu}q_\nu]\tau_\pm]u(p) \quad \boxed{\text{CVC}}$$

$$< N \,|\, \gamma_\mu\gamma_5\tau_\pm \,|\, N > = \bar{u}(p')[F_A\gamma_5\gamma_\mu - iF_P\gamma_5 q_\nu]\tau_\pm]u(p)$$

$$F_A(0) = -1.26 \qquad F_P = \frac{2mF_A}{q^2 + m_\pi^2}$$

$$\vec{J} = F_A\vec{\sigma} - (F_1^V + F_2^V)\frac{1}{2m}i\vec{\sigma}\times\vec{q} + F_1^V\frac{1}{2m}(\vec{p} + \vec{p}\,')$$

$$J_0 = F_1^V + F_A\frac{1}{2m}\vec{\sigma}\cdot(\vec{p} + \vec{p}\,')$$

Neutral Current

$$J_\mu^{(0)} = \frac{i}{2} < N \,|\, [\bar{u}\gamma_\mu(1+\gamma_5)u - \bar{d}\gamma_\mu(1+\gamma_5)d - \bar{s}\gamma_\mu(1+\gamma_5)s \,|\, N > -2\sin^2\theta_W J_\mu^\gamma$$

$$= \frac{i}{2} < N \,|\, [(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d) + (\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d) - \bar{s}\gamma_\mu s - \bar{s}\gamma_\mu\gamma_5 s \,|\, N >$$

isovector-V isovector-AV strange-V strange-AV

$$= V_\mu^3 + A_\mu^3 + V_\mu^S + A_\mu^S - 2\sin^2\theta_W J_\mu^\gamma$$

$$V_\mu^3 = \frac{i}{2} < N \,|\, (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d) \,|\, N >= \frac{i}{2}\bar{u}(p')[F_1^V\gamma_\mu + \frac{1}{2m}F_2^V\sigma_{\mu\nu}q]\tau_3 u(p)$$

$$A_\mu^3 = \frac{i}{2} < N \,|\, (\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d) \,|\, N >= \frac{i}{2}\bar{u}(p')[F_A\gamma_5\gamma_\mu - iF_P\gamma_5 q_\nu]\tau_3 u(p)$$

$$V_\mu^S = -\frac{i}{2} < N \,|\, \bar{s}\gamma_\mu s \,|\, N >= -\frac{i}{2}\bar{u}(p')[F_1^S\gamma_\mu + \frac{1}{2m}F_2^S\sigma_{\mu\nu}q]u(p)$$

$$A_\mu^S = -\frac{i}{2} < N \,|\, \bar{s}\gamma_\mu\gamma_5 s \,|\, N >= -\frac{i}{2}G_1^S\bar{u}(p')\gamma_\mu\gamma_5 u(p)$$

$$F_1^S(0) = 0 \qquad F_1^S(q_\mu^2 \neq 0), \quad F_2^S, \quad G_1^S \quad not \quad well \quad \det er\min ed$$

Note

$$j_\mu^3 = \frac{1}{2}(\bar{u}\Gamma u - \bar{d}\Gamma d)$$

$$j_\mu^8 = \frac{1}{2\sqrt{3}}(\bar{u}\Gamma u + \bar{d}\Gamma d - 2\bar{s}\Gamma s)$$

$$j_\mu^0 = \frac{1}{3}(\bar{u}\Gamma u + \bar{d}\Gamma d + \bar{s}\Gamma s)$$

$$j_\mu^0 \quad not \quad well \quad \mathrm{det}er\,\mathrm{min}ed$$

Asumption $\quad A_\mu^S = V_\mu^S = 0$

$$J_\mu^{(0)} = A_\mu^3 + V_\mu^3 - 2\sin^2\theta_W J_\mu^\gamma$$

Note: vector part $\quad V_\mu^{(0)} = V_\mu^3 - 2\sin^2\theta_W J_\mu^\gamma$

C0 $\quad (G_E^{IV} - 2\sin^2\theta_W G_E) < 0 \,|\, j_0(qr) Y^{(0)} \,|\, 0 >$

$$proton: \quad \frac{1}{2} G_E^p (1 - 4\sin^2\theta_W)\rho_p(r) = \frac{0.08}{2} G_E^p \rho_p(r)$$

$$neutron: \quad (\frac{1}{2} G_E^p - 2\sin^2\theta_W G_E^n)\rho_n(r) \cong \frac{1}{2} G_E^p \rho_n(r)$$

Cross sections for $(\nu, e^-), (\bar{\nu}, e^+), (\nu, \nu), (\bar{\nu}, \bar{\nu})$

$$d\sigma = 2\pi \sum_h |< f | H_W | i >|^2 \delta(W_f - W_i) \frac{d^3 k}{(2\pi)^3}$$

$$\frac{d\sigma}{d\Omega_{\mp}} = \frac{G^2}{2\pi^2} \frac{1}{\sqrt{(k_1 \cdot p)^2}} \frac{d^3 k}{2\varepsilon_2} \eta_{\mu\nu} W_{\mu\nu}$$

$$\eta_{\mu\nu} = \frac{1}{4} Tr[\gamma_\mu (1 + \gamma_5) \not{k}_1 \gamma_\nu (1 + \gamma_5) \not{k}_2]$$

$$= 2(k_{1\mu} k_{2\nu} + k_{1\nu} k_{2\mu} - (k_1 \cdot k_2)\delta_{\mu\nu} \pm \varepsilon_{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma})$$

$$W_{\mu\nu} = W_{\mu\nu}^{VV} + W_{\mu\nu}^{VA} + W_{\mu\nu}^{AA}$$

$$W_{\mu\nu}^{VV} = W_1 \delta_{\mu\nu} + \frac{W_2}{M_T^2}\left(p_\mu - \frac{p \cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p \cdot q}{q^2} q_\nu\right)$$

$$W_{\mu\nu}^{AA} = W_3 \delta_{\mu\nu} + \frac{W_4}{M_T^2} p_\mu p_\nu + \frac{W_5}{M_T^2} q_\mu q_\nu$$

$$+ \frac{W_6}{M_T^2}(p_\mu q_\nu + p_\nu q_\mu) + \frac{W_7}{M_T^2}(p_\mu q_\nu - p_\nu q_\mu)$$

$$W_{\mu\nu}^{VA} = \frac{W_8}{M_T^2} \varepsilon_{\mu\nu\rho\sigma} p_\rho q_\sigma$$

Current Conservation

$$q_\mu W_{\mu\nu}{}^{VV} = W_{\mu\nu}{}^{VV} q_\nu = 0, \quad q_\mu \eta_{\mu\nu}{}^{VV} = \eta_{\mu\nu}{}^{VV} q_\nu = 0$$

$$\frac{d^2\sigma}{d\Omega d\varepsilon_2}_{\mp} = \frac{G^2 \varepsilon_2^2}{2\pi^2} \frac{1}{M_T} \{(W_2 + W_4)\cos^2\frac{\theta}{2} + 2(W_1 + W_3)\sin^2\frac{\theta}{2}$$

$$\mp 2\frac{W_8}{M_T} \sin\frac{\theta}{2} (q^2 \cos^2\frac{\theta}{2} + \vec{q}^{\,2} \sin^2\frac{\theta}{2})^{1/2} \}$$

Multipole expansions

$$\frac{d^2\sigma}{d\Omega}_{\mp} = \frac{G^2 \varepsilon^2}{2\pi^2} \frac{4\pi}{2J_i + 1} \{\cos^2\frac{\theta}{2} |< J_f \,||\, M_J(q) - \frac{q_0}{q} L_J(q) \,||\, J_i >|^2$$

$$+[\frac{q_\mu^2}{2q^2}\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}]\sum_{J=1}^{\infty} [|< J_f \,||\, T_J^{mag}(q) \,||\, J_i >|^2$$

$$+ |< J_f \,||\, T_J^{el}(q) \,||\, J_i >|^2] \mp \frac{\sin\frac{\theta}{2}}{q} (q_\mu{}^2 \cos^2\frac{\theta}{2} + \vec{q}^{\,2} \sin^2\frac{\theta}{2})^{1/2}$$

$$\sum_{J=1}^{\infty} 2\,\mathrm{Re}[< J_f \,||\, T_J^{mag}(q) \,||\, J_i >< J_f \,||\, T_J^{el}(q) \,||\, J_i >^*]\}$$

$$M_{JM}(q) = \int d^3x\, j_J(qx) Y_{JM}(\hat{x})\, \rho(\vec{x})$$

$$L_{JM}(q) = \frac{i}{q} \int d^3x \vec{\nabla}[\, j_J(qx) Y_{JM}(\hat{x})] \vec{J}(\vec{x})$$

$$T^{mag}_{JM}(q) = \int d^3x [\, j_J(qx) \vec{Y}^{M}_{JJ1}(\hat{x})] \vec{J}(\vec{x})$$

$$T^{el}_{JM}(q) = \frac{1}{q} \int d^3x [\vec{\nabla} \times\, j_J(qx) \vec{Y}^{M}_{JJ1}(\hat{x})] \vec{J}(\vec{x})$$

$q \to 0$

$$M_{JM} = \frac{q^J}{(2J+1)!!} \int d^3x Y_{JM} J_0$$

$$L_{JM} = \frac{1}{i} \frac{q^{J-1}}{(2J+1)!!} \int d^3x Y_{JM} \vec{\nabla} \cdot \vec{J}(\vec{x})$$

$$T^{mag}_{JM} = -\frac{1}{i} \frac{q^J}{(2J+1)!!} \sqrt{\frac{J+1}{J}} \int d^3x \frac{1}{J+1} [\vec{x} \times \vec{J}(\vec{x})] \cdot \vec{\nabla} x^J Y_{JM}$$

$$T^{el}_{JM} = \frac{1}{i} \frac{q^{J-1}}{(2J+1)!!} \sqrt{\frac{J+1}{J}} \int d^3x\, x^J Y_{JM} \vec{\nabla} \cdot \vec{J}(\vec{x})$$

Ref: J. D. Walecka, Theoretical Nuclear and Subnuclear Physics, Oxford (1995)

$$Vector:\ T_{1M}^{mag} = \frac{i\sqrt{2}}{3}\frac{\hbar q}{2mc}\sqrt{\frac{3}{4\pi}}\sum_i(\vec{\ell}_i + \mu_i\vec{\sigma}_i)$$

$$< f\,|T_{JM}^{el}(q)|\,i > \underset{q\to 0}{\rightarrow} -\frac{E_f - E_i}{q}\sqrt{\frac{J+1}{J}} < f\,|M_{JM}(q)|\,i >$$

$$AV:\quad T_{1M}^{el5} = \sqrt{2}L_{1M} = \frac{i}{\sqrt{6\pi}}F_A\sum_i \vec{\sigma}_i\tau_{\pm i} \qquad GT$$

$$M_{00}^{5} = \frac{1}{\sqrt{4\pi}}F_1\sum_i \tau_{\pm i} \qquad Fermi$$

Spin-dependent excitations

- Gamow-Teller (1^+): $\quad \vec{\sigma}\ \tau_{\pm}$
- Spin-diole (0^-, 1^-, 2^-): $\quad [\vec{\sigma} \times \vec{r}\,]^J\ \tau_{\pm}$

Multipoles

1^+: $E_5 1$, $M1$, $C_5 1$, $L_5 1$

2^-: $E_5 2$, $M2$, $C_5 2$, $L_5 2$

1^-: $M_5 1$, $E1$, $C1$

0^-: $C_5 0$, $L_5 0$

Probe of nuclear structure

(e,e) Frois & Papanicolas, Ann. Rev. Nucl. Part. 37 (1987)

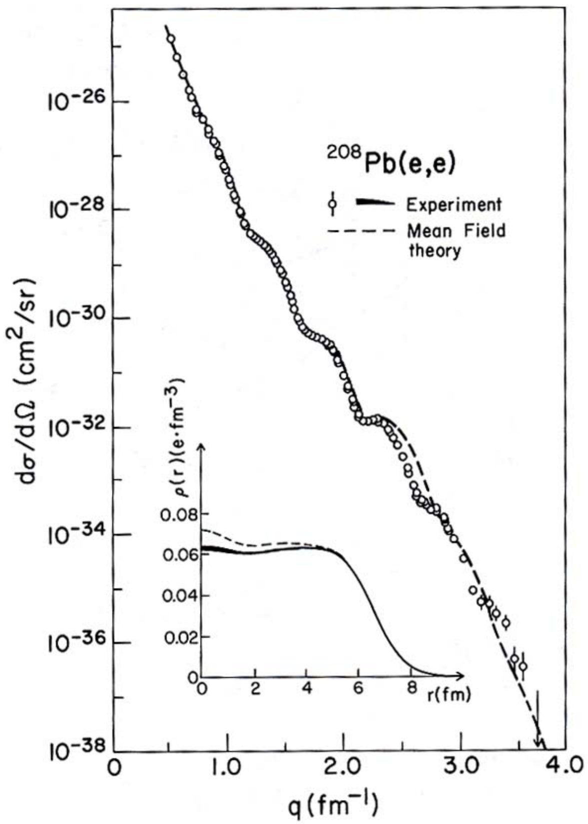
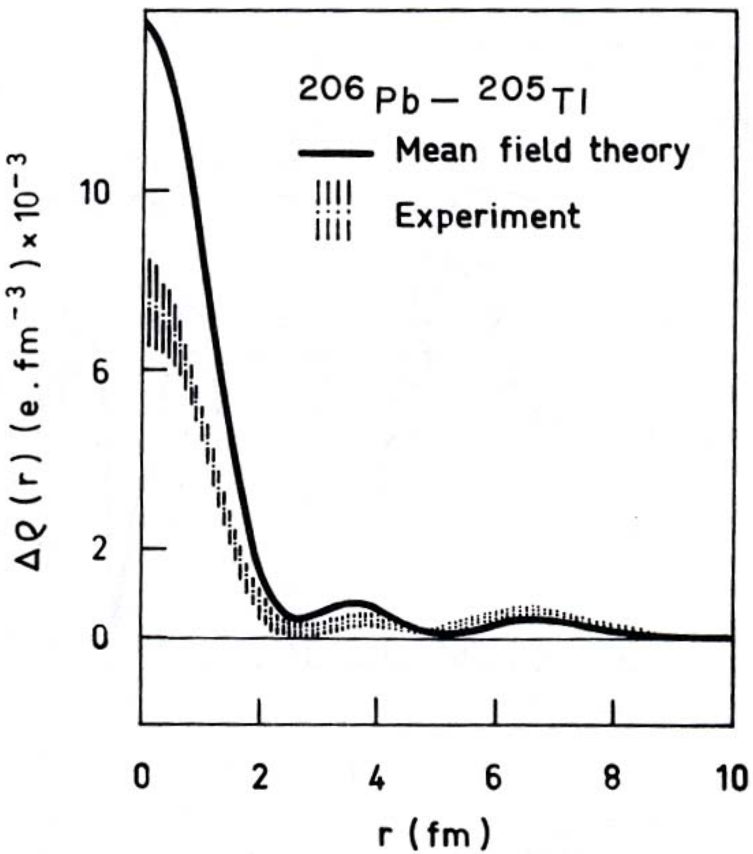


Figure 7 Elastic cross section from ^{208}Pb (47). The dynamic range of the measurements allows the reconstruction of the charge density distribution (*insert*). The thickness of the line in $\rho(r)$ depicts the experimental uncertainty. The mean field result in that of Dechargé & Gogny (53).



gure 11 The charge density difference of ^{206}Pb and ^{205}Tl (63, 64).

Charge density

Charge density of $2s_{1/2}$ orbit

M1 $2s_{1/2}$ $2p_{1/2}$

M7 $0f_{7/2}$ $<r_n^2(f_{7/2})>$

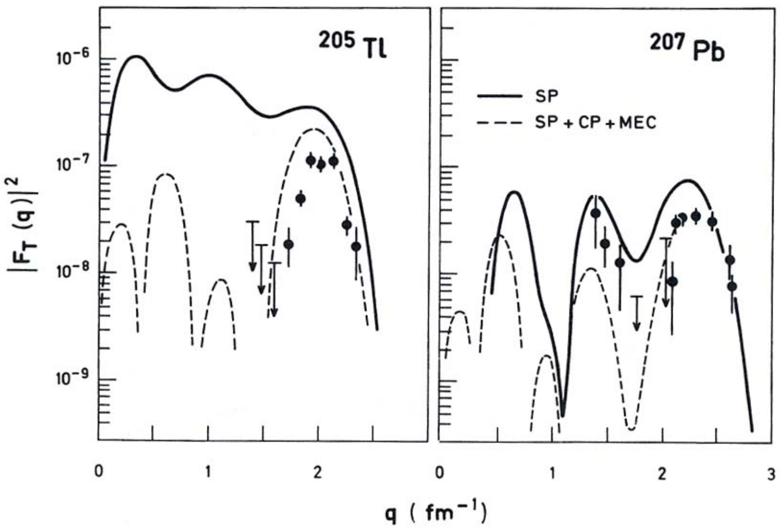
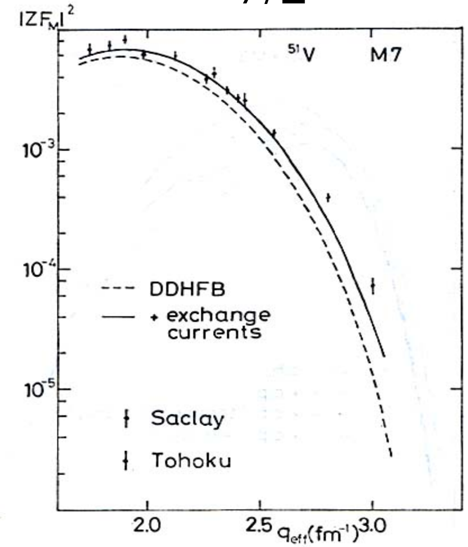


Figure 12 The elastic magnetic form factors of ^{207}Pb and ^{205}Tl (65). The solid curve depicts the single-particle predictions (53). The dashed curve includes in addition the effects of core



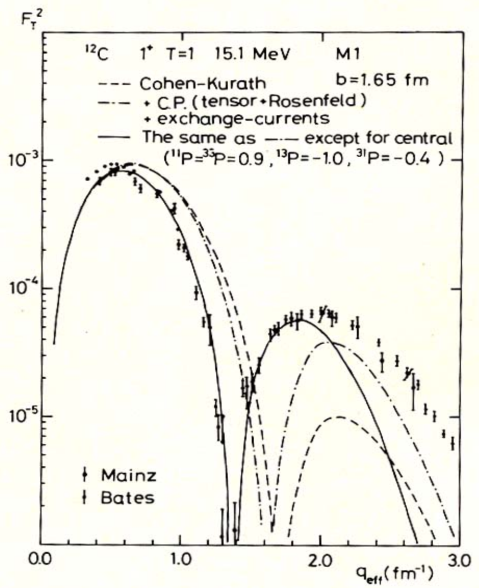


Fig. 2. M1 form factors in $^{12}C(e,e')^{12}C$ (1^+, T=1, 15.1 MeV) scattering.

Core-Polarization

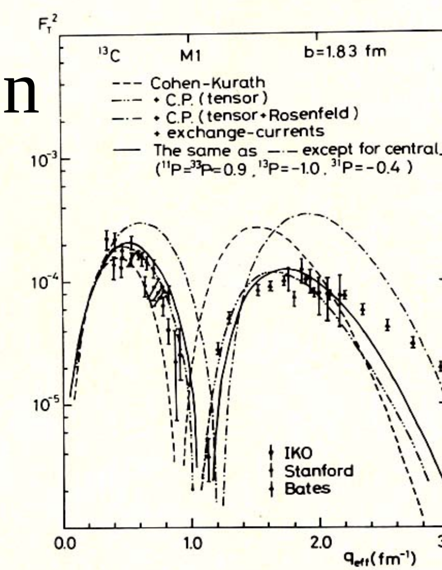


Fig. 1. M1 form factors in $^{13}C(e,e)^{13}C$ scattering.

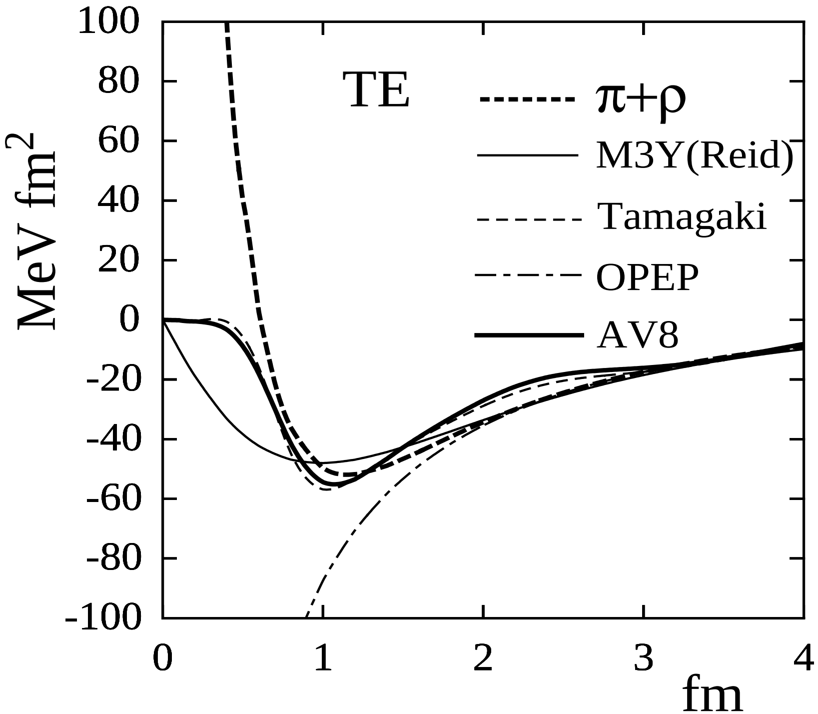
Shell-model interactions

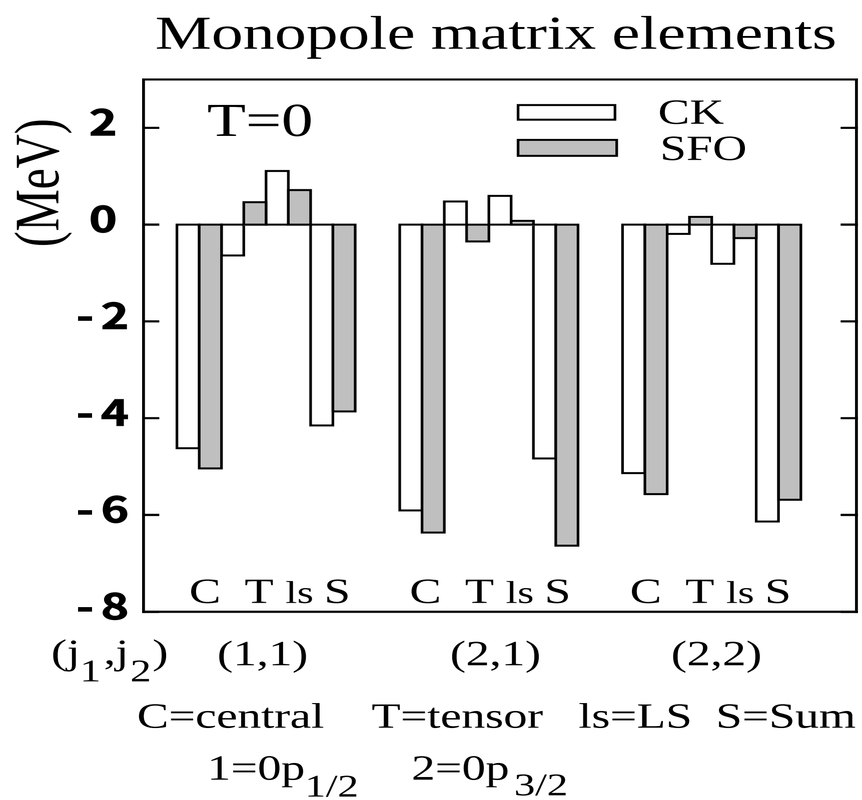
$$V = V_C + V_{Tensor} + V_{LS}$$

$$V_T = \vec{\tau}_1 \cdot \vec{\tau}_1 \{3(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_2 \cdot \hat{r}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2\} Y(r)$$

Monopole terms in p-shell

$$\bar{V}^{T}_{j_1 j_2} = \sum_J (2J+1) < j_1 j_2 : JT \,|\, V \,|\, j_1 j_2 : JT > / \sum_J (2J+1)$$





Effects of Tensor Force on Shell Evolution

Otsuka, Suzuki, Fujimoto, Grawe, Akaishi, PRL 69 (2005)

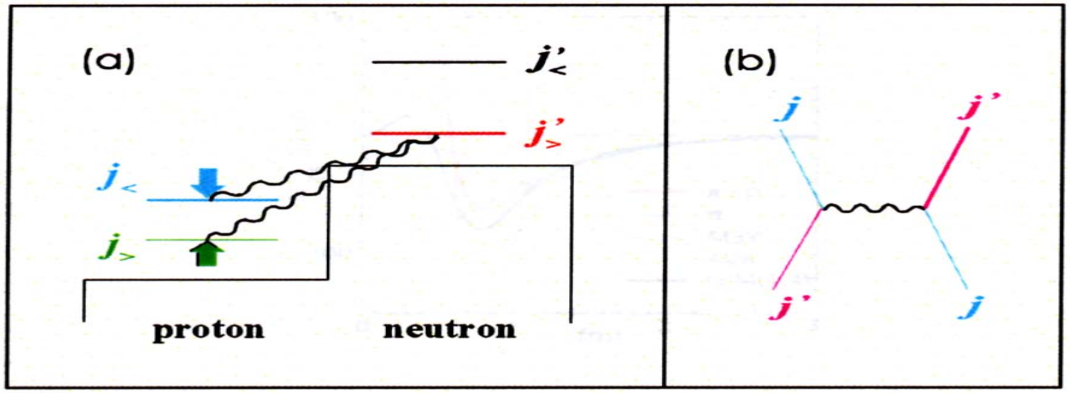


FIG. 1: (a) Schematic picture of the monopole interaction produced by the tensor force between a proton in $j_{>,<} = l \pm 1/2$ and a neutron in $j'_{>,<} = l' \pm 1/2$. (b) Exchange processes contributing to the monopole interaction of the tensor force.

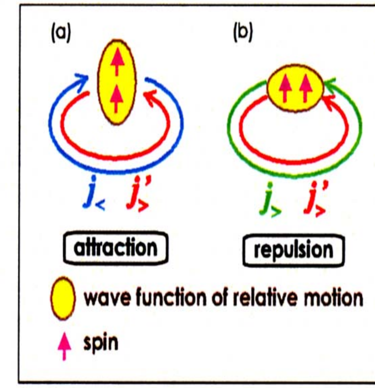
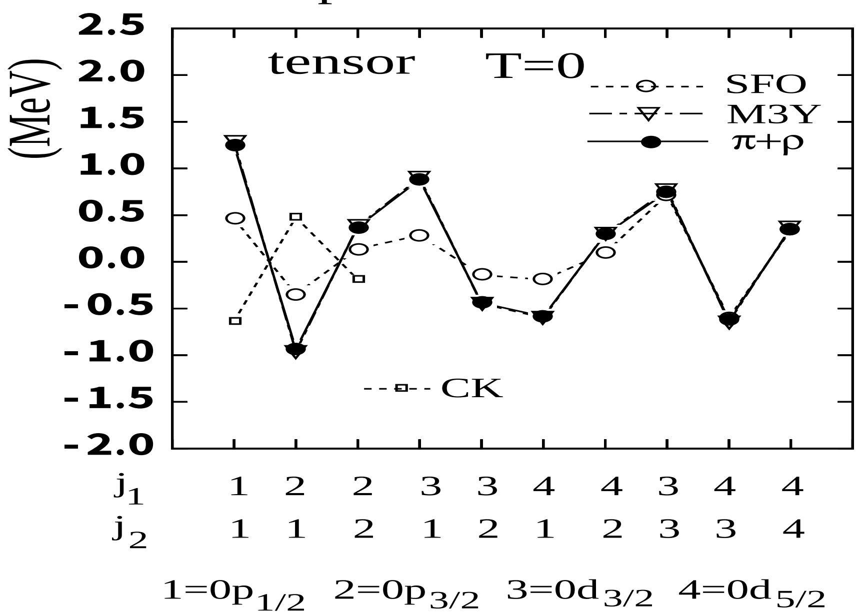
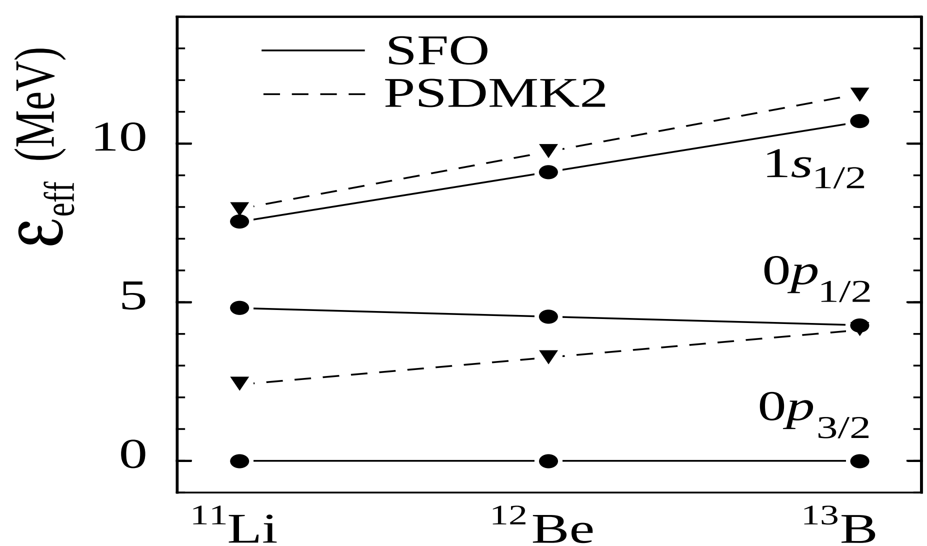


FIG. 2: Intuitive picture of the tensor force acting two nucleons on orbits j and j'.

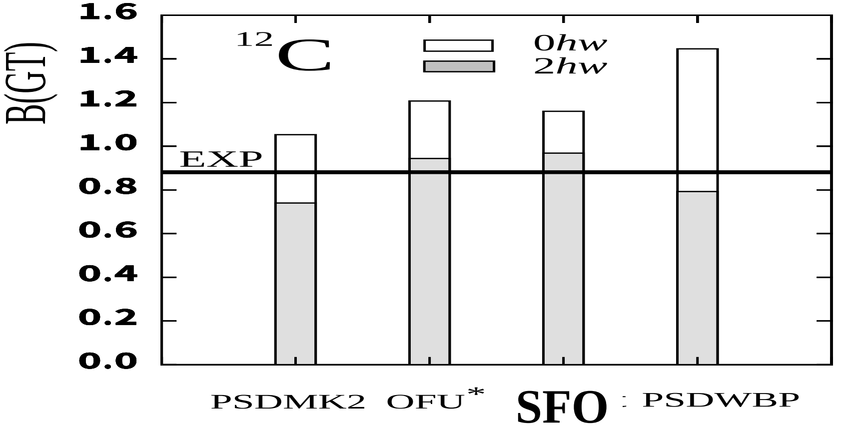
Monopole matrix elements



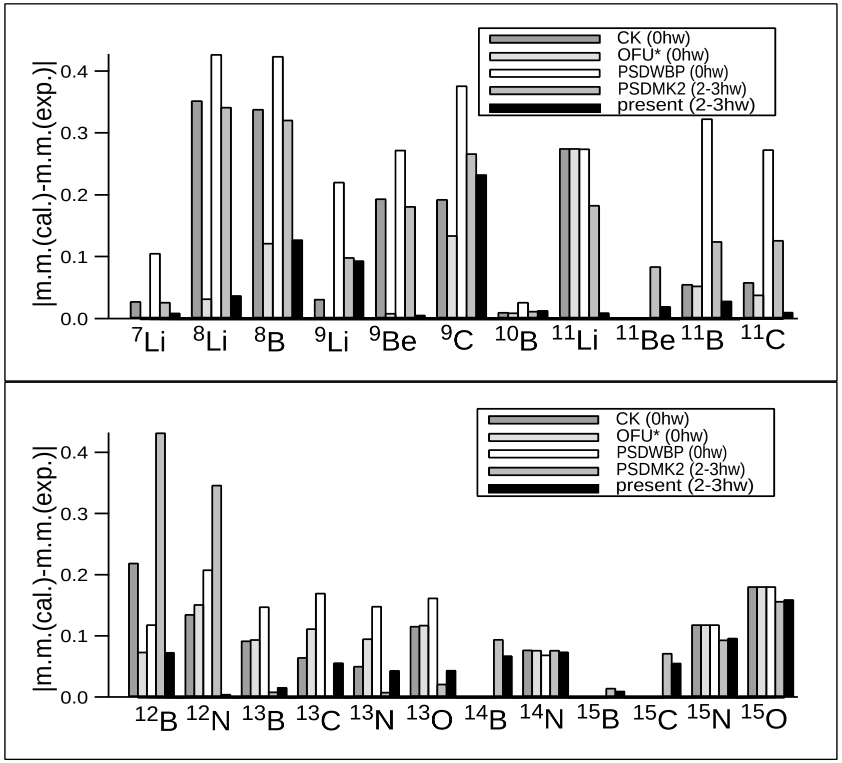
Effective Single-Particle Energy for N=8 Nuclei



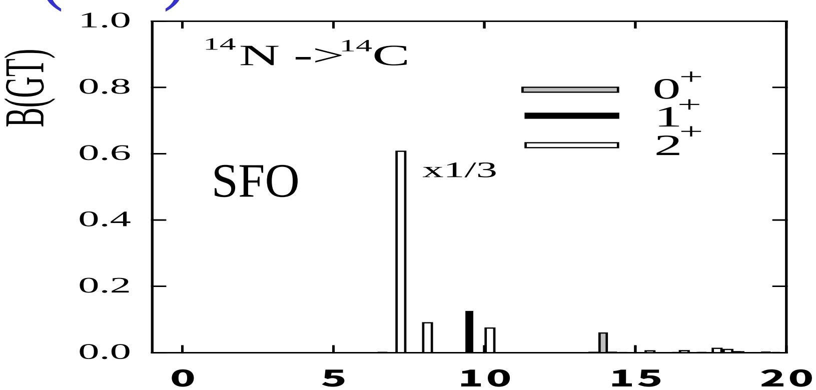
B(GT) values for ^{12}C -> ^{12}N



Magnetic moments of p-shell nuclei



B(GT) values for ^{14}N -> ^{14}C



present = SFO **Suzuki, Fujimoto, Otsuka, PR C67 (2003)**

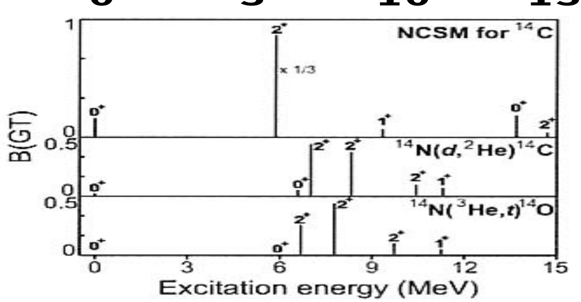


FIG. 3. Experimental B(GT) distributions, compared to the theoretical result of Aroua *et al.* [14], where the B(GT) to the 2^+ state was scaled down by a factor of 3.

Negret et al., PRL 97 (2006)

KVI

RCNP

SFO*: g_A^{eff}/g_A=0.95

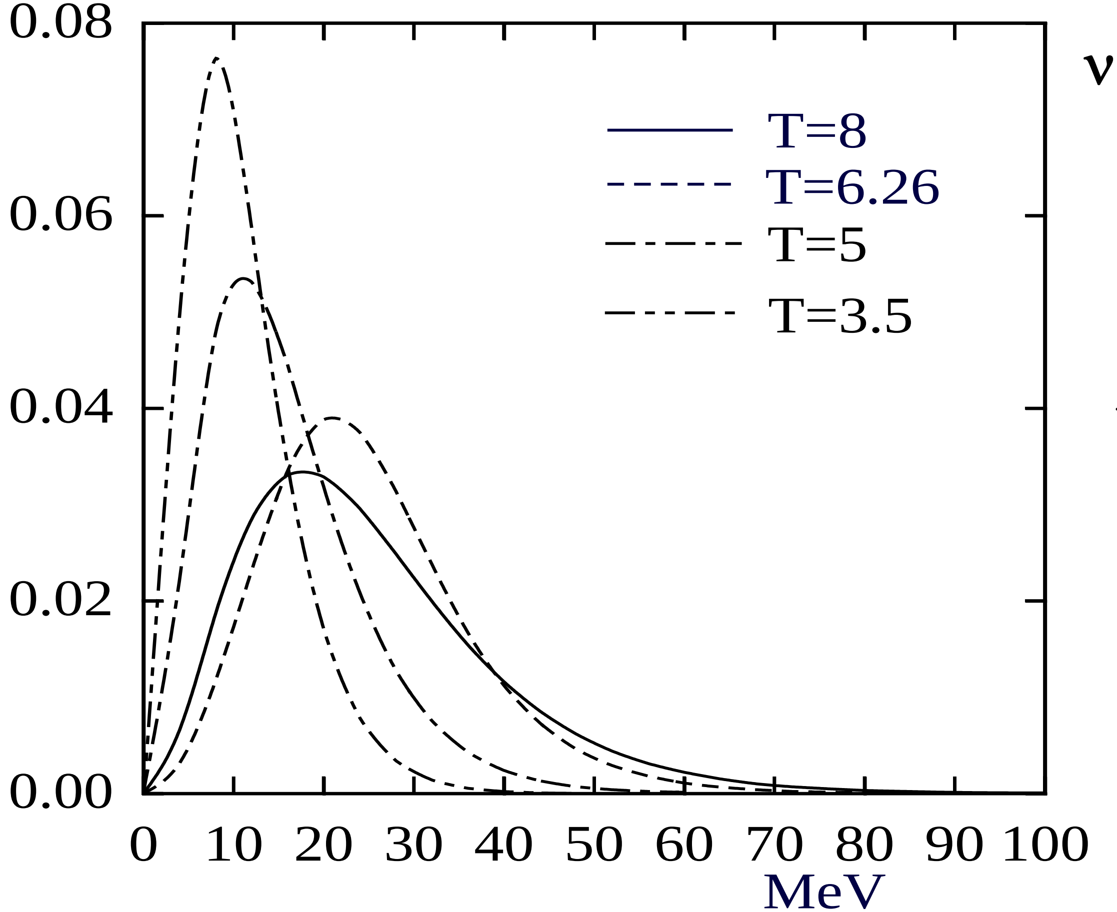
B(GT: ^{12}C) fitted to experiment

Supernovae ν Spectra

$\sigma \propto E^2$

<E> & tail part

$$f(E) = N \frac{E^2 / T^3}{1 + \mathbf{exp}[E/T - \alpha]}$$



$\nu_\mu, \nu_\tau : \langle E \rangle = 25\ MeV$

$(T, \alpha) = (8 MeV, 0)$

$(6.26 MeV, 3)$

$\nu_e : \langle E \rangle = 11\ MeV$

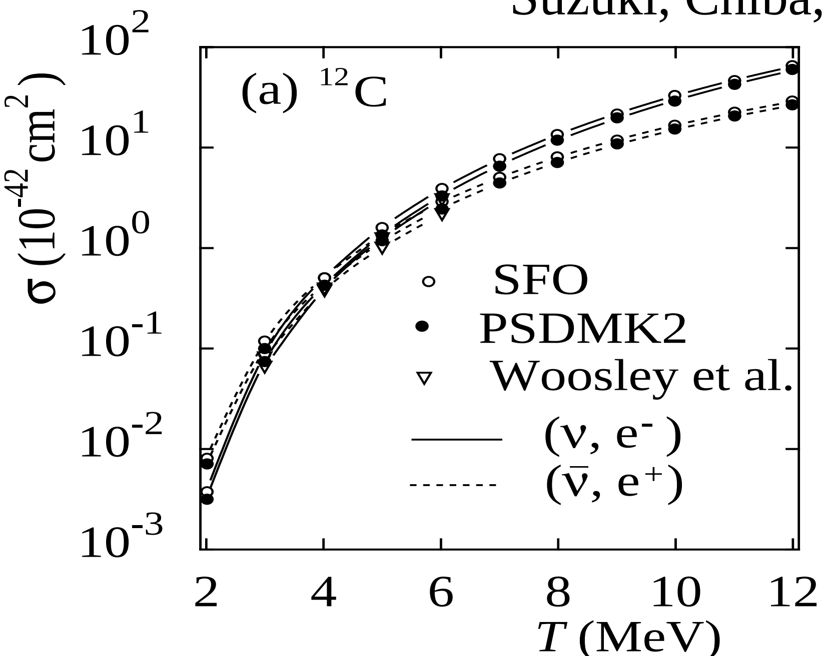
$(T, \alpha) = (3.5 MeV, 0)$

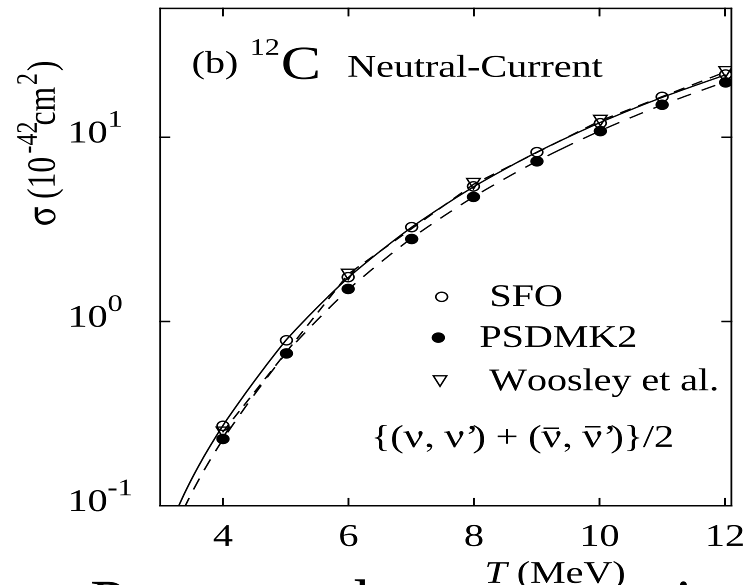
$\overline{\nu}_e : \langle E \rangle = 16\ MeV$

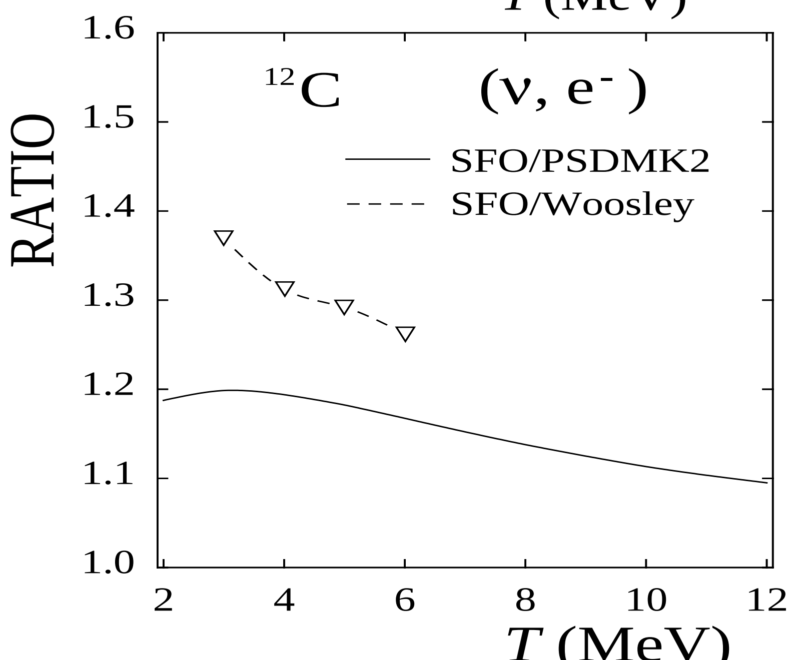
$(T, \alpha) = (5 MeV, 0)$

Cross sections for Supernova Neutrinos with temperature T

Suzuki, Chiba, Yoshida , Kajino, Otsuka, PR C74 (2006)

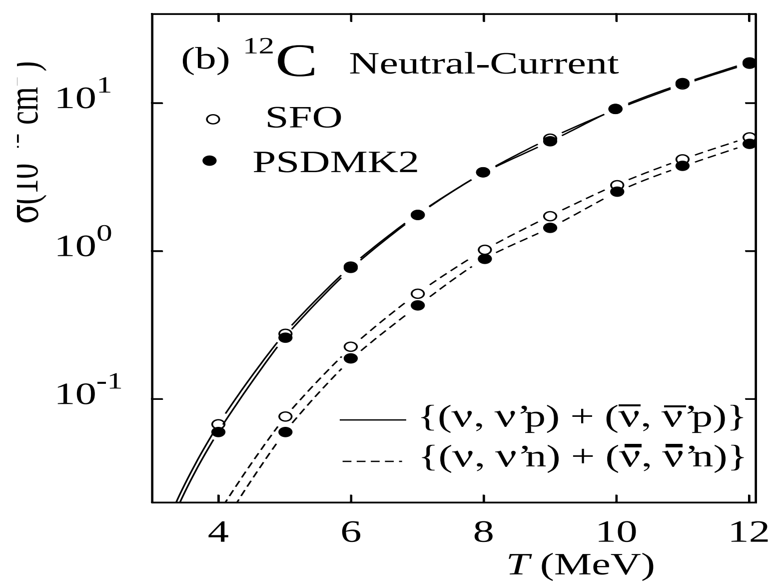






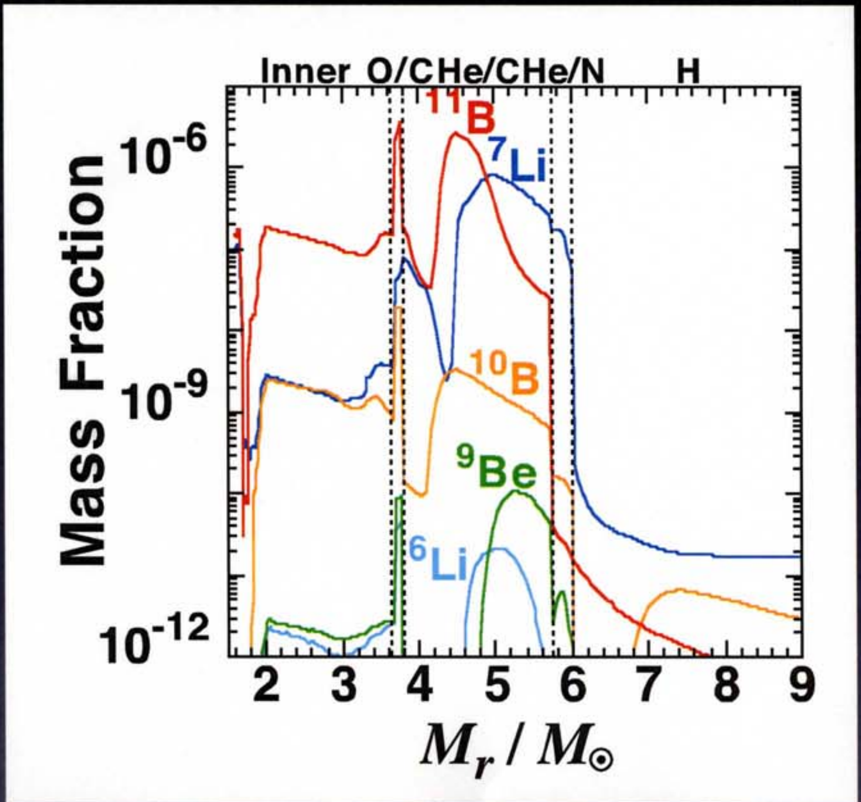
Proton and neutron emissions

BR: Hauser-Feshbach model

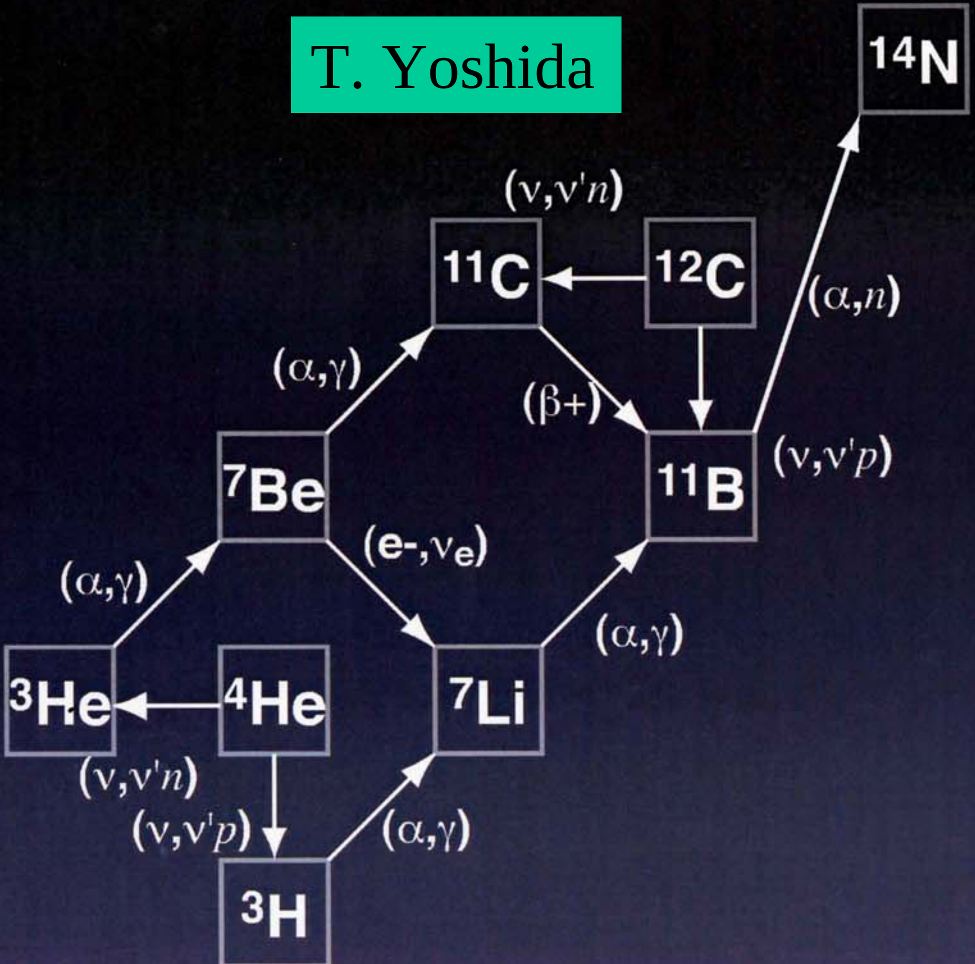


Light Element Abundances and Nucleosynthesis Processes

T. Yoshida



E_{ν}=300 foe, τ_{ν} = 3 s, $T_{\nu\mu,\tau}$= 8 MeV

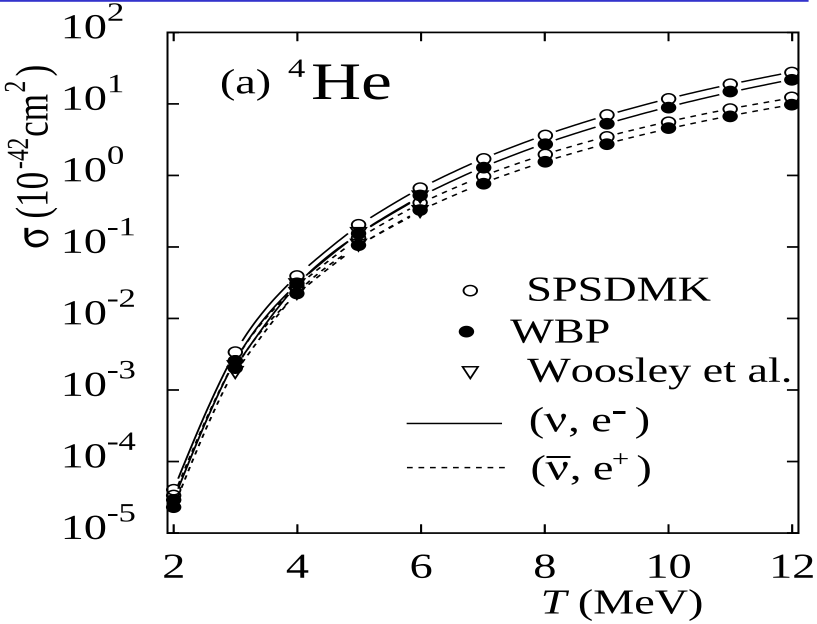


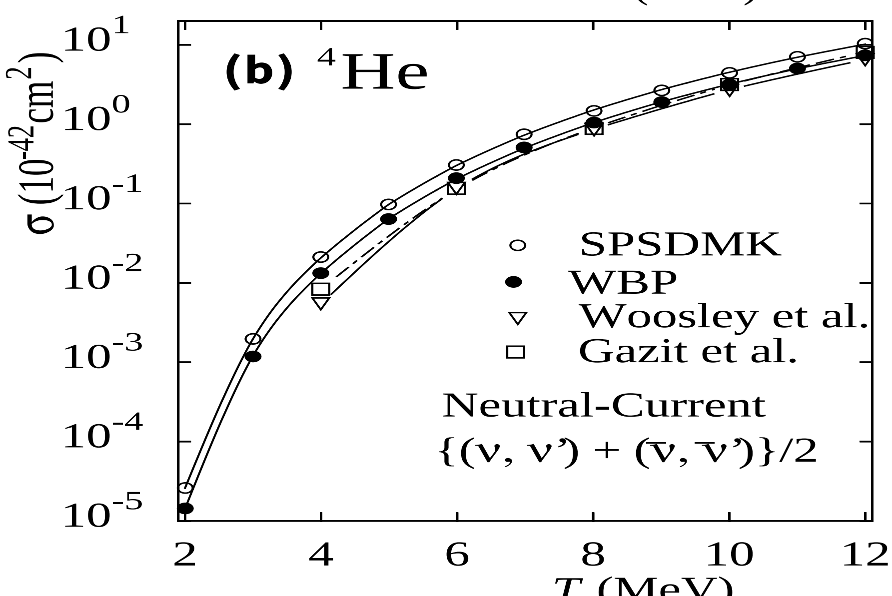
^{7}Li & ^{11}B production in He/C layer

^{4}He$(\nu,\nu'p)^{3}$H, ^{4}He$(\nu,\nu'n)^{3}$He, ^{12}C$(\nu,\nu'p)^{11}$B

^{4}He$(\bar{\nu}_e,e^{+}n)^{3}$H, ^{4}He$(\nu_e,e^{-}n)^{3}$He

ν - ^{4}He reaction cross sections





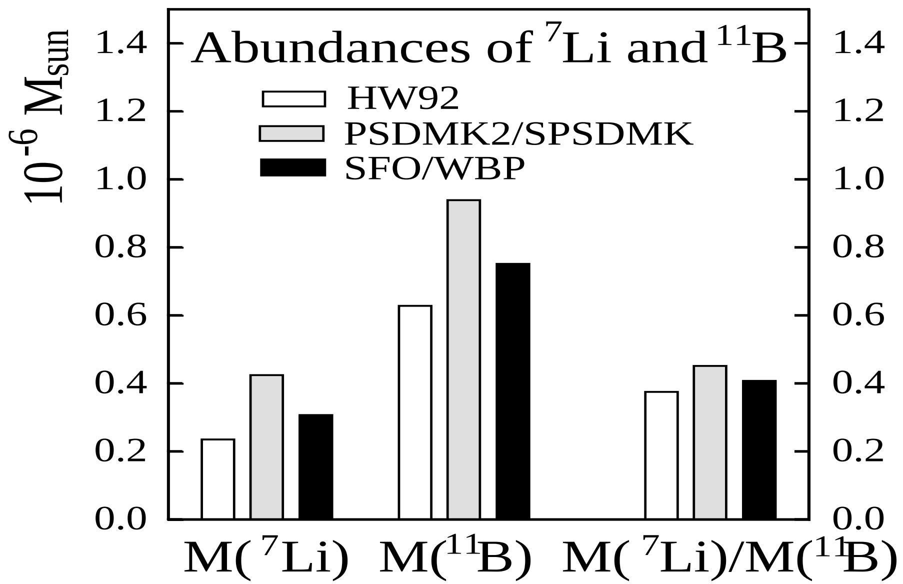
Abundances of ^{7}Li and ^{11}B produced in supernova explosion processes

M=16.2 M☼ (SN 1987A)

$$T_{\nu_e} = 3.2\ MeV,\quad T_{\bar{\nu}_e} = 5.0\ MeV$$

$$T_{\nu_\mu,\nu_\tau} = 6.0\ MeV$$

$$(\nu,\nu' p),(\nu,\nu' n) \quad \nu = \nu_{\mu,\tau}, \bar{\nu}_{\mu,\tau}$$



cf. Woosley-Haxton: Sussex potential by Elliott et al.

Nucleosynthesis through neutrino-induced reactions

- Production of rare elements by ν - reactions

$$^{138}Ba(\nu, e^-)^{138}La$$

$$^{180}Hf(\nu, e^-)^{180}Ta$$

Calculation by Heger et al.

GT exp. RCNP (^{3}He, t)
More GT strength than RPA (Heger et al.)
Byelikov et al., PRL 98 (2007)

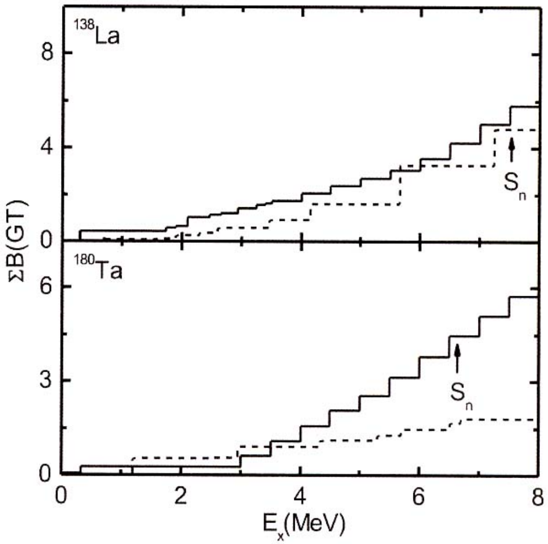


FIG. 4. Summed B(GT) strength in ^{138}La and ^{180}Ta as a function of excitation energy. The neutron emission thresholds (S_n) are indicated by arrows. Solid lines: present experiment. Dashed lines: RPA calculations from Ref. [2].

- Role of ν in r-process nucleosynthesis

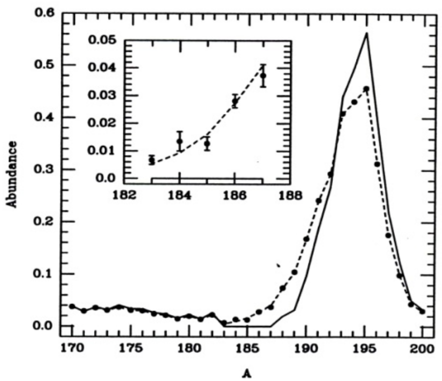


Figure 5. Effects of neutrino-induced neutron emission for the region near the abundance peak at $A \sim 195$ [48, 50]. The abundances before and after neutrino-induced neutron emission (following freeze-out of the r-process) are given by the solid and dashed curves, respectively. The filled circles (some with error bars) give the solar r-process abundances. The region with $A = 183$–187 is highlighted in the inset.

N=82, 126 regions
With ν -induced n emission → solar abundances

Qian, Haxton, Langanke, Vogel, PR C55 (1997)

Probe of Weak Neutral Current

$$G_1^S(<N|\bar{s}\gamma_\mu\gamma_5 s|N>) \quad via \quad A(\nu,\nu')A(T=0)$$

$$\frac{d\sigma}{d\Omega} \propto \quad |V^S + A^S - 2\sin^2\theta_W J^\gamma|^2 \approx |A^S|^2 \propto \quad (G_1^S)^2$$

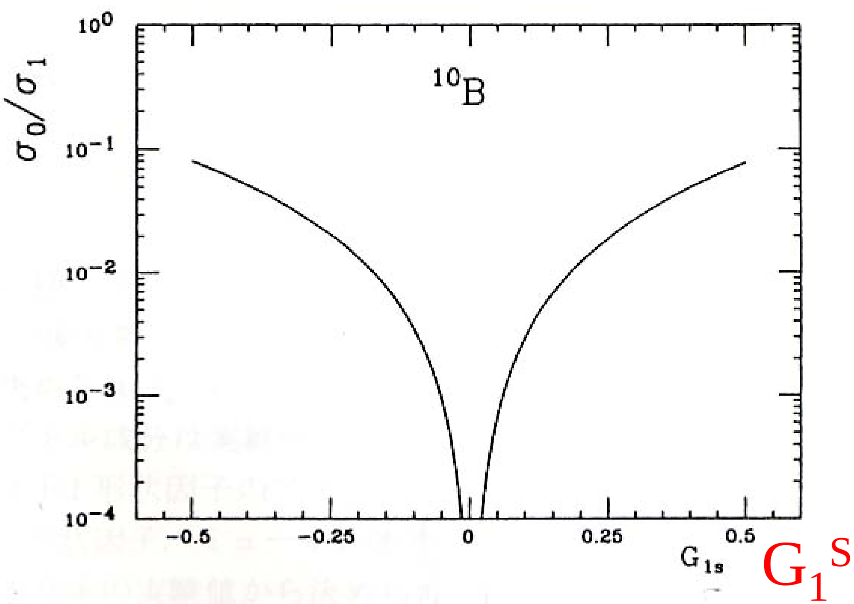


Fig. 5 Ratio of the total cross section of $^{10}B(\nu, \nu')$ $^{10}B(2^+, T=0, 3.59\,\mathrm{MeV})$ over that of $^{10}B(\nu, \nu')$ $^{10}B(2^+, T=1, 5.16\,\mathrm{MeV})$.

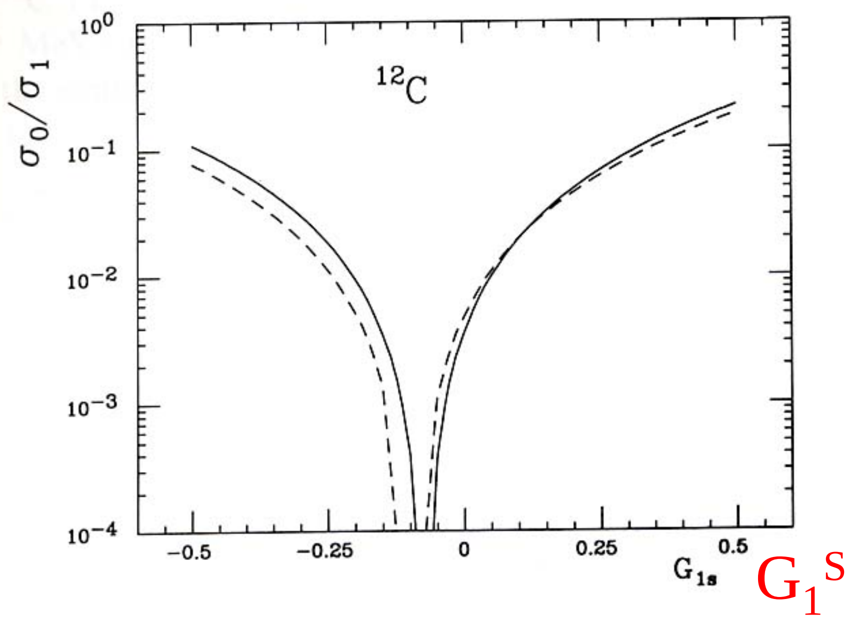


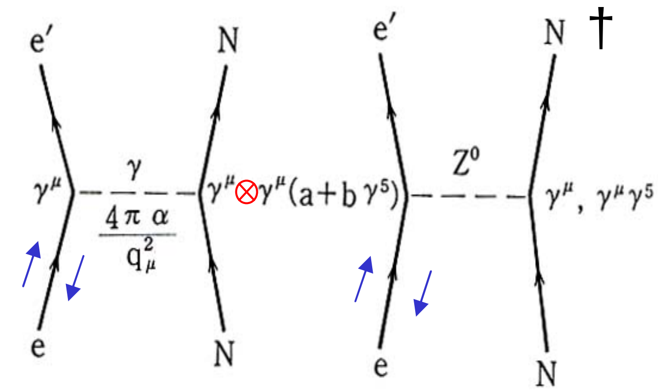
Fig. 1. Ratios of the total cross section of $^{12}C(\nu, \nu')^{12}C$ (1^+, 12.71 MeV) over that of $^{12}C(\nu, \nu')^{12}C$ (1^+, 15.11 MeV). The solid curve is obtained by using the Cohen–Kurath wave function, while the dashed curve includes the effects of the admixtures of $p^6(sd)^2$ components.

Suzuki, Kohyama, Yazaki

Polarized electron scattering

$$A(\vec{e},e)A$$

$$A=\frac{d\sigma_{\rightarrow}-d\sigma_{\leftarrow}}{d\sigma_{\rightarrow}+d\sigma_{\leftarrow}} \propto \quad \mathrm{Re}\{$$



$$\}$$

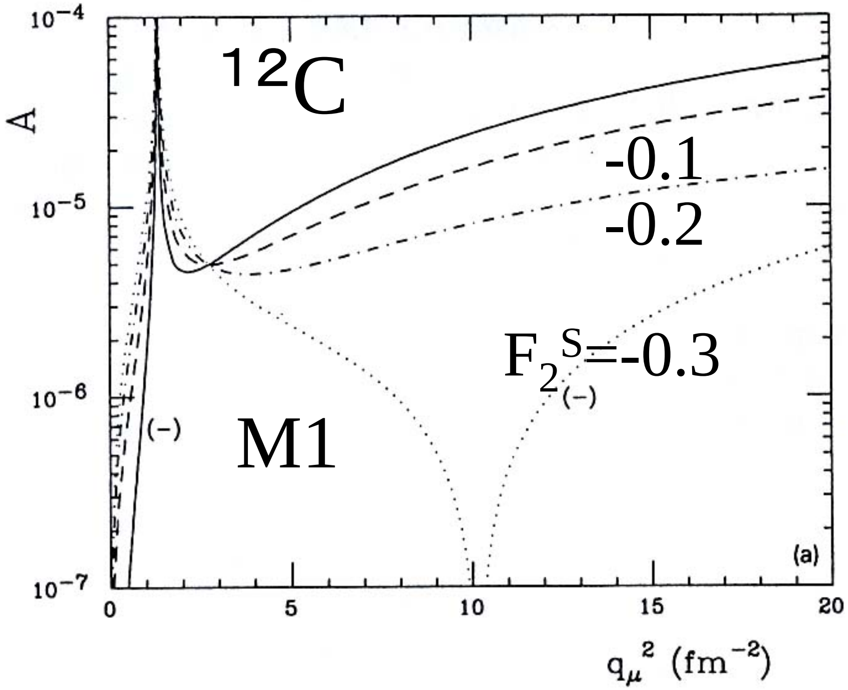
$$R-L=\frac{1}{2}(1-\gamma_5)-\frac{1}{2}(1+\gamma_5)=-\gamma_5$$

$$A \propto \mathrm{Re}\{Tr[\not{k}_2\gamma_\mu\gamma_5\not{k}_1\gamma_\nu(1-4\sin^2\theta_W+\gamma_5)J_\mu^\gamma J_\nu^{W*}\}$$

$$\approx \mathrm{Re}\{(1-4\sin^2\theta_W)J_\mu^\gamma J_\mu^{A*}+J_\mu^\gamma J_\mu^{V*}\}$$

$$\approx J_\mu^\gamma\{0.08<N|\bar{s}\gamma_\mu\gamma_5 s|N>+<N|\bar{s}\gamma_\mu s|N>\}$$

$$\boxed{F_2^S}$$



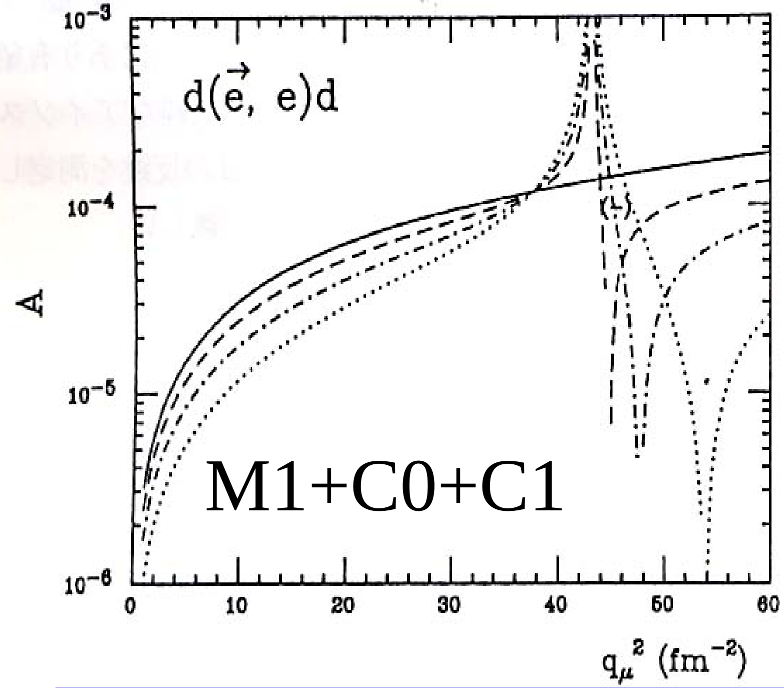
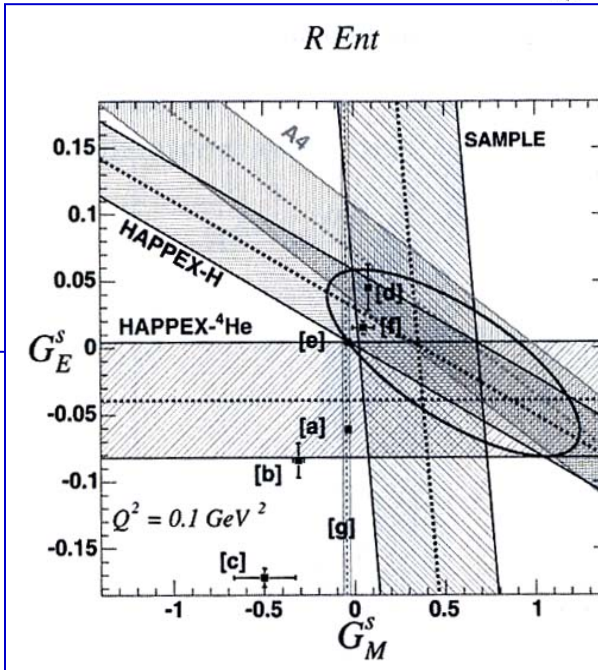


Fig. 3a
Calculated asymmetries of the ^{12}C $(\vec{e}, e')$ ^{12}C $(1^+, 12.71\,\mathrm{MeV})$ reaction at Ee= 1 GeV. Adopted values of $F_2^s(0)$ are 0.0 (solid), −0.1 (dashed), −0.2 (dash-dotted) and −0.3 (dotted curve). Vector form factors of the nucleon are taken from ref. 26). $G_1^s(0)$ is fixed to be 0.0.



SAMPLE etc

Ent, in *EM Int. and Hadron Structure (2007)*

$$A(\vec{e},e)A \quad \text{Isoscalar Coulomb scatt.} \quad 0^+ \rightarrow 0^+$$

$$A \approx \frac{G q_\mu^2}{4\pi\alpha\sqrt{2}} \frac{\rho_n(q)}{\rho_p(q)}$$

$$\left(G_E^S = F_1^S - \frac{q_\mu^2}{4m^2} F_2^S \qquad (F_1^S(0) = 0)\right)$$

$$A_S \equiv \pi\alpha\sqrt{2}A/(G q_\mu^2 \sin^2\theta_W)$$

$$\approx \frac{1}{4\sin^2\theta_W} \frac{N}{Z} \left\{1 - \frac{q^2}{6} \boxed{(< r_n^2 > - < r_p^2 >)}\right\}$$

Ca isotopes

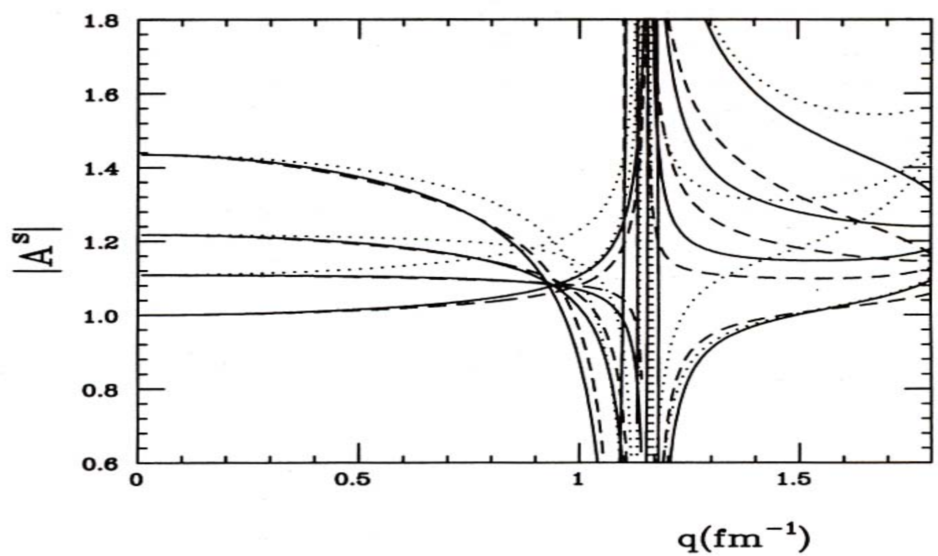


FIG. 1. Calculated asymmetries for Ca isotopes. Solid and dashed curves are obtained by using the SGII-HF and HF-T wave functions, respectively. The dotted curve is obtained by using the modified HF-T wave functions which reproduce the experimental values of r_n and r_p.

Beyond standard model
Neutrino oscillations

$\alpha = e、\mu、\tau$

$$|\nu_\alpha\rangle = \sum_a U_{\alpha a}^* |\nu_a\rangle \qquad a=1,2,3$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \times \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$c_{ij} = \cos\theta_{ij}, \quad s_{ij} = \sin\theta_{ij}$$

Present status of parametrization

$1.5\times10^{-3} < |\Delta m_{23}^2| < 3.4\times10^{-3}\ \mathrm{eV}^2$

$\theta_{23} = 45 \pm 8^\circ$ (90%) (K2K)

$|\Delta m_{12}^2| = 8.2^{+0.6}_{-0.5}\times10^{-5}\ \mathrm{eV}^2$

$\theta_{12} = 32.3^{+2.7^\circ}_{-2.4^\circ}$ (1σ) (KamLAND, SN)

$\sin^2 2\theta_{13} < 0.1$ (Chooz)

ν oscilltions in supernova explosion

$\rightarrow$ Possible constraint on lower limit of θ_{13}