
Nuclear Matter Equation of State

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- **Introduction**

Why do we study EOS ?

- **Relativistic Mean Field description of EOS**

$\sigma\omega$ model, Non-linear terms, Chiral RMF

- **Dense Matter EOS with Hyperons**

Hyperon potentials in nuclear matter, EOS with hyperons,

- **Collective flow and EOS in High-Energy Heavy-Ion Collisions**

Semi-classical transport model, Collective flows at AGS & SPS

- **Summary**

Introduction

Why do we study Nuclear Matter EOS ?

Why do we study Nuclear Matter EOS ?

- Answer 1: Since bulk nuclear properties are mainly determined by nuclear matter EOS, it is important for nuclear physics.

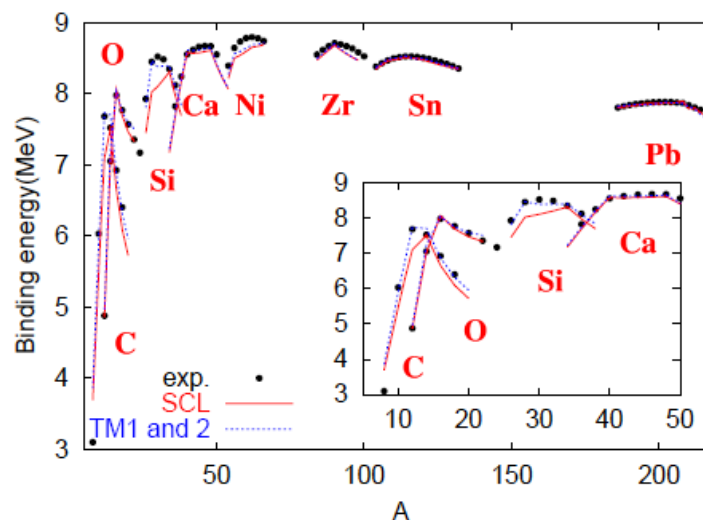
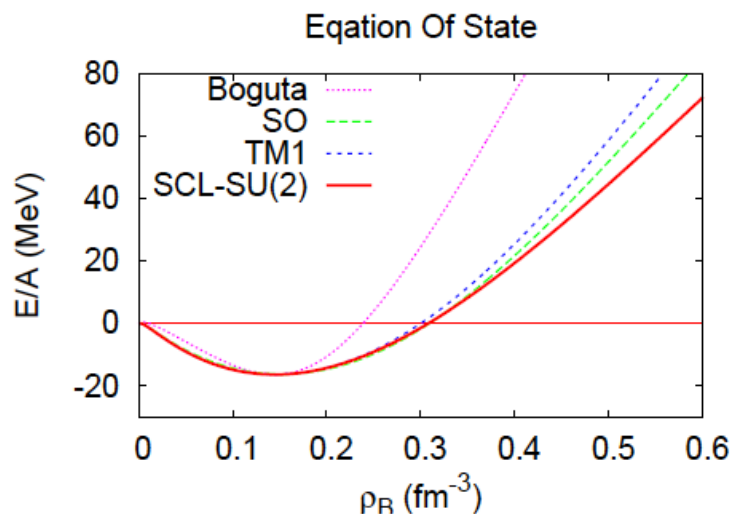
- Nuclear Radius → Saturation of Density

$$R_A = r_0 A^{1/3} \quad (r_0 = 1.2 \text{ fm})$$

- Nuclear Binding Energy (Bethe-Weizsacker Formula)

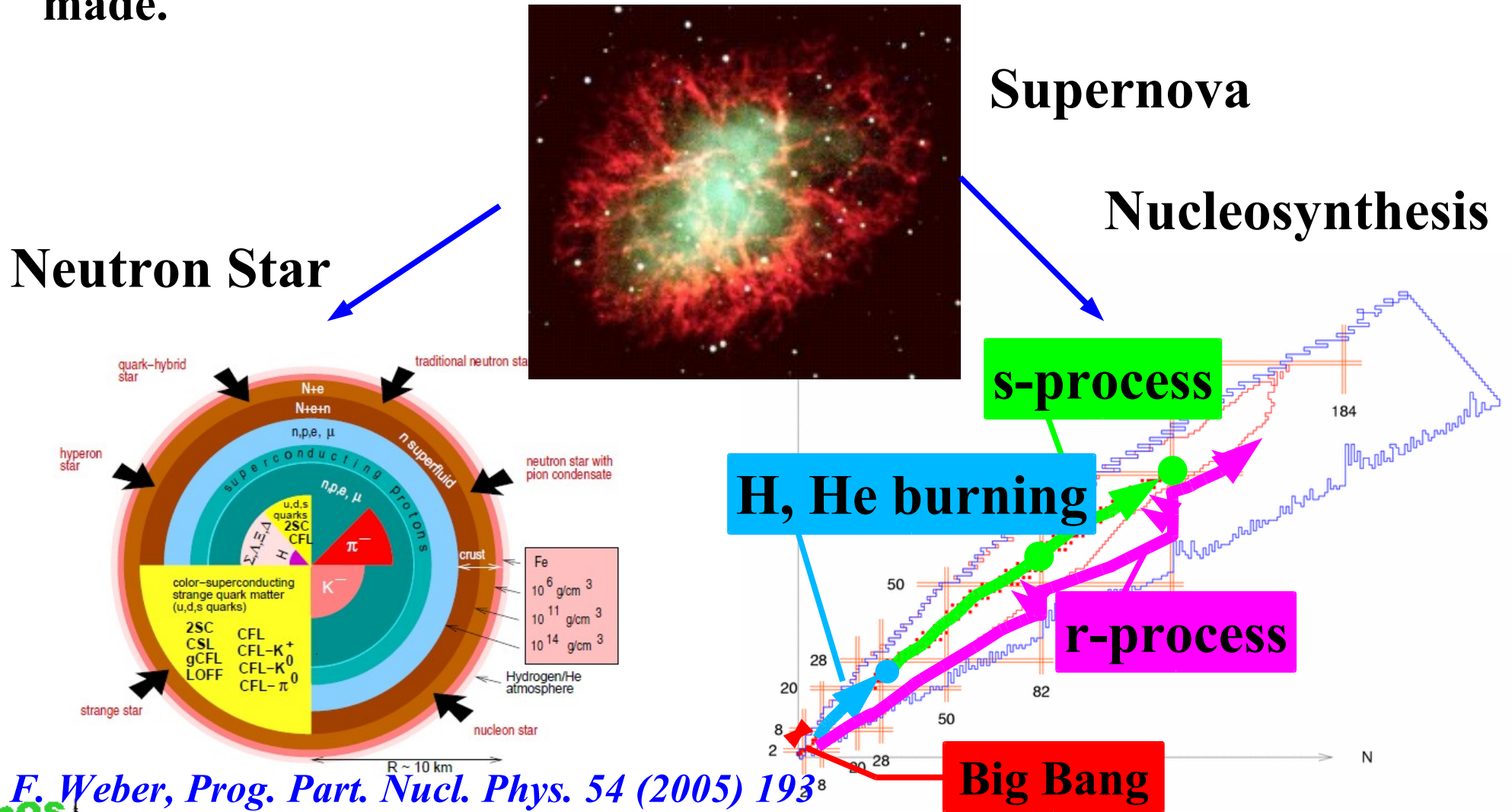
$$B(A, Z) = a_{vol} A - a_{surf} A^{2/3} - a_{Coulomb} \frac{Z^2}{A^{1/3}} - a_{sym} \frac{(N-Z)^2}{A} + a_{pair} \delta(A, Z) A^{-3/4}$$

Nuclear Matter



Why do we study Nuclear Matter EOS ?

- Answer 2: Since nuclear matter EOS is decisive in compact astrophysical objects such as neutron stars, supernovae, and black hole formation, EOS is important to understand where atomic elements are made.



F. Weber, Prog. Part. Nucl. Phys. 54 (2005) 193

Why do we study Nuclear Matter EOS ?

- **Answer 3: Since the EOS should have singularity (or at least sudden change) at phase boundary, it would be possible to catch the signal of phase transition in nuclear collisions.**

- **Pressure and Energy Density of Free Massless Gas**

$$P = \frac{\pi^2}{90} N_B T^4, \quad \epsilon = \frac{\pi^2}{30} N_B T^4$$

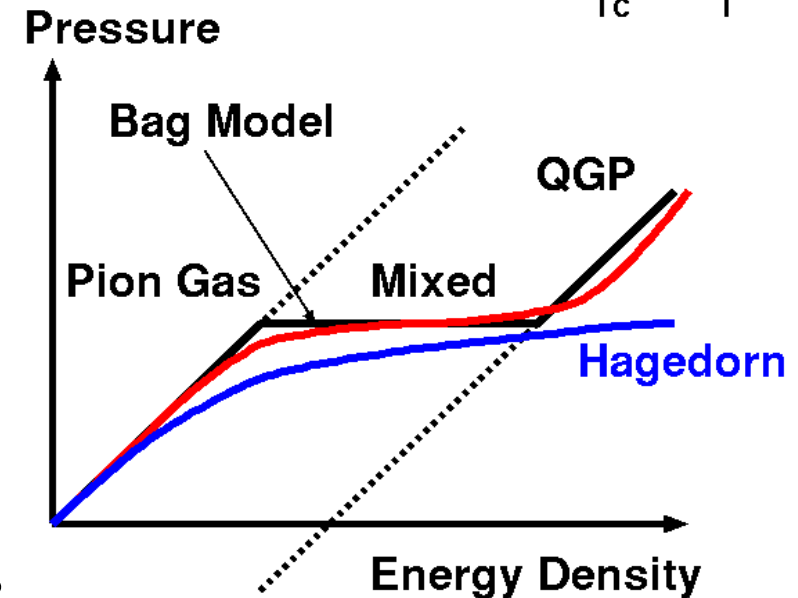
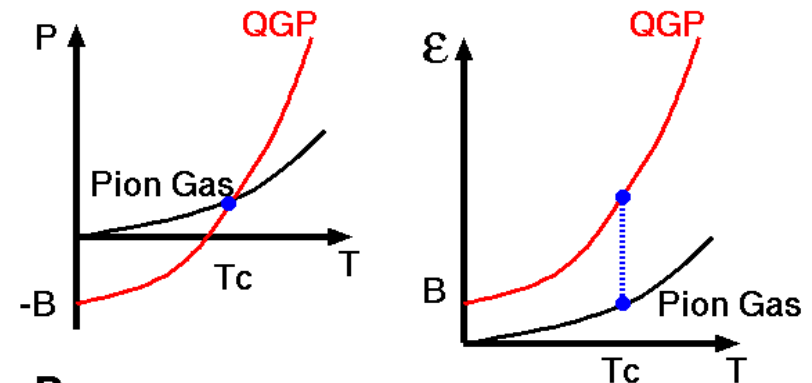
$N_B =$ **Bosonic DOF (7/8 for Fermions)**

- **Hadron Gas ~ 3 pions ($N_B=3$)**

$$P_\pi = \frac{\pi^2}{30} T^4, \quad \epsilon_\pi = \frac{\pi^2}{10} T^4$$

- **QGP $N_B=16$ (gluon)+ $24 \times 7/8$ (quarks) and Bag Pressure**

$$P_{QGP} = \frac{37\pi^2}{90} T^4 - B, \quad \epsilon_{QGP} = \frac{37\pi^2}{30} T^4 + B$$



Nuclear Matter EOS

- In this lecture, I discuss several aspects of Nuclear Matter EOS

Lecture 1

- (1) EOS and Mean Field in Finite Nuclei and Nuclear Matter**
→ **Relativistic Mean Field**

Lecture 2

- (2) Dense Matter EOS and Compact Astrophysical Objects**
→ **EOS with Hyperons, Neutron Stars, Supernovae, Black Hole Formation**
- (3) Hot and Dense Matter EOS and High-Energy Heavy-Ion Collisions**
→ **Nuclear Transport Model, Collective Flows**

Relativistic Mean Field

Theories/Models for Nuclear Matter EOS

■ Ab initio Approach

- LQCD, GFMC, Variational, DBHF, G-matrix
→ Not easy to handle, Not satisfactory for phen. purposes

■ Mean Field from Effective Interactions ~ Nuclear Density Functionals

● Skyrme Hartree-Fock(-Bogoliubov)

- ◆ Non.-Rel., Zero Range, Two-body + Three-body (or ρ -dep. two-body)
- ◆ In HFB, Nuclear Mass is very well explained (Total B.E. $\Delta E \sim 0.6$ MeV)
- ◆ Causality is violated at very high densities.

● Relativistic Mean Field

- ◆ Relativistic, Meson-Baryon coupling, Meson self-energies
- ◆ Successful in describing pA scattering (Dirac Phenomenology)

Relativistic Mean Field (1)

■ Relativistic Mean Field

= Nuclear scalar and vector mean field generated by mesons

→ Why do we use relativistic framework ?

- Nuclear Force is mediated by mesons

→ Let's consider meson-baryon system !
(Entrance of Hadron Physics)

- We are also interested in Dense Matter EOS

→ Sound velocity exceeds the Speed of Light (=c) with Non.-Rel. MF

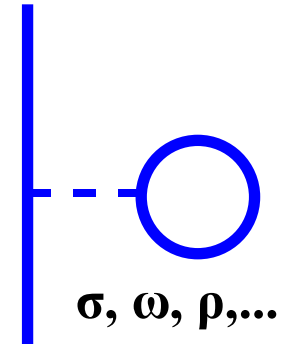
- Success of “Dirac Phenomenology”

(Dirac Eq. for pA scattering → Spin Observables)

→ Strong Scalar and Vector Mean Fields are preferable to explain Spin Observables

- DBHF (Dirac-Brueckner-Hatree-Fock)

→ Successful description of nuclear matter saturation point based on bare NN interactions

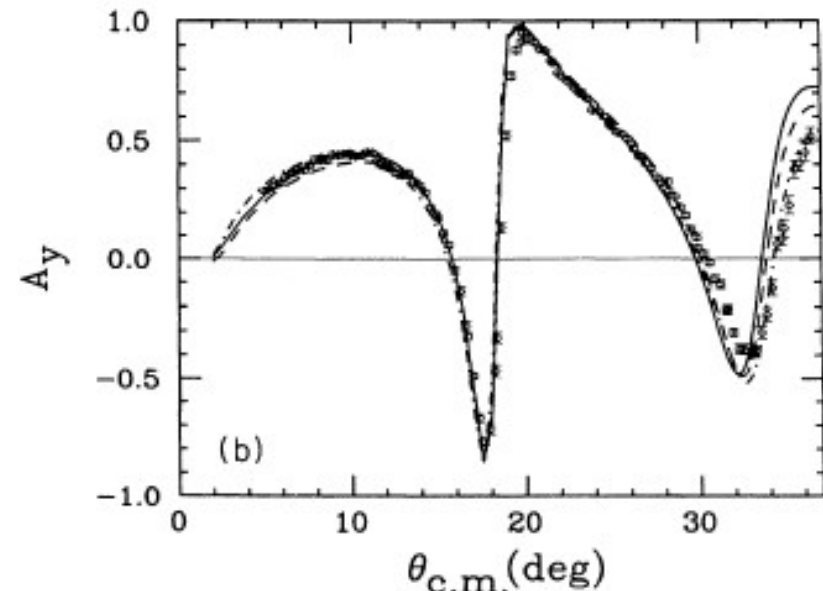
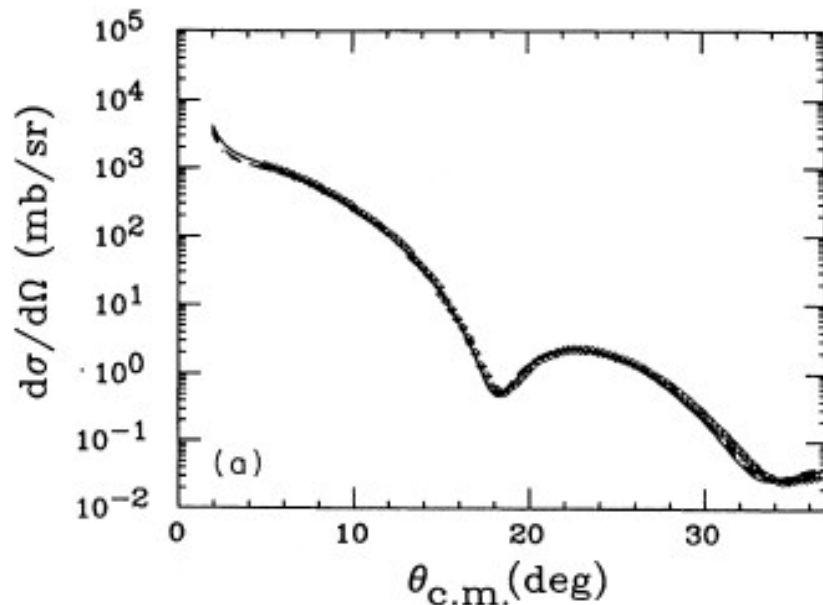
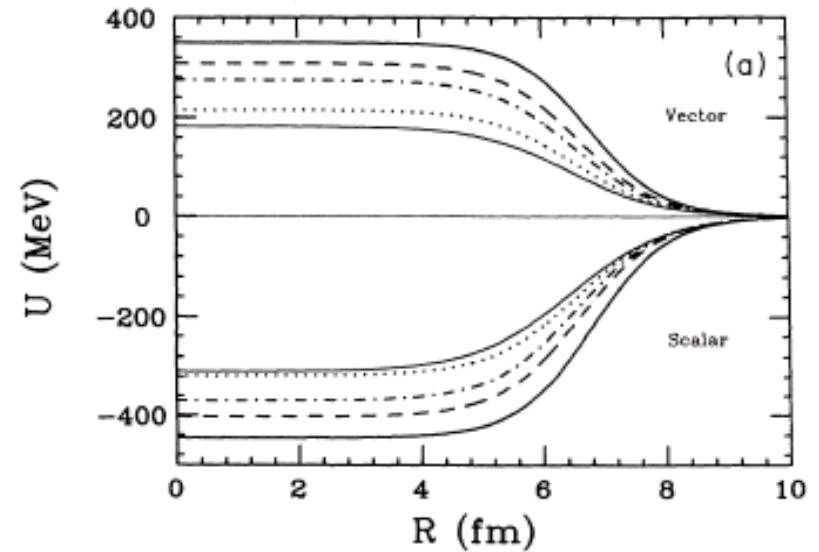


RMF is a good starting point as a framework of hadronic system including Nuclei and Nuclear Matter

Dirac Phenomenology

E.D. Cooper, S. Hama, B.C. Clark, R.L. Mercer, PRC47('93),297

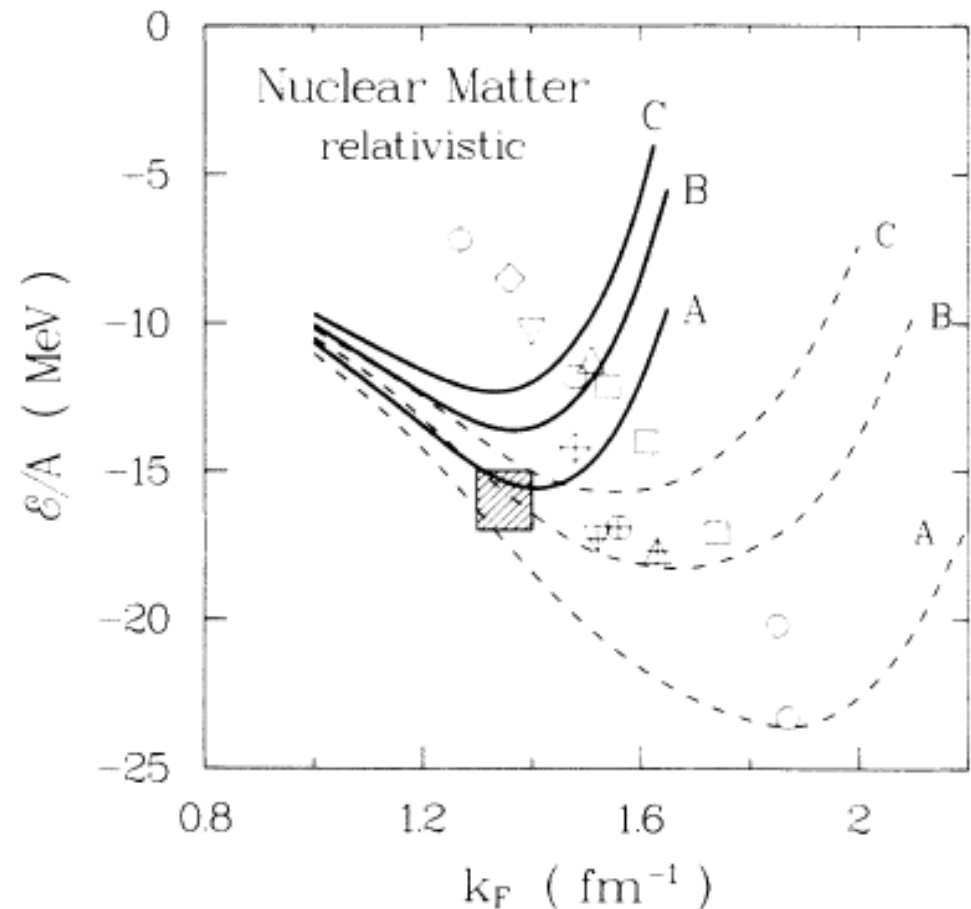
- Dirac Eq. with
Scalar + Vector pA potential
(-400 MeV + 350 MeV)
→ Cross Section, Spin Observables



EOS in Dirac-Brueckner-Hartree-Fock

R. Brockmann, R. Machleidt, PRC42('90),1965

- **Non Relativistic Brueckner Calculation**
→ **Nuclear Saturation Point cannot be reproduced (Coester Line)**
- **Relativistic Approach (DBHF)**
→ **Relativity gives additional repulsion, leading to successful description of the saturation point.**



Relativistic Mean Field (2)

- **Mean Field treatment of meson field operator**
= Meson field operator is replaced with its expectation value

$$\varphi(\mathbf{r}) \rightarrow \langle \varphi(\mathbf{r}) \rangle$$

Ignoring fluctuations compared with the expectation value may be a good approximation at strong condensate.

- **Which Hadrons should be included in RMF ?**

- **Baryons (1/2+)** $p, n, \Lambda, \Sigma, \Xi, \Delta, \dots$
- **Scalar Mesons (0+)** $\sigma(600), f_0(980), a_0(980), \dots$
- **Vector Mesons (1-)** $\omega(783), \rho(770), \phi(1020), \dots$
- **Pseudo Scalar (0-)** $\pi, K, \eta, \eta', \dots$
- **Axial Vector (1+)** a_1, \dots

We require that the meson field can have uniform expectation values in nuclear matter.

→ Scalar and Time-Component of Vector Mesons ($\sigma, \omega, \rho, \dots$)

$\sigma\omega$ Model (1)

Serot, Walecka, Adv.Nucl.Phys.16 (1986),1

- Consider only σ and ω mesons
- Lagrangian

$$L = \bar{\psi} (i \gamma^\mu \partial_\mu - M + g_s \sigma - g_v \omega) \psi + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_s^2 \sigma^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_v^2 \omega_\mu \omega^\mu$$
$$(F_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu)$$

- Equation of Motion

- Euler-Lagrange Equation

$$\frac{\partial}{\partial x^\mu} \left[\frac{\partial L}{\partial (\partial_\mu \phi_i)} \right] - \frac{\partial L}{\partial \phi_i} = 0$$

$$\sigma : \left[\partial_\mu \partial^\mu + m_s^2 \right] \sigma = g_s \bar{\psi} \psi$$

$$\omega : \partial_\mu F^{\mu\nu} + m_v^2 \omega^\nu = g_v \bar{\psi} \gamma^\nu \psi \quad \rightarrow \quad \left[\partial_\mu \partial^\mu + m_v^2 \right] \omega^\nu = g_v \bar{\psi} \gamma^\nu \psi$$

$$\psi : \left[\gamma^\mu (i \partial_\mu - g_v V_\mu) - (M - g_s \sigma) \right] \psi = 0$$

EOM of ω (for beginners)

■ Euler-Lagrange Eq.

$$\partial_\mu F^{\mu\nu} + m_\nu^2 \omega^\nu = g_\nu \bar{\psi} \gamma^\nu \psi$$

■ Divergence of LHS and RHS

$$\partial_\nu \partial_\mu F^{\mu\nu} + m_\nu^2 (\partial_\nu \omega^\nu) = m_\nu^2 (\partial_\nu \omega^\nu) = g_\nu (\partial_\nu \bar{\psi} \gamma^\nu \psi) = 0$$

LHS: derivatives are sym. and $F_{\mu\nu}$ is anti-sym.

RHS: Baryon Current = Conserved Current

■ Put it in the Euler-Lagrange Eq.

$$\partial_\mu F^{\mu\nu} = \partial_\mu (\partial^\mu \omega^\nu - \partial^\nu \omega^\mu) = \partial_\mu \partial^\mu \omega^\nu - \partial^\nu (\partial_\mu \omega^\mu) = \partial_\mu \partial^\mu \omega^\nu$$

Schroedinger Eq. for Upper Component

Dirac Equation for Nucleons

$$(i \gamma \partial - \gamma^0 U_v - M - U_s) \psi = 0 \quad , \quad U_v = g_\omega \omega \quad , \quad U_s = -g_\sigma \sigma$$

Decompose 4 spinor into Upper and Lower Components

$$\begin{pmatrix} E - U_v - M - U_s & i \sigma \cdot \nabla \\ -i \sigma \cdot \nabla & -E + U_v - M - U_s \end{pmatrix} \begin{pmatrix} f \\ g \end{pmatrix} = 0 \quad \begin{aligned} g &= \frac{-i}{E + M + U_s - U_v} (\sigma \cdot \nabla) f \\ (E - M - U_v - U_s) f &= -i (\sigma \cdot \nabla) g \end{aligned}$$

Erase Lower Component (assuming spherical sym.)

$$\begin{aligned} -i (\sigma \cdot \nabla) g &= -(\sigma \cdot \nabla) \frac{1}{X} (\sigma \cdot \nabla) f = -\frac{1}{X} \nabla^2 f - \frac{1}{r} \left[\frac{d}{dr} \frac{1}{X} \right] (\sigma \cdot r) (\sigma \cdot \nabla) f = -\nabla \frac{1}{X} \nabla f + \frac{1}{r} \left[\frac{d}{dr} \frac{1}{X} \right] (\sigma \cdot l) f \\ (\sigma \cdot r) (\sigma \cdot \nabla) &= (r \cdot \nabla) + i \sigma \cdot (r \times \nabla) = r \cdot \nabla - \sigma \cdot l \end{aligned}$$

“Schroedinger-like” Eq. for Upper Component

$$-\nabla \frac{1}{E + M + U_s - U_v} \nabla f + (U_s + U_v + U_{LS} (\sigma \cdot l)) f = (E - M) f$$

$$U_{LS} = \frac{1}{r} \left[\frac{d}{dr} \frac{1}{E + M + U_s - U_v} \right] < 0 \quad \text{on surface}$$

$(U_s, U_v) \sim (-350 \text{ MeV}, 280 \text{ MeV}) \rightarrow$ Small Central $(U_s + U_v)$, Large LS $(U_s - U_v)$

Various Ways to Evaluate Non.-Rel. Potential

■ From Single Particle Energy

$$\left(\gamma^0 (E - U_v) + i \boldsymbol{\gamma} \cdot \nabla - (M + U_s) \right) \psi = 0 \rightarrow (E - U_v)^2 = p^2 + (M + U_s)^2$$

$$\rightarrow E = \sqrt{p^2 + (M + U_s)^2} + U_v \approx E_p + \frac{M}{E_p} U_s + U_v + \frac{p^2}{2 E_p^3} U_s^2$$

$$(E_p = \sqrt{p^2 + M^2})$$

■ Schroedinger Equivalent Potential (Uniform matter)

$$-\frac{\nabla^2}{2M} f + \left[U_s + \frac{E}{M} U_v + \frac{U_s^2 - U_v^2}{2M} \right] f = \frac{E + M}{2M} (E - M) f$$

$$U_{\text{SEP}} \approx U_s + \frac{E}{M} U_v$$

Anyway, slow baryons feel Non.-Rel. Potential,

$$U \approx U_s + U_v = -g_s \sigma + g_v \omega$$

Nuclear Matter in $\sigma\omega$ Model

Serot, Walecka, *Adv.Nucl.Phys.*16 (1986),1

Uniform Nuclear Matter

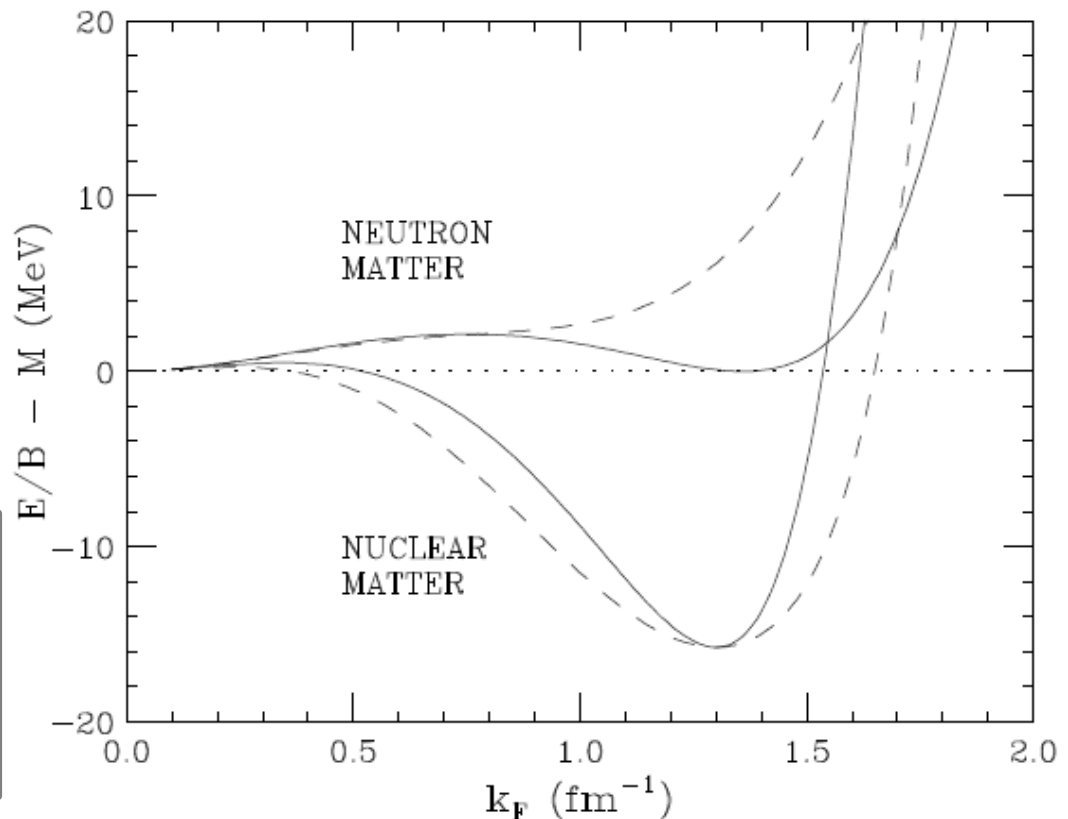
$$E/V = \gamma_N \int^{P_F} \frac{d^3 p}{(2\pi)^2} E^* + \frac{1}{2} m_s^2 \sigma^2 - \frac{1}{2} m_v^2 \omega^2 + g_v \rho_B \omega$$

$$\sigma = \frac{g_s}{m_s^2} \rho_s = \frac{g_s}{m_s^2} \int^{P_F} \frac{\gamma_N d^3 p M^*}{(2\pi)^2 E^*} \quad (M^* = M + U_s = M - g_s \sigma, \quad E^* = \sqrt{p^2 + M^{*2}})$$

$$\omega = \frac{g_v}{m_v^2} \rho_B = \gamma_N \frac{g_v}{m_v^2} \int^{P_F} \frac{d^3 p}{(2\pi)^3}$$

$\gamma_N =$ Nucleon degeneracy
(=4 in sym. nuclear matter)

Problem: EOS is too stiff
K ~ (500-600) MeV !
→ How can we solve ?



RMF with Non-Linear Meson Int. Terms

Boguta, Bodmer ('77), Reinhardt, Rufa, Maruhn, Greiner, Friedrich ('86), Lalazissis, Konig, Ring ('97), Sugahara, Toki ('94), Brockmann, Toki ('92)

- Too stiff EOS in the simplest RMF ($\sigma\omega$ model) is improved by introducing non-linear terms (σ^4 , ω^4)
 - Fit B.E. of Stable as well as Unstable (n-rich) Nuclei
 - Three Mesons (σ, ω, ρ) are included
 - Meson Self-Energy Term (σ, ω)

$$\begin{aligned} \mathcal{L} = & \bar{\psi}_N (i\partial - M - g_\sigma \sigma - g_\omega \not{\omega} - g_\rho \tau^a \not{\rho}^a) \psi_N \\ & + \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 \\ & - \frac{1}{4} W^{\mu\nu} W_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4} R^{a\mu\nu} R_{\mu\nu}^a + \frac{1}{2} m_\rho^2 \rho^{a\mu} \rho_\mu^a + \frac{1}{4} c_3 (\omega_\mu \omega^\mu)^2 \\ & + \bar{\psi}_e (i\partial - m_e) \psi_e + \bar{\psi}_\nu i\partial \psi_\nu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} , \\ W_{\mu\nu} = & \partial_\mu \omega_\nu - \partial_\nu \omega_\mu , \\ R_{\mu\nu}^a = & \partial_\mu \rho_\nu^a - \partial_\nu \rho_\mu^a + g_\rho \epsilon^{abc} \rho^{b\mu} \rho^{c\nu} , \\ F_{\mu\nu} = & \partial_\mu A_\nu - \partial_\nu A_\mu . \end{aligned} \tag{2}$$

RMF with Non-Linear Meson Int. Terms

- Are the Lagrangian parameters are well determined ?

$$\mathcal{L} = \mathcal{L}_{\text{free}}(\psi, \sigma, \omega, \rho, \dots) + \bar{\psi} [g_{\sigma}\sigma - g_{\omega}\gamma^0\omega - g_{\rho}\tau_z\gamma^0\rho] \psi + c_{\omega}\omega^4/4 - V_{\sigma}(\sigma), \quad (3)$$

$$V_{\sigma} = \begin{cases} \frac{1}{3}g_3\sigma^3 + \frac{1}{4}g_4\sigma^4 & (\text{NL1, NL3, TM1}) \\ -a_{\sigma}f_{\text{SCL}}(\sigma/f_{\pi}) & (\text{SCL}) \end{cases}, \quad (4)$$

- Linear terms, Meson-Nucleon Coupling → Well determined
- Meson interaction terms → Different in RMF parameterization
- Negative Coef. of $\sigma^4 < 0$ → Vacuum is unstable

Is there any way to “Derive” RMF Lagrangian ?
→ Symmetry in QCD

TABLE II. RMF parameters

	$g_{\sigma N}$	$g_{\omega N}$	$g_{\rho N}$	$g_3(\text{MeV})$	g_4	c_{ω}	$m_{\sigma}(\text{MeV})$	$m_{\omega}(\text{MeV})$	$m_{\rho}(\text{MeV})$
NL1[18]	10.138	13.285	4.976	2401.9	-36.265	0	492.25	795.359	763
NL3[19]	10.217	12.868	4.474	2058.35	-28.885	0	508.194	782.501	763
TM1[6]	10.0289	12.6139	4.6322	1426.466	0.6183	71.3075	511.198	783	770
SCL[20>(*1)	10.08	13.02	4.40	1255.88	13.504	200	502.63	783	770

(*1): g_3 and g_4 are from the expansion of $f_{\pi}(\sigma)$.

AO, Jido, Sekihara, Tsubakihara, in prep.

Chiral RMF

Nuclear Many-Body Theory preserving Chiral Sym. ?

■ Chiral Symmetry

$$\psi_R = \frac{1}{2}(1 + \gamma_5)\psi \rightarrow U_R \psi_R, \quad \psi_L = \frac{1}{2}(1 - \gamma_5)\psi \rightarrow U_L \psi_L$$

● Symmetry in QCD with small quark mass

Kinetic term = invariant: $\bar{\psi} i \gamma^\mu D_\mu \psi = \bar{\psi}_R i \gamma^\mu D_\mu \psi_R + \bar{\psi}_L i \gamma^\mu D_\mu \psi_L$

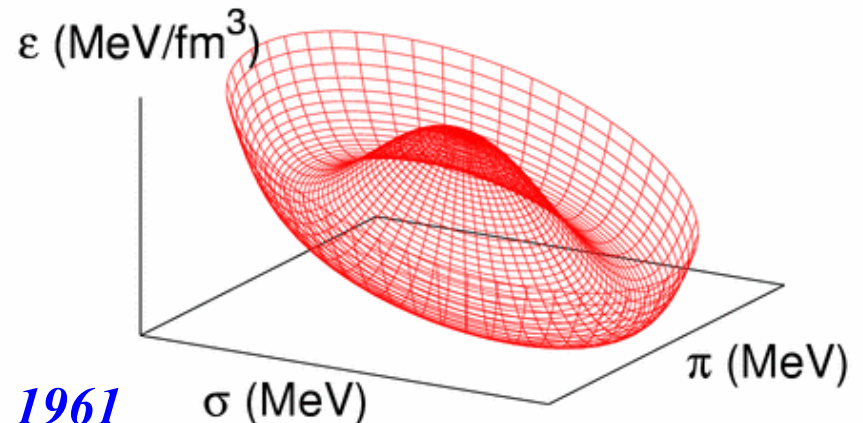
mass term \neq invariant: $\bar{\psi} \psi = \bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R$

● Should be kept in Nuclear Lagrangian

Problem: Nucleon cannot have mass !

Solution: Spontaneous breaking of Chiral Sym.

$$L_{L\sigma M} = \frac{1}{2} \left(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \pi \partial^\mu \pi \right) - \frac{\lambda}{4} (\sigma^2 + \pi^2)^2 + \frac{\mu^2}{2} (\sigma^2 + \pi^2) + c \sigma + \bar{N} i \partial_\mu \gamma^\mu N - g_\sigma \bar{N} (\sigma + i \pi \tau \gamma_5) N$$



Gell-Mann, Levy, 1960; Nambu, Jona-Lasino, 1961

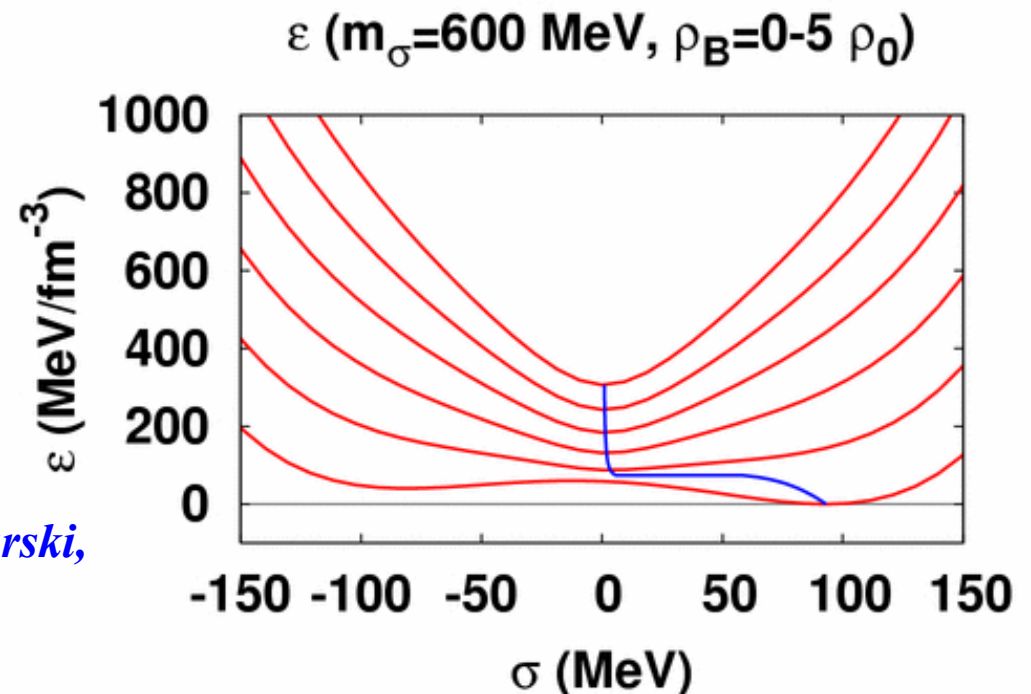
Chiral Collapse Problem (Lee-Wick Vacuum)

- At finite ρ_B , Nucleon Fermi Integral favors smaller σ
→ Chiral Sym. is restored below ρ_0 (Chiral Collapse) *Lee, Wick, 1974*

■ Prescriptions

- $\sigma\omega$ coupling (too stiff EOS) (*Boguta 1983, Ogawa et al. 2004*)
- Loop effects (unstable at large σ)
(*Matsui-Serot, 1982, Glendenning 1988, Prakash-Ainsworth 1987, Tamenaga et al. 2006*)
- Higher order terms (unstable at large σ) (*Hatsuda-Prakash 1989, Sahu-Ohnishi 2000*)
- **Dielectric (Glueball) Field representing scale anomaly**
(*Furnstahl-Serot 1993, Heide-Rudaz-Ellis 1994, Papazoglou et al. (SU(3)) 1998*)
- Different Chiral partner assignment
(*DeTar-Kunihiro 1989, Hatsuda-Prakash 1989, Harada-Yamawaki 2001, Zschesche-Tolos-Schaffner-Bielich-Pisarski, nucl-th/0608044*) → $SU_f(3)$ extention ?

but



Chiral Collapse Problem (Lee-Wick Vacuum)

■ Many of the proposed prescriptions fail

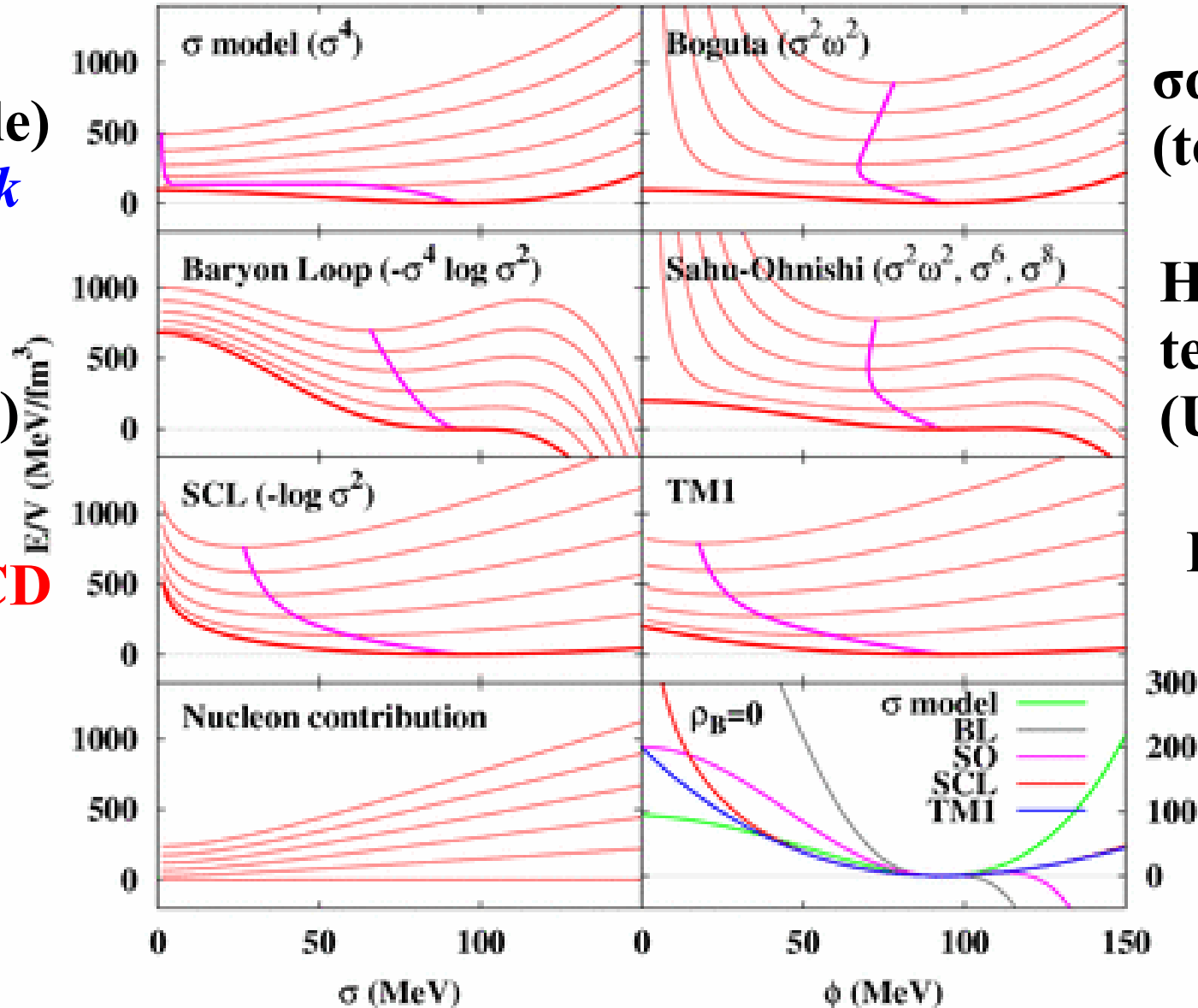
Tsubakihara, Ohnishi, 2007

Energy Density at $\rho_B = (0-5) \rho_0$

Naïve ϕ^4
(Unstable)
Lee-Wick

Fermion
Loop
(Unstable)

SCL-LQCD



$\sigma\omega$ coupling
(too stiff EOS)

Higher order
terms
(Unstable)

Phenom.

Chiral RMF based on SCL-LQCD

■ Relativistic Mean Field model

Tsubakihara, Ohnishi, 2007

- Effective Lagrangian consisting Baryons and Mesons
- Attractive Scalar Field (σ) + Repul. Vector Field (ω) \rightarrow Matter Saturation

$$\begin{aligned} \mathcal{L}_X = & \bar{\psi}_N [i\cancel{\partial} - g_\sigma(\sigma + i\gamma_5\boldsymbol{\tau} \cdot \boldsymbol{\pi}) - g_\omega\cancel{\not{\omega}} - g_\rho\boldsymbol{\tau} \cdot \boldsymbol{\rho}] \psi_N \\ & + \frac{1}{2} (\partial^\mu\sigma\partial_\mu\sigma + \partial^\mu\boldsymbol{\pi} \cdot \partial_\mu\boldsymbol{\pi}) - V_\sigma(\sigma, \boldsymbol{\pi}) \\ & - \frac{1}{4} W^{\mu\nu}W_{\mu\nu} + \frac{1}{2} m_\omega^2\omega^\mu\omega_\mu + \frac{c_\omega}{4} (\omega^\mu\omega_\mu)^2 - \frac{1}{4} R^{\mu\nu} \cdot R_{\mu\nu} + \frac{1}{2} m_\rho^2\rho^\mu \cdot \rho_\mu \end{aligned}$$

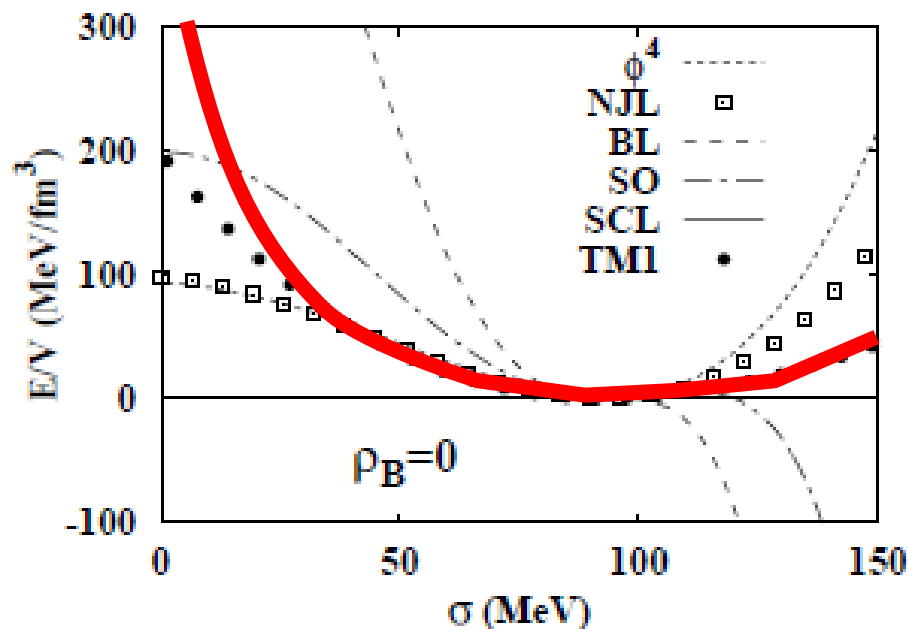
● Linear σ Model

$$V_\sigma = -\frac{1}{2}\mu\sigma^2 + \frac{1}{4}\lambda\sigma^4$$

● SCL-LQCD (Zero T)

$$V_\sigma = \frac{1}{2}a_\sigma\sigma^2 - b_\sigma\log\sigma$$

\rightarrow Agree with Phen. RMF results at large chiral condensates.



Chiral RMF based on SCL-LQCD

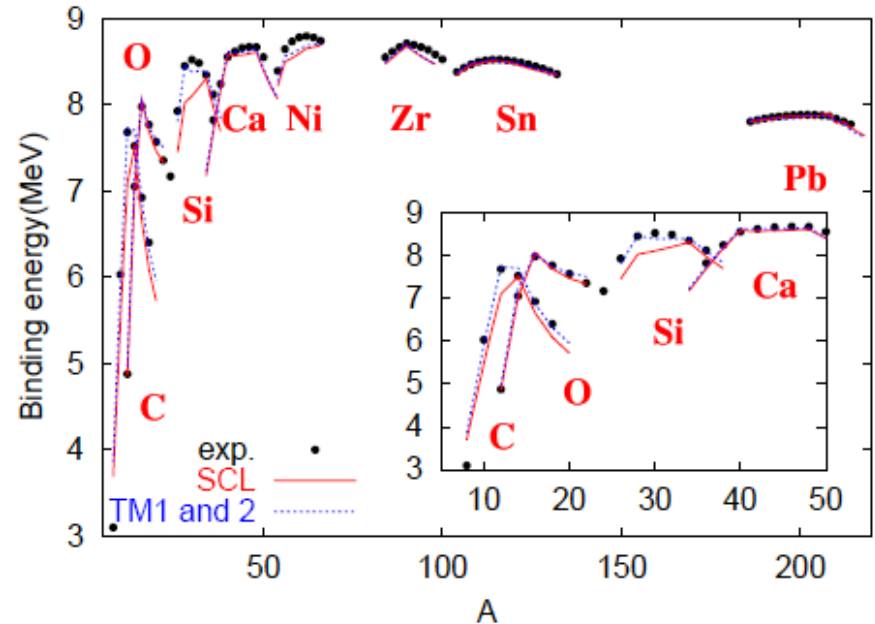
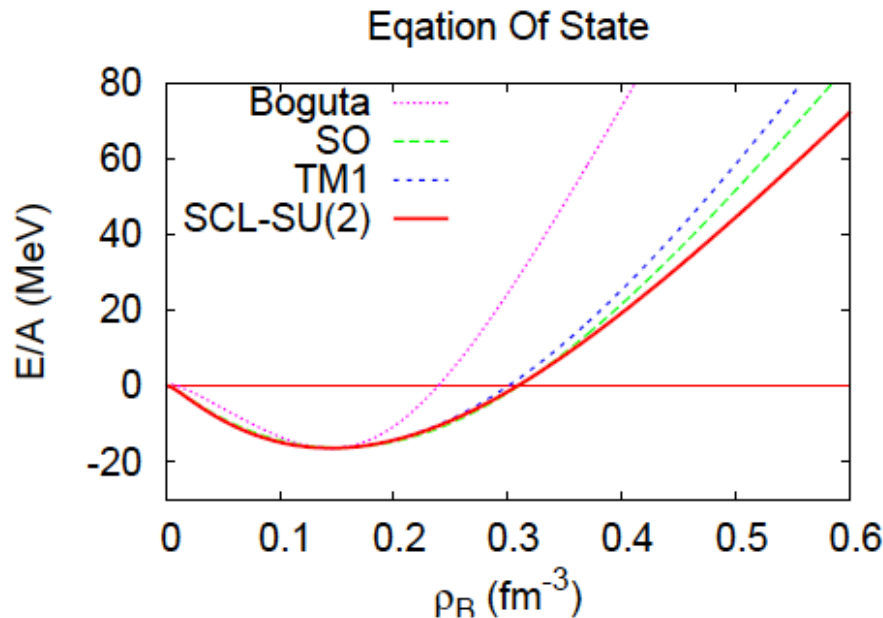
Tsubakihara, AO, PTP 117('07)903 [nucl-th/0607046]

■ Nuclear Matter EOS

- Gives “Medium” EOS ($K \sim 280$ MeV), Comparable to Phen. RMF

■ Bulk properties of nuclei

- B.E./Nucleon, Charge radii → Comarable to High Quality Phen. RMF



This would be the first step of giving
Nuclear Density Functional from QCD

Binding Energies in Chiral and Non-Chiral RMF

- **Non-Chiral High Precision RMF: TM1 & 2, NL1, NL3**
(Sugahara, Toki, 1994; Reinhard et al., 1986; Lalazissis, Koenig, Ring, 1997)
- **Log term from Scale Anomaly: I/110, IF/110, VIIF/100**
Chiral Symmetric, No Instability, with Glueball $V_\sigma = -\chi^4 \log \sigma^2$
(Heide, Rudas, Ellis, 1994)
- **Quark Meson Coupling model**

Nucleus	<i>B/A</i> (MeV)									
	exp.	SCL	TM1	TM2	NL1	NL3	I/110	IF/110	VIIF/100	QMC-I
¹² C	7.68	7.09	-	7.68	-	-	-	-	-	-
¹⁸ O	7.98	8.06	-	7.92	7.95	8.05	7.35	7.86	7.18	5.84
²⁸ Si	8.45	8.02	-	8.47	8.25	-	-	-	-	-
⁴⁰ Ca	8.55	8.57	8.62	8.48	8.56	8.55	7.96	8.35	7.91	7.36
⁴⁸ Ca	8.67	8.62	8.65	8.70	8.60	8.65	-	-	-	7.26
⁸⁸ Ni	8.73	8.54	8.64	-	8.70	8.68	-	-	-	-
⁹⁰ Zr	8.71	8.69	8.71	-	8.71	8.70	-	-	-	7.79
¹¹⁸ Sn	8.52	8.51	8.53	-	8.52	8.51	-	-	-	-
¹⁹⁸ Pb	7.87	7.87	7.87	-	7.89	-	-	-	-	-
²⁰⁸ Pb	7.87	7.87	7.87	-	7.89	7.88	7.33	7.54	7.44	7.25

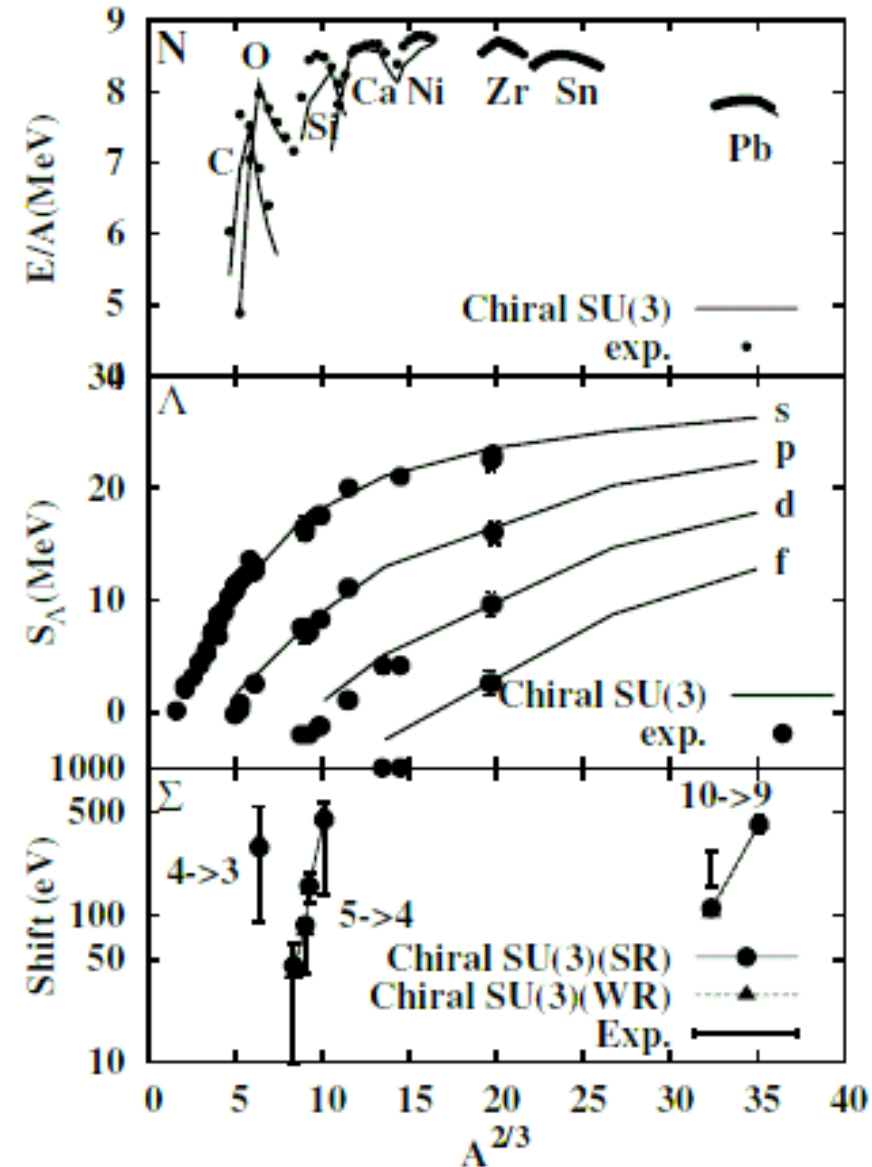
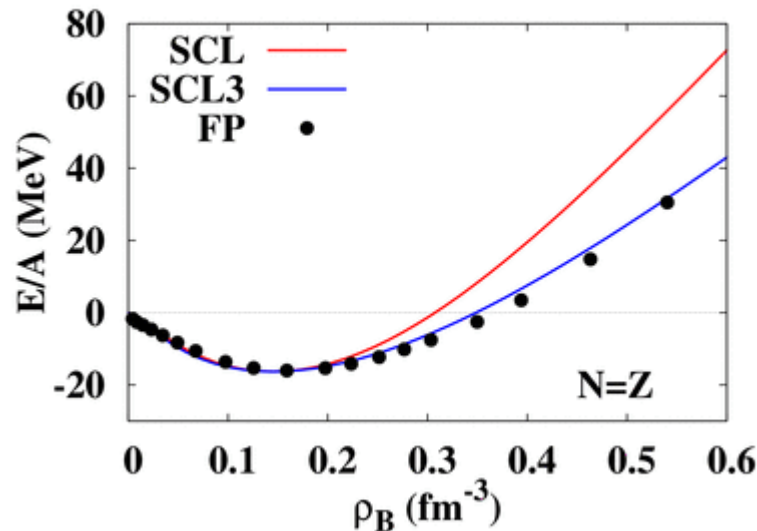
Chiral $SU_f(3)$ RMF

Tsubakihara, Maekawa, AO, EPJA33('07)295 [nucl-th/0702008]

- Extention to Flavor $SU(3)$
 - Chiral Potential from SCL-LQCD
 - + Determinant Int. ($U_A(1)$ anomaly)
 - + Explicit breaking term

$$U_{\sigma\zeta} = -a \log(\det MM^\dagger) + b \text{tr}(MM^\dagger) + c_\sigma \sigma + c_\zeta \zeta + d(\det M + \det M^\dagger),$$

- Normal, Single & Double Λ , Σ atom, EOS (\sim FP),



Summary of Lecture 1

- **Nuclear Matter EOS is important in various aspects of Nuclear Physics**
- **Relativistic Mean Field may be a good starting point to describe hadronic (baryon and meson) systems.**
 - **Relativistic → Saturation, Causality**
 - **Based on successes of Dirac Phenomenology and DBHF**
 - **Covariant Density Functional**
→ It is desirable to obtain E/V (energy density) in fundamental theories. (Renormalizability is not required.)
 - **We can re-write RMF equations in Schroedinger-like eqs. We should consider it as a method to parameterize DF in a transparent manner.**
 - **Higher order terms / Density dependence of the coupling constants (not mentioned) → Necessary for precise description of nuclei, but need foundations of extension.**

*Relativistic EOS
of Supernova Matter with Hyperons*

Supernova Explosion from Nucl. Phys. Point of View

Multi Dim.
Instability
Magnetic Field
Acoustic Revival
....

Hydro.

EOS

ν physics

or my

r-process

EOS tables
Lattimer-Swesty (1981)
Rel. (Shen) EOS (1998)
→ How to extend ?

Realistic ν -A int.
Nuclear Dist.
Exact ν transfer
.....

- Supernovae **DO NOT EXPLODE** in theor. calculation at present with realistic microphysics inputs. → How can we succeed ?

Multi-Dim. Hydro (Instability)+*Additional Energy Release* (10 %-factor 10)

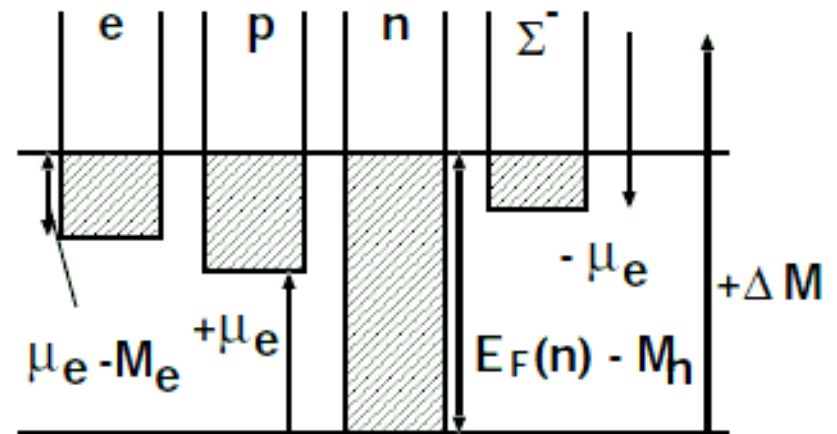
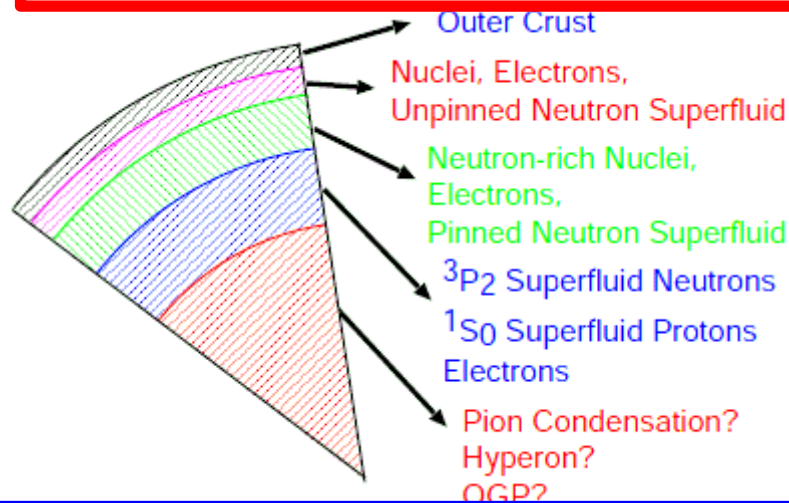
Hyperons in Dense Matter

What appears at high density ?

- Nucleon superfluid (3S_1 , 3P_2)
- Pion condensation, Kaon condensation, Baryon Rich QGP, Color SuperConductor (CSC), Quarkyonic Matter,

Hyperons

Tsuruta, Cameron (66); Langer, Rosen (70); Pandharipande (71); Itoh(75); Glendenning; Weber, Weigel; Sugahara, Toki; Schaffner, Mishustin; Balberg, Gal; Baldo et al.; Vidana et al.; Nishizaki, Yamamoto, Takatsuka; Kohno, Fujiwara et al.; Sahu, Ohnishi; Ishizuka, Ohnishi, Sumiyoshi, Yamada; ...



Nobody says “Hyperons do not appear in neutron star core” !

Y appears when $\mu_B = E_F(n) + U(n) \geq M(Y) + U(Y) + Q_Y \mu_e$

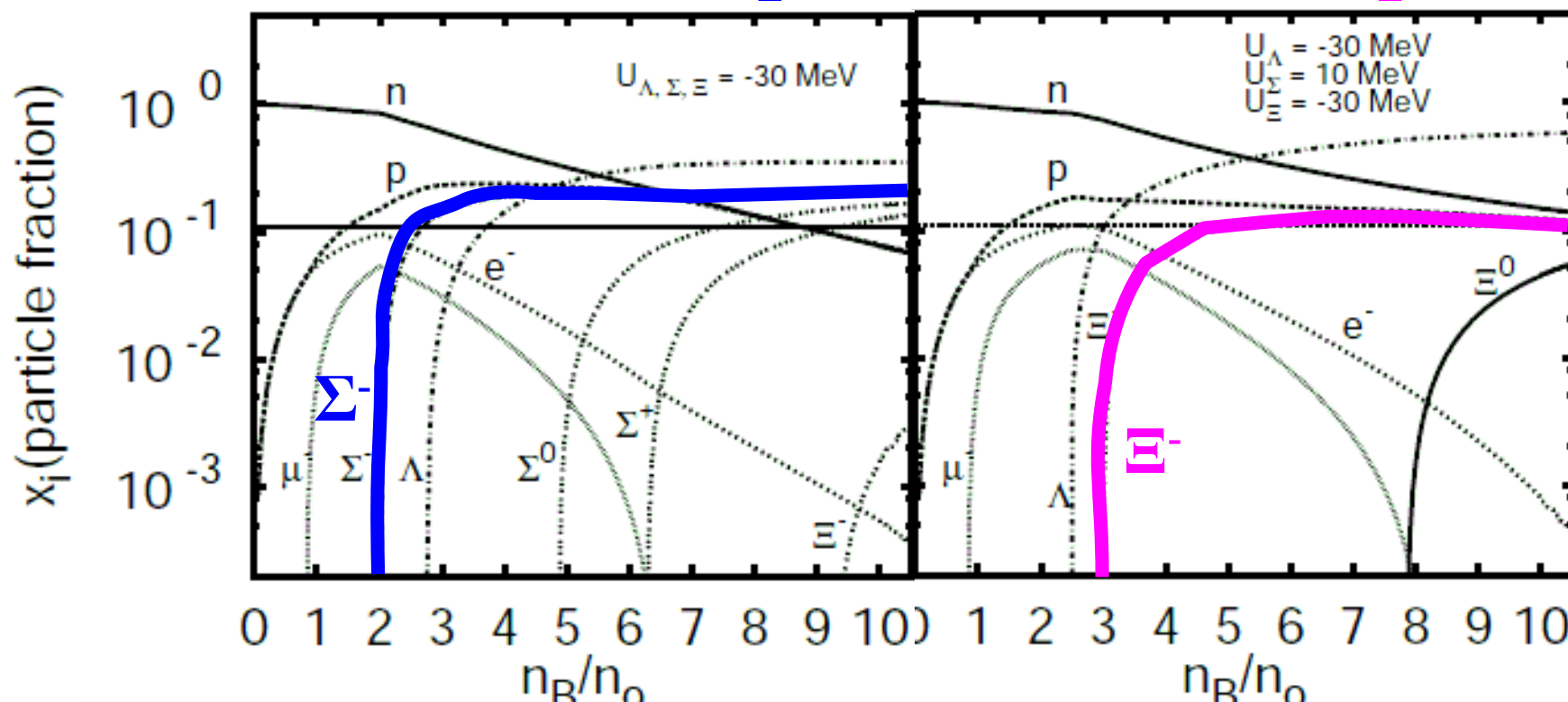
Hyperons in Supernova Matter

■ Problems to include hyperons in Supernova Matter EOS

- Uncertainties of hyperon potentials $U_Y(\rho) \rightarrow$ *Recent Hypernuclear Phys.* (e.g. Balberg, Gal, 1997)
- Density may not be very high in supernova \rightarrow *Needed in cooling stage*

Attractive U_Σ

Repulsive U_Σ



Sahu,
AO, 2003

We include recent hyperon info. in supernova matter EOS

Σ Potential in Nuclear Matter

■ $U_{\Lambda}(\rho_0) \sim -30$ MeV: Well known from single particle energies

■ Naïve expectation

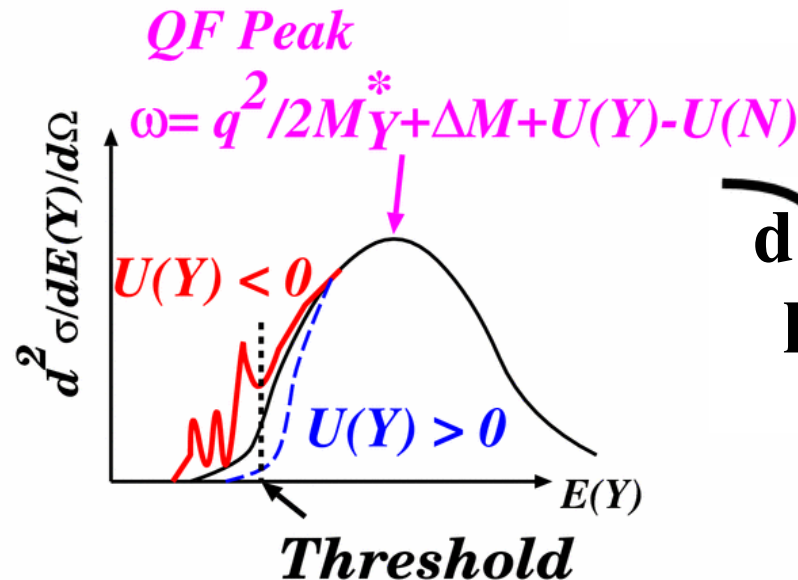
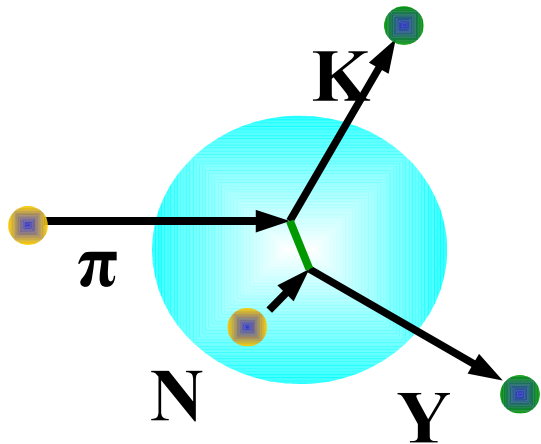
= Quark Number (ud number) Scaling

$$U_{\Lambda} \sim 2/3 U_N \rightarrow U_{\Sigma} \sim 2/3 U_N \sim -30 \text{ MeV}$$

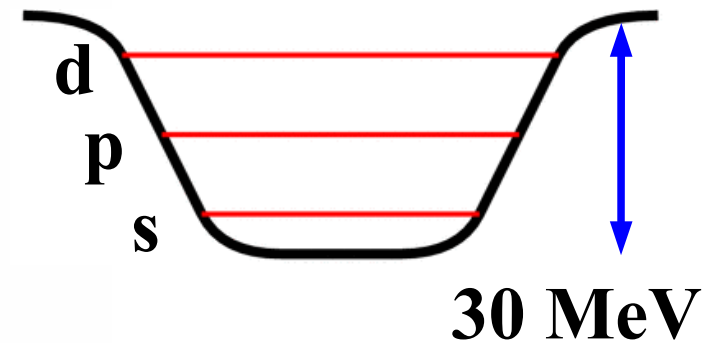
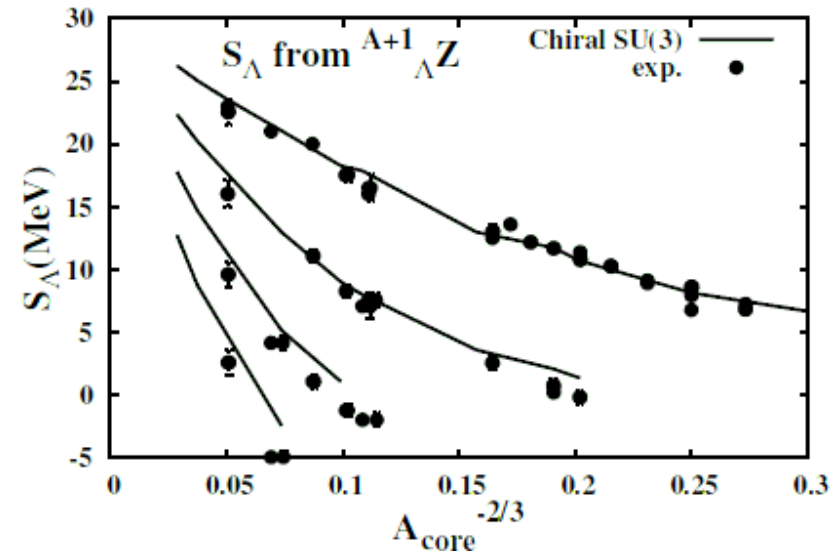
■ Problems with Σ

● Only one bound state $^4_{\Sigma}\text{He}$ (Too light !)

→ Continuum (Quasi-Free) Spectroscopy is necessary



Tsubakihara, Maekawa, AO, EPJA33('07),295.



Σ Potential in Nuclear Matter

- Cont. Spec. Theory = Distorted Wave Impulse Approx. (DWIA)

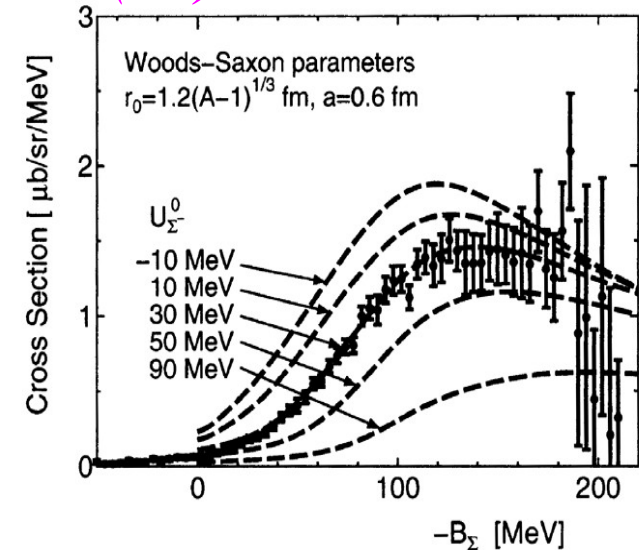
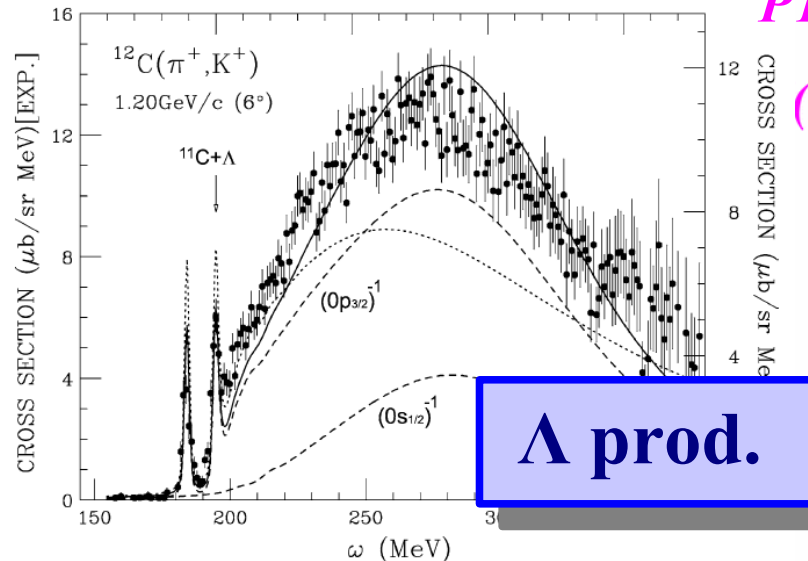
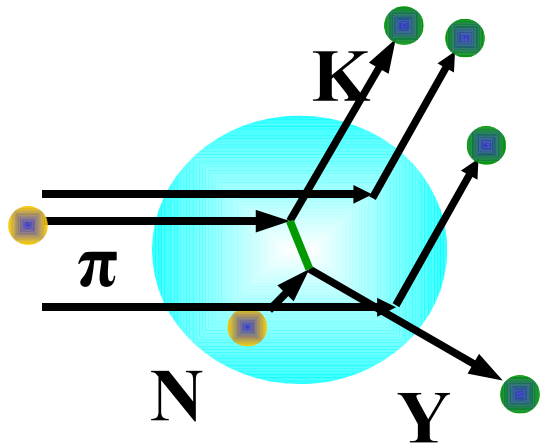
$$\frac{d^2 \sigma}{dE_K d\Omega_K} = \beta \left(\frac{d\sigma}{d\Omega} \right)_{N\pi \rightarrow KY}^{Elem.} S(E, q) \text{--- Strength Func.}$$

Kinematical Factor

Elem. Cross Sec.

- Large (ω, q) range \rightarrow Important to respect **On-Shell Kinematics**
- Kinematics depends on Reaction Point with Hyperon Potential

Harada, Hirabayashi, NPA744('04),323. Kohno, Fujiwara, Kawai, et al. PTP112('04)895



Σ Potential in Nuclear Matter

Maekawa, Tsubakihara, AO, EPJA 33(2007),269.

Maekawa, Tsubakihara, Matsumiya, AO, in preparation.

■ DWIA with Local Optimal Fermi Averaging t-matrix (DWIA-LOFAt)

● Green's Func. Method + Reaction Point Deps. of t-matrix

$$\frac{d^2 \sigma}{d E_K d \Omega_K} = \frac{p_K E_K}{(2\pi)^2 v_{\text{inc}}} R_Y(E_Y) \quad R_Y(E_Y) = -\frac{1}{\pi} \text{Im} \left\langle \bar{t}(\mathbf{r})^+ \frac{1}{E_Y - H_Y + i\epsilon} \bar{t}(\mathbf{r}') \right\rangle$$

Response Func. Local t-mat. Green's Func.

$$\bar{t}(\mathbf{r}, \omega, \mathbf{q}) = \frac{\int d \mathbf{p}_N t(s, t) \rho(p_N) \delta^{(4)}(p_1(\mathbf{r}) + p_2(\mathbf{r}) - p_3(\mathbf{r}) - p(\mathbf{r}))}{\int d \mathbf{p}_N \rho(p_N) \delta^{(4)}(p_1(\mathbf{r}) + p_2(\mathbf{r}) - p_3(\mathbf{r}) - p(\mathbf{r}))} \quad E_i = \sqrt{p_i^2 + m_i^*(r)^2} \simeq m_i + \frac{p_i^2}{2m_i} + V_i, \quad m_i(r)^2 = m_i^2 + 2m_i V_i(r)$$

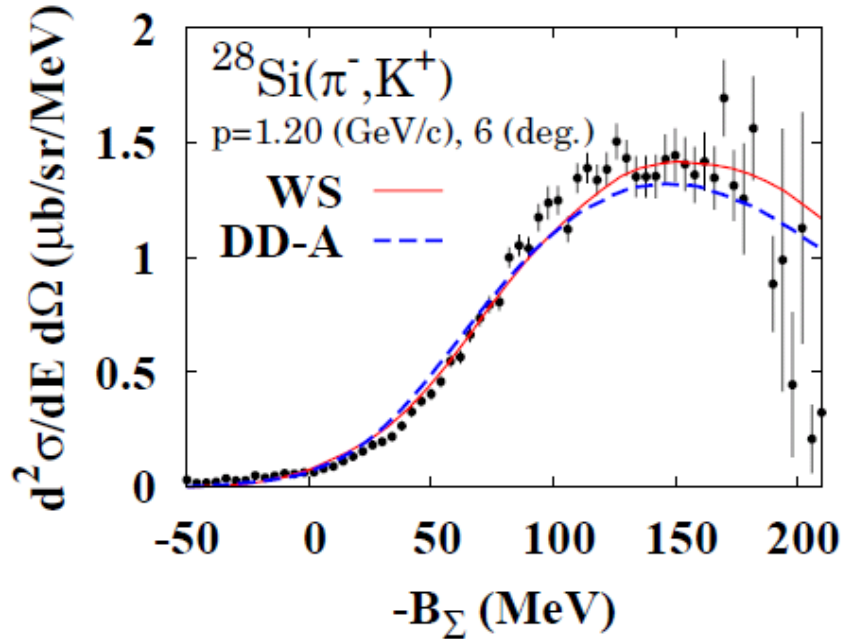
● After careful treatment of

K⁺ potential, Elementary cross section, Angular distribution, ...
we analyze the recently measured Σ^- production spectrum
(Saha, Noumi et al. (KEK-E438), PRC70('04)044613)

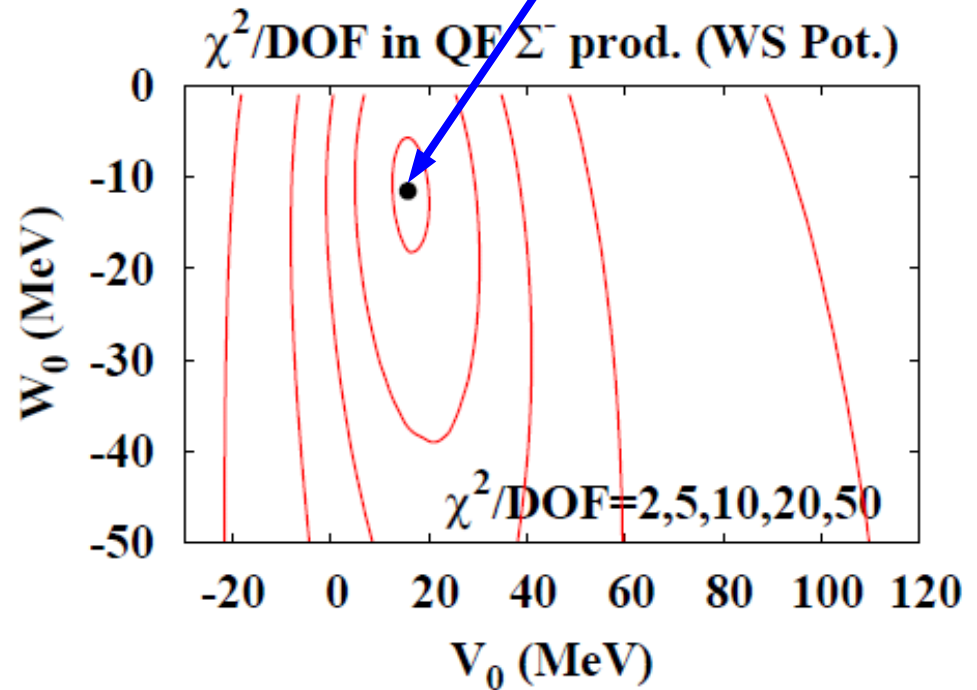
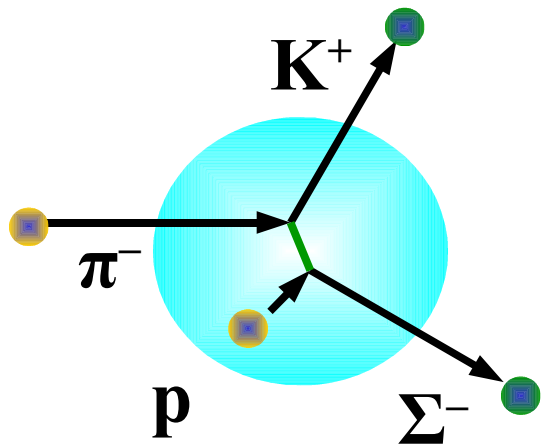
Σ Potential in Nuclear Matter

$$\frac{d^2\sigma}{dE_K d\Omega_K}$$

Maekawa, Tsubakihara, AO, EPJA 33(2007),269.
 Maekawa, Tsubakihara, Matsumiya, AO, in preparation.



$U_\Sigma(\rho_0) \sim +15$ MeV $-i$ 10 MeV
 with Woods-Saxon potential,
 no Atomic shift fit

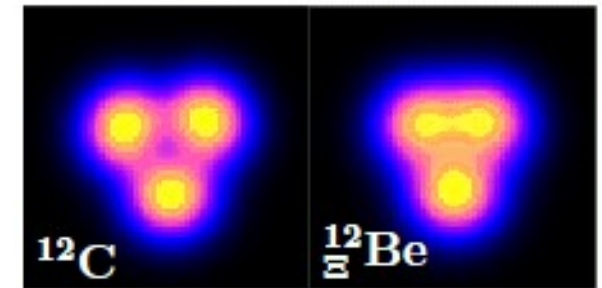
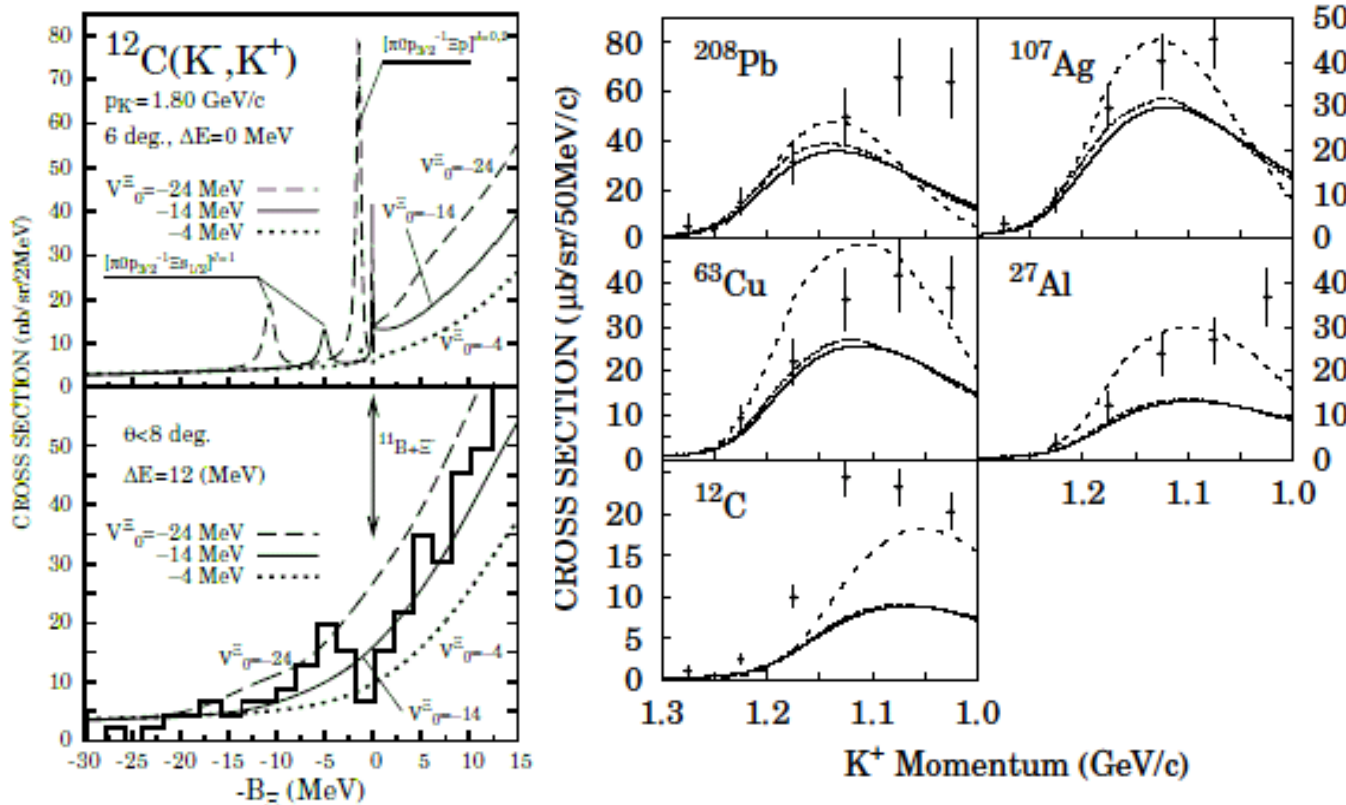


Ξ Potential in Nuclear Matter

- Currently accepted value: $U_{\Xi} \sim -14$ MeV

Twin hypernuclear form., Spectrum shape in the bound state region
 (Aoki et al. PLB355('95),45; Fukuda et al. PRC58('98),1306; Khaustov et al. PRC61('00), 054603)

- Absolute values of $^{12}\text{C}(K^-,K^+)$ spectra \rightarrow Still Difficult to Understand
- Large $q \rightarrow$ Spectrum may depend on detailed nuclear structure



Matsumiya, et al.
 (Coupled Channel AMD)

Let's wait for
 J-PARC results

Maekawa, Tsubakihara, Matsumiya, AO, arXiv:0704.3929.

“Stars” of Hyperon Potentials (A la Michelin)

- $U_{\Lambda}(\rho_0) \sim -30 \text{ MeV}$ * * *
- *Bound State Spectroscopy + Continuum Spectroscopy*
- $U_{\Sigma}(\rho_0) > +15 \text{ MeV}$ * *
- **Continuum (Quasi-Free) spectroscopy with *Local Optimal Fermi Averaging t-matrix* (LOFAt)**
- **Atomic shift data (attractive at surface) should be respected.**
- $U_{\Xi}(\rho_0) \sim -14 \text{ MeV}$ *
- **No confirmed bound state, No atomic data, High mom. transf., → Small Potential Deps.**
- **Continuum low-res. spectrum shape → -14 MeV**
- **Spin-Isospin deps. (π exch.) → Deformation → Spectrum shape may be modified.**



Relativistic EOS of Supernova Matter with Hyperons

- Extention of the Relativistic (Shen) EOS to $SU_f(3)$ with updated Hyperon Potentials in Nuclear Matter (*Ishizuka, Ohnishi, Tsubakihara, Sumiyoshi, Yamada, J. Phys. G 35 (2008), 085201*)
 - Relativistic (Shen) EOS (*Shen, Toki, Oyamatsu, Sumiyoshi, PTP 100('98), 1013*)
Rel. Mean Field (RMF) + Local Density Approx.(Nuclear Formation)
 - $SU_f(3)$ Extention of RMF (*Schaffner, Mishustin, PRC53 (1996), 1416*)
Coupling ~ Quark Number Counting

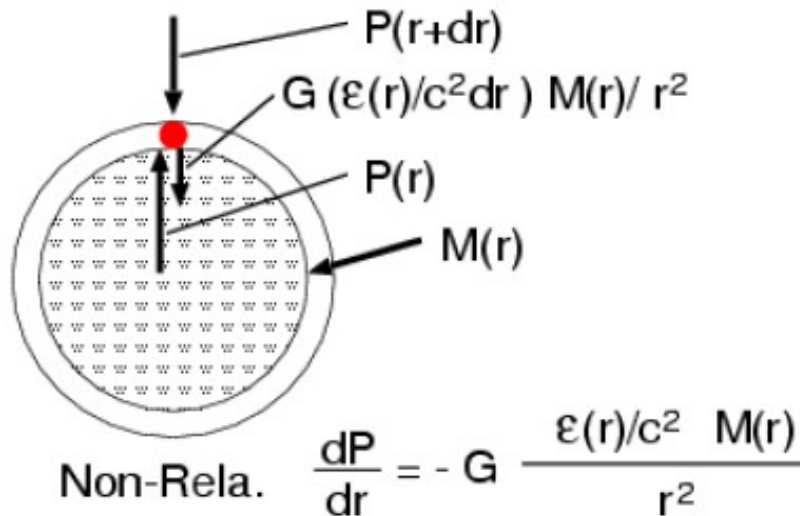
g_{MB}	σ	ζ	ω	ρ	ϕ
N	10.0289	0	12.6139	4.6783	0
Λ	6.21	6.67	8.41	0	-5.95
Σ	4.36 (6.21)	6.67	8.41	$2g_{\rho N}$	-5.95
Ξ	3.11 (3.49)	12.35	4.20	4.63	-11.89

SM
IOTSY

- $g_{\sigma Y}$ is tuned to fit Hyperon Potential in Nuclear Matter
 $U_{\Lambda} = -30 \text{ MeV}, U_{\Sigma} = +30 \text{ MeV}, U_{\Xi} = -15 \text{ MeV}$
- Nuclear Formation is included using Shen EOS table

Tolman-Oppenheimer-Volkoff (TOV) equation

- TOV Eq. = General Relativistic Balance of pressure and gravity



$$\frac{dP}{dr} = -G \frac{(\epsilon/c^2 + P/c^2)(M + 4\pi r^3 P/c^2)}{r^2(1 - 2GM/rc^2)}$$

$$\frac{dM}{dr} = 4\pi r^2 \epsilon/c^2, \quad \frac{dP}{dr} = \frac{dP}{d\epsilon} \frac{d\epsilon}{dr}$$

$$P = P(\epsilon), \quad \frac{dP}{d\epsilon} = \frac{dP}{d\epsilon}(\epsilon) \quad (\text{EOS})$$

Neutron Star Mass = M(R) where P(R)=0

When you make a new EOS, please check the NS mass !

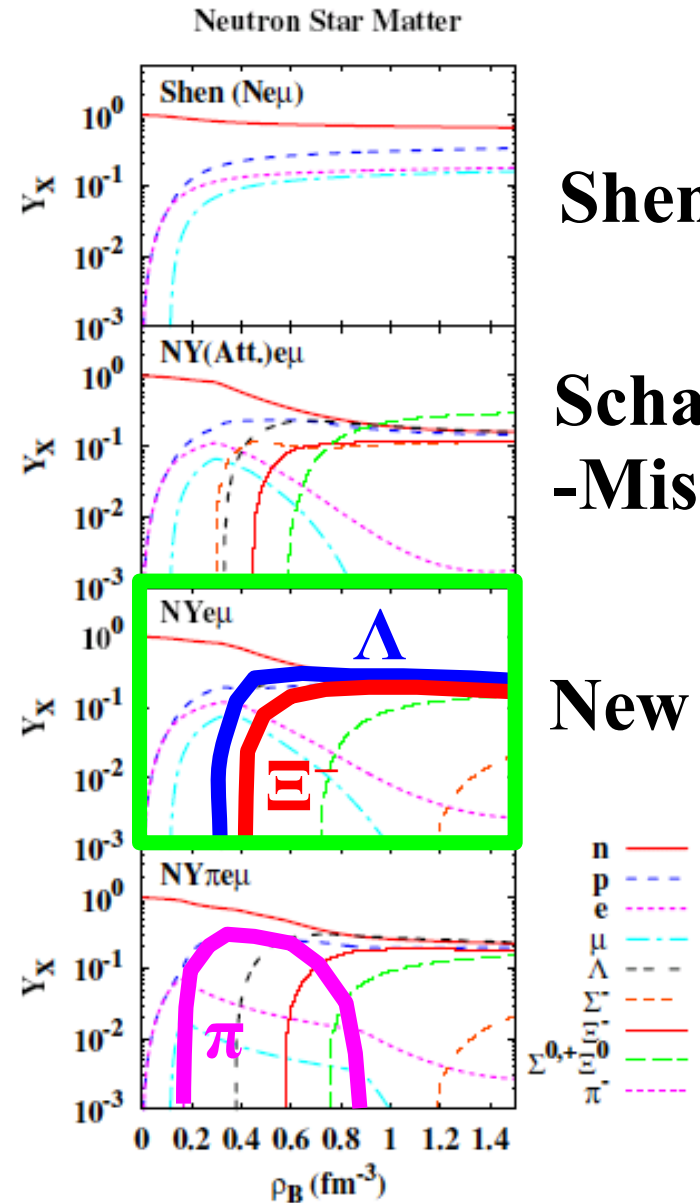
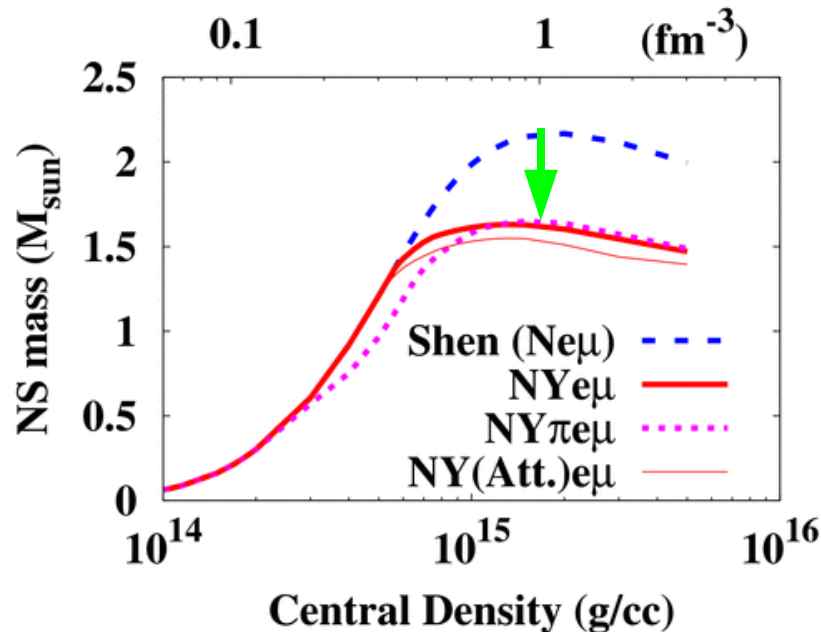
Neutron Star

Ishizuka, AO, Tsubakihara, Sumiyoshi, Yamada, *J. Phys. G 35 (2008), 085201*

Hyperon Effect is DRASTIC

- $M_{\text{max}} = 2.1 M_{\text{sun}} \rightarrow 1.56 M_{\text{sun}}$
- Composition $Y_{\Lambda} \sim Y_n$
- Large fraction of Ξ

Thermal (free) pions can admix at $\rho > 1.5 \rho_0$



Shen

Schaffner
-Mishustin

New

Finite Temperature and Supernova

Ishizuka, AO, Tsubakihara, Sumiyoshi, Yamada, *J. Phys. G* 35 (2008), 085201

■ Example: $T=10$ MeV, $Y_e = 0.4$

- Λ starts to increase at $\rho \sim 2\rho_0$, becomes significant at $\rho \sim 3\rho_0$.

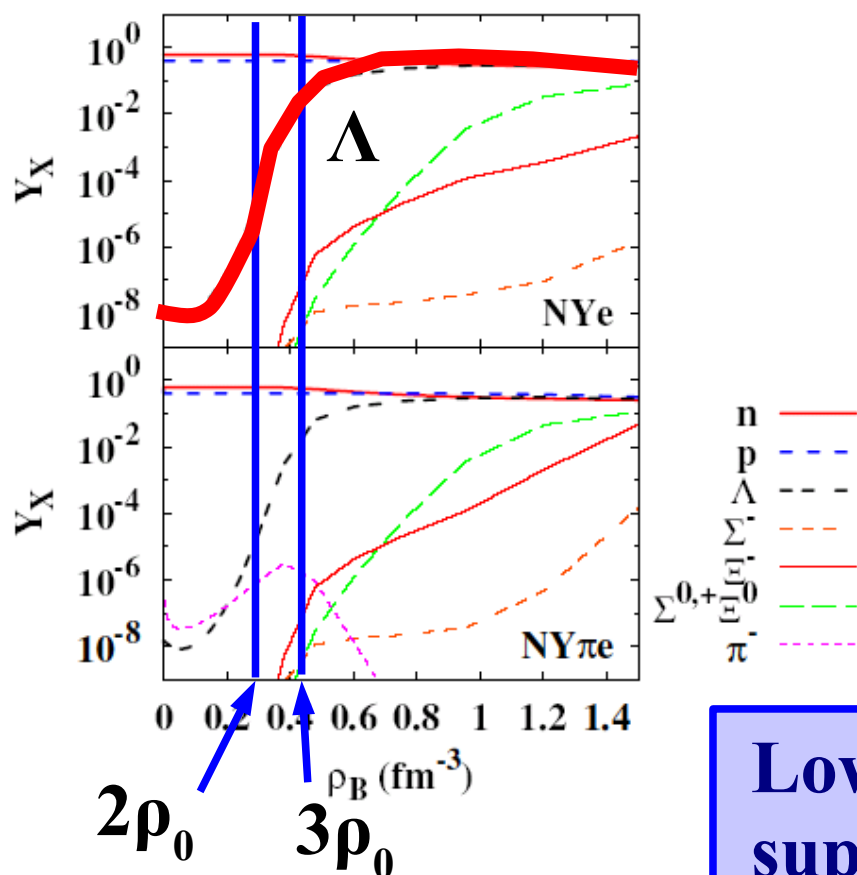
■ Prompt explosion

(without ν transport)

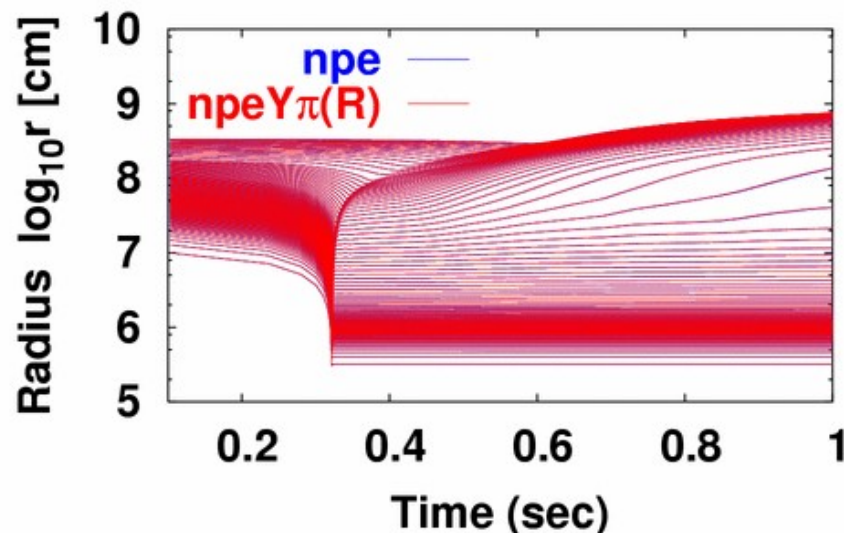
→ Almost no change

(Expl. E. increase $\sim (0.1-0.5 \%)$)

$T=10$ MeV, $Y_C=0.4$



$15 M_{\text{solar}}$



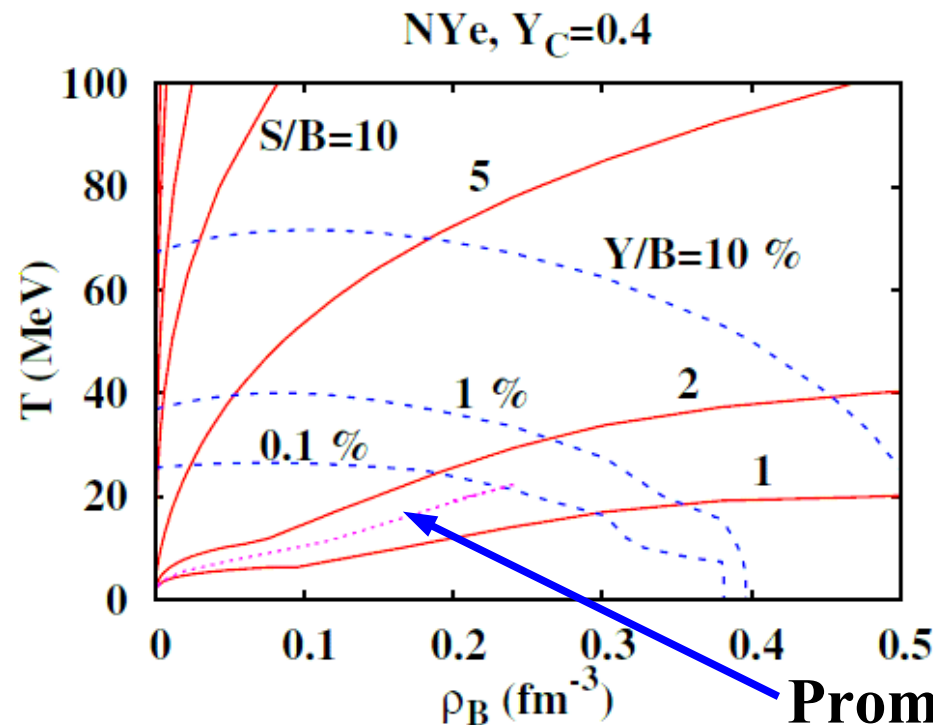
WW95 + 1 Dim. Hydro. (Sumiyoshi, Yamada)

Low density and High Y_e suppresses Hyperons in the Early Stage

Where Do We See Hyperons ?

- Hyperon Fraction is sensitive to Y_e , T , and ρ_B .
 - $Y_v \sim 0$ (Neutron Star) $\rightarrow \rho_B > 2 \rho_0$
 - $Y_e \sim 0.4$ (Supernova, early stage) $\rightarrow T > 40$ MeV or $\rho_B > 3 \rho_0$

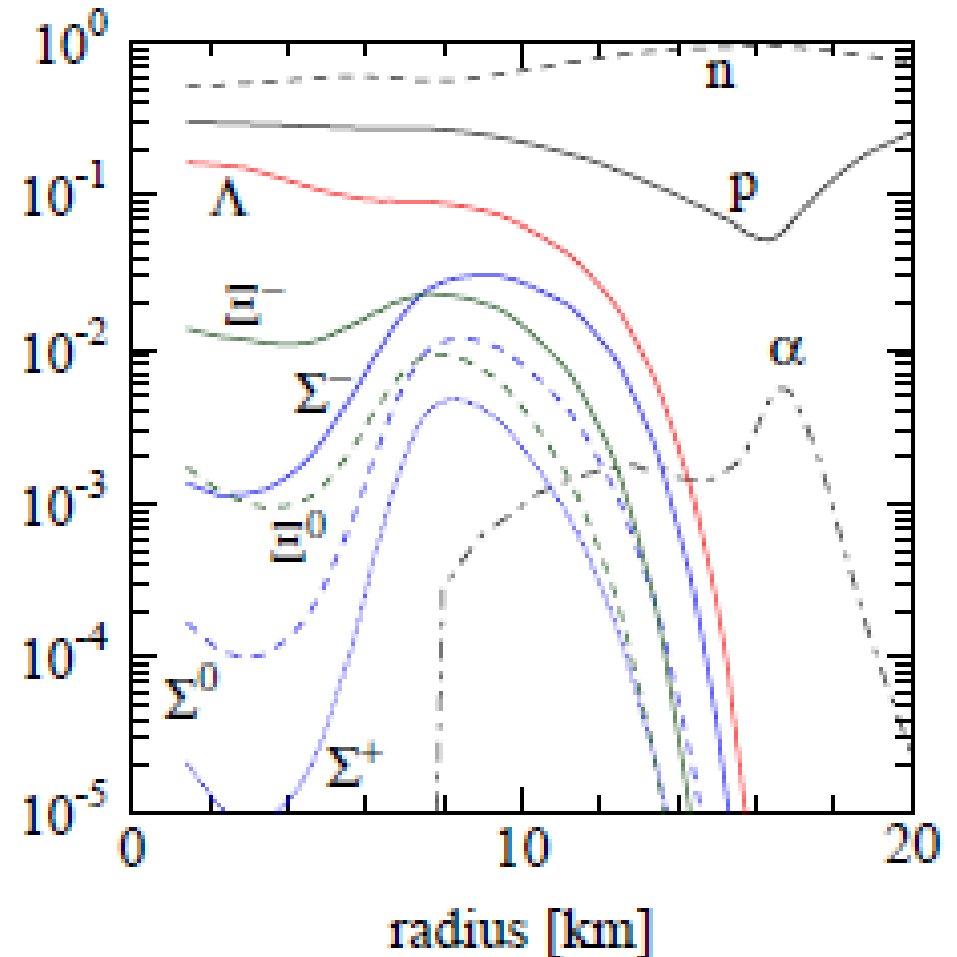
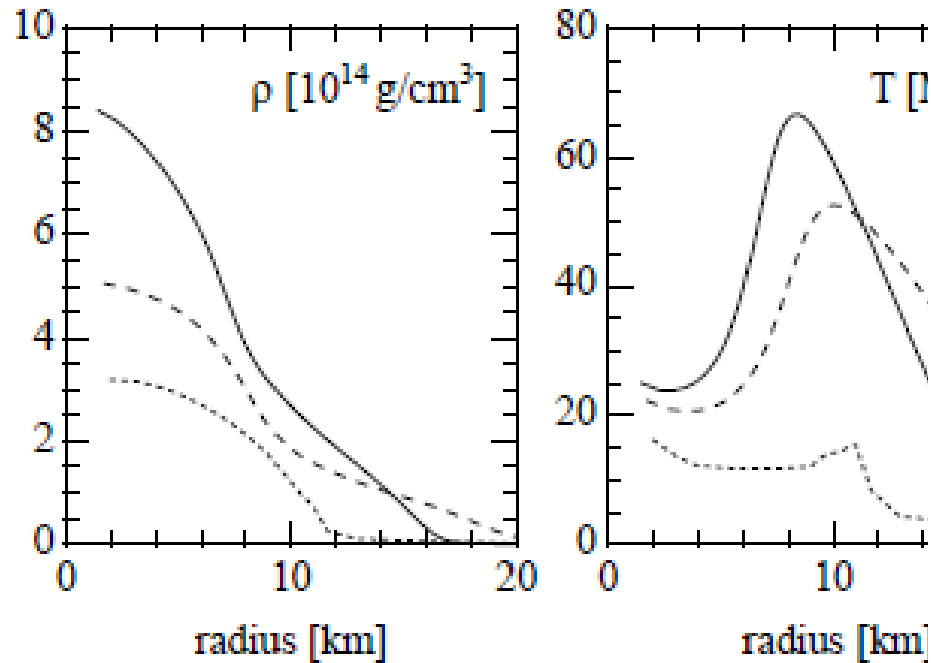
Hyperons would be important in Late Stages
Proto neutron star cooling, Black Hole Formation



Prompt Expl. (15 Msun)

Hyperons during Black Hole Formation

- Hyperons appears abundantly during Black Hole Formation Processes
 - Off-Center: Large $T \rightarrow \Sigma > \Xi$
 - Center: Large $\rho_B \rightarrow \Sigma < \Xi$



Sumiyoshi, Ishizuka, AO, Yamada, Suzuki, ApJ Lett., submitted

Summary (1) of Lecture 2

- Hyperons are included in the Relativistic (Shen) EOS with recently accepted Hyperon Potentials in Nuclear Matter,

$$U_{\Lambda} = -30 \text{ MeV}, U_{\Sigma} = +30 \text{ MeV}, U_{\Xi} = -15 \text{ MeV}$$

<http://nucl.sci.hokudai.ac.jp/~chikako/EOS>

$$\rho = 10^{10} \text{ (5.1-15.4) g/cc}, T = 0-100 \text{ MeV}, Y_e = 0-0.56$$

(Ishizuka, AO, Tsubakihara, Sumiyoshi, Yamada, *J. Phys. G* 35 (2008), 085201)

EOSY by IOTSY

- Hyperon effects:
 - Decisive in Nstar
 - Small in SNe (early)
 - Significant in BH formation.
- Japan Proton Accelerator Research Complex (J-PARC) data will come soon.
Stay Tuned !

Relativistic EOS table including hyperons and pions

*** INTRODUCTION ***
As you know, baryons having strangeness (hyperons) exist in dense matter like high density supernova explosion environment, neutrons stars, or early stage of blackhole. Today, we can obtain the basic information on hyperon-nucleon (YN) interaction at around normal nuclear density through pion induced heavy ion collision at KEK etc. Then we know Lambda-N, Xi-N interaction at the normal density from such a recent progress in strangeness nuclear physics. However, unfortunately, Sigma-N interaction has a large ambiguity even at present. This difference of Sigma-N interaction results in different components of dense matter and the stiffness of EOS. Therefore, we provide various EOS tables within this Sigma-N ambiguity as follows in this site. We wish this EOS tables will be helpful to your study.

*** RELATIVISTIC EOS TABLE ***
We adopt these YN interactions: Λ -N = -30MeV, Ξ -N = -15MeV, Σ -N = (-30 to +90)MeV. **The most recommended Sigma-N interaction is +30 MeV at normal density.** These EOS tables contain the same information as [Shen EOS table](#), physical quantities such as pressure, energy, or something like that, follow the Shen EOS notation and units. Therefore if you have already used Shen EOS table, you can apply these EOS tables to your calculations. The following compressed directories are made of two files --- "###.tbl" and "###.urt".
"###.tbl" means EOS table in Shen EOS table style, while you can see particle ratios at each (Ye, rhoB, T) in "###.urt". Here, the (Ye, rhoB, T) conditions are decided by Shen EOS tables. The former four files consist of only nucleons and hyperons, thermal pion contributions are added to the latter four files.

- Shen EOS+Hyperons(Sigma-N=-30MeV)
- Shen EOS+Hyperons(Sigma-N=0MeV)
- Shen EOS+Hyperons(Sigma-N=+30MeV)
- Shen EOS+Hyperons(Sigma-N=+90MeV)
- Shen EOS+Hyperons+pions(Sigma-N=-30MeV)
- Shen EOS+Hyperons+pions(Sigma-N=0MeV)
- Shen EOS+Hyperons+pions(Sigma-N=+30MeV)
- Shen EOS+Hyperons+pions(Sigma-N=+90MeV)
- Shen EOS+Y(log(rhoB)=5, Tto17, T=0to400MeV) updated at 2007/9/8

I also open a [power point file](#) which was prepared for the APJ spring meeting held at Tokyo, 2005. This power point file give a detailed explanation for construction method of our EOS table, its importance and effects on supernova explosion.

*** README ***
I'm sorry to be late making README... it's now under construction.

If you have any questions and comments, please let me know!
=====

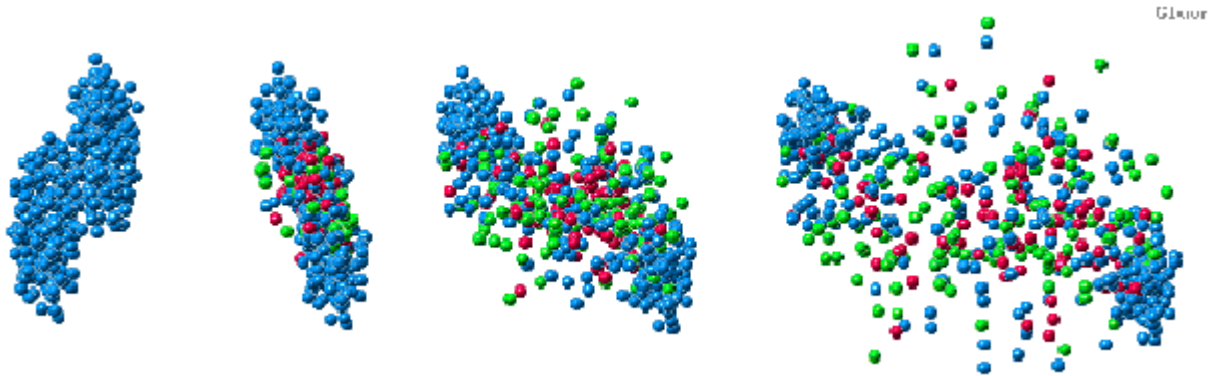
Chikako ISHIZUKA
Grad. Sch. of Sci., Hokkaido Univ.
E-mail:chikako@nucl.sci.hokudai.ac.jp

*Nuclear Transport Models
for Heavy-Ion Collisions
and Collective Flows*

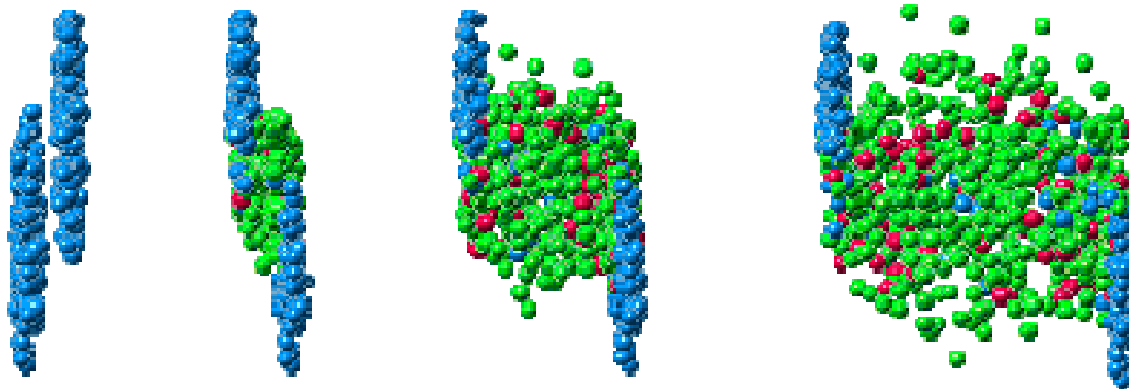
Heavy-Ion Collisions at $E_{\text{inc}} \sim (1-100) A \text{ GeV}$

- Study of Hot and Dense Hadronic Matter
 - Particle Yield, Collective Dynamics (Flow), EOS,

AGS



SPS



JAMming on the Web, linked from <http://www.jcprg.org/>

Nuclear Mean Field

- MF has on both of ρ and p -deps.

- ρ dep.: $(\rho_0, E/A) = (0.15 \text{ fm}^{-3}, -16.3 \text{ MeV})$ is known

Stiffness is not known well

- p dep.: Global potential up to $E=1 \text{ GeV}$ is known from pA scattering

$$U(\rho_0, E) = U(\rho_0, E=0) + 0.3 E$$

- Ab initio Approach; LQCD, GFMC, DBHF, G-matrix,

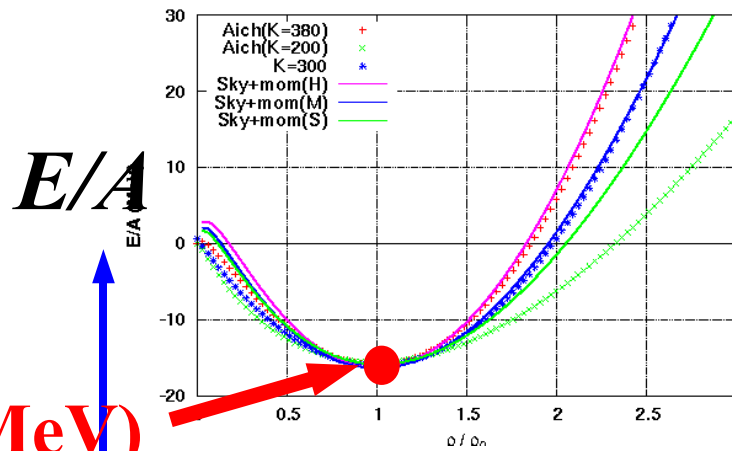
→ Not easy to handle, Not satisfactory for phen. purposes

- Effective Interactions (or Energy Functionals):

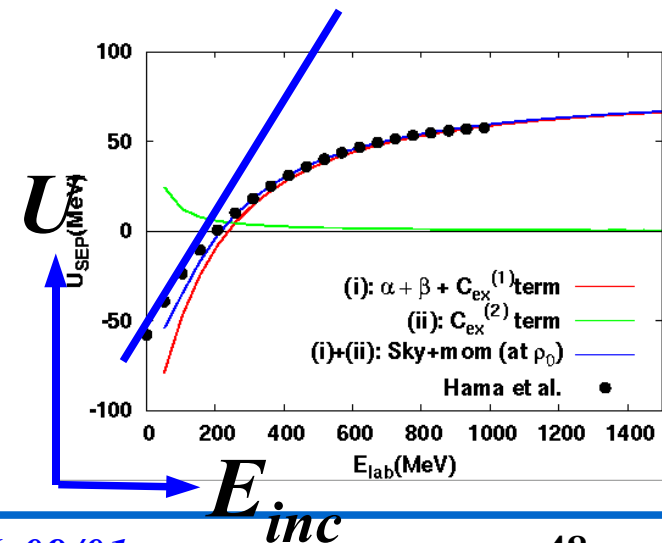
Skyrme HF, RMF, ...

$(\rho_0, E/A)$

$= (0.15 \text{ fm}^{-3}, -16.3 \text{ MeV})$



$$U(E) = U(0) + 0.3E$$



HIC Transport Models: Major Four Origins

■ *Nuclear Mean Field Dynamics*

- **Basic Element of Low Energy Nuclear Physics, and Critically Determines High Density EOS / Collective Flows**
- **TDHF → Vlasov → BUU**

■ *NN two-body (residual) interaction*

- **Main Source of Particle Production**
- **Intranuclear Cascade Models**

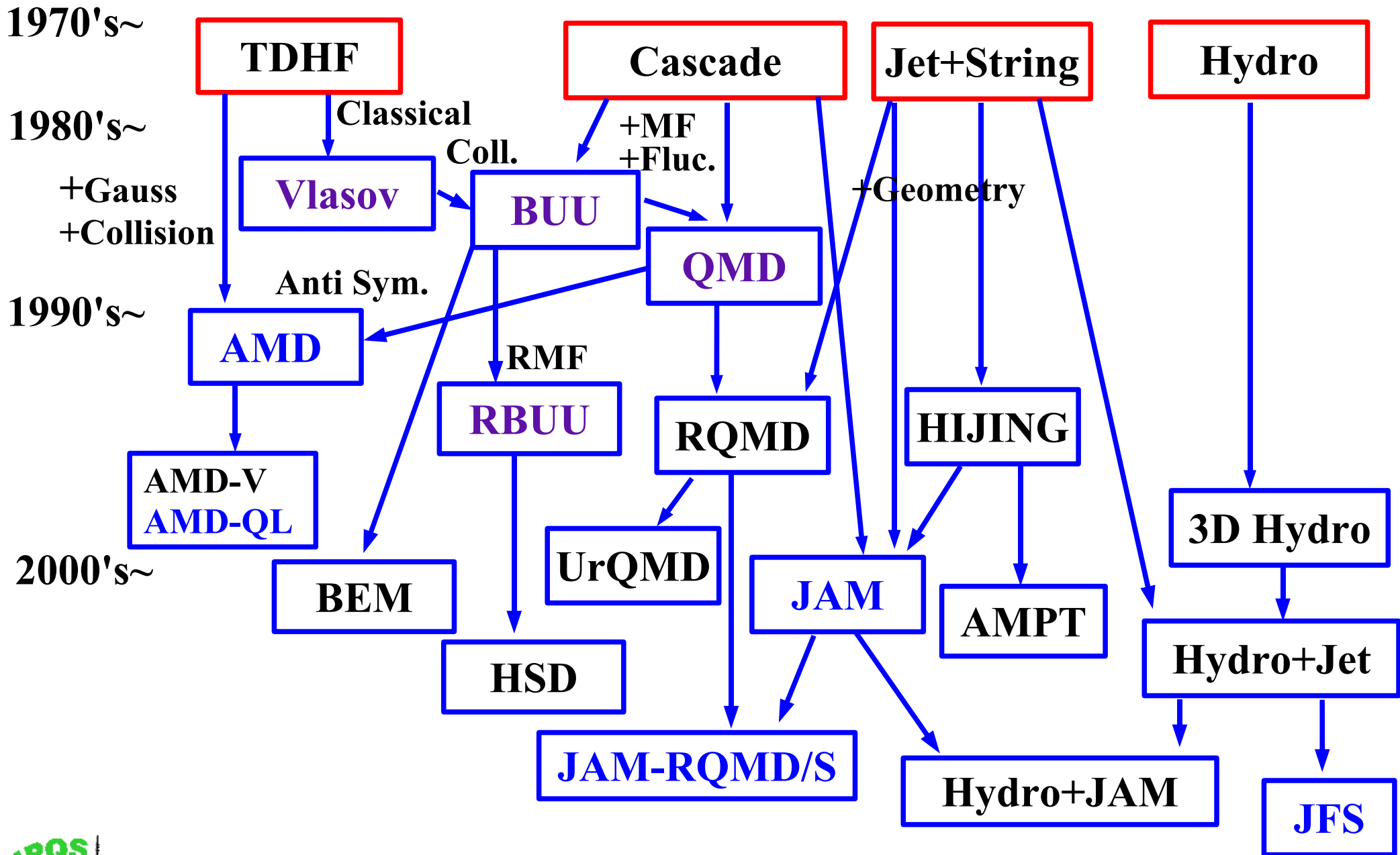
■ *Partonic Interaction and String Decay*

- **Main Source of high pT Particles at Collider Energies**
- **JETSET + (previous) PYTHIA (Lund model) → (new) PYTHIA**

■ *Relativistic Hydrodynamics*

- **Most Successful Picture at RHIC**

HIC Models: History



TDHF and Vlasov Equation

- Time-Dependent Mean Field Theory (e.g., TDHF)

$$i \hbar \frac{\partial \phi_i}{\partial t} = h \phi_i$$

- Density Matrix

$$\rho(r, r') = \sum_i^{\text{Occ}} \phi_i(r) \phi_i^*(r') \rightarrow \rho_W = f \text{ (phase space density)}$$

- TDHF for Density Matrix

$$i \hbar \frac{\partial \rho}{\partial t} = [h, \rho] \rightarrow \frac{\partial f}{\partial t} = \{h_W, f\}_{P.B.} + O(\hbar^2)$$

- Wigner Transformation and Wigner-Kirkwood Expansion

(Ref.: Ring-Schuck)

$$O_W(r, p) \equiv \int d^3 s \exp(-i p \cdot s / \hbar) \langle r + s/2 | O | r - s/2 \rangle$$

$$(AB)_W = A_W \exp(i \hbar \Lambda) B_W \quad \Lambda \equiv \nabla'_r \cdot \nabla_p - \nabla'_p \cdot \nabla_r \quad (\nabla' \text{ acts on the left})$$

$$[A, B]_W = 2i A_W \sin(\hbar \Lambda / 2) B_W = i \hbar \{A_W, B_W\}_{P.B.} + O(\hbar^3)$$

Test Particle Method

■ Vlasov Equation

$$\frac{\partial f}{\partial t} - \{h_W, f\}_{P.B.} = \frac{\partial f}{\partial t} + v \cdot \nabla_r f - \nabla U \cdot \nabla_p f = 0$$

■ Classical Hamiltonian

$$h_W(r, p) = \frac{p^2}{2m} + U(r, p)$$

■ Test Particle Method (C. Y. Wong, 1982)

$$f(r, p) = \frac{1}{N_0} \sum_i^{AN_0} \delta(r - r_i) \delta(p - p_i) \quad \rightarrow \quad \frac{dr_i}{dt} = \nabla_p h_w, \quad \frac{dp_i}{dt} = -\nabla_r h_w,$$

Mean Field Evolution can be simulated

by Classical Test Particles

**→ Opened a possibility to Simulate High Energy HIC
including Two-Body Collisions in Cascade**

BUU (Boltzmann-Uehling-Uhlenbeck) Equation

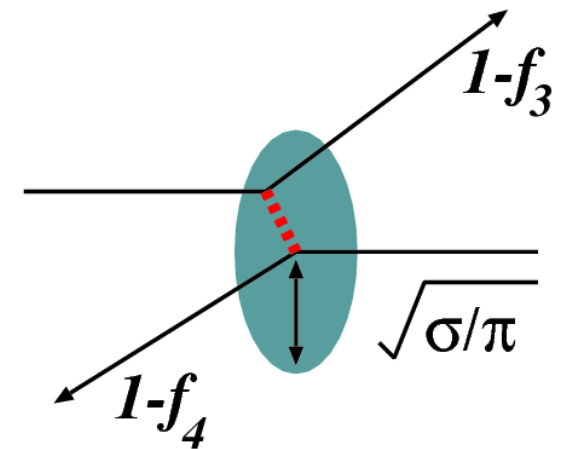
- **BUU Equation** (Bertsch and Das Gupta, Phys. Rept. 160(88), 190)

$$\frac{\partial f}{\partial t} + v \cdot \nabla_r f - \nabla U \cdot \nabla_p f = I_{coll}[f]$$

$$I_{coll}[f] = -\frac{1}{2} \int \frac{d^3 p_2 d\Omega}{(2\pi\hbar)^3} v_{12} \frac{d\sigma}{d\Omega} \\ \times [f f_2 (1-f_3)(1-f_4) - f_3 f_4 (1-f)(1-f_2)]$$

- **Incorporated Physics in BUU**

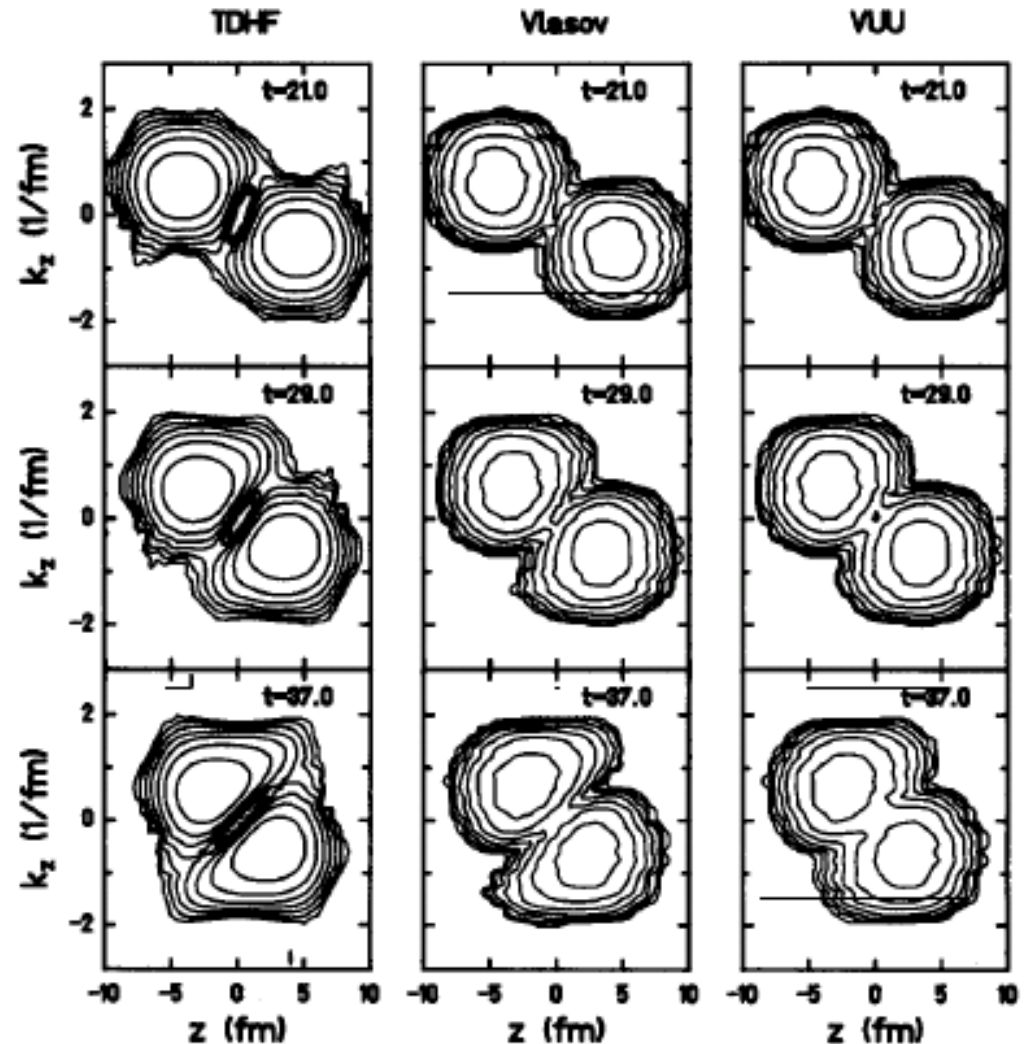
- Mean Field Evolution
- (Incoherent) Two-Body Collisions
- Pauli Blocking in Two-Body Collisions



- One-Body Observables (Particle Spectra, Collective Flow, ..)
- ✗ Event-by-Event Fluctuation (Fragment, Intermittency, ...)

Comparison of TDHF, Vlasov and BUU(VUU)

- Ca+Ca, 40 A MeV
(Cassing-Metag-Mosel-Niita, Phys. Rep. 188 (1990) 363).



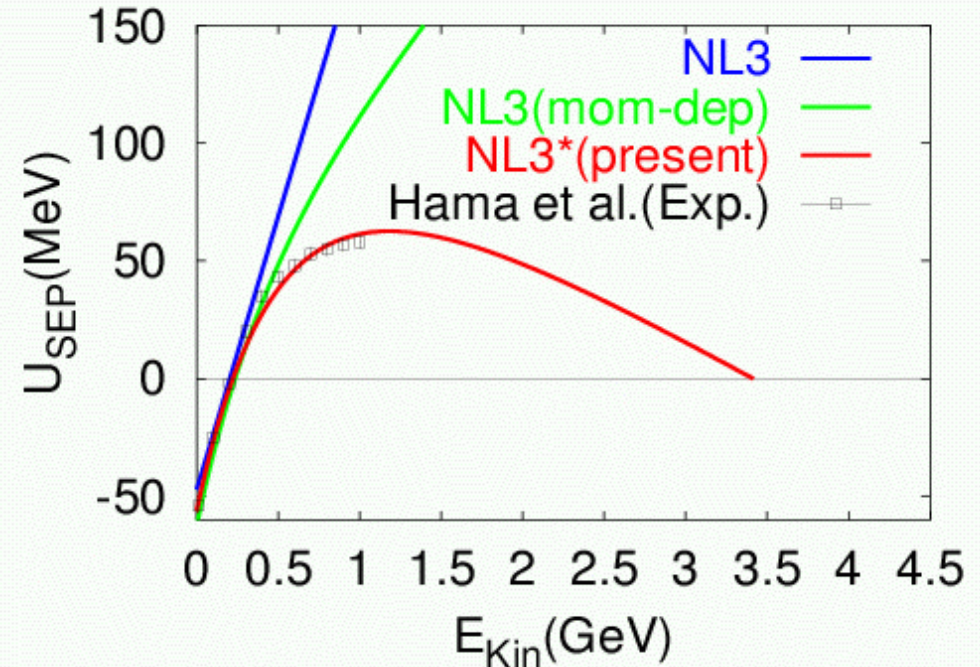
Relativistic Mean Field (II)

■ **Dirac Equation** $(i\gamma\partial - \gamma^0 U_v - M - U_s)\psi = 0$, $U_v = g_\omega \omega$, $U_s = -g_\sigma \sigma$

■ **Schroedinger Equivalent Potential**

$$\begin{pmatrix} E - U_v - M - U_s & i\sigma \cdot \nabla \\ -i\sigma \cdot \nabla & -E + U_v - M - U_s \end{pmatrix} \begin{pmatrix} f \\ g \end{pmatrix} = 0$$

$$\begin{aligned} U_{sep} &\sim U_s + \frac{E}{m} U_v = -g_\sigma \sigma + \frac{E}{m} g_\omega \omega \\ &= -\frac{g_\sigma^2}{m_\sigma^2} \rho_s + \frac{E}{m} \frac{g_\omega^2}{m_\omega^2} \rho_B \end{aligned}$$



Saturation: -Scalar+Baryon Density

Linear Energy Dependence: Good at Low Energies,

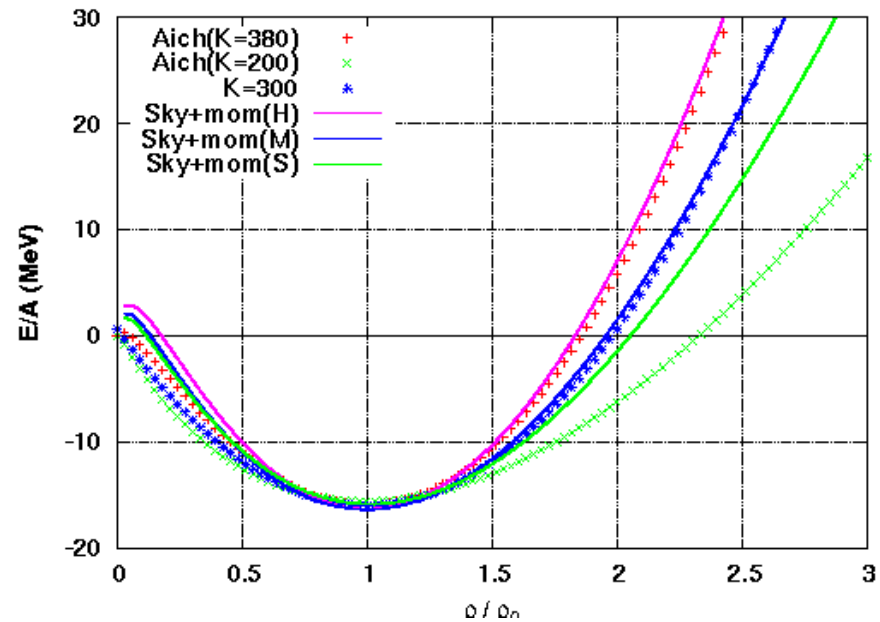
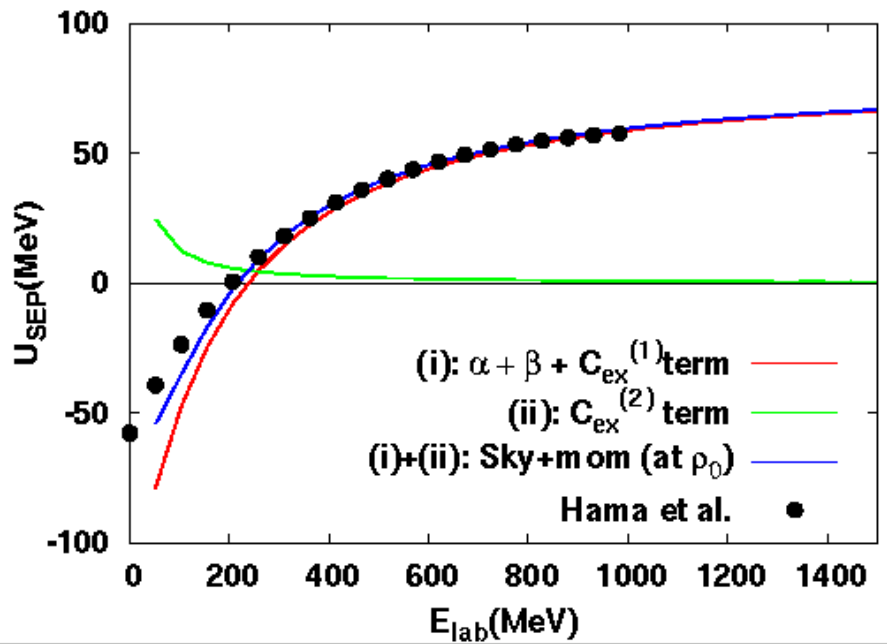
Bad at High Energies (We need cut off !)

(Sahu, Cassing, Mosel, AO, Nucl. Phys. A672 (2000), 376.)

Phenomenological Mean Field

■ Skyrme type ρ -Dep. + Lorentzian p -Dep. Potential

$$V = \sum_i V_i = \int d^3 r \left[\frac{\alpha}{2} \left(\frac{\rho}{\rho_0} \right)^2 + \frac{\beta}{\gamma + 1} \left(\frac{\rho}{\rho_0} \right)^{\gamma + 1} \right] + \sum_k \int d^3 r d^3 p d^3 p' \frac{C_{ex}^{(k)}}{2\rho_0} \frac{f(r, p) f(r, p')}{1 + (p - p')^2 / \mu_k^2}$$



Isse, AO, Otuka, Sahu, Nara, *Phys.Rev. C* 72 (2005), 064908

Collective Flow and EOS: Old Problem ?

- **1970's-1980's: First Suggestions and Measurement**
 - Hydrodynamics suggested the Existence of Flow.
 - Strong Collective Flow suggests Hard EOS
- **1980's-1990's: Deeper Discussions in Wider E_{inc} Range**
 - Momentum Dep. Pot. can generate Strong Flows.
 - E_{inc} deps. implies the importance of Momentum Deps.
 - Flow Measurement up to AGS Energies.
- **2000's: Extention to SPS and RHIC Energies**
 - EOS is determined with Mom. AND Density Dep. Pot. ?

Old but New (Continuing) Problem !

What is Collective Flow ?

(Directed) Flow (dP_x/dY)

Stiffness (Low E)
+ Time Scale (High E)

Elliptic Flow (V_2)

Thermalization
& Pressure Gradient

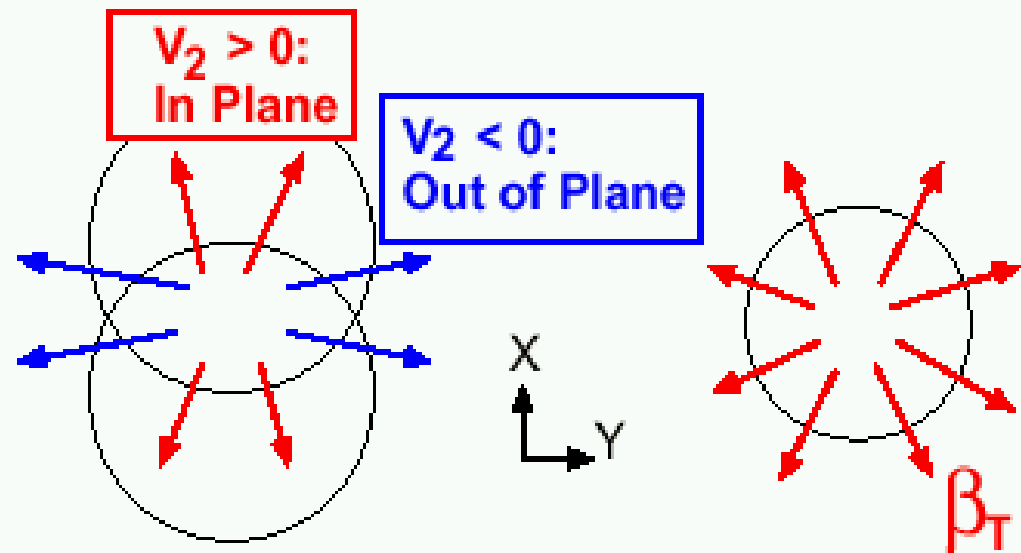
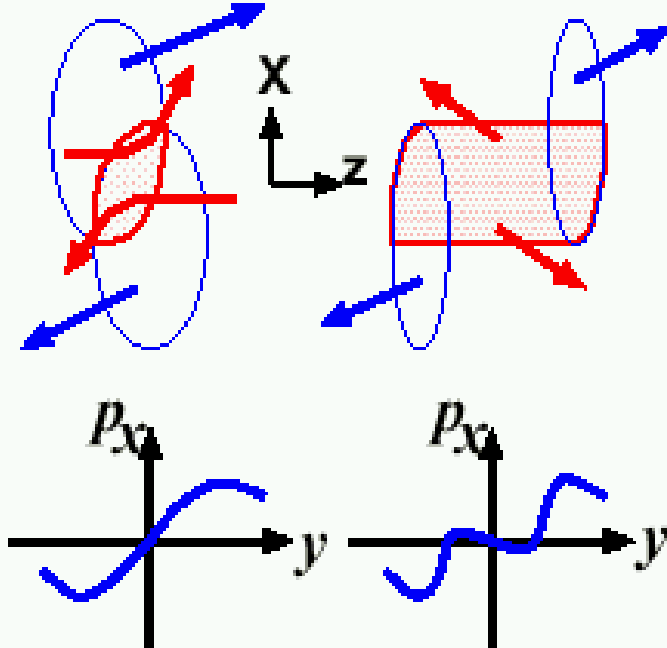
Radial Flow (β_T)

Pressure History

$$\epsilon \frac{DV}{Dt} = -\nabla P$$

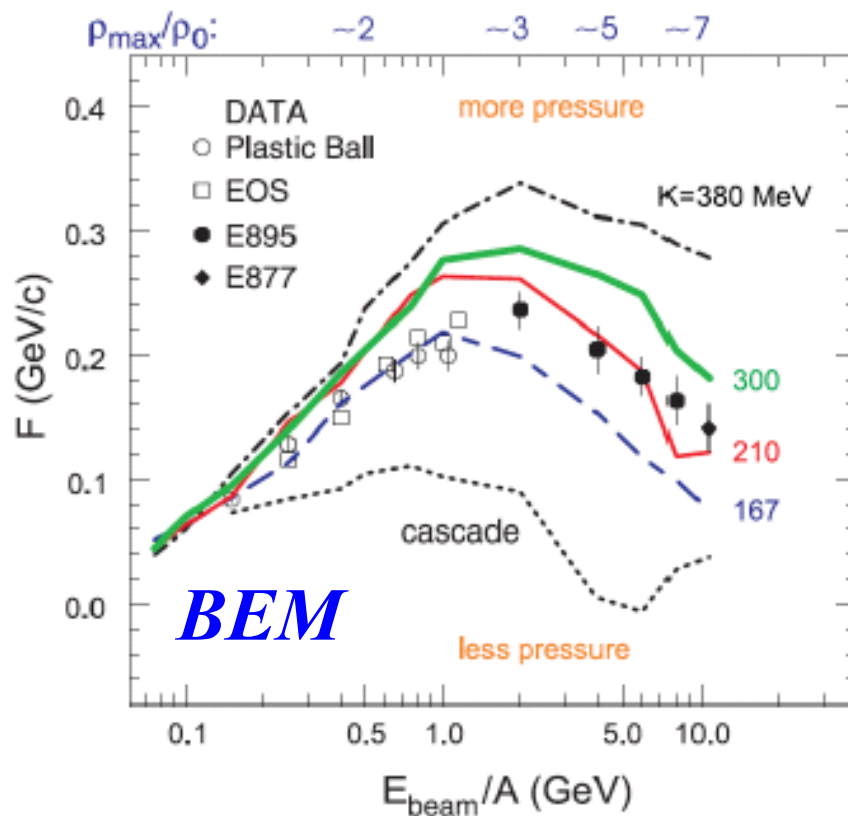
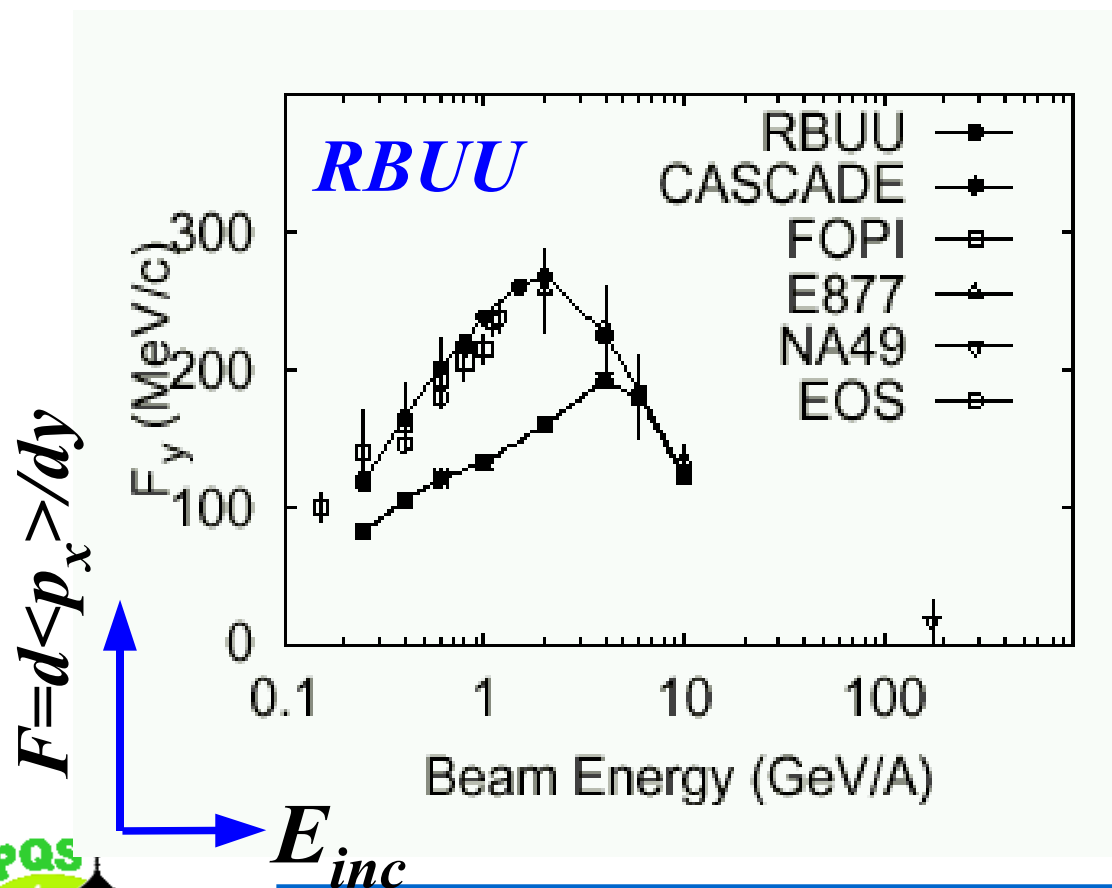
$$\rightarrow V = \int_{path} \frac{-\nabla P dt}{\epsilon}$$

Until AGS **Above SPS**



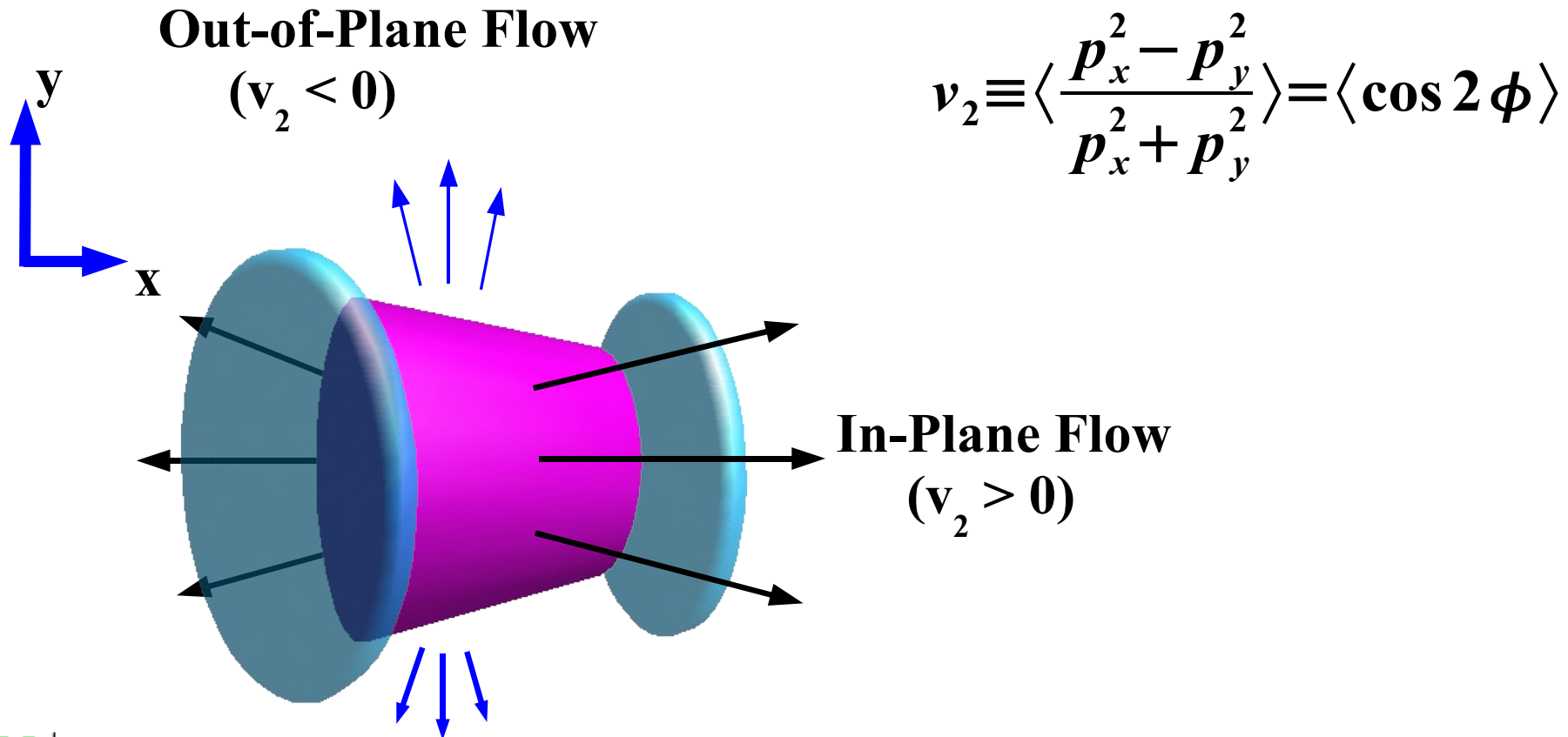
Side Flow at AGS Energies

- Relativistic BUU (RBUU) model: $K \sim 300 \text{ MeV}$
(Sahu, Cassing, Mosel, AO, Nucl. Phys. A672 (2000), 376.)
- Boltzmann Equation Model (BEM): $K=167\sim 210 \text{ MeV}$
(P. Danielewicz, R. Lacey, W.G. Lynch, Science 298(2002), 1592.)



Elliptic Flow

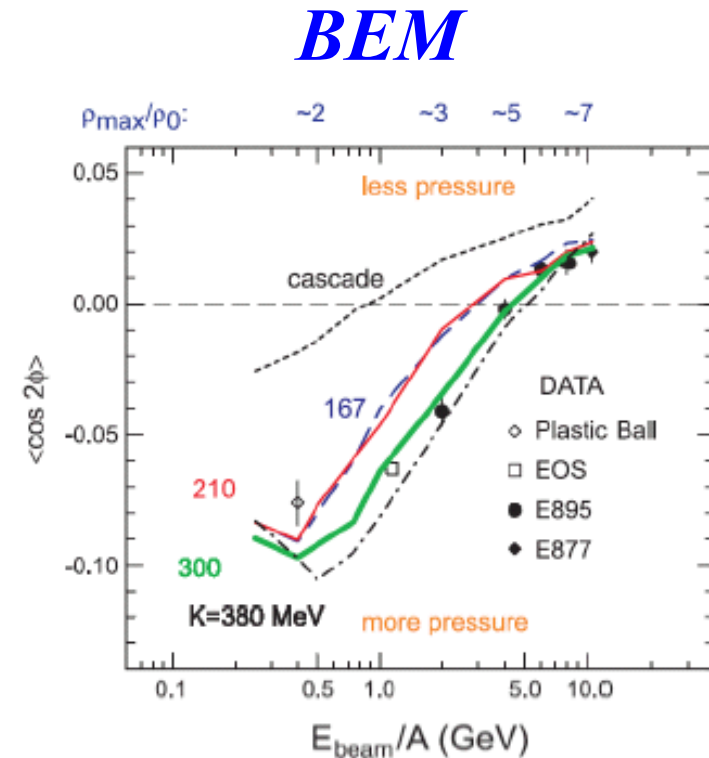
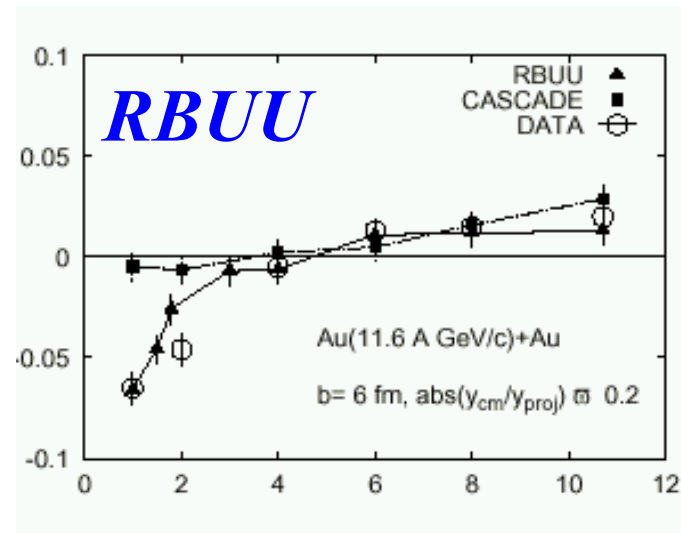
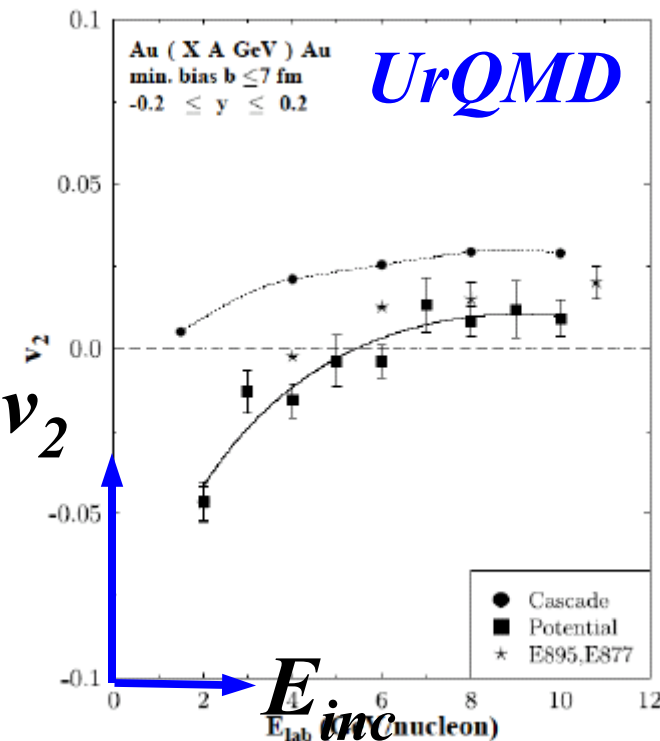
- What is Elliptic Flow ? → Anisotropy in P space
- Hydrodynamical Picture
 - Sensitive to the Pressure Anisotropy in the Early Stage
 - Early Thermalization is Required for Large v_2



Elliptic Flow at AGS

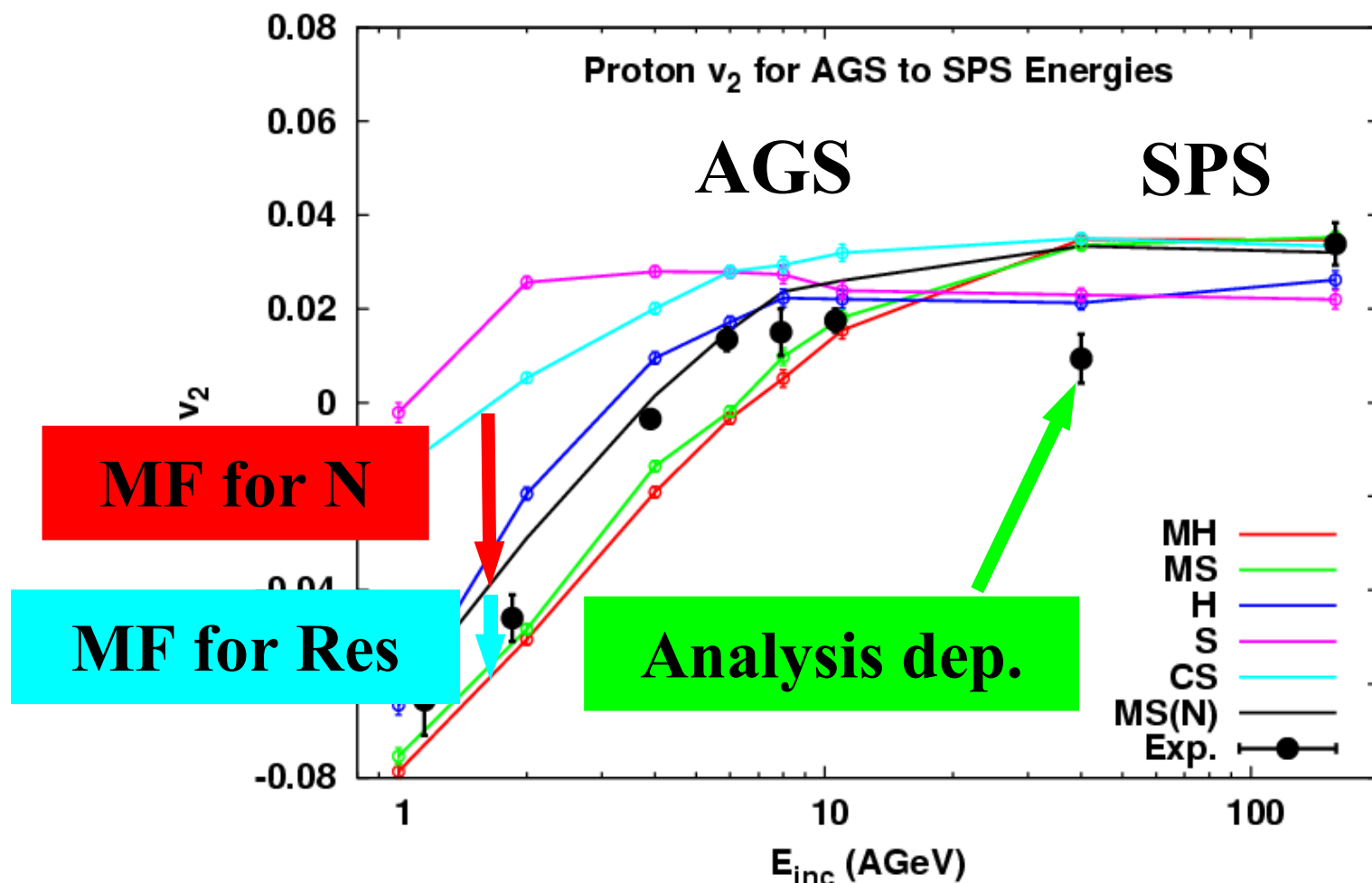
Strong Squeezing Effects at low E (2-4 A GeV)

- UrQMD: Hard EOS (S.Soff et al., nucl-th/9903061)
- RBUU (Sahu-Cassing-Mosel-AO, 2000): $K \sim 300$ MeV
- BEM(Danielewicz2002): $K = 167 \rightarrow 300$ MeV



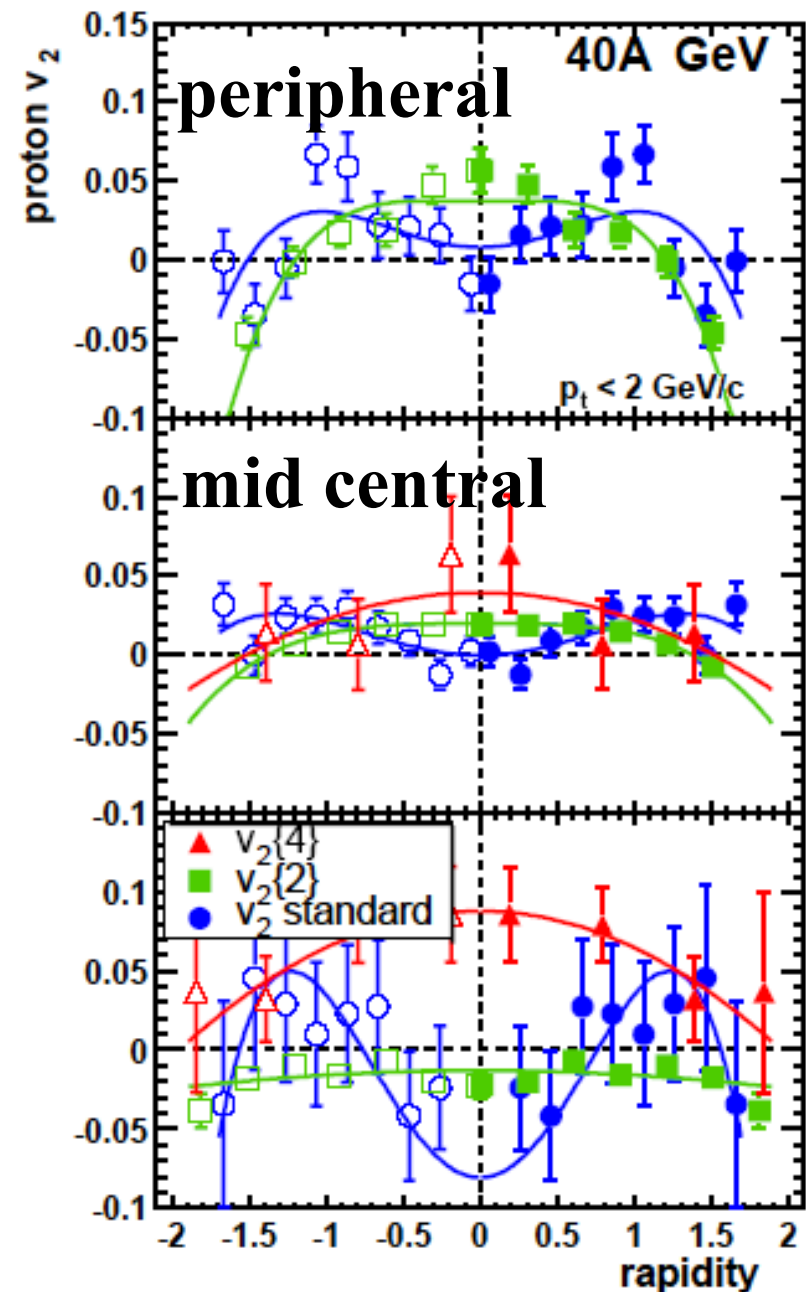
Elliptic Flow from AGS to SPS

- JAM-MF with p dep. MF explains proton v_2 at 1-158 A GeV
 - v_2 is not very sensitive to K (incompressibility)
 - Data lies between MS(B) and MS(N)



Dip of V_2 at 40 A GeV: Phase Transition ?

- Dip of V_2 at 40 A GeV may be a signal of QCD phase transition at high baryon density.
(Cassing et al.)
- However, the data is too sensitive to the way of the analysis (reaction plane/two particle correlation).
 - We have to wait for better data.



Flow and EOS; to be continued

- In addition to the ambiguities in in-medium cross sections, Res.-Res. cross sections, we have model dependence.

- RBUU (*e.g. Sahu, Cassing, Mosel, AO, Nucl. Phys. A672 (2000), 376.*)

- ◆ In RMF, Strong cut-off for meson-N coupling in RMF
→ Smaller EOS dep.

- Scalar potential interpretation in BUU

Larionov, Cassing, Greiner, Mosel, PRC62,064611('00), Danielewicz, NPA673,375('00)

$$\varepsilon(\mathbf{p}, \rho) = \sqrt{[m + U_s(\mathbf{p}, \rho)]^2 + \mathbf{p}^2} = \sqrt{m^2 + \mathbf{p}^2} + U(\mathbf{p}, \rho)$$

- ◆ Due to the Scalar potential nature, EOS dependence is smaller.

- Scalar/Vector Combination *Danielewicz, Lacey, Lynch, Science 298('02), 1592*

$$\varepsilon(p, \rho) = m + \int_0^p dp' v^*(p', \rho) + \tilde{U}(\rho), \quad v^*(p, \rho) = \frac{p}{\sqrt{p^2 + [m^*(p, \rho)]^2}}$$

- ◆ Relatively Strong EOS dependence even at high energy

- JAM-RQMD/S *Isse, AO, Otuka, Sahu, Nara, PRC 72 (2005), 064908*

- ◆ Similar to the Scalar model BUU