Giant Resonances – Wavelets, Scales and Level Densities

- Giant resonances
- Damping mechanisms, time and energy scales
- Fine structure
- Wavelets and characteristic scales
- Application: GQR
- Many-body nuclear models and damping mechanisms
- Relevance of scales: GTR
- Level densities of $J^{\pi} = 1^+, 2^+, 2^-$ states

Supported by DFG under SFB 634, 446 JAP 113 / 267 / 0-1 und 445 SUA-113 / 6 / 0-1

S-DALINAC
Giant Resonances

Monopole
$\Delta L = 0$

Dipole
$\Delta L = 1$

Quadrupole
$\Delta L = 2$

$\Delta T = 0$  $\Delta T = 1$  $\Delta T = 0$  $\Delta T = 1$
$\Delta S = 0$  $\Delta S = 0$  $\Delta S = 1$  $\Delta S = 1$
Giant Resonances

<table>
<thead>
<tr>
<th>Monopole</th>
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<td>$\Delta L = 0$</td>
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</table>

$\Delta T = 0$
$\Delta S = 0$

$\Delta T = 1$
$\Delta S = 0$

→ interplay between collective and s.p. motion

Courtesy of P. Adrich
Isoscalar Quadrupole Mode

- Pitthan and Walcher (Darmstadt, 1972)

- Centroid energy: $E_x \sim 63 A^{-1/3} \text{ MeV}$
- Width
- Damping mechanisms

$^{140}\text{Ce}(e,e')$

$E_0 = 65 \text{ MeV}$

$\theta = 93^\circ$

DALINAC

$\Delta E = 200 \text{ keV}$

$\theta = 129^\circ$

$\theta = 165^\circ$
Early Work on GR’s: $^{90}$Zr

Bertrand et al. (1981)

$\Delta E$ (FWHM) $\sim$ 1 MeV

$^{90}$Zr $(p,p')$

$E_p = 200$ MeV
Microscopic Picture of Giant Resonances: $^{90}\text{Zr}$

Giant vibration $\hat{\Psi}$ is coherent superposition of elementary p-h excitations

$$\Psi^{J\pi} = \sum_{\alpha} c_\alpha | (p - h)^{J\pi}_\alpha \rangle$$

$$H(1, \ldots A) = H_0(1, \ldots A) + \sum_{\substack{i,k}} V(i, k)$$
Excitation and Decay of Giant Resonances

\[ \Gamma = \Delta \Gamma + \Gamma^\uparrow + \Gamma^\downarrow \]

- Resonance width
- Landau damping
- Escape width
- Spreading width

Direct decay
Pre-equilibrium and statistical decay
Doorway State Model

- $|1p-1h>$
- $|2p-2h>$
- $|np-nh>$

**Scales**

- $t \sim 10^{-22} \text{ s}$  \quad $\Gamma \sim \text{MeV}$
- $t \sim 10^{-21} \text{ s}$  \quad $\Gamma \sim \text{MeV} - \text{keV}$
- $t \sim 10^{-16} \text{ s}$  \quad $\Gamma \sim \text{keV} - \text{eV}$
High resolution is crucial

Possible probes: electrons and hadrons
Fine Structure of Giant Resonances

Different probes but similar structures

→ physical information content is the same
Fine Structure of Giant Resonances

- Global phenomenon?
  - Other nuclei
  - Other resonances

- Methods for characterization of fine structure

- Goal: dominant damping mechanisms
**Recent Experiments**

<table>
<thead>
<tr>
<th>Place:</th>
<th>iThemba LABS, South Africa</th>
<th>RCNP, Osaka</th>
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</thead>
<tbody>
<tr>
<td>Reaction:</td>
<td>ISGQR from ((p,p'))</td>
<td>GT from ((^3\text{He},t))</td>
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<table>
<thead>
<tr>
<th>Element</th>
<th>Mass Number</th>
<th>Place</th>
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<tbody>
<tr>
<td>Ni</td>
<td>58</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y</td>
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<tr>
<td>Zr</td>
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<tr>
<td>Sn</td>
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<tr>
<td>Nd</td>
<td>142</td>
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<tr>
<td>Er</td>
<td>166</td>
<td></td>
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<tr>
<td>Pb</td>
<td>208</td>
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<tr>
<td>Zr</td>
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<tr>
<th>Beam energy:</th>
<th>200 MeV</th>
<th>140 MeV/u</th>
</tr>
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<tbody>
<tr>
<td>Scattering angles:</td>
<td>8° – 10° ((\Delta L = 2))</td>
<td>0° ((\Delta L = 0))</td>
</tr>
<tr>
<td>Energy resolution:</td>
<td>(\Delta E = 35 - 50) keV</td>
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**Place:**

- IThemba LABS, South Africa
- RCNP, Osaka

**Reaction:**

- ISGQR from \((p,p')\)
- GT from \((^3\text{He},t)\)

**Beam energy:**

- 200 MeV
- 140 MeV/u

**Scattering angles:**

- 8° – 10° (\(\Delta L = 2\))
- 0° (\(\Delta L = 0\))

**Energy resolution:**

- \(\Delta E = 35 - 50\) keV
- \(\Delta E = 50\) keV
Fine Structure of the ISGQR

\[ \Delta E \approx 1 \text{ MeV} \]
TRIUMF (1981)

\[ \Delta E \approx 40 \text{ keV} \]
iThemba (2004)

Fluctuations of different strengths and scales
Not a Lorentzian
Fine structure of the ISGQR is a global phenomenon
Fine Structure in Deformed Nuclei?

\[^{166}\text{Er}(p,p')\]

\[\Theta_p = 8^\circ\]

\[E_0 = 200\text{ MeV}\]
Wavelet Analysis

\[ \int_{-\infty}^{\infty} \Psi^*(x) \, dx = 0 \quad \text{and} \quad \int_{-\infty}^{\infty} \left| \Psi^*(x) \right|^2 \, dx < \infty \]

\[ \Psi(x) = \cos(2\pi \omega x) \, e^{-x^2/2} \]

Wavelet coefficients:

\[ C(\delta E, E_x) = \frac{1}{\sqrt{\delta E}} \int \sigma(E) \, \Psi^*(\frac{E_x - E}{\delta E}) \, dE \]

Continuous: \( \delta E, E_x \) are varied continuously
Wavelet Analysis

Intensity

Spectrum

Location

Scale

Wavelet transform

Location
Wavelet Analysis

Intensity

Spectrum

Location

Scale

Wavelet transform

Location
Wavelet Analysis

Intensity

Spectrum

Location

Scale

Wavelet transform

Location
Wavelet Analysis
$^{208}\text{Pb}(p,p')$ at iThemba LABS

Power Spectrum

Counts ($\times 10^3$)

Wavelet Scale (MeV)

Excitation Energy (MeV)

Wavelet Power (a.u.)

Excitation Energy (MeV)

Wavelet Transform
$^{208}\text{Pb}(e,e^-)$ at DALINAC

Power Spectrum

Counts ($\times 10^3$)

Wavelet Scale (MeV)

Wavelet Transform

Excitation Energy (MeV)

Wavelet Power (a.u.)

$^{208}\text{Pb}(e,e^-)$

$E_e = 50\, \text{MeV}$

$\theta = 93^\circ$
Summary of Scales

- Scales are found in all nuclei

- Three classes
  
  - I: ~ 100 keV all nuclei
  
  - II: ~ 200 – 900 keV changes with mass number
  
  - III: ~ 1.2 – 4.7 MeV gross width
<table>
<thead>
<tr>
<th>Model</th>
<th>Author(s)</th>
<th>Year</th>
</tr>
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<tbody>
<tr>
<td>RPA</td>
<td>Wambach et al.</td>
<td>2000</td>
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<td>SRPA</td>
<td>Wambach et al.</td>
<td>2000</td>
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<tr>
<td>QPM</td>
<td>Ponomarev</td>
<td>2003</td>
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<tr>
<td>ETDHF</td>
<td>Lacroix et al.</td>
<td>1997</td>
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<tr>
<td>1p – 1h ⊗ phonon ETFFS</td>
<td>Kamerdziev et al.</td>
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\( ^{208}\text{Pb} \text{ RPA} \)

- No scales from 1p-1h states
Coupling to 2p-2h generates fine structure and scales
Microscopic Models: Case of $^{208}\text{Pb}$

- Large differences between model predictions

- No a priori judgement possible which model should be preferred

- Use wavelet analysis for a quantitative measure in comparison with the experimental observations
<table>
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<th>Scales (keV)</th>
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<td>Exp / keV</td>
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- Three classes of scales as in the experiment on a qualitative level
- But strong variations of class II and class III scales
- Take QPM for semi-quantitative analysis of damping mechanisms
Semi-Quantitative Attempt of Interpretation: $^{208}\text{Pb}$ as Example

Two types of dissipation mechanisms:

- **collective damping**
  - low-lying surface vibrations
  - $1p - 1h \otimes \text{phonon}$

- **non-collective damping**
  - background states
  - coupling to $2p - 2h$ states
How Can the Two Mechanisms Be Separated: Distribution of the Coupling Matrix Elements

QPM: distribution for \( \langle 1p1h \mid V_{1p1h}^{2p2h} \mid 2p2h \rangle \)

RMT: deviations at large and at small m.e. \( \rightarrow 4\text{nd Lecture} \)

Large m.e. define the collective damping mechanism

Small m.e. are responsible for the non-collective damping
Collective vs. Non-Collective Damping in $^{208}\text{Pb}$

- **Collective part:** all scales
- **Non-collective part:** no prominent scales

Stochastic coupling
Spreading of a GR due to the Coupling to Doorway States and Decay into Compound Nucleus States

- First step of coupling hierarchy $1p-1h \rightarrow 2p-2h$ has been tested
- Further steps require improvement in resolution
IVGDR in Exclusive Electron Scattering
\[ ^{40}\text{Ca}(e,e'p_0)^{39}\text{K} \]
$^{48}\text{Ca}(e,e'\text{n})$
Are All Scales Equally Relevant for the Fine Structure?

Example: Gamow-Teller Giant Resonance

\[ \Delta L = 0, \Delta T = 1, \Delta S = 1 \]
Fine Structure of the Spin-Flip GTR

$^{90}\text{Zr}(^{3}\text{He},t)^{90}\text{Nb}$

$E_0 = 140$ MeV/u

RCNP

$0^\circ < \theta < 0.9^\circ$

- High energy resolution
- Asymmetric fluctuations
Wavelet coefficients:

\[ C(\delta E, E_x) = \frac{1}{\sqrt{\delta E}} \int \sigma(E) \Psi^* \left( \frac{E_x - E}{\delta E} \right) dE \]

Discrete: \( \delta E = 2^j \) and \( E_x = k \delta E \) with \( j, k = 1, 2, 3, \ldots \)

Orthogonal basis of wavelet functions (e.g. biorthogonal form)

Exact reconstruction of the spectrum is possible and it is fast

Relevance of scales

\[ \int_{-\infty}^{+\infty} E^n \Psi^* \left( \frac{E_x - E}{\delta E} \right) dE = 0, \quad n = 0, 1 \ldots m - 1 \]

vanishing moments

this defines the shape and magnitude of the background

* http://www.mathworks.com/products/wavelet/
Decomposition

\[ \sigma(E) = A1 + D1 \]

\[ \sigma(E) = A2 + D2 + D1 \]

\[ \sigma(E) = A3 + D3 + D2 + D1 \]
Decomposition of $^{90}$Zr($^3$He,$t$)$^{90}$Nb Spectrum

Reconstruct the spectrum using important scales

Model-independent background determination + fluctuations $\rightarrow$ level densities
Discrete Wavelet Transform: Reconstructed Spectra

\[ \sigma_R(E) = A_8 + D_6 + D_5 + D_4 + D_3 \]

\[ \sigma_R(E) = A_8 + D_8 + D_7 + D_2 + D_1 \]
Level Densities

- Astrophysical network calculations

- Back-shifted Fermi gas model
  - semiempirical approach
  - shell and pairing effects
  
  \[ \rho(E_x, J) \sim e^{2\sqrt{a(E_x - \delta)}} \cdot e^{-\frac{J(J+1)}{2\sigma^2}} \]

- Many-body density of states
  - two-component Fermi gas
  - shell effects, deformations

- HF-BCS
  - microscopic statistical model (partition function, MSk7 force, local renormalization)
  - shell effects, pairing correlations, deformation effects, collective excitations

- Monte-Carlo shell model calculations
  - parity dependence?
Monte-Carlo Shell Model Predictions: pf + g\(_{9/2}\) Shell

- Total level density (not spin projected) shows strong parity dependence*
- Questioned by recent experiments (\(^{45}\)Sc)**

\[ \Delta_{pf-g_{9/2}} \text{ small} \]
\[ \rho_- \text{ important at low energies} \]


Experimental Techniques

Selectivity
hadron scattering at extremely forward angles and intermediate energies
electron scattering at 180° and low momentum transfers

High resolution
lateral and angular dispersion matching
faint beam method*

Level density
fluctuation analysis**

Background
discrete wavelet transform***

* H. Fujita et al., NIM A484 (2002) 17
*** Y. Kalmykov et al., PRL 96 (2006) 012502
Selective excitation of $2^+$ states

A. Shevchenko et al., PRL 93 (2004) 122501
**Fine Structure of the M2 Resonance: A = 90**

- Selective excitation of 2- states

P. von Neumann-Cosel et al., PRL 82 (1999) 1105
Fluctuations and Level Densities

\[ \frac{D}{\langle D \rangle} \quad \text{Wigner} \]

\[ \frac{I}{\langle I \rangle} \quad \text{Porter-Thomas} \]

\[ \Gamma < \langle D \rangle \]

\[ \Gamma < \langle D \rangle < \Delta E \]

4th Lecture
Fluctuation Analysis

- Background from wavelet analysis
- Statistics, local features
- Local fluctuations
- Autocorrelation function

**\( ^{90}\text{Zr}(^3\text{He},t)^{90}\text{Nb} \)**

Spectrum

- Background

Stationary Spectrum

- Autocorrelation function

Counts \( \times 10^3 / 10 \text{ keV} \)

Excitation Energy (MeV)

\( C(e) - 1 \)
**Autocorrelation Function and Mean Level Spacing**

- \( C(\varepsilon) = \frac{\langle d(E_x) d(E_x + \varepsilon) \rangle}{\langle d(E_x) \rangle \langle d(E_x + \varepsilon) \rangle} \)  
  autocorrelation function

- \( C(\varepsilon = 0) - 1 = \frac{\langle d^2(E_x) \rangle - \langle d(E_x) \rangle^2}{\langle d(E_x) \rangle^2} \)  
  variance

- \( C(\varepsilon) - 1 = \frac{\alpha \langle D \rangle}{2\sigma \sqrt{\pi}} \times f(\sigma, \varepsilon) \)  
  level spacing \( \langle D \rangle \)

- \( \alpha = \alpha_{PT} + \alpha_W \)  
  selectivity

- \( \sigma \)  
  resolution


Results and Model Predictions: $A = 90, J^\pi = 1^+$

Y. Kalmykov et al., PRL 96 (2006) 012502
Fine Structure of Level Density: $A = 90$, $J^\pi = 1^+$

$^{90}$Zr($^3$He,t)$^{90}$Nb

$J^\pi = 1^+$

RCNP

[Graph showing level density vs. excitation energy with data points and fitted lines for BSFG and HFB models.]
Phenomenological and Microscopic Models

Different quality of model predictions

- BSFG, MB DOS
  - parameters fitted to experimental data
  - no distinction of parity

- HF-BCS
  - microscopic
  - no distinction of parity

- HFB, SMMC
  - fully microscopic calculation of levels
  - with spin and parity

- HFB
  - fine structure of level densities
Ingredients of HFB

- Nuclear structure: HFB calculation with a conventional Skyrme force
  - single particle energies
  - pairing strength for each level
  - quadrupole deformation parameter
  - deformation energy

- Collective effects
  - rotational enhancement
  - vibrational enhancement
  - disappearance of deformation at high energies
Ingredients of SMMC

- Partition function of many-body states with good $J^\pi$
  \[ Z^\pi_J(\beta) = \text{Tr}_{J,\pi} e^{-\beta H} \]

- Expectation values at inverse temperature $\beta = 1/kT$
  \[ E^\pi_J(\beta) = \frac{\int dE' e^{-\beta E'} E' \rho^\pi_J(E')} {Z^\pi_J(\beta)} \]

- Level density from inverse Laplace transform in the saddle-point approximation
  \[ \rho^\pi_J(\beta) = \frac{e^{\beta E^\pi_J} + \ln Z^\pi_J(\beta)} {\sqrt{-2\pi} \frac{dE^\pi_J(\beta)} {d\beta}} \]
Level Density of $2^+$ and $2^-$ States: $^{90}$Zr

$^{90}$Zr(p,p$'$)
$J^\pi = 2^+$
iThemba LABS

$^{90}$Zr(e,e$'$)
$J^\pi = 2^-$
S-DALINAC

Y. Kalmykov et al., PRL 96 (2006) 012502
Equilibration of Parity-Projected Level Densities

$^{58}$Ni

$\rho_- \approx \rho_+ \text{ at } E_x \approx 20 \text{ MeV}$

$^{90}$Zr

$\rho_- \approx \rho_+ \text{ at } E_x \approx 5 \text{ – } 10 \text{ MeV}$

Two energy scales which determine $\rho_-/\rho_+$

- pair-breaking
  - 5 – 6 MeV for intermediate mass nuclei
  - shell gap between opposite-parity states near the Fermi level
    - depends strongly on the shell structure, e.g. $^{68}$Zn $\Delta_{pf-g9/2}$ is small

Core breaking

- e.g. near shell closure $^{58}$Ni $\Delta_{sd-pf}$ transitions are important
  - $\rho_-$ would be enlarged
Summary

Fine structure is a general phenomenon of many-body systems (particles, nuclei, atoms, molecules, clusters, condensates,...)

Quantitative analysis of nuclear giant resonances with wavelets

Origin of scales in GR’s:
- collective damping: low-lying surface vibrations
- non-collective damping: stochastic coupling

New method for level densities
Outlook

$^{208}\text{Pb}(\bar{p},\bar{p}')$

$E_p = 295$ MeV

$\Theta = 0^\circ - 2.5^\circ$

$\Delta E = 25$ keV

→ 3rd Lecture