Some basic properties and excitations in nuclei
—— β-stable nuclei vs. unstable nuclei.

1) Shape and shell-structure.

One-body potentials and occupied levels;

Effect of neutron excess on density and potential;

Change of shell-structure as $\varepsilon_\nu = -10\text{ MeV} \rightarrow 0\text{ MeV}$;

Deformation (Jahn-Teller effect, and new shell-structure at large deformation);

One-particle motion in deformed potentials — Nilsson diagram.

2) Collective modes.

Giant resonances;

Low-energy threshold strength in drip line nuclei;

Model-independent sum-rules.

H. Geissel, G. Münzenberg and K. Riisager, Annu. Rev. Nucl. Part. Sci., 45 (1995) 163.

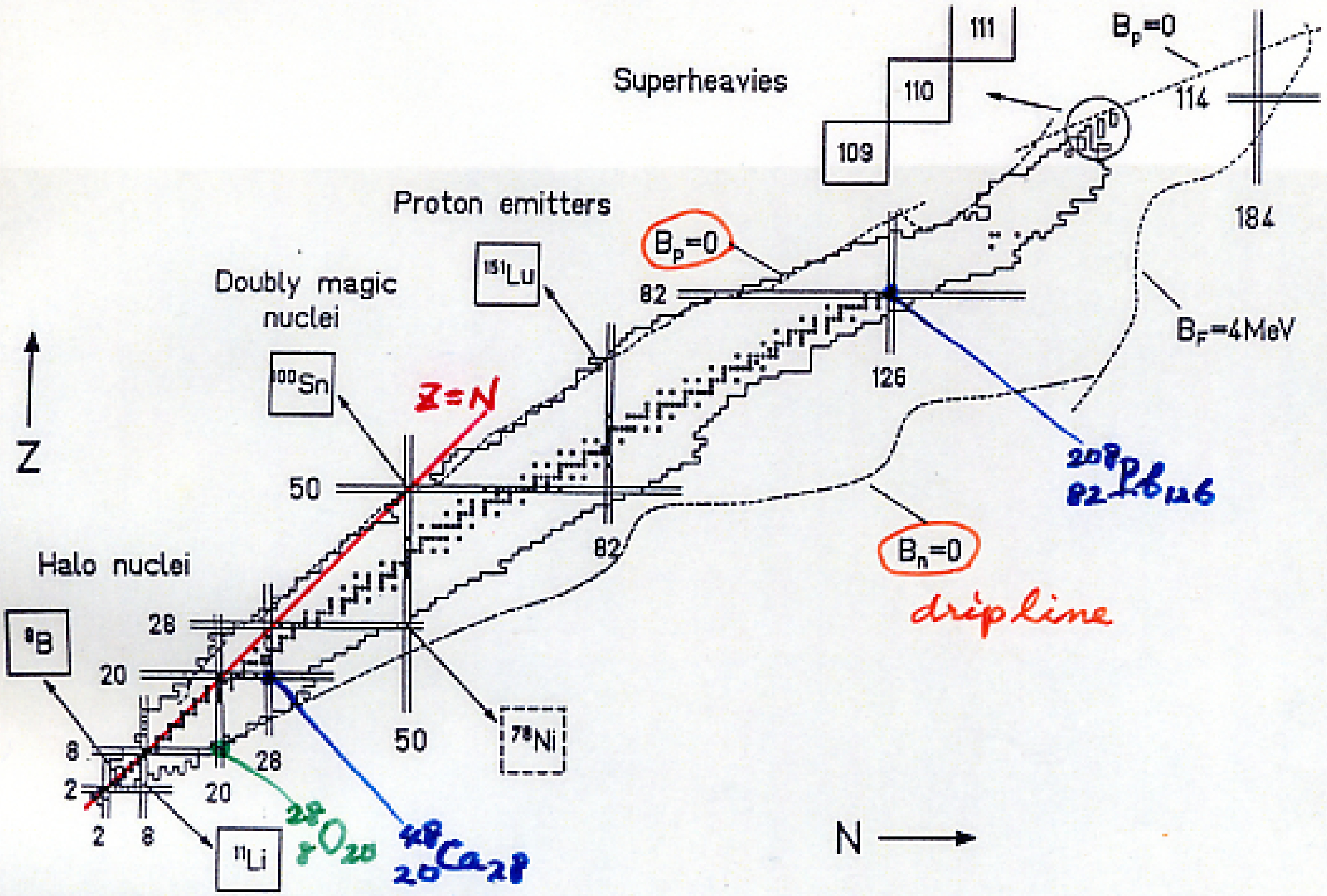


Figure 1 Chart of nuclei with nuclear driplines (*dashed lines*) and selected landmarks of exotic nuclear structure. The full lines indicate the present limits of discovered nuclei.

§ Shape and Shell-structure

β-stable nuclei :

one-body potential ~ harmonic-oscillator potential
+ $(\vec{l} \cdot \vec{s})$ potential
+ surface effect.

separation energy of nucleons, $S_n \sim S_p \sim$ 7–10 MeV.

magic numbers (N or Z) = 2, 8, 20, 28, 50, 82, 126,

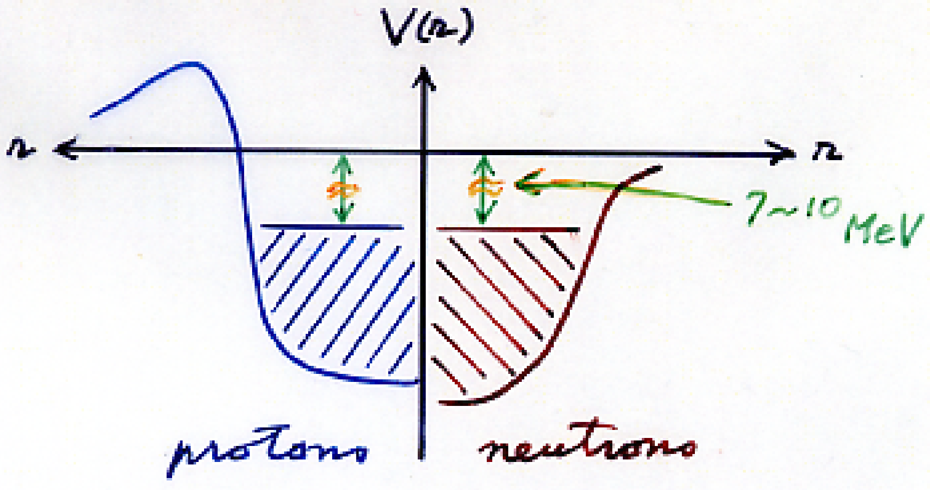
ground states of nuclei with magic numbers are spherical.

- - - - - - - - - - - - -

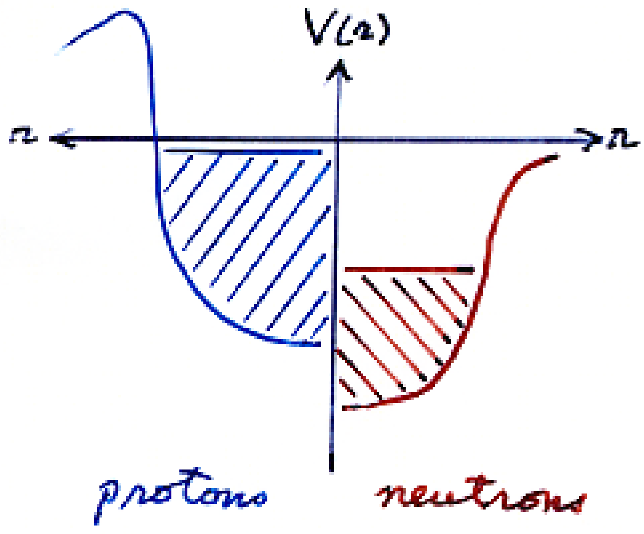
Hartree-Fock approximation (= mean-field approximation of nuclear many-body problems)

$$H = \underbrace{\sum_i t_i}_{\text{kinetic energy}} + \underbrace{\sum_{i<j} v_{ij}}_{\text{two-body interaction}} \longrightarrow \sum_i t_i + \underbrace{\sum_i V_i}_{\text{one-body (HF) potential calculated self-consistently.}}$$

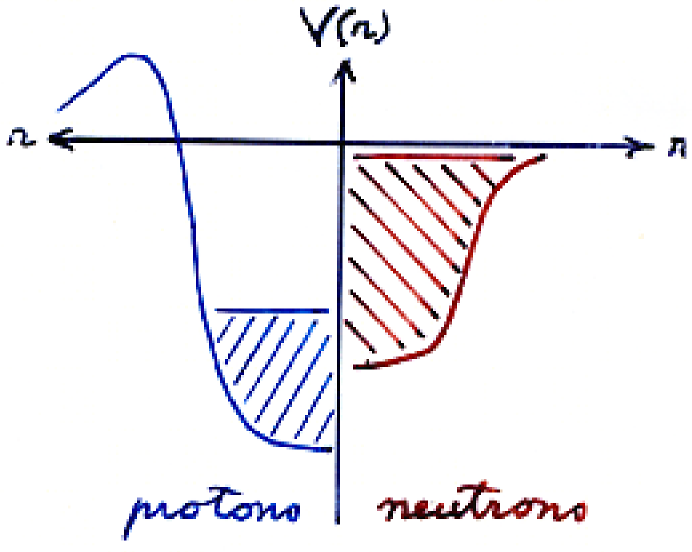
β-stable nuclei

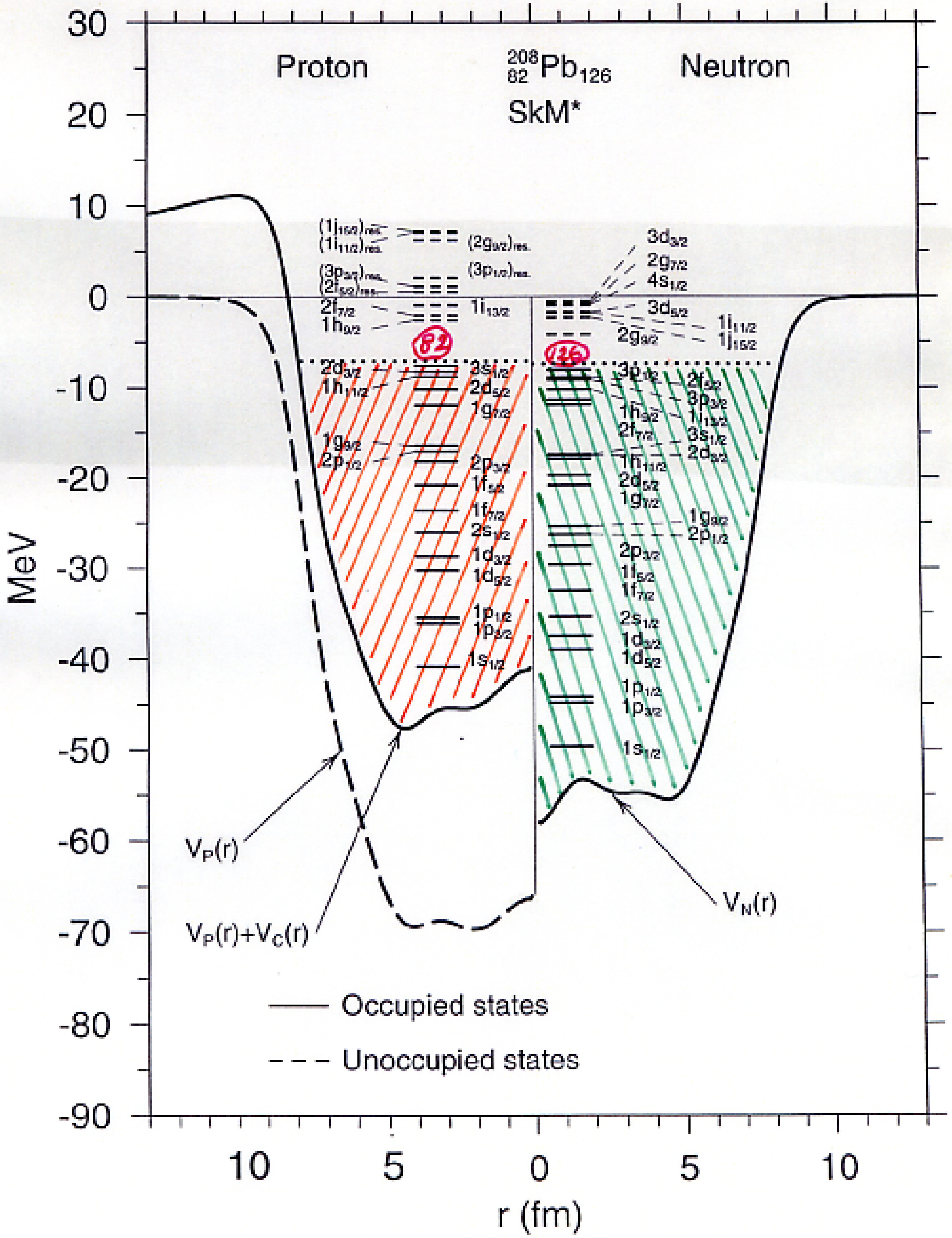


p drip line nuclei

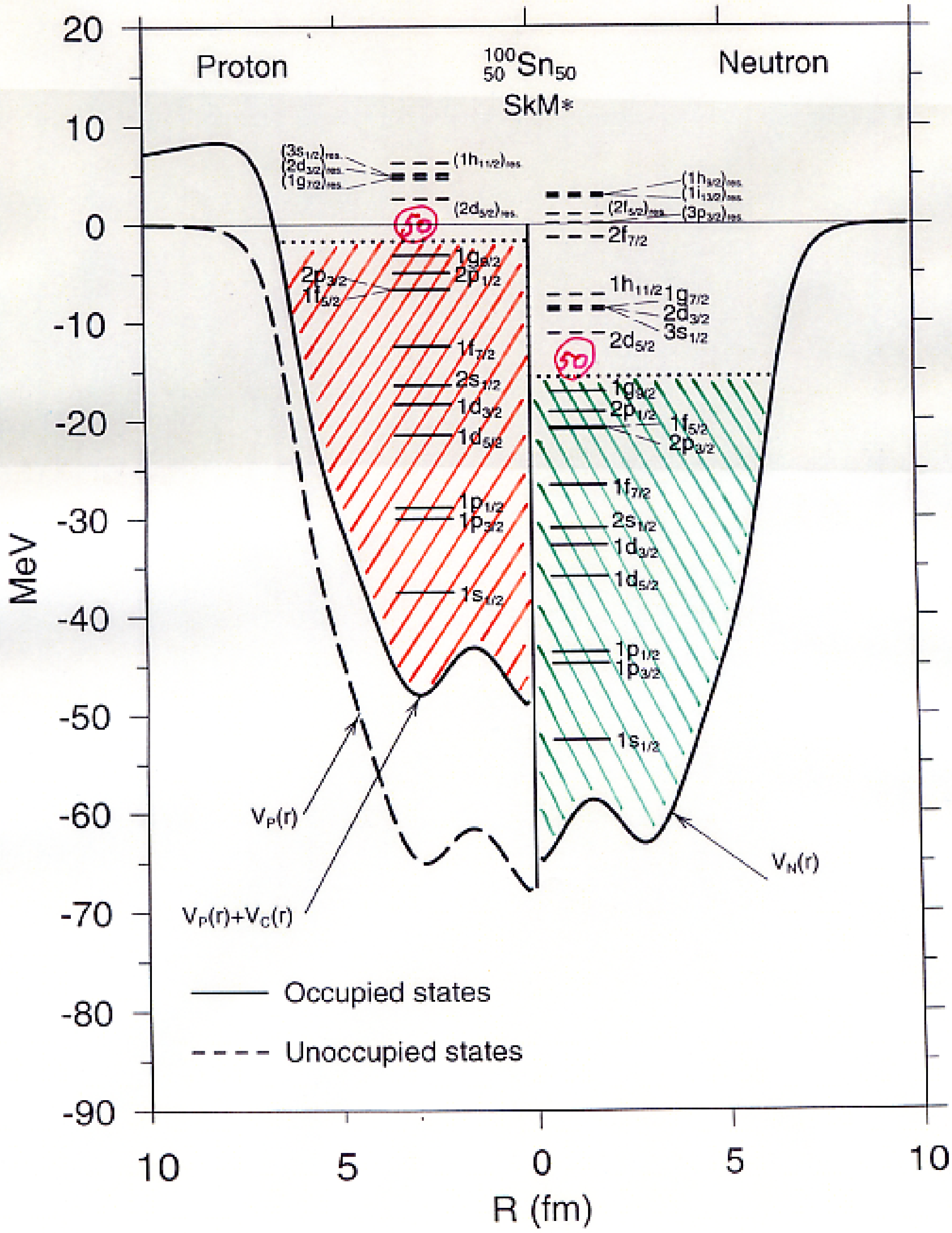


n drip line nuclei

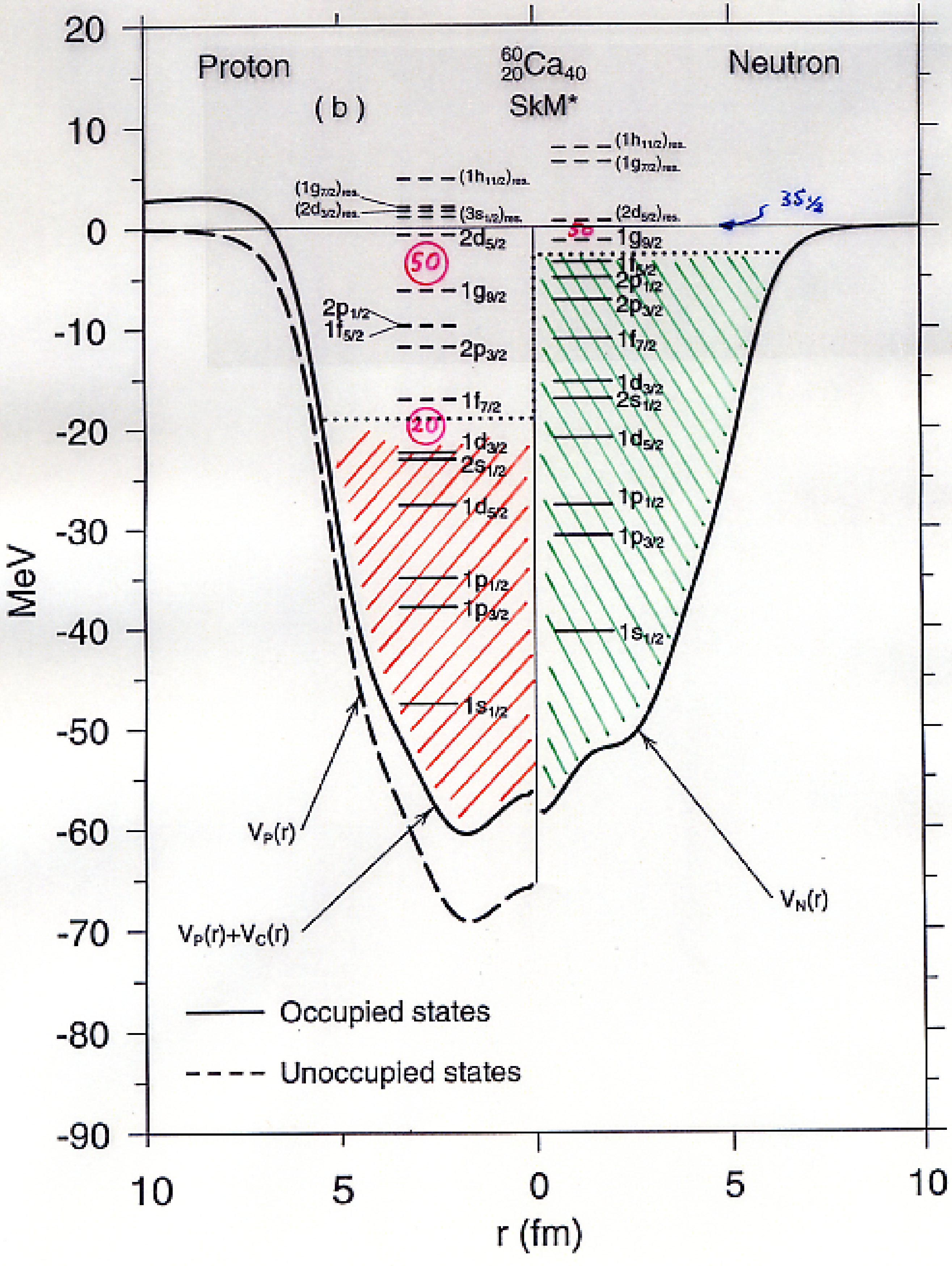




Proton
$^{208}_{82}Pb_{126}$
SkM*
Neutron
MeV
r (fm)
82
126
$V_P(r)$
$V_P(r)+V_C(r)$
$V_N(r)$
Occupied states
Unoccupied states
$(1j_{15/2})_{res.}$
$(1i_{11/2})_{res.}$
$(2g_{9/2})_{res.}$
$(3p_{3/2})_{res.}$
$(3p_{1/2})_{res.}$
$(2f_{5/2})_{res.}$
$2f_{7/2}$
$1h_{9/2}$
$1i_{13/2}$
$2d_{3/2}$
$3s_{1/2}$
$1h_{11/2}$
$2d_{5/2}$
$1g_{7/2}$
$1g_{9/2}$
$2p_{1/2}$
$2p_{3/2}$
$1f_{5/2}$
$1f_{7/2}$
$2s_{1/2}$
$1d_{3/2}$
$1d_{5/2}$
$1p_{1/2}$
$1p_{3/2}$
$1s_{1/2}$
$3d_{3/2}$
$2g_{7/2}$
$4s_{1/2}$
$3d_{5/2}$
$1j_{11/2}$
$2g_{9/2}$
$1j_{15/2}$
$3p_{1/2}$
$2f_{5/2}$
$3p_{3/2}$
$1h_{9/2}$
$1i_{13/2}$
$2f_{7/2}$
$3s_{1/2}$
$2d_{3/2}$
$1h_{11/2}$
$2d_{5/2}$
$1g_{7/2}$
$1g_{9/2}$
$2p_{1/2}$
$2p_{3/2}$
$1f_{5/2}$
$1f_{7/2}$
$2s_{1/2}$
$1d_{3/2}$
$1d_{5/2}$
$1p_{1/2}$
$1p_{3/2}$
$1s_{1/2}$



Proton
$^{100}_{50}Sn_{50}$
SkM*
Neutron
MeV
R (fm)
$(3s_{1/2})_{res.}$
$(2d_{3/2})_{res.}$
$(1g_{7/2})_{res.}$
$(1h_{11/2})_{res.}$
$(2d_{5/2})_{res.}$
50
$1g_{9/2}$
$2p_{1/2}$
$2p_{3/2}$
$1f_{5/2}$
$1f_{7/2}$
$2s_{1/2}$
$1d_{3/2}$
$1d_{5/2}$
$1p_{1/2}$
$1p_{3/2}$
$1s_{1/2}$
$(1h_{9/2})_{res.}$
$(1i_{13/2})_{res.}$
$(2f_{5/2})_{res.}$
$(3p_{3/2})_{res.}$
$2f_{7/2}$
$1h_{11/2}$
$1g_{7/2}$
$2d_{3/2}$
$3s_{1/2}$
$2d_{5/2}$
50
$1g_{9/2}$
$2p_{1/2}$
$1f_{5/2}$
$2p_{3/2}$
$1f_{7/2}$
$2s_{1/2}$
$1d_{3/2}$
$1d_{5/2}$
$1p_{1/2}$
$1p_{3/2}$
$1s_{1/2}$
$V_P(r)$
$V_P(r)+V_C(r)$
$V_N(r)$
Occupied states
Unoccupied states



Proton
(b)
60 20 Ca 40
SkM*
Neutron
(1h11/2)res.
(1g7/2)res.
(2d3/2)res.
(3s1/2)res.
2d5/2
1g9/2
2p1/2
1f5/2
2p3/2
1f7/2
1d3/2
2s1/2
1d5/2
1p1/2
1p3/2
1s1/2
(2d5/2)res.
1f5/2
50
20
VP(r)
VP(r)+VC(r)
VN(r)
MeV
r (fm)
Occupied states
Unoccupied states

Effect of neutron excess

ex.

Keeping $Z=50$, $N=50 \rightarrow$ larger

i.e.

$$\underbrace{{}^{100}_{50}Sn_{50}}_{\substack{\| \\ \text{proton drip} \\ \text{line nucleus}}} \longrightarrow \underbrace{{}^{132}_{50}Sn_{82}}_{\substack{\| \\ \text{neutron-rich nucleus,} \\ \text{spectroscopy is experimentally possible.}}} \longrightarrow {}^{150}_{50}Sn_{100} \longrightarrow$$

$$\begin{cases} V_p(r) \longrightarrow \text{deeper}, \quad \rho_p(r=0) \longrightarrow \text{smaller}, \quad R_p \longrightarrow \text{larger}, \\ \rho_n(r=0) \longrightarrow \text{larger}, \quad R_n \longrightarrow \text{larger}, \quad a_n \longrightarrow \text{larger}, \end{cases}$$

Dobaczewski, Hamamoto, Nazarewicz and Sheikh, Acta. Phys. Pol. B25 (1994) 541.

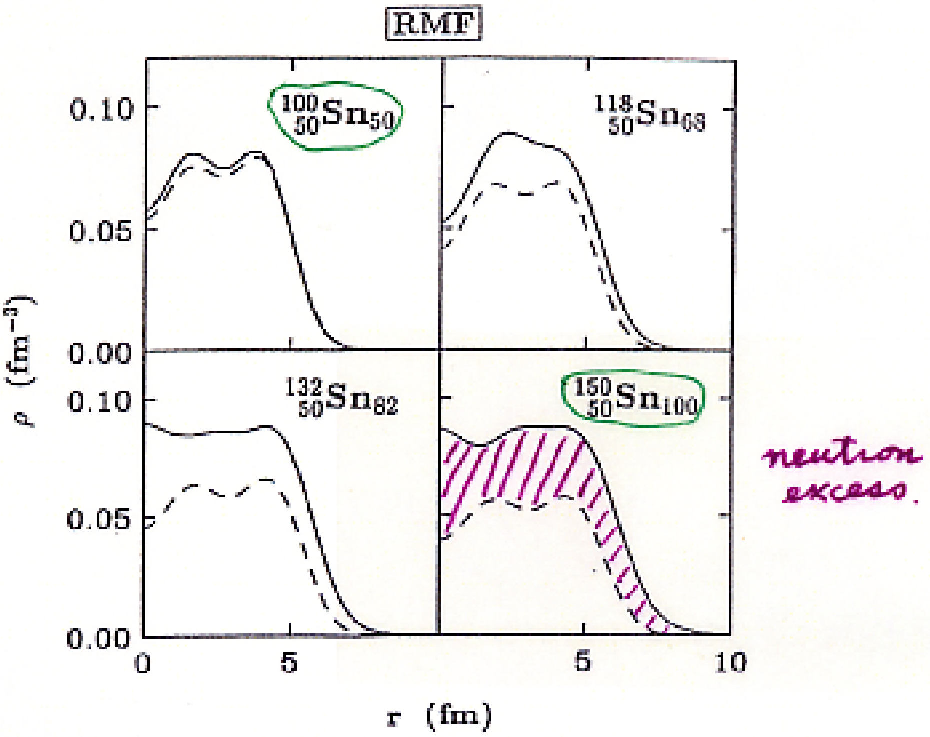
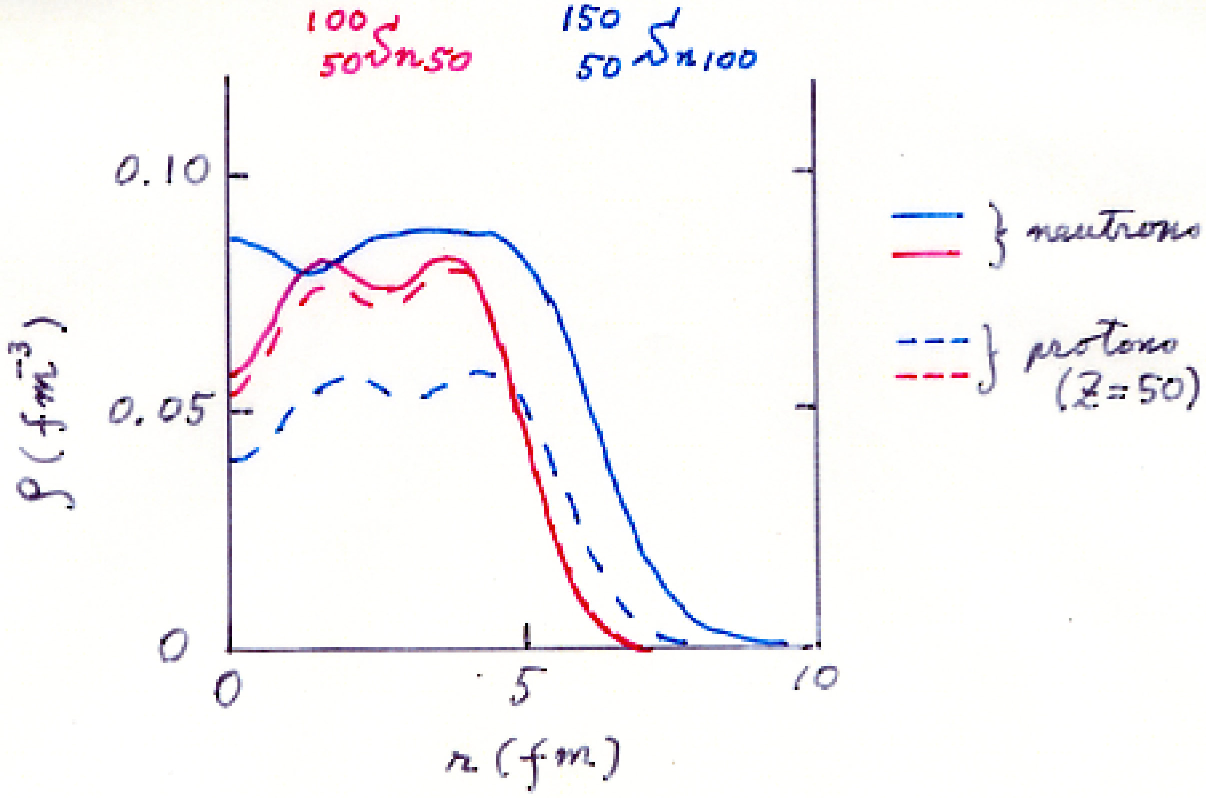
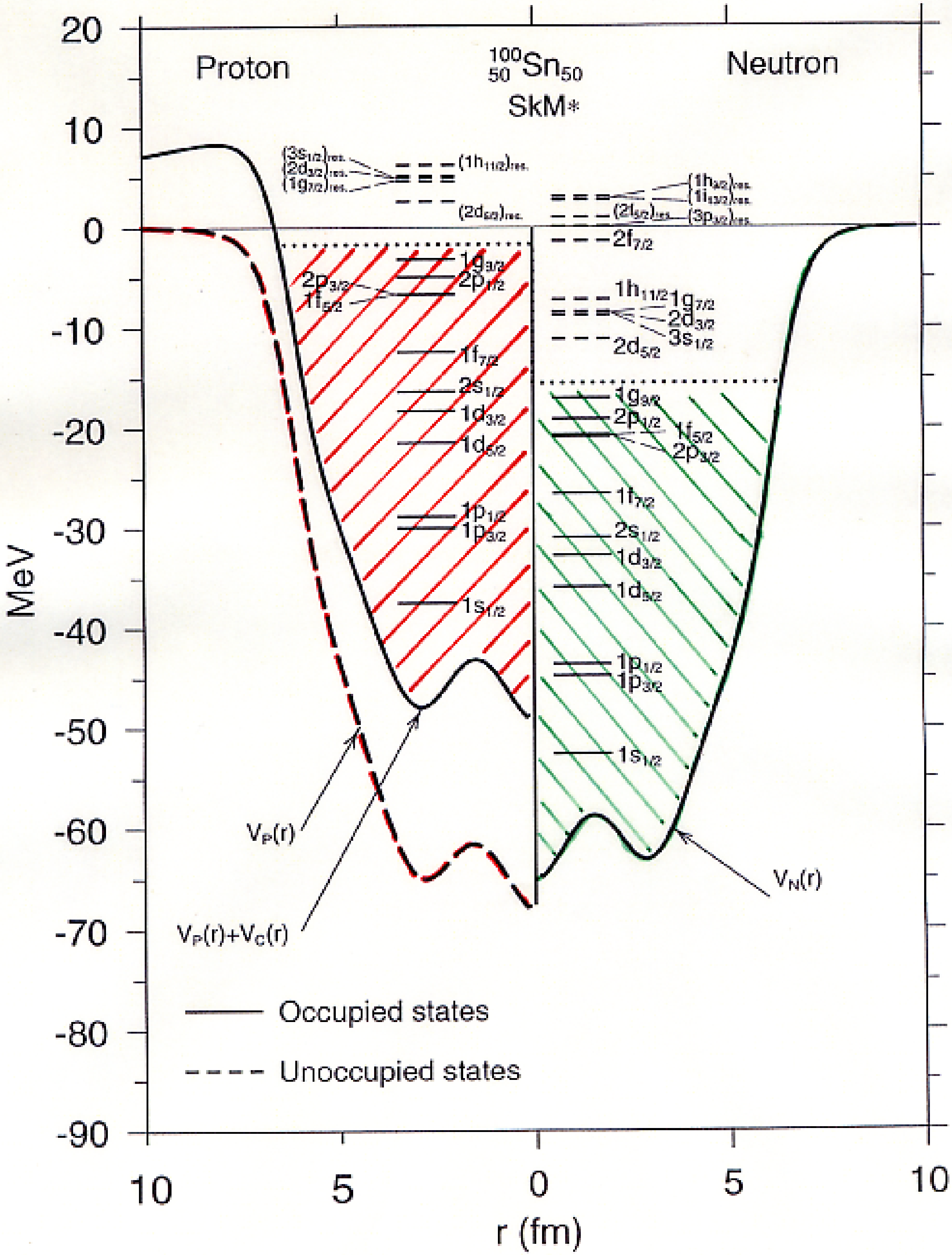
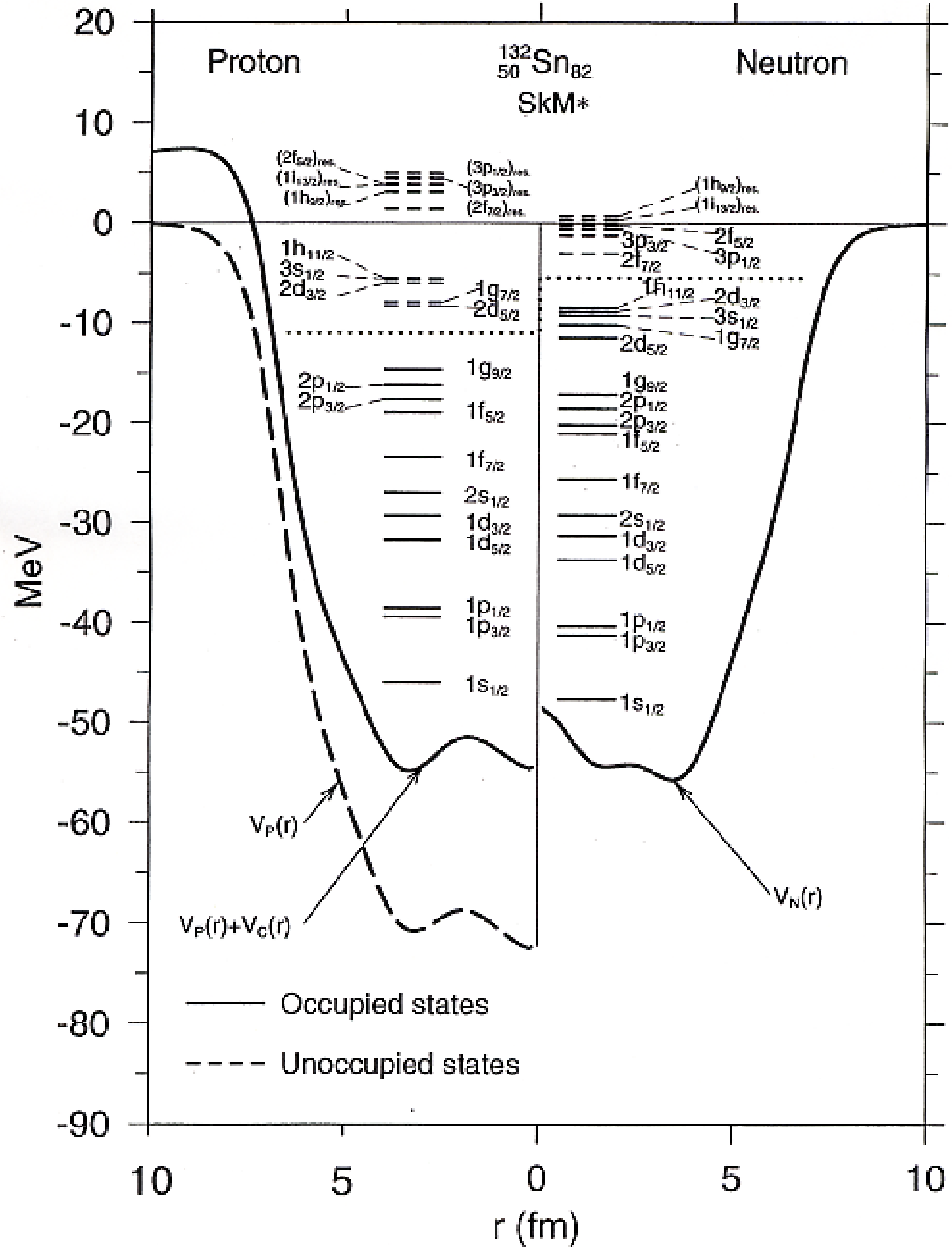


Fig. 3. Single-particle nucleonic densities of the NL-SH RMF model for $^{100,118,132,150}Sn$ isotopes (neutrons: solid line, protons: dashed line).

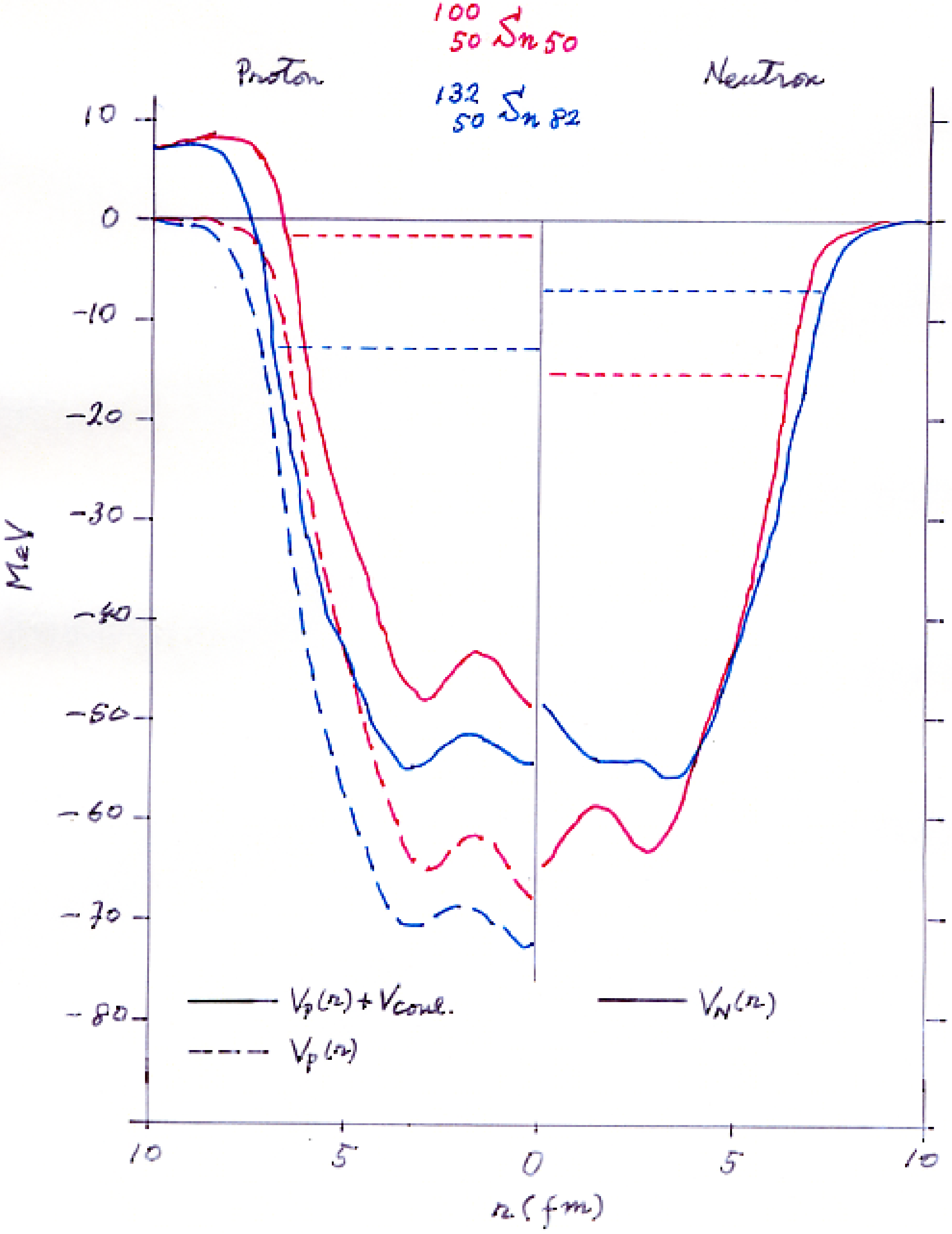




Proton
$^{100}_{50}Sn_{50}$
SkM*
Neutron
MeV
r (fm)
$(3s_{1/2})_{res.}$
$(2d_{3/2})_{res.}$
$(1g_{7/2})_{res.}$
$(1h_{11/2})_{res.}$
$(2d_{5/2})_{res.}$
$1g_{9/2}$
$2p_{1/2}$
$2p_{3/2}$
$1f_{5/2}$
$1f_{7/2}$
$2s_{1/2}$
$1d_{3/2}$
$1d_{5/2}$
$1p_{1/2}$
$1p_{3/2}$
$1s_{1/2}$
$(1h_{9/2})_{res.}$
$(1i_{13/2})_{res.}$
$(2f_{5/2})_{res.}$
$(3p_{3/2})_{res.}$
$2f_{7/2}$
$1h_{11/2}$
$1g_{7/2}$
$2d_{3/2}$
$3s_{1/2}$
$2d_{5/2}$
$1g_{9/2}$
$2p_{1/2}$
$1f_{5/2}$
$2p_{3/2}$
$1f_{7/2}$
$2s_{1/2}$
$1d_{3/2}$
$1d_{5/2}$
$1p_{1/2}$
$1p_{3/2}$
$1s_{1/2}$
$V_P(r)$
$V_P(r)+V_C(r)$
$V_N(r)$
Occupied states
Unoccupied states



Proton
$^{132}_{50}Sn_{82}$
SkM*
Neutron
MeV
r (fm)
$(2f_{5/2})_{res.}$
$(1i_{13/2})_{res.}$
$(1h_{9/2})_{res.}$
$(3p_{1/2})_{res.}$
$(3p_{3/2})_{res.}$
$(2f_{7/2})_{res.}$
$1h_{11/2}$
$3s_{1/2}$
$2d_{3/2}$
$1g_{7/2}$
$2d_{5/2}$
$1g_{9/2}$
$2p_{1/2}$
$2p_{3/2}$
$1f_{5/2}$
$1f_{7/2}$
$2s_{1/2}$
$1d_{3/2}$
$1d_{5/2}$
$1p_{1/2}$
$1p_{3/2}$
$1s_{1/2}$
$(1h_{9/2})_{res.}$
$(1i_{13/2})_{res.}$
$2f_{5/2}$
$3p_{3/2}$
$3p_{1/2}$
$2f_{7/2}$
$1h_{11/2}$
$2d_{3/2}$
$3s_{1/2}$
$1g_{7/2}$
$2d_{5/2}$
$1g_{9/2}$
$2p_{1/2}$
$2p_{3/2}$
$1f_{5/2}$
$1f_{7/2}$
$2s_{1/2}$
$1d_{3/2}$
$1d_{5/2}$
$1p_{1/2}$
$1p_{3/2}$
$1s_{1/2}$
$V_P(r)$
$V_P(r)+V_C(r)$
$V_N(r)$
Occupied states
Unoccupied states



$^{100}_{50}Sn_{50}$
$^{132}_{50}Sn_{82}$
Proton
Neutron
MeV
10
0
-10
-20
-30
-40
-50
-60
-70
-80
$V_p(r) + V_{coul.}$
$V_p(r)$
$V_N(r)$
10
5
0
5
10
r (fm)

Change of shell-structure, especially for $\varepsilon_{n\ell j} = -10 \to 0$ MeV

ex. neutron one-particle energies in Woods-Saxon potential.

Lower centrifugal barrier for particles with smaller ℓ

$\Longrightarrow$ wave functions can extend beyond the nucleus as $|\varepsilon_{n\ell j}| \to 0$.

$\Longrightarrow$ { kinetic energy decreases $\overset{\text{virial theorem}}{\propto} \langle r \frac{dU(r)}{dr} \rangle$

spin-orbit splitting decreases $\propto \langle \frac{1}{r} \frac{dU(r)}{dr} \rangle$

$\Longrightarrow$ Shell structure may change drastically for $|\varepsilon_{n\ell j}| \to 0$.

For nuclear potentials with realistic diffuseness the drastic change of shell-structure occurs for

$$\varepsilon_{n\ell j} \approx \underbrace{\boxed{-10 \text{ MeV}}}_{\parallel} \to 0$$

S_n or S_p for β-stable nuclei.

Recent exp. data on light nuclei towards n-drip-line show: magic numbers 8 and 20 disappear, while 16 is a new magic number.

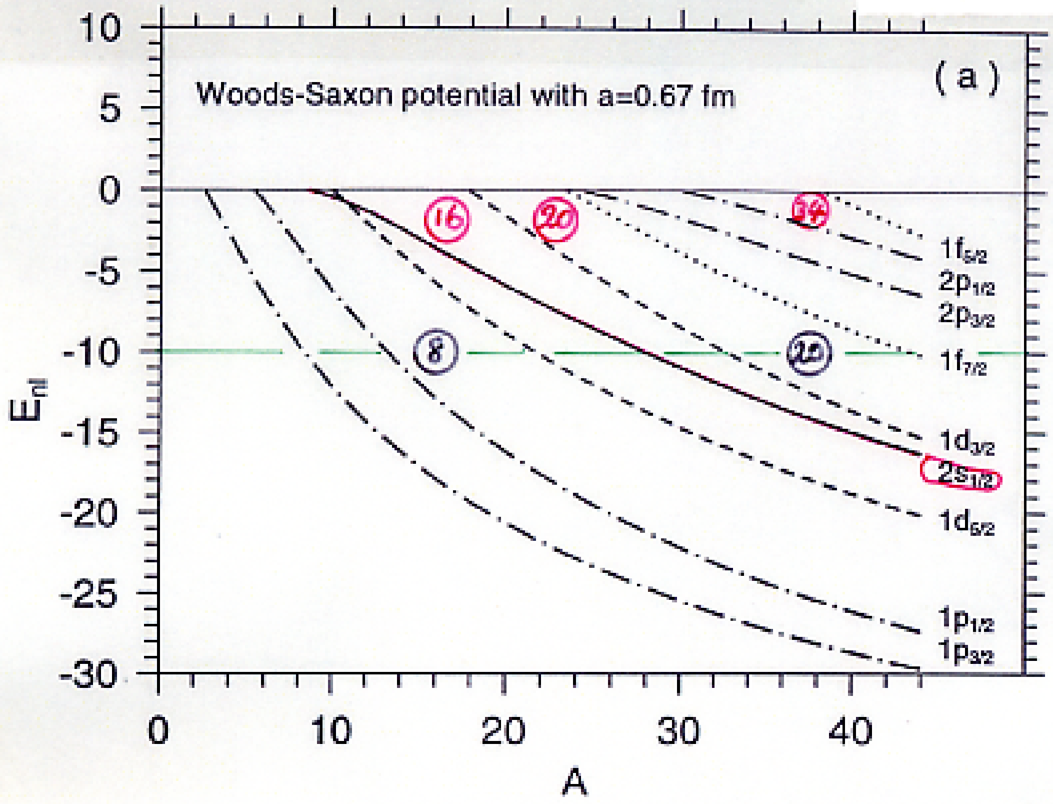
References:

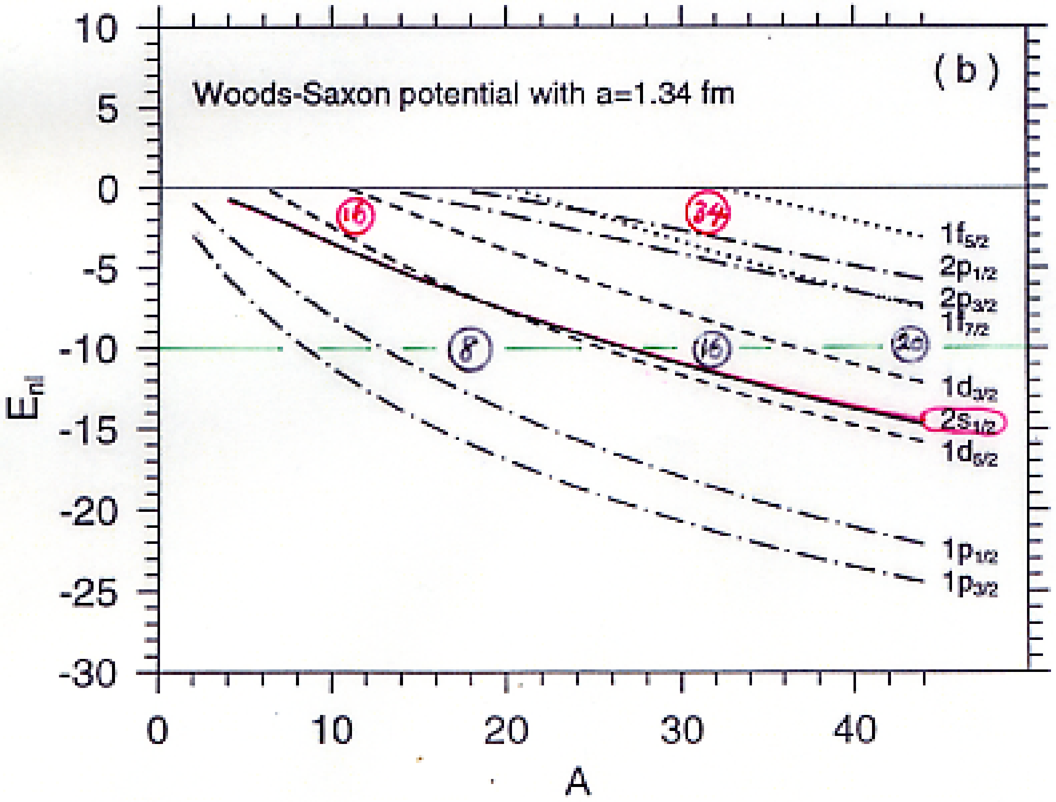
$^{32}_{12}Mg_{20}$ T. Motobayashi et al, Phys. Lett. B346 (1995) 9.
D. Guillemaud-Mueller et al, Nucl. Phys. A426 (1984) 37.

$^{12}_{4}Be_{8}$ H. Iwasaki et al, Phys. Lett. B481 (2000) 7; Phys. Lett. B491 (2000) 8.

N=16 A. Ozawa et al, Phys. Rev. Lett. 84 (2000) 5493.

Neutron one-particle energies

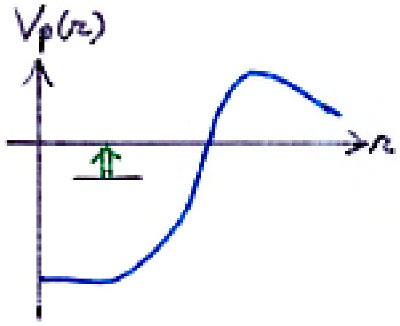




figs taken from
I. Hamamoto, S.V. Lukyanov and X. Z. Zhang,
Nucl. Phys. A683 (2001) 255.

Close to p-drip-line :

In medium-heavy nuclei the large Coulomb barrier hinders wave-functions from extending beyond the nucleus.



$N \approx Z$ nuclei with $A \approx 80$

ex. $^{80}_{40}Zr_{40}$ is deformed.

OBS.
All nuclei with N (or Z) $= 40$ around β-stability line are spherical.

Deformation

Some nuclei are deformed in the ground states.

ex. ${}^{12}_{6}C_6$, ${}^{20}_{10}Ne_{10}$, rare-earth ($90 \leq N \leq 112$) nuclei , ...,

1) Jahn-Teller effect

In the presence of many particles outside of closed-shell in spherical potentials, [the degeneracy in quantum spectra]

$\longrightarrow$ possibility of gaining energy by breaking away from spherical symmetry.

ex. medium-size deformation of ground states of rare-earth nuclei.

2.) New shell-structure (and new magic numbers) at large deformation.

ex. superdeformation $\left\{ \begin{array}{l} \text{fission isomers in actinide nuclei.} \\ \text{high-spin states in rare-earth nuclei.} \\ \text{..........} \end{array} \right.$

strongly deformed bands in closed-shell (of spherical potentials) nuclei.

$\left\{ \begin{array}{l} \text{excited states in } {}^{40}_{20}Ca_{20} \\ \text{ground states of } N=Z \text{ nuclei around } A \sim 76. \end{array} \right.$

${}^{72}_{36}Kr_{36}$, ${}^{76}_{38}Sr_{38}$, ${}^{80}_{40}Zr_{40}$, Anisotropic oscillator potential

$$V = \frac{1}{2}M\left(\omega_3^2 x_3^2 + \omega_\perp^2 (x_1^2 + x_2^2)\right),$$

$$\varepsilon(n_3\, n_\perp) = (n_3 + \tfrac{1}{2})\hbar\omega_3 + (n_\perp + 1)\hbar\omega_\perp$$

where $n_3, n_\perp = 0, 1, 2, 3, \ldots\ldots$

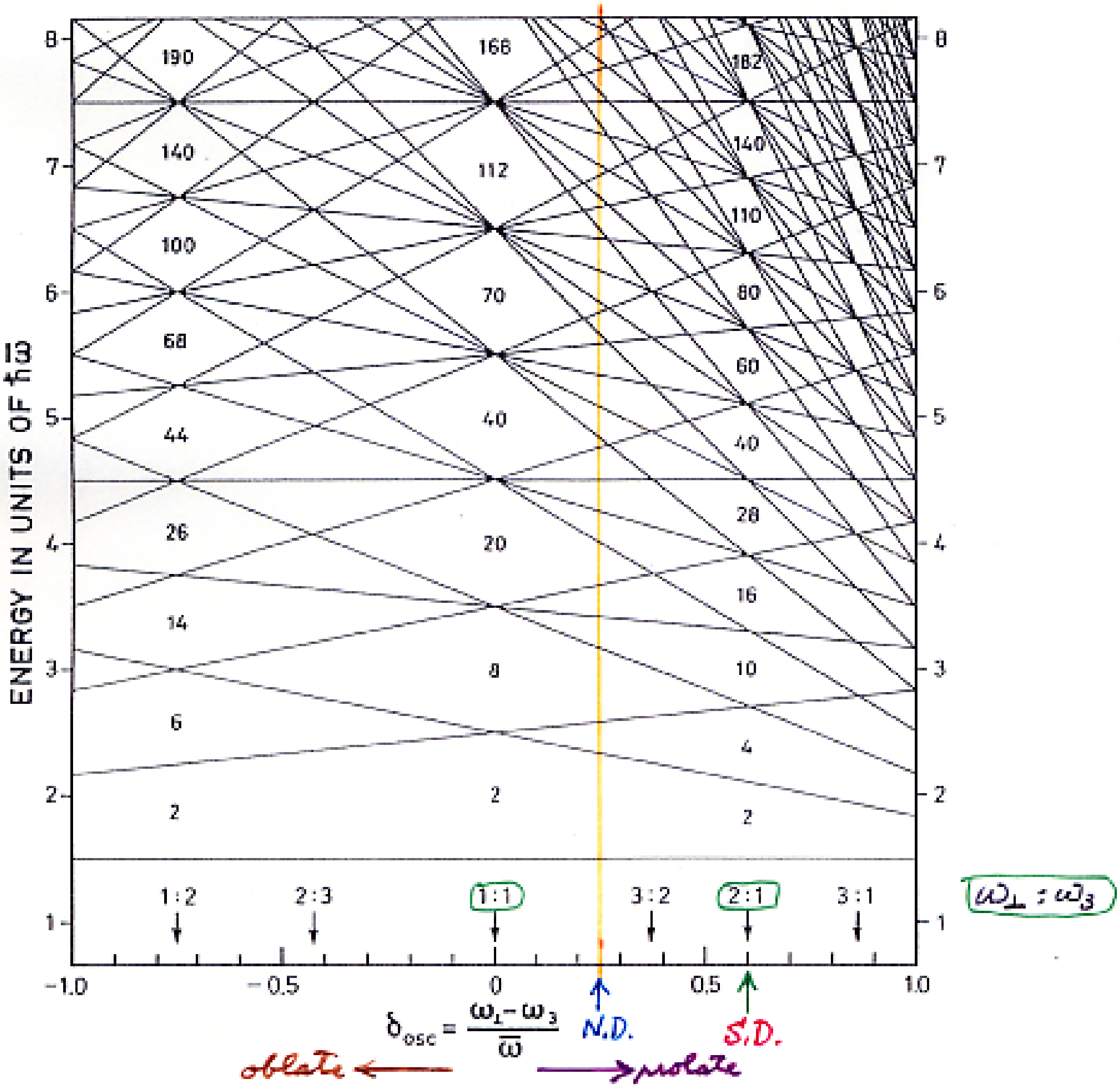


Figure 6-48 Single-particle spectrum for axially symmetric harmonic oscillator potentials. The eigenvalues are measured in units of $\overline{\omega} = (2\omega_\perp + \omega_3)/3$, and the deformation parameter δ_{osc} is that defined by Eq. (5-11). The arrows mark the deformations corresponding to the indicated rational ratios of frequencies $\omega_\perp : \omega_3$.

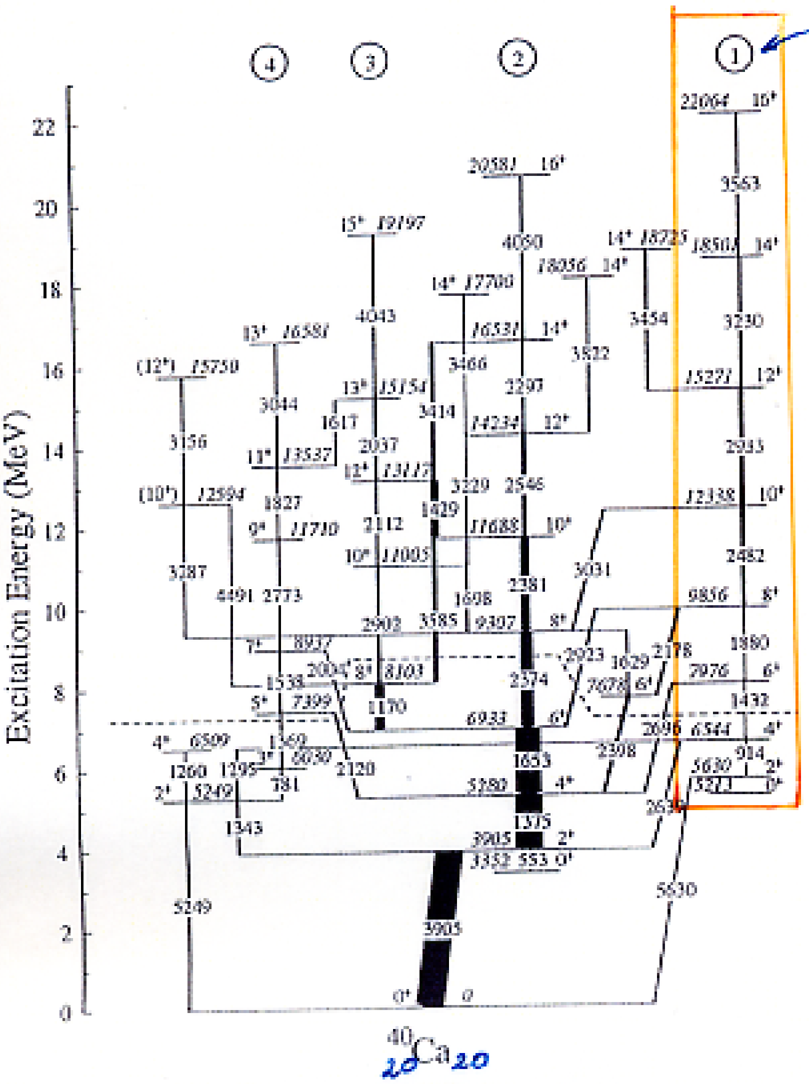
A. Bohr and B.R. Mottelson, Nuclear Structure, vol II.

Strongly Deformed band.

$\left[Q_t = 1.80^{+0.39}_{-0.29}\ eb \right.$

from Doppler shift measurement. $\left. \right]$

$\Rightarrow \beta_2 = 0.59^{+0.11}_{-0.07}$



E. Ideguchi (井出口) et al.,
Phys. Rev. Lett. 87 (2001) 222501

FIG. 1. Partial level scheme of ^{40}Ca; the energy labels are given in keV, and the widths of the arrows are proportional to the relative intensities of the γ rays. Only the levels below the dashed line were known prior to this work.

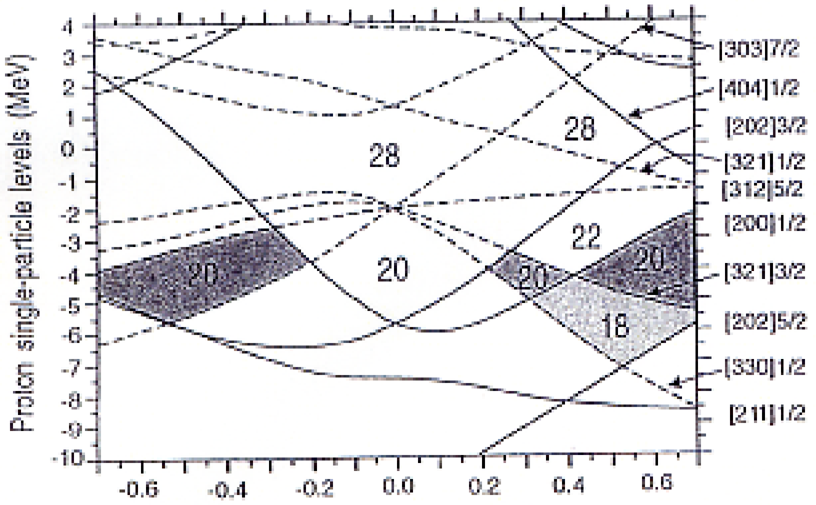
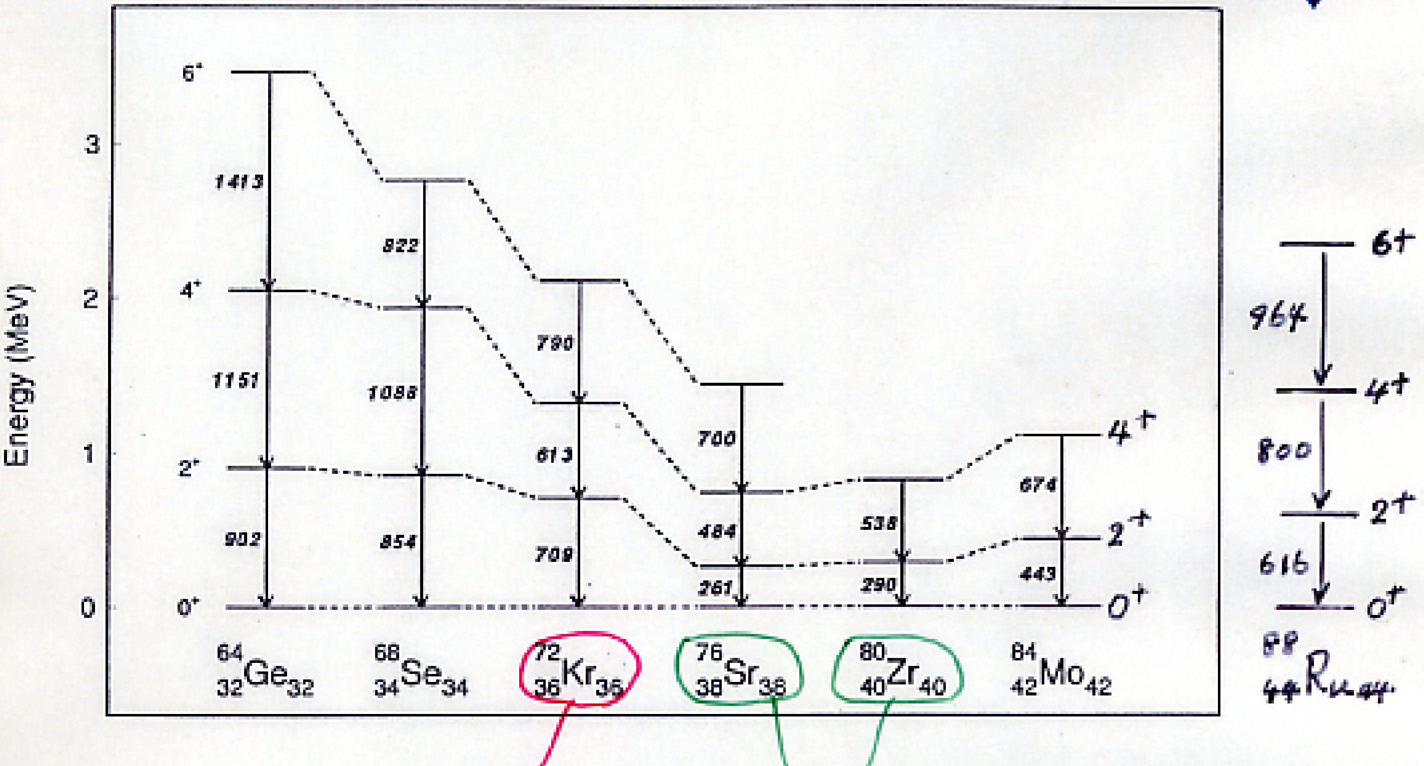


FIG. 4. Woods-Saxon orbitals as a function of the deformation, β_2, for particle numbers near $N = Z = 20$. The solid and dashed lines are for positive and negative parity orbitals, respectively. Here $\beta_4 = 0$ was assumed. The dark shaded areas indicate the gaps responsible for the larger deformation of band 1 and the smaller deformation of band 2 in ^{40}Ca.

from $^{58}Ni(^{32}S, 2n)^{88}Ru$ at GASP

N = Z nuclei



The evolution of the yrast levels in $N=Z$ even-even nuclei.

Oblate? Prolate?

Fig.10.4 Observed excitation spectra of even-even N=Z nuclei. The energy differences, $E(2^+)-E(0^+)$, $E(4^+)-E(2^+)$ and $E(6^+)-E(4^+)$, are expressed in units of keV. The figure is taken from D.Bucurescu et al., Phys.Rev.C56(1997) 2497.

$^{80}_{40}Zr_{40}$ is deformed.

[OBS. all nuclei with N (or Z) = 40 along β-stability line are spherical.]

One-particle motion in deformed potential — Nilsson diagram.

Spectroscopy of deformed nuclei is much simpler than that of spherically-vibrating nuclei, since the major part of two-body interaction is already taken into account in the deformation.

$\boxed{\text{Nilsson diagram}}$ for axially-symmetric quadrupole deformation:

One-particle orbitals are named by asymptotic quantum numbers $[N\, n_z\, \Lambda\, \Omega]$.

$\Omega (\Leftarrow j_z)$: always good quantum number.

$N,\ n_z,\ \Lambda (\Leftarrow l_z)$: become good quantum number for large β_2.

Each orbital has double degeneracy with $\pm\Omega$.

Low-lying states in deformed odd-A nuclei may well be interpreted in terms of one-particle

$[N\, n_z\, \Lambda\, \Omega]$ orbitals, where $\Omega = K\ (\Leftarrow I_z)$,

together with rotational spectra

$$E(I) = A\left[I(I+1) + a\,(-1)^{I+1/2}\,(I+\tfrac{1}{2})\cdot\delta(K,\tfrac{1}{2}) \right]$$

$$+ \text{------}$$

decoupling parameter, which depends on $(N,\ n_z,\ \Lambda)$.

[home work]

Try to understand experimental information on ${}^{33}_{12}Mg_{21}$, using Nilsson diagram.

deformed nucleus.

Slopes are determined by (N, n_z).

Selection rules in $[N\, n_z\, \Lambda\, \Omega]$ for various operators.

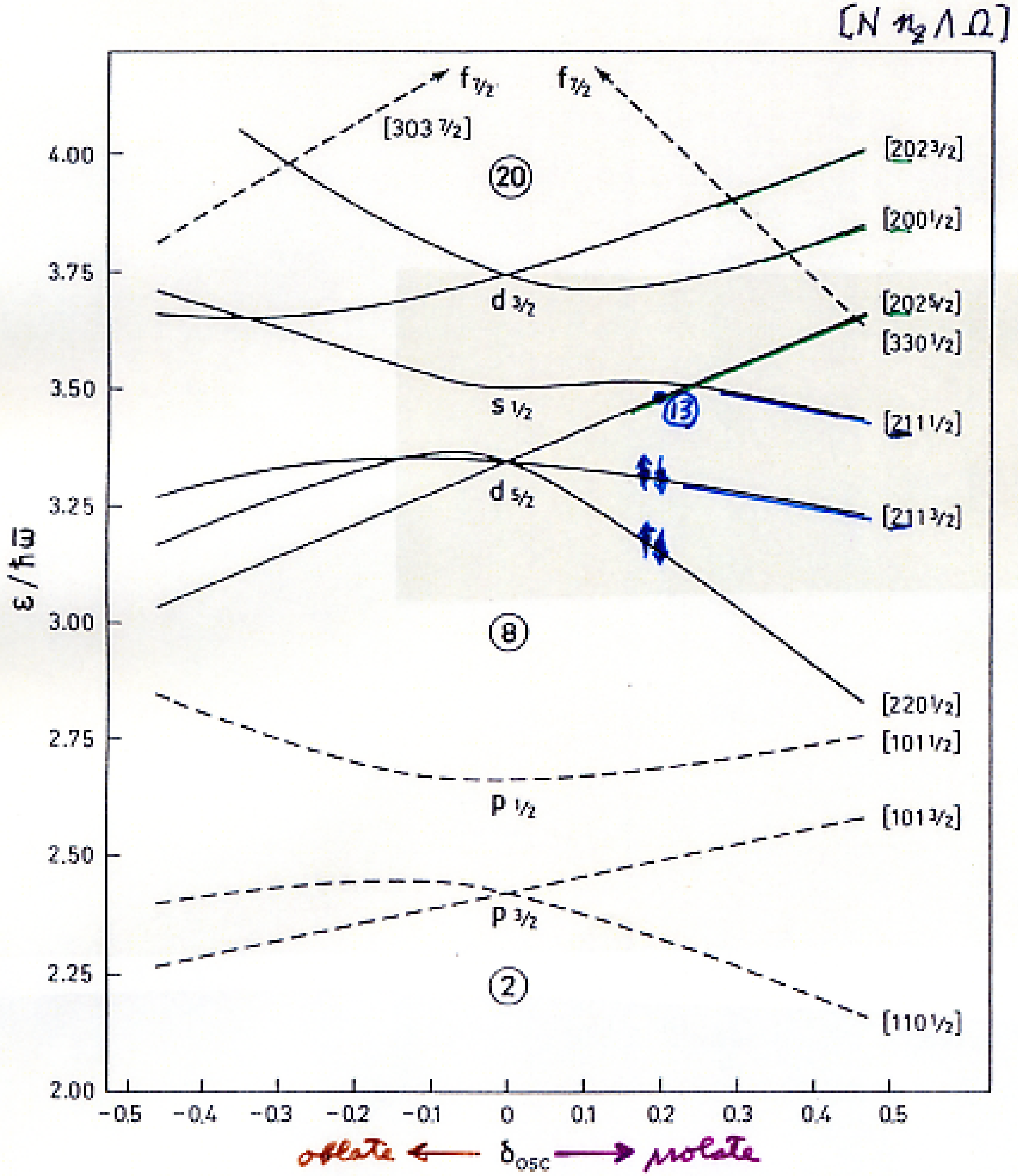


Figure 5-1 Spectrum of single-particle orbits in spheroidal potential (N and $Z < 20$). The spectrum is taken from B. R. Mottelson and S. G. Nilsson, *Mat. Fys. Skr. Dan. Vid. Selsk.* **1**, no. 8 (1959). The orbits are labeled by the asymptotic quantum numbers $[Nn_3\Lambda\Omega]$ referring to large prolate deformations. Levels with even and odd parity are drawn with solid and dashed lines, respectively.

[A. Bohr and B. R. Mottelson, Nuclear Structure, vol II.]

A. Bohr and B.R. Mottelson, Nuclear Structure, vol. II.

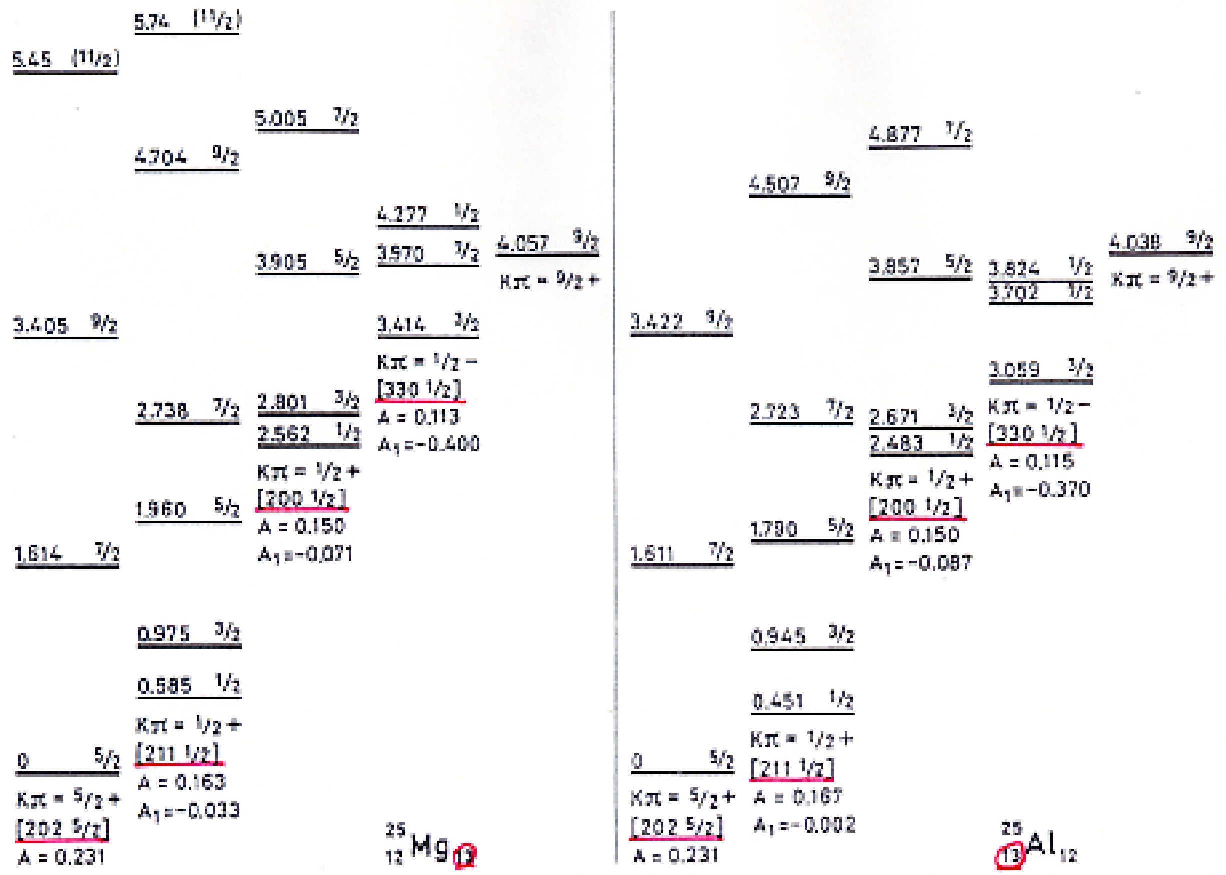


Figure 5-15 Spectra of ^{25}Mg and ^{25}Al. The recognition of rotational band structure in the $A = 25$ system followed from the extensive series of experiments at Chalk River (see the survey by A. E. Litherland, H. McManus, E. B. Paul, D. A. Bromley, and H. E. Gove, *Can. J. Phys.* 36, 378, 1958). The figure is based on the experimental data summarized by Endt and van der Leun, 1967, and by Litherland, 1968. The tentative assignment of $I = 11/2$ for the 5.45 and 5.74 MeV states in ^{25}Mg is based on the data of S. Hinds, R. Middleton, and A. E. Litherland, *Proceedings of the Rutherford Jubilee Intern. Conf.*, p. 305, ed. J. B. Birks, Heywood and Co., London 1961. Energies are in units of MeV.

The 13th nucleon occupies the orbital $[N\, n_z\, \Lambda\, \Omega]$.

§ Collective modes $\quad\quad \mathcal{O}|0\rangle$

(The best collective motion in nuclei is rotation of deformed nuclei.

Giant Resonances ($\boxed{}$: experimentally well-established)

1) Charge-exchange ($t_\pm$) modes.

	operator	I^π of the operator
$\boxed{\text{Fermi (IAS)}}$	$\sum_i t_\pm(i)$	0^+
$\boxed{\text{Gamow-Teller (GTGR)}}$	$\sum_i t_\pm(i)\,\sigma_\mu(i)$	1^+
Spin dipole (SDR)	$\sum_i t_\pm(i)\, r_i (Y_1\sigma)^I_\mu$	$0^-, 1^-, 2^-$
Spin monopole* (SMR)	$\sum_i t_\pm(i)\, r_i^2\, \sigma_\mu(i)$	1^+

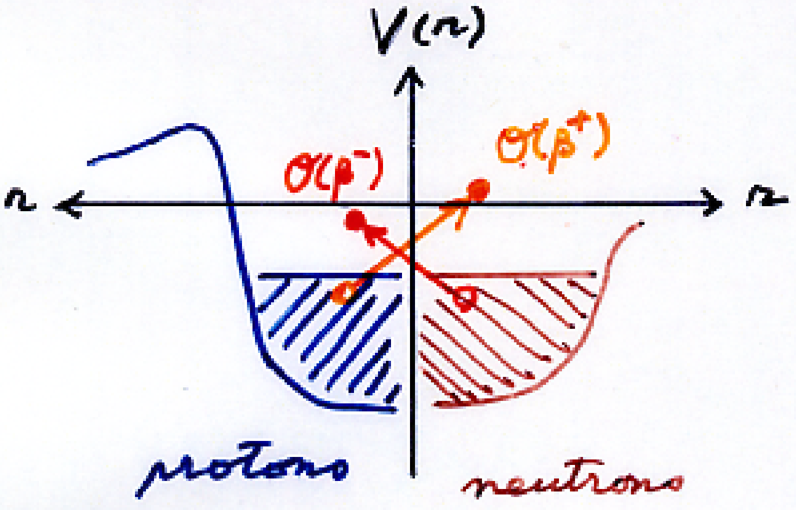
2) (1 or t_z) modes

1 ↑ IS, t_z ↑ IV

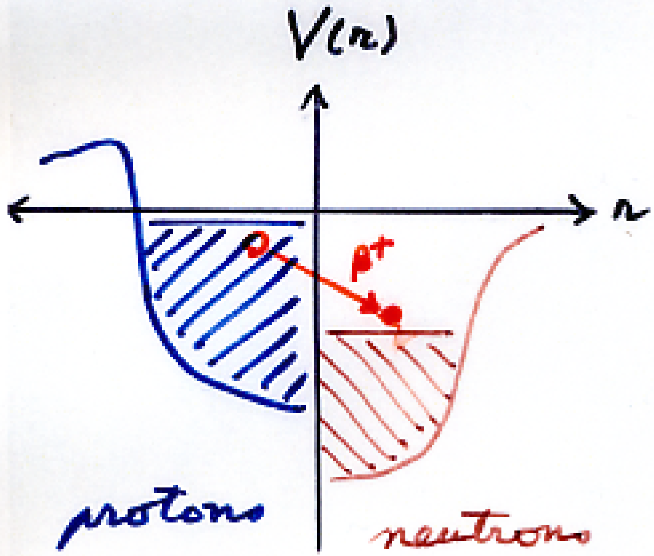
	operator	I^π of the operator	Observed energy for $A \gtrsim 60$
$\boxed{\text{IV dipole (IVGDR)}}$	$\sum_i t_z(i)\, r_i\, Y_{1\mu}(i)$	1^-	$79\, A^{-1/3}$ MeV
$\boxed{\text{IS quadrupole (ISGQR)}}$	$\sum_i r_i^2\, Y_{2\mu}(i)$	2^+	$58\, A^{-1/3}$ MeV
$\boxed{\text{IS monopole* (ISGMR)}}$	$\sum_i r_i^2$	0^+	$80\, A^{-1/3}$ MeV
IV quadrupole (IVGQR)	$\sum_i t_z(i)\, r_i^2\, Y_{2\mu}(i)$	2^+	
IS dipole* (ISGDR)	$\sum_i r_i^3\, Y_{1\mu}(i)$	1^-	

* : compression mode

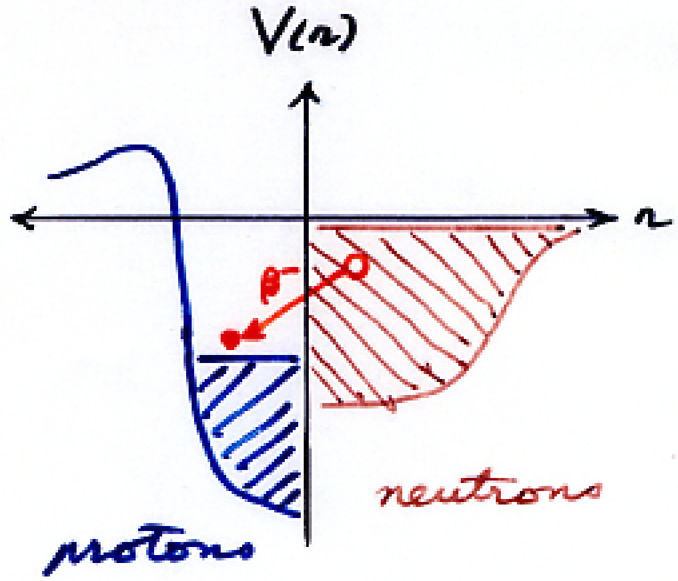
β-stable nuclei



p drip line nuclei



n drip line nuclei



§ Giant resonances and low-energy threshold strength.

Quadrupole mode

$$Q = \sum_i r_i^2 Y_{2\mu}(i) \qquad \text{for isoscalar}$$

$$Q = \sum_i \tau_z(i)\, r_i^2 Y_{2\mu}(i) \qquad \text{for isovector}$$

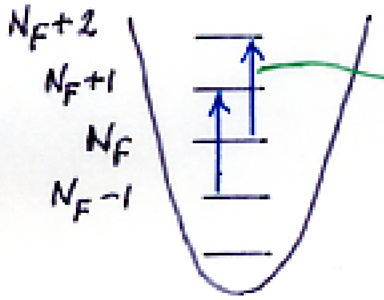
---

ex. ℓ-s closed-shell nuclei, ${}^{40}_{20}\mathrm{Ca}_{20}$ & ${}^{60}_{20}\mathrm{Ca}_{40}$:

In the harmonic-oscillator model,

unperturbed p-h excitations at $2\hbar\omega_0$

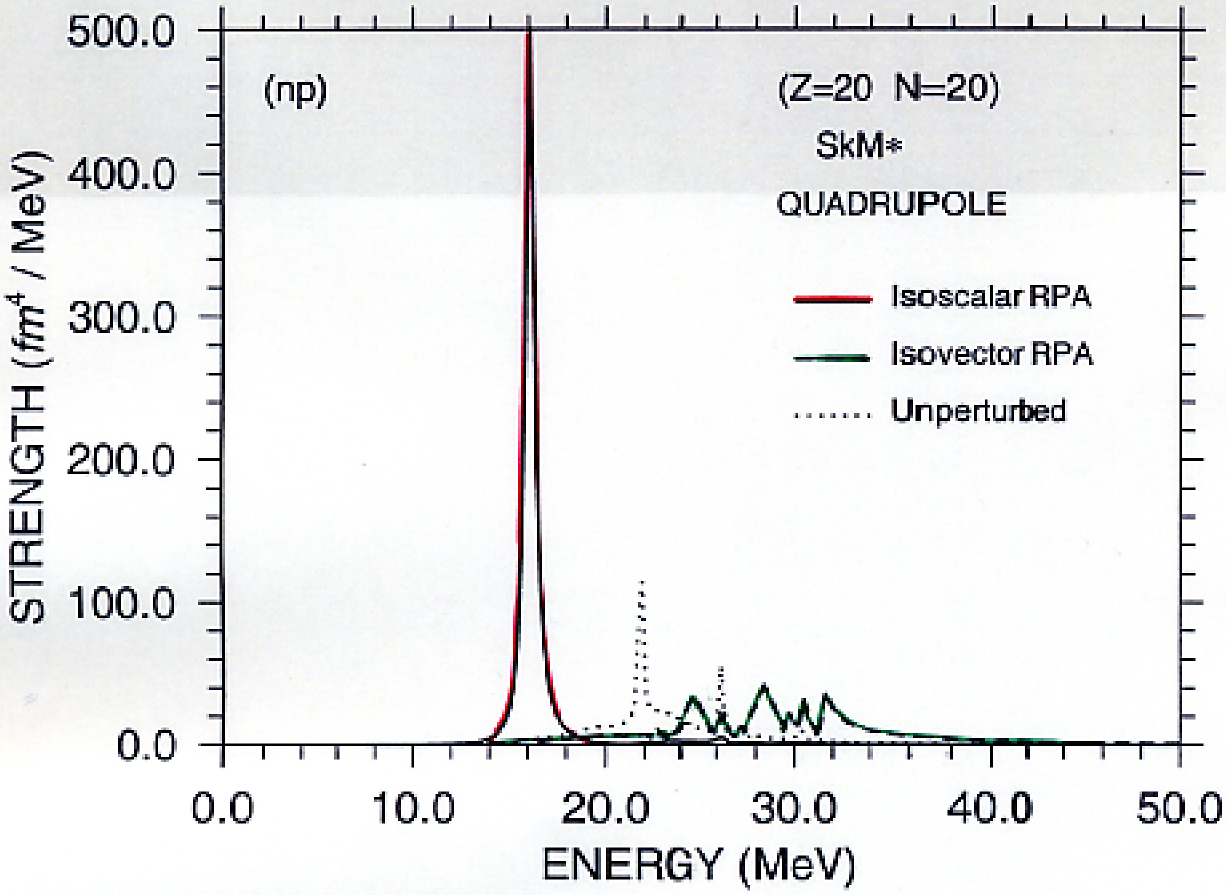
(no low-lying excitations !)

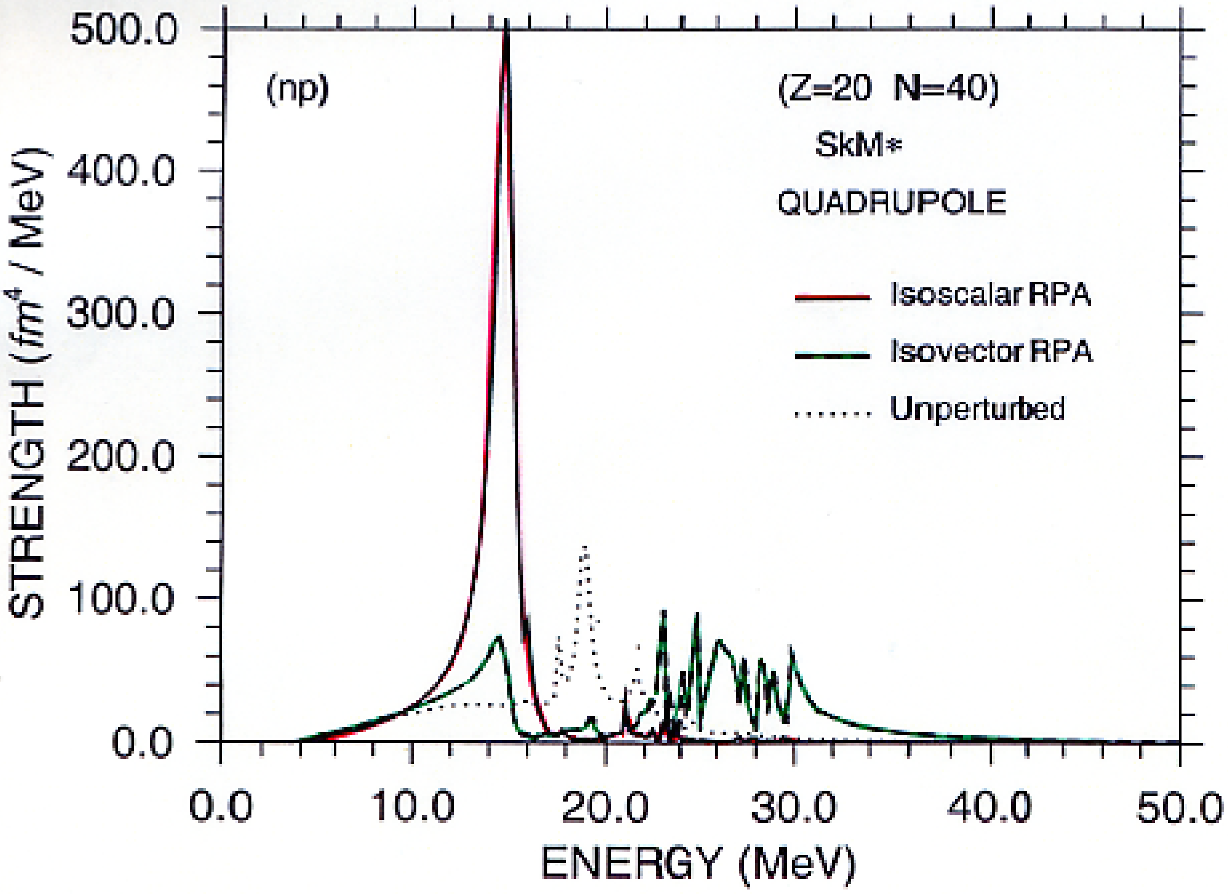


$2\hbar\omega_0$ for both n and p.

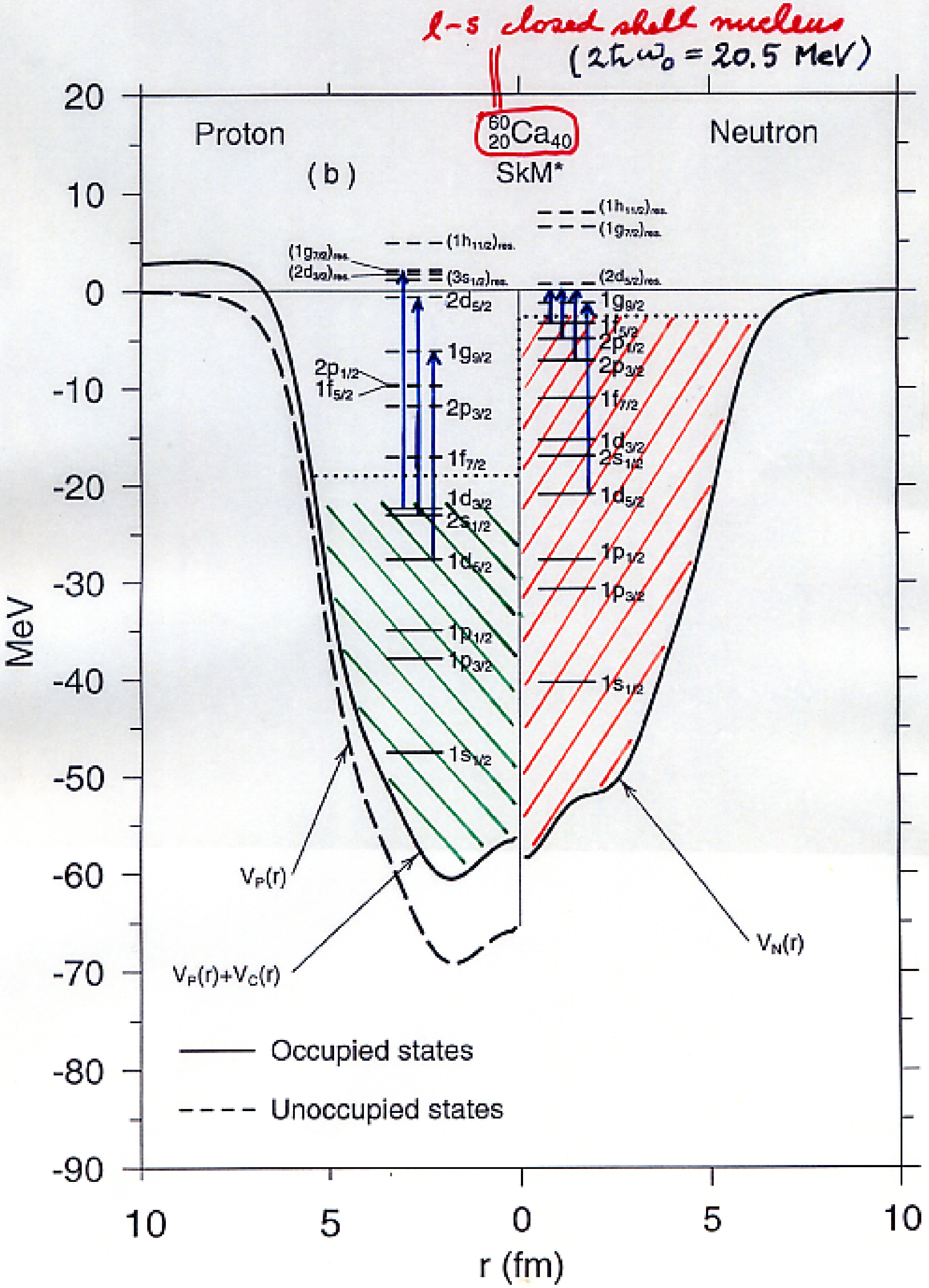
Quadrupole (2^+) excitation

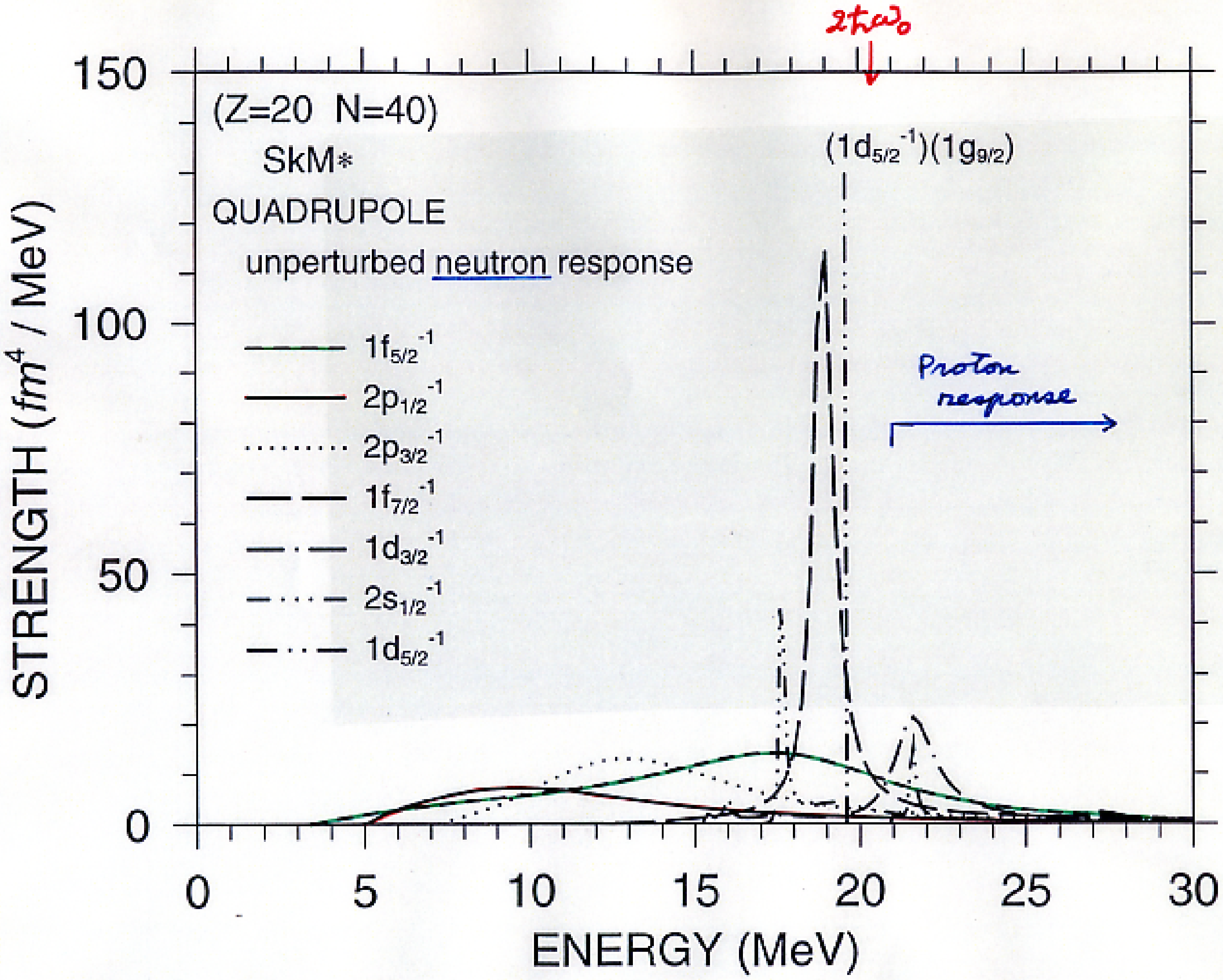
self-consistent HF+RPA with Skyrme interaction





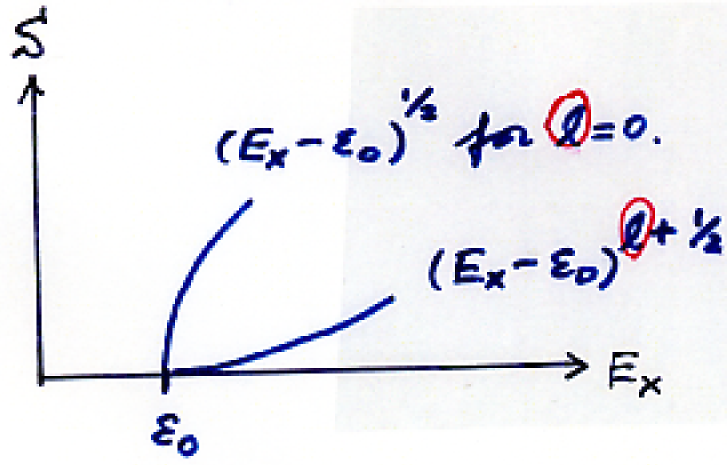
Quadrupole (2^+) excitation

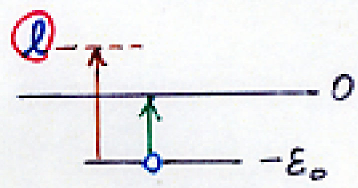


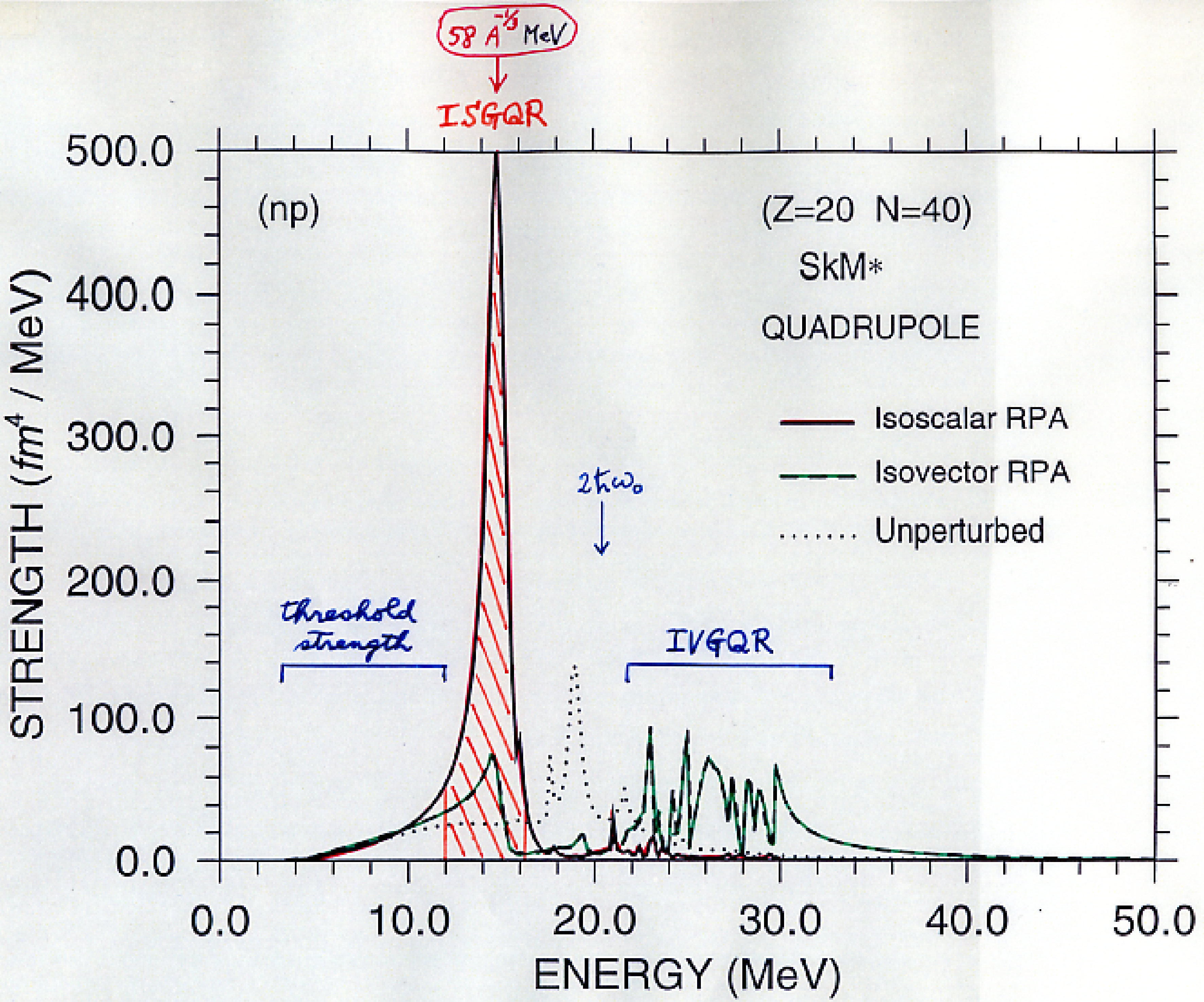


$2\hbar\omega_0$
(Z=20 N=40)
SkM*
QUADRUPOLE
unperturbed neutron response
$1f_{5/2}^{-1}$
$2p_{1/2}^{-1}$
$2p_{3/2}^{-1}$
$1f_{7/2}^{-1}$
$1d_{3/2}^{-1}$
$2s_{1/2}^{-1}$
$1d_{5/2}^{-1}$
$(1d_{5/2}^{-1})(1g_{9/2})$
Proton response
STRENGTH (fm^4 / MeV)
ENERGY (MeV)
0
50
100
150
0
5
10
15
20
25
30

For neutrons







58 A^-1/3 MeV
ISGQR
500.0
400.0
300.0
200.0
100.0
0.0
STRENGTH (fm^4 / MeV)
(np)
(Z=20 N=40)
SkM*
QUADRUPOLE
Isoscalar RPA
Isovector RPA
Unperturbed
2ħω₀
threshold strength
IVGQR
0.0
10.0
20.0
30.0
40.0
50.0
ENERGY (MeV)

Model-independent SUM-RULES

Giant Resonances carry a major part of sum-rules.

---

1) Charge-exchange ($t_\pm$) modes on $|0\rangle$;

$$\boxed{\left[\sum_i t_+(i),\ \sum_j t_-(j)\right] = 2\sum_k t_z(k)}$$

$$2\langle 0|\sum_i t_z(i)|0\rangle = N-Z,$$

(a) Fermi transitions ; $\mathcal{O} = \sum_i t_\pm(i)$,

$$\sum_m \left|\langle m|\sum_i t_-(i)|0\rangle\right|^2 - \sum_n \left|\langle n|\sum_i t_+(i)|0\rangle\right|^2 = N-Z$$

(b) Gamow-Teller ; $\mathcal{O}_\mu = \sum_i t_\pm(i)\,\sigma_\mu(i)$,

$$\sum_\mu \sum_m \left|\langle m|\sum_i t_-(i)\,\sigma_\mu(i)|0\rangle\right|^2 - \sum_\mu \sum_n \left|\langle n|\sum_i t_+(i)\,\sigma_\mu(i)|0\rangle\right|^2$$

$$= 3(N-Z), \qquad \boxed{\sum_\mu \sigma_\mu^2 = 3}$$

(c) $\mathcal{O} = \sum_i t_\pm(i)\, f(r_i)$,

$$\sum_m \left|\langle m|\sum_i t_-(i) f(r_i)|0\rangle\right|^2 - \sum_n \left|\langle n|\sum_i t_+(i)\, f(r_i)|0\rangle\right|^2$$

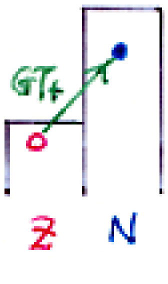
$$= N\cdot\langle (f(r))^2\rangle_n - Z\cdot\langle (f(r))^2\rangle_p .$$

---

ex. If $\left|\langle n|\sum_i t_+(i)\,\sigma_\mu(i)|0\rangle\right|^2 = 0$ due to a sufficient neutron excess,

then,

$$\sum_\mu \sum_m \left|\langle m|\sum_i t_-(i)\sigma_\mu(i)|0\rangle\right|^2 = 3\,(N-Z).$$



2) (1 or t_z) modes

For one-body operator $F_{\lambda\mu} = \sum_i f(r_i) \cdot Y_{\lambda\mu}(i)$

$$[H, F_{\lambda\mu}] \Rightarrow [\underset{\text{kinetic energy}}{T}, F_{\lambda\mu}], \quad \text{if } [v_{ij}, F_{\lambda\mu}] = 0$$

2-body interaction does **not** explicitly depend on momenta of particles.

then,
Energy-weighted sum-rule

$$\boxed{S^1(F) \equiv \sum_\mu \sum_a (E_a - E_0) \left| \langle a | F_{\lambda\mu} | 0 \rangle \right|^2}$$

$$= \frac{1}{2} \sum_\mu \langle 0 | [F_{\lambda\mu}, [H, F_{\lambda\mu}]] | 0 \rangle$$

$$\Rightarrow \frac{1}{2} \sum_\mu \langle 0 | [F_{\lambda\mu}, [T, F_{\lambda\mu}]] | 0 \rangle$$

$$= \sum_\mu \langle 0 | \frac{\hbar^2}{2M} \sum_k | \nabla_k F_{\lambda\mu}(\vec{r}_k) |^2 | 0 \rangle$$

$$= \frac{2\lambda+1}{4\pi} \frac{\hbar^2}{2M} A \boxed{\left\langle \left(\frac{df}{dr}\right)^2 + \lambda(\lambda+1)\left(\frac{f}{r}\right)^2 \right\rangle}$$

expectation value in the ground state

$\boxed{E\lambda \sim (\frac{1}{2} - t_z) \cdot \bigcirc}$

ex. $F_{\lambda\mu} = r^\lambda Y_{\lambda\mu}$

$$S^1(F) = \frac{\lambda(2\lambda+1)^2}{4\pi} \frac{\hbar^2}{2M} A \langle r^{2\lambda-2} \rangle$$

$$S^1(E\lambda)_{cl} = \frac{\lambda(2\lambda+1)^2}{4\pi} \frac{\hbar^2}{2M} Z e^2 \langle r^{2\lambda-2} \rangle_{\text{proton}} \quad \underline{\text{for } \lambda \geq 2.}$$

ex. monopole mode $\quad F = r^2$

$$S^1(E0)_{cl} = \frac{2\hbar^2}{M} Z e^2 \langle r^2 \rangle_{\text{proton}}$$

ex. dipole mode

$$F(E1)_{1\mu} = e \sum_k \left[\left(\frac{1}{2} - t_z\right) r Y_{1\mu} \right]_k \rightarrow e \sum_k \left[\boxed{\left(\frac{N-Z}{2A} - t_z\right)} r Y_{1\mu} \right]_k$$

$$\begin{cases} N/A & \text{for protons} \\ -Z/A & \text{for neutrons} \end{cases}$$

E1 operator must be referred to center-of-mass motion.

$$S^1(E1)_{cl} = \frac{9}{4\pi} \frac{\hbar^2}{2M} e^2 \left[Z\left(\frac{N}{A}\right)^2 + N\left(\frac{-Z}{A}\right)^2 \right]$$

$$= \frac{9}{4\pi} \frac{\hbar^2}{2M} e^2 \frac{NZ}{A} \quad = 14.8 \frac{NZ}{A} \quad e^2 fm^2 MeV$$

The motion of a particle is associated with a recoil of the rest of the nucleus, since the total center of mass remains at rest.

——— This is especially important in E1 transitions.

$$e \rightarrow (e)_{E1} = \boxed{\left[\left(\frac{1}{2} - t_z\right) - \frac{Z}{A}\right]} e = \begin{cases} \frac{N}{A} e & \text{for protons.} \\ -\frac{Z}{A} e & \text{for neutrons.} \end{cases}$$

$$\boxed{\frac{N-Z}{2A} - t_z}$$

ex. IS quadrupole modes, $F_{2\mu} = r^2 Y_{2\mu}$

$$S'(r^2 Y_{2\mu}) = \frac{2\cdot 5^2}{4\pi}\frac{\hbar^2}{2M} A\cdot\langle r^2\rangle ,$$

$\langle r^2\rangle$: ground-state expectation-value

Threshold strength, which comes from loosely-bound nucleons, has extra contribution to $\langle r^2\rangle$.

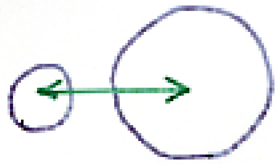
Thus, $S(r^2 Y_{2\mu})$-values are larger for nuclei with threshold strength.

ex. $S(E1)$ is unique.

Both possible threshold strength and so-called pygmy resonance are a part of

$$S(E1)_{cl} = \frac{9}{4\pi}\frac{\hbar^2}{2M} e^2 \frac{NZ}{A}$$

pygmy resonance



A_1 (N_1, Z_1) A_2 (N_2, Z_2)

The contribution to $S(E1)$ is

$$\frac{9}{4\pi}\frac{\hbar^2}{2M} e^2 \frac{(Z_1 A_2 - Z_2 A_1)^2}{A A_1 A_2}$$

T. Nakamura et al. / Nuclear Phys. A588 (1995) 81c.

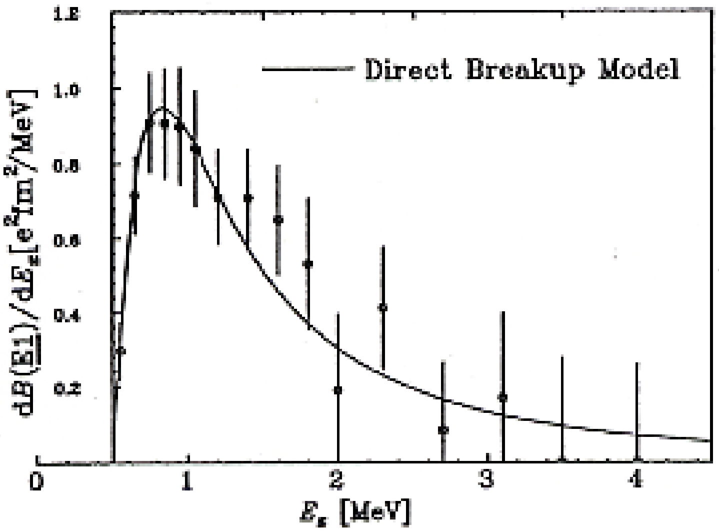


Figure 1. Dipole strength distribution of ^{11}Be. The solid line represents the direct break up model (eq.(1)), after taking the detector resolution and the post acceleration into consideration.

One-neutron halo nucleus, $^{11}_{4}Be_{7}$.

$$S(E\lambda) = S(E\lambda)_{cl}\,(1 + x)$$

$\left\{ \begin{array}{l} \text{charge-exchange } (\tau\cdot\tau) \\ \text{velocity-dependent} \end{array} \right\}$ component in the nucleonic interactions.

---

velocity-dependent component of **Skyrme** interactions ($\propto \delta(\vec{r}_1 - \vec{r}_2)$) do not contribute to $S(\tau = 0)$ such as $S(r^{\lambda} Y_{\lambda\mu})$.