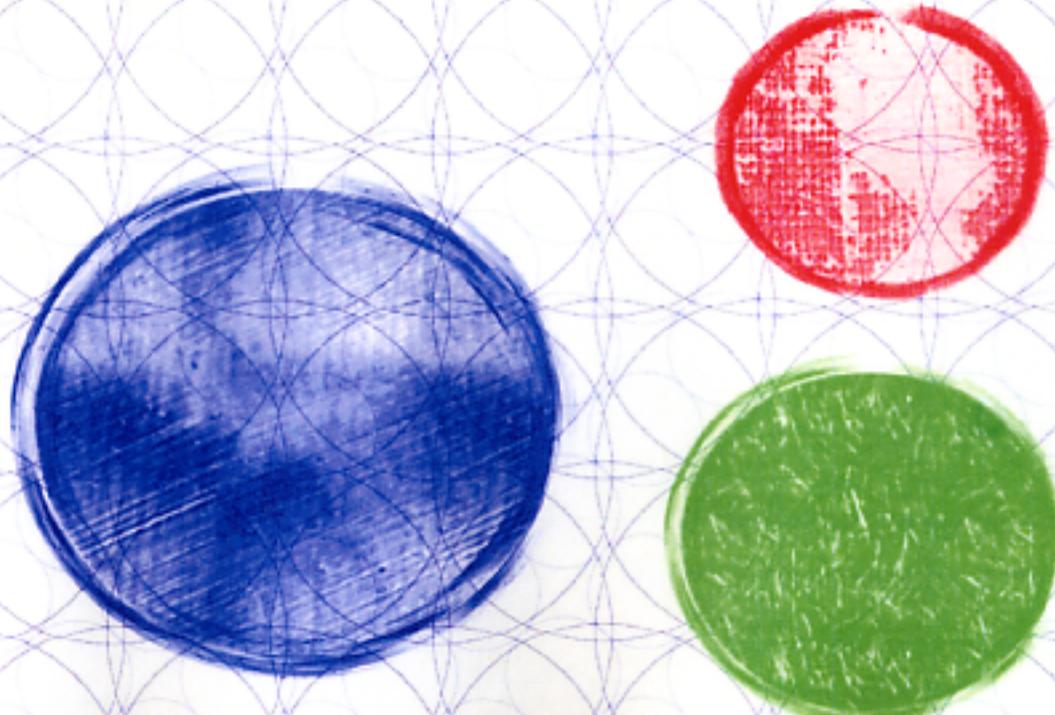


<http://www.cns.s.u-tokyo.ac.jp/summerschool/>

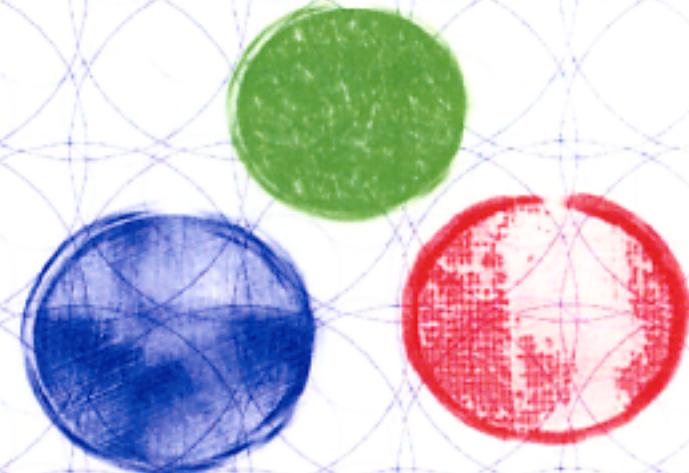
# Exotic cluster structure in light nuclei

Naoyuki Itagaki      University of Tokyo



# **Outline of the talk**

- 1) Basic idea**
- 2) History of cluster theory in  $4N$  nuclei**
- 3) Cluster structure in neutron-rich nuclei**
- 4) Conclusion and future perspective**



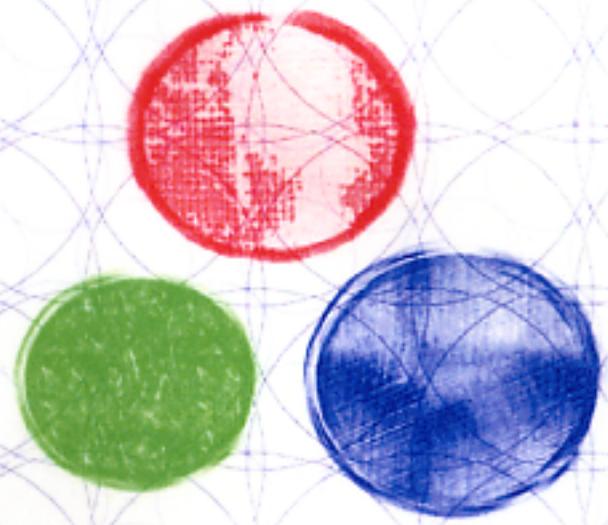
**Nuclei ---**

**Finite Quantum Many–body System  
consisting of Protons and Neutrons**

**Structure of nuclei ---**

**Nucleons construct self–consistent  
mean field and occupy  
single–particle orbits**

**Mean–field Model  
Shell Model**



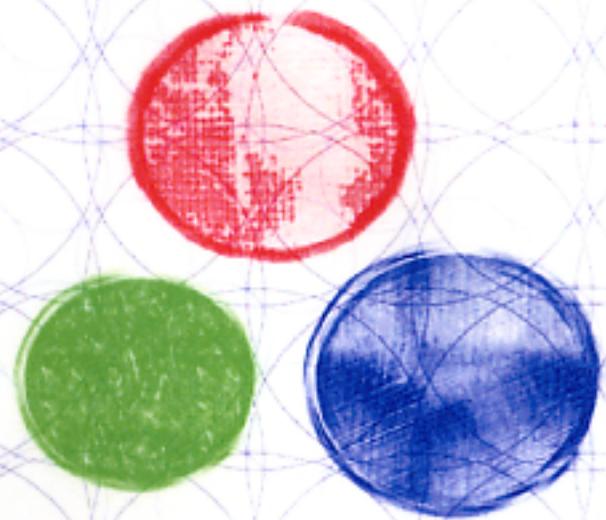
**Nuclei ---**

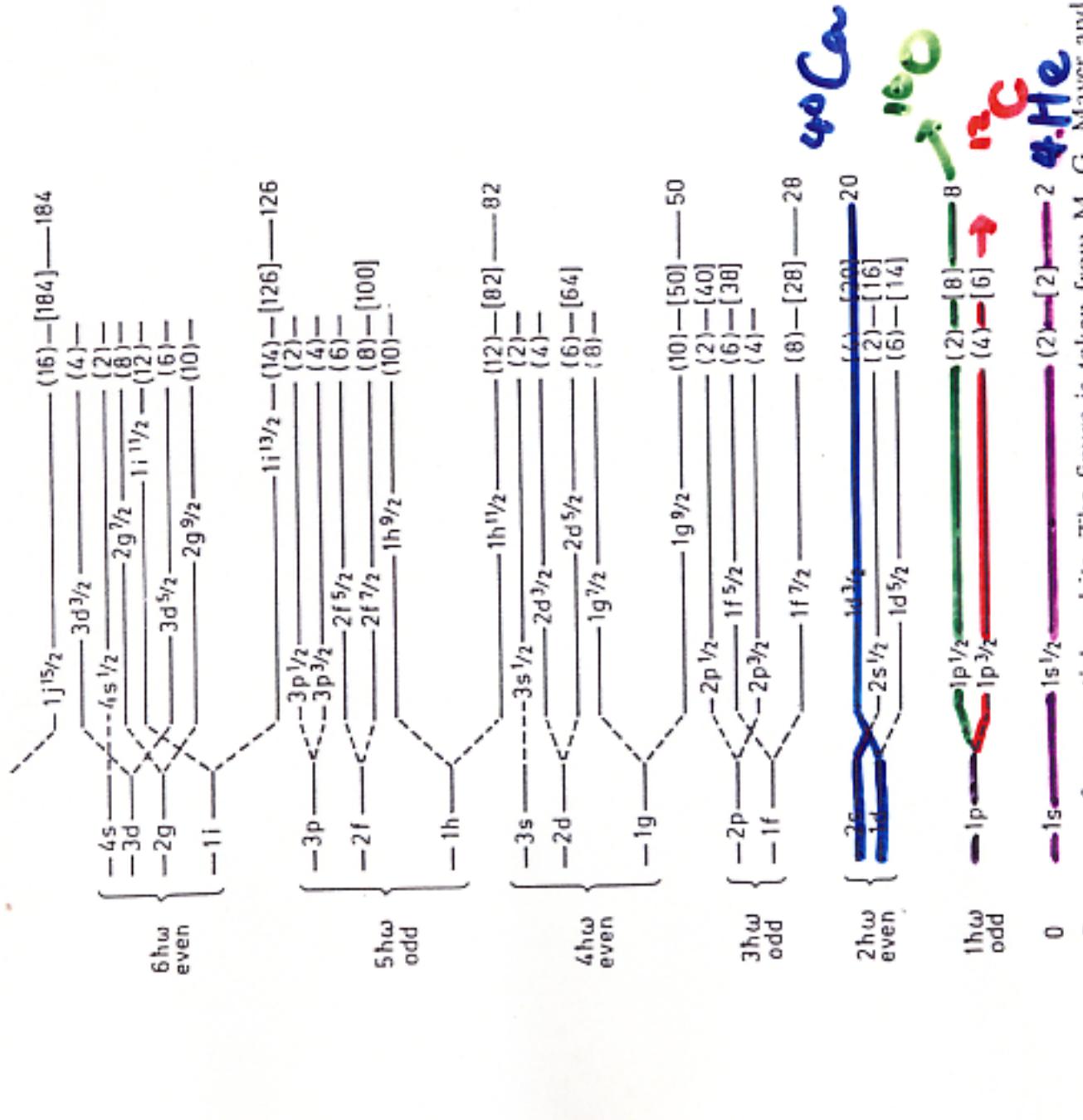
**Finite Quantum Many–body System  
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**Nucleons construct self–consistent  
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**Mean–field Model  
Shell Model**





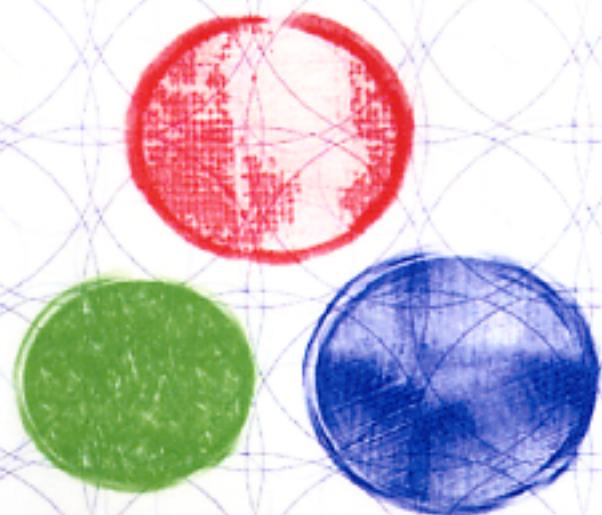
J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure*, p. 58, Wiley, New York, 1955.

# **Cluster structure ---**

**Another phase of the nuclear structure**

**How it appears?**

**Why it appears?**



$Z^{\pi}: I=1$  ————— 0 838 ms



$$\Delta: 4941.66 \quad S_u: 18899.25 \quad S_p: 17255.05 \quad Q_\alpha: 91.844$$

*Populating Reactions and Decay Modes*

- A  ${}^8\text{Li}$   $\beta^-$  decay
- B  ${}^8\text{B}$   $\beta^+$  decay
- C  ${}^4\text{He}(\alpha, \gamma)$
- D  ${}^4\text{He}(\alpha, \alpha)$
- E  ${}^6\text{Li}(\text{t}, \eta)$
- F  ${}^6\text{Li}({}^3\text{He}, \text{p})$ ,  $({}^3\text{He}, 2\alpha)$
- G  ${}^6\text{Li}(\alpha, \text{d})$ ,  $(\alpha, 2\alpha)$
- H  ${}^6\text{Li}({}^6\text{Li}, \alpha)$ ,  $({}^6\text{Li}, 2\text{d})$
- I  ${}^7\text{Li}(\text{p}, \gamma)$
- J  ${}^7\text{Li}(\text{p}, \eta)$
- K  ${}^7\text{Li}(\text{p}, \text{p})$ ,  $(\text{p}, \text{p}')$
- L  ${}^9\text{Be}({}^3\text{He}, \alpha)$ ,
- M  ${}^6\text{Li}(\text{d}, \alpha)$ ,  $(\text{d}, \alpha\text{p})$

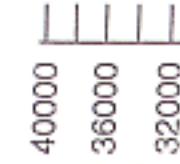
Levels and  $\gamma$ -ray branchings:

- $0, 0^+, \Gamma=6.8 \text{ keV}, [ABCDEFGHIL], \%2\alpha=100, T=0$
- $3040 \text{ } 30, 2^+, \Gamma=1.502 \text{ MeV}, [ABCDEFGHIL], \%2\alpha=100, T=0$
- $11400 \text{ } 300, 4^+, \Gamma \approx 3.5 \text{ MeV}, [DGHL], T=0$
- $16626 \text{ } 3, 2^+, \Gamma=108.15 \text{ keV}, [BCDEFHIL], \%2\alpha=100, T=0+1 \quad \gamma_{3040} \quad 1$   
 $(\dagger_{\gamma} \text{ } 100) \text{ M1}$
- $16922 \text{ } 3, 2^+, \Gamma=74.04 \text{ keV}, [CDEFHIL], T=0+1 \quad \gamma_{3040} \text{ } 13882 \text{ } (\dagger_{\gamma} \text{ } 100)$
- $17640 \text{ } 10, 1^+, \Gamma=10.75 \text{ keV}, [FIK], T=1 \quad \gamma_{16922} \text{ } 718 \text{ } (\dagger_{\gamma} \text{ } 0.0082) \text{ M1}$   
 $(\dagger_{\gamma} \text{ } 0.0192) \text{ M1+E2: } \delta=-0.014 \text{ } 13 \quad \gamma_{3040} \text{ } 14586 \text{ } (\dagger_{\gamma} \text{ } 49.74) \text{ M1+E2: } \delta=\zeta$   
 $\gamma_0 \text{ } 17619 \text{ } (\dagger_{\gamma} \text{ } 100) \text{ M1}$
- $18150 \text{ } 4, 1^+, \Gamma=138.6 \text{ keV}, [FIK], T=0 \quad \gamma_{16922} \text{ } 1228 \text{ } (\dagger_{\gamma} \text{ } 1.62) \text{ M1} \quad \gamma_{1662}$   
 $(\dagger_{\gamma} \text{ } 2.05) \text{ M1} \quad \gamma_{3040} \text{ } 15095 \text{ } (\dagger_{\gamma} \text{ } 100) \text{ M1} \quad \gamma_0 \text{ } 18128 \text{ } (\dagger_{\gamma} \text{ } 79) \text{ M1}$
- $18910, 2^-, \Gamma=122 \text{ keV}, [FIJK] \quad \gamma_{16922} \text{ } 1988 \text{ } (\dagger_{\gamma} \text{ } 59) \text{ E1} \quad \gamma_{16626} \text{ } 2284 \text{ } (\dagger_{\gamma} \text{ } 19070 \text{ } 30, 3^+, \Gamma=270.20 \text{ keV}, [FIK], T=(1) \quad \gamma_{3040} \text{ } 16013 \text{ } (\dagger_{\gamma} \text{ } 100) \text{ M1}$
- $19240 \text{ } 25, 3^+, \Gamma=230.30 \text{ keV}, [JKL], T=(0) \quad \gamma_{16922} \text{ } 19400, 1^-, \Gamma \approx 650 \text{ keV}, [FJK]$
- $19860 \text{ } 50, 4^+, \Gamma=0.71 \text{ MeV}, [DFL], T=0 \quad \gamma_{16922} \text{ } 20100, 2^+, \Gamma \approx 1.1 \text{ MeV}, [DJK], T=0$
- $20200, 0^+, \Gamma < 1 \text{ MeV}, [D], T=0 \quad \gamma_{16922} \text{ } 20900, 4^-, \Gamma=1.62 \text{ MeV}, [K]$
- $21500, 3(^+), \Gamma=1 \text{ MeV}, [IJ]$
- $22000, 1^-, \Gamma \approx 4 \text{ MeV}, [IJ], T=1$
- $22050 \text{ } 100, \Gamma=270.70 \text{ keV}, [L]$

$$\sigma = 0, 2, 4$$



$$E_x = \frac{\hbar^2}{2I} \tau (\tau + 1)$$



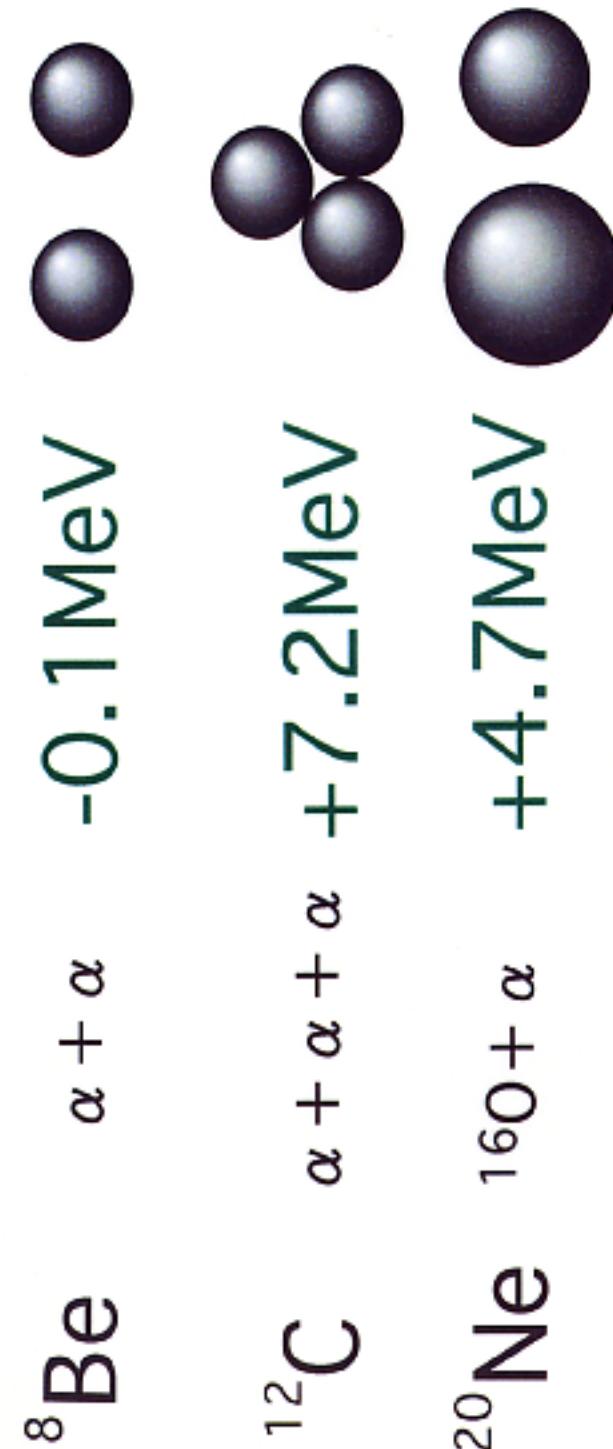
# Mechanisms of $(\alpha)$ -clustering

${}^4\text{He}$  -- strongly bound system

$$\text{B.E.}/A = 7.1 \text{ MeV}$$

${}^4\text{He}-{}^4\text{He}$  -- weakly interacting

Unit of structures

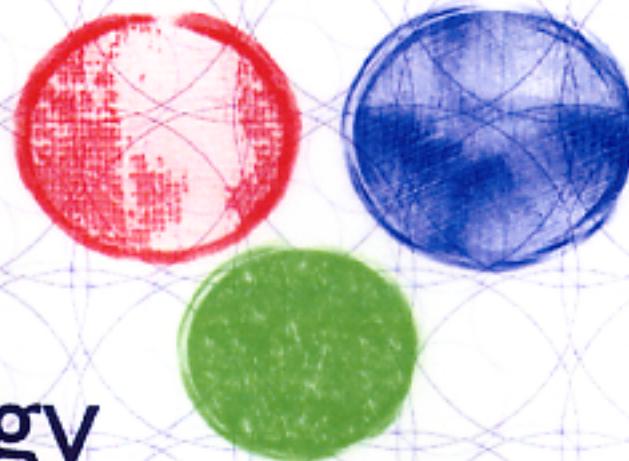


Threshold Rule

# Cluster structure ---

**Why it appears?**

It is another way for nuclei  
to be excited with low cost (energy)



sometimes  
the excitation energy  
is lower than the ph excitation case

# Mysterious $0^+$ state in $^{16}\text{O}$

The ground  $0^+$  state --- closed p-shell

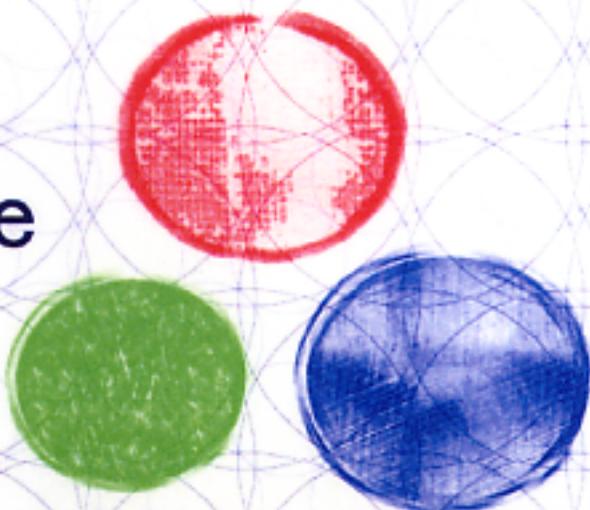
The first excited state --  $0^+$  at 6.049 MeV

2p-2h excitation to the sd-shell?

c.f.  $^{15}\text{O}$

$1/2^-$  state is the ground state

$1/2^+$  state is at 5.183 MeV



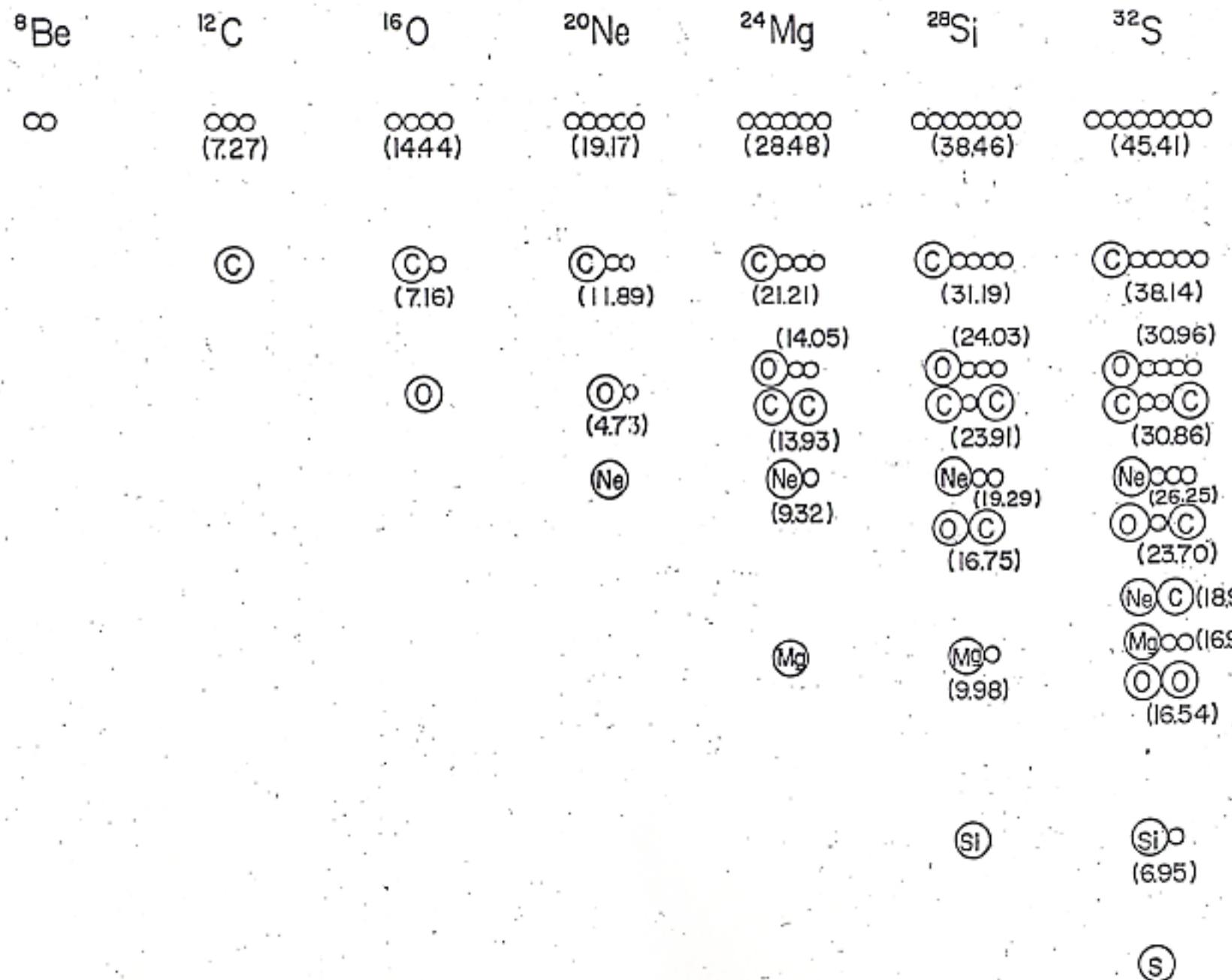


Fig. 1. The Ikeda diagram which shows the subunits of the possible molecule-like structures expected to appear near the respective threshold energies for the break-up into subunit nuclei. The threshold energies are written in parentheses.

# Inversion doublet (parity doublet) structure (by Horiuchi, Ikeda)

Cluster – Cluster interaction is weak



Cluster – Cluster potential is shallow



relative wave function can increase  
the number of node with low energy

rotational bands of positive-parity  
and negative-parity almost degenerate

energy in MeV.

eight negative-parity  $T=0$  states below the  
and many  $T=0$  states above this as shown in

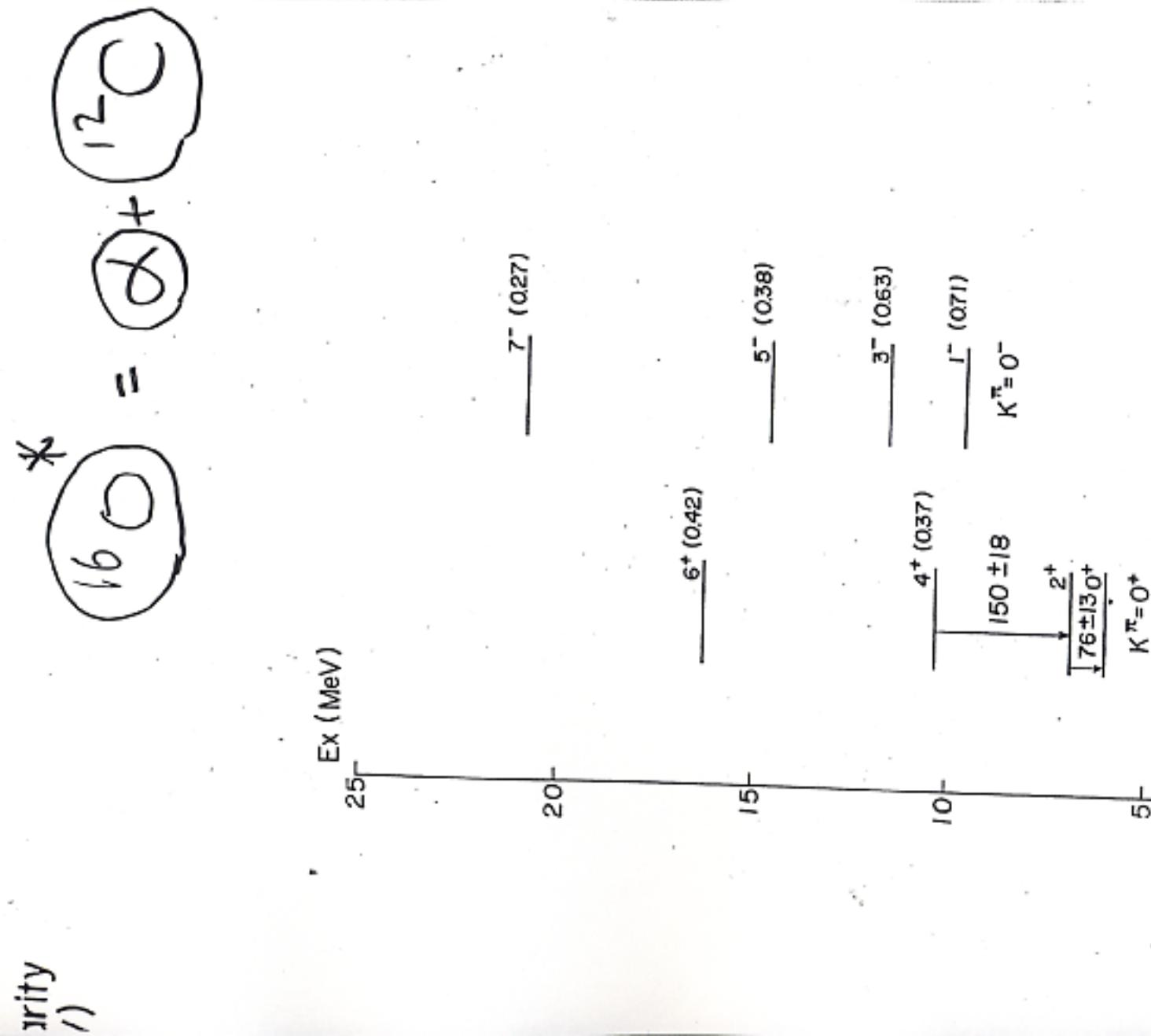


Fig. 1-3. The rotational bands in  $^{16}\text{O}$ .  
The values in parentheses are the  
dimensionless reduced  $\alpha$ -widths at  
a channel radius 5.2 fm. The  
 $B(E2)$  values are in unit of  $e^2 \text{fm}^4$ .

$^{20}\text{Ne}$				
$^{12}\text{C} + ^8\text{Be}$ thresh.	(0.0009 ± 0.0003)	(0.09 ± 0.01)	(0.0002 ± 0.0001)	14.31 $6^+$ 13.90 $6^+$ (0.0045 ± 0.0009) 12.44 $0^+$
11.98	<u>11.95</u> $8^+_1$	<u>12.59</u> $6^+_3$	<u>12.13</u> $6^+_2$	(~0.058)
11.89 $^{12}\text{C} + 2\alpha$ thresh.				
10.78		9.99 $4^+_3$ (0.009)	(0.23) 10.79 $4^+_5$ 10.97 $0^+_6$ (0.012) (0.14) 10.55 $4^+_4$ 10.61 $6^-$	
$^{16}\text{O}^*(6.05\text{MeV} 0^+) + \alpha$ thresh.	8.78 $6^-_1$ (0.010 ± 0.002)	9.03 $4^+_2$ (0.005)	>0.59 9.87 $3^+$ ~8.8 $2^+_4$ ~8.3 $0^+_4$ (0.014 ± 0.007) 8.45 $5^-_1$	10.26 $5^-_2$ (0.30)
	7.42 $2^+_2$ (0.04)	7.83 $2^+_3$	(>0.50) 7.00 $4^-$	7.17 $3^-_2$ (0.27)
	6.72 $0^+_2$ (0.007)	7.19 $0^+_3$		5.62 $3^-_1$ 5.78 $1^-$
	(0.15 ± 0.07)			4.97 $2^-$ (>0.13)
4.73 $0^+_1$ thresh.	4.25 $4^+_1$		$(\Theta_\alpha(a)^2 a=6\text{ fm})$	

1.63  $2^+_1$

0.  $0^+_1$

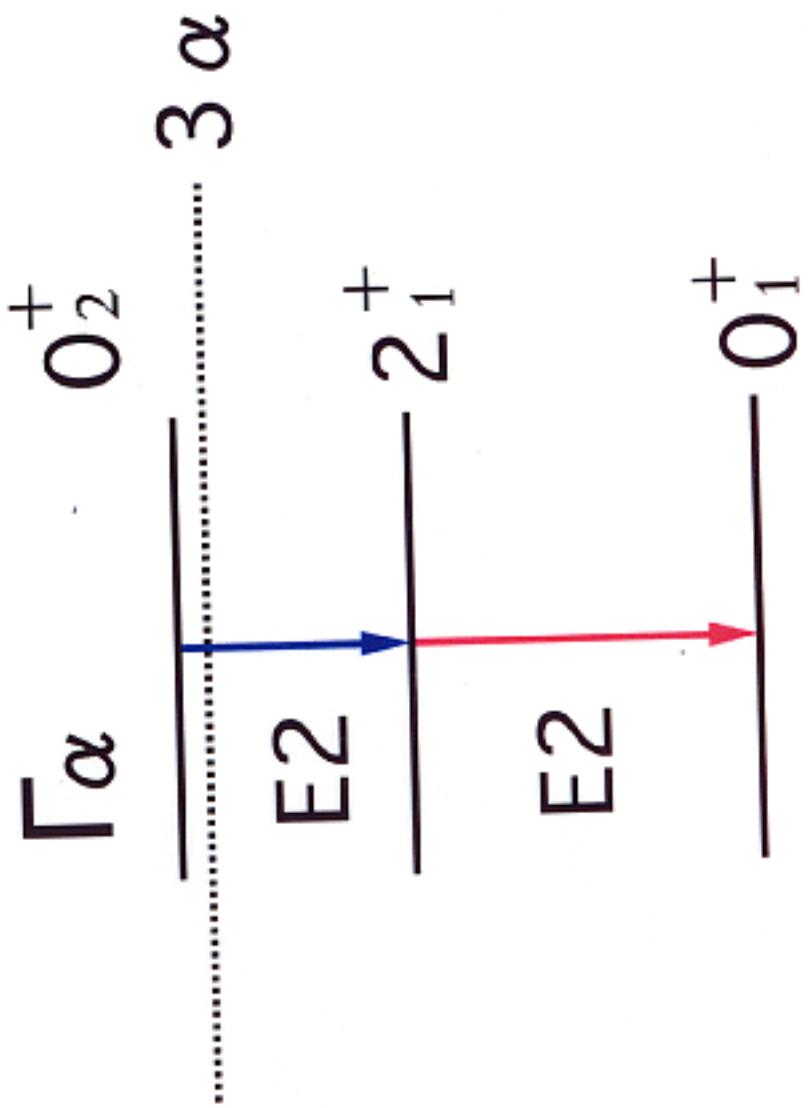
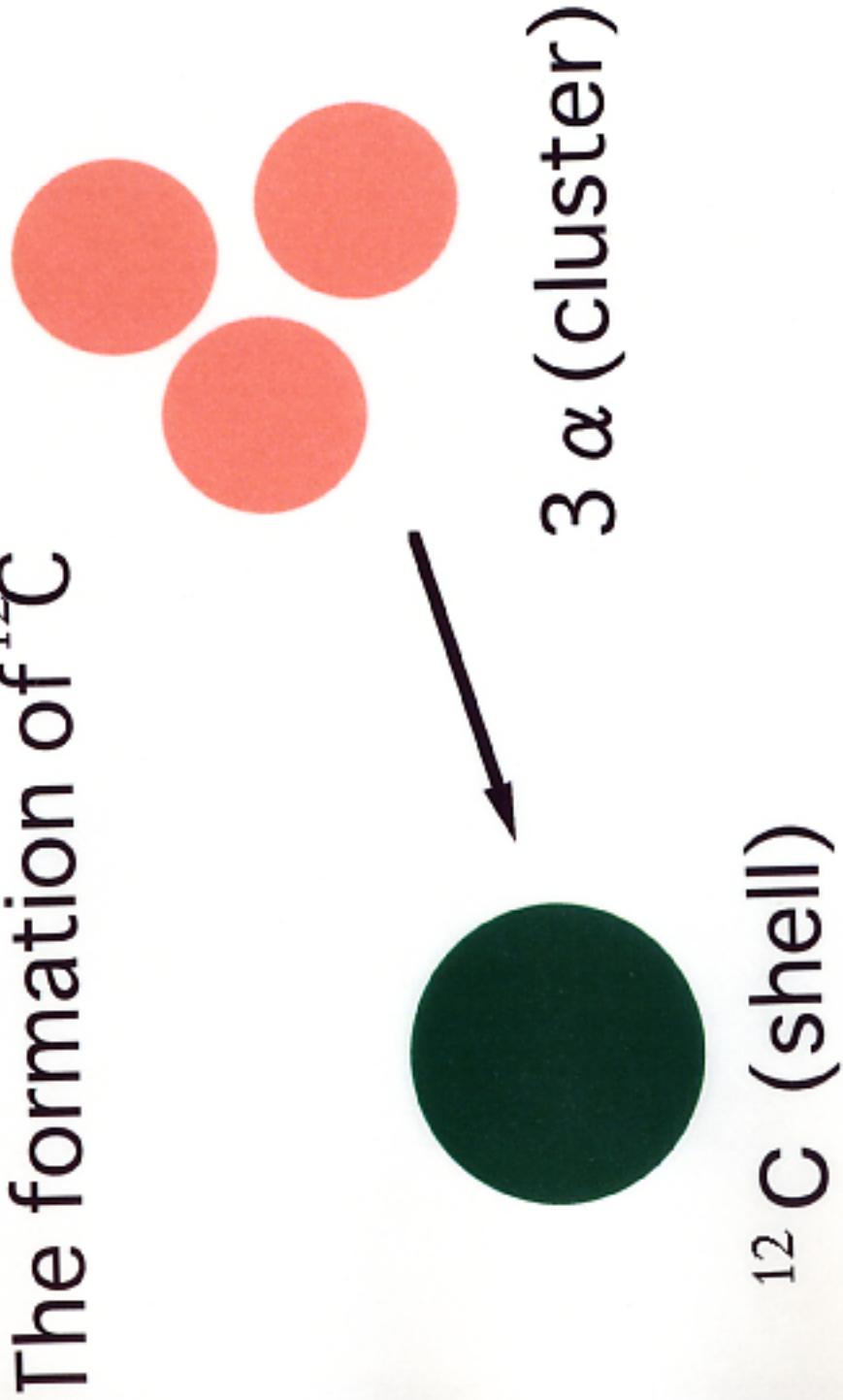
$K^\pi = 0^+_1$   $K^\pi = 0^+_2$   $K^\pi = 0^+_3$   $K^\pi = 0^+_4$

## $^{20}\text{Ne}$

$(K^\pi = 2^+)$   $K^\pi = 2^-$   $K^\pi = 0^-$

Fig. 1-5. Low-lying  $T=0$  energy levels of  $^{20}\text{Ne}$  with excitation energies in MeV and with reduced  $\alpha$ -widths ( $\Theta_\alpha(a)^2$ ) at the channel radius  $a=6\text{ fm}$ . The excitation energies and the  $\alpha$ -decay widths are taken from Ref. 91) except those of the 12.44 MeV  $0^+_6$  state (Ref. 92)). Almost all the levels below about 10 MeV excitation are classified into rotational bands. The dashed bars denote tentative band assignment. The  $(sd)^4$  shell model reproduces the  $K^\pi = 0^+_1$  and  $0^+_2$  bands, the  $^{16}\text{O} + \alpha$  cluster model the  $K^\pi = 0^+_1$ ,  $0^+_4$  and  $0^-$  bands, the ESM (MRGFM) and the  $\alpha$ - $^{12}\text{C}$ - $\alpha$  GCM the  $K^\pi = 0^+_1$ ,  $0^+_2$ ,  $0^+_4$  and  $0^-$  bands, and the  $(^{16}\text{O}-\alpha) + (^{12}\text{C}-^8\text{Be})$  CCOCM and the  $^{12}\text{C} + 2\alpha$  OCM the  $K^\pi = 0^+_1$ ,  $0^+_2$ ,  $0^+_3$ ,  $0^+_4$ ,  $2^-$  and  $0^-$  bands.

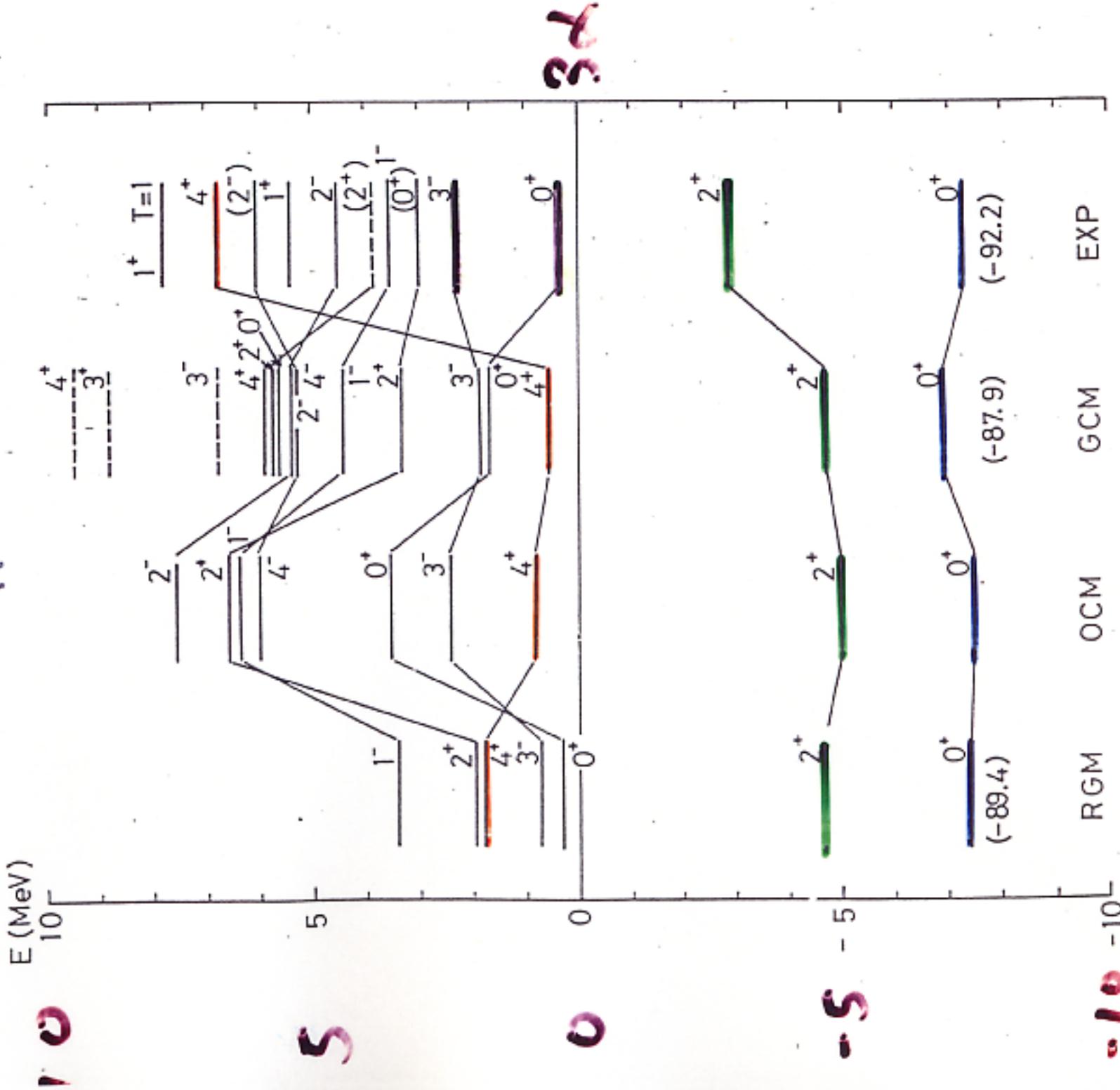
# The formation of $^{12}\text{C}$



CM calculations and  $0.2515 \text{ fm}^{-2}$  ( $b = 1.41 \text{ fm}$ ) in the GCM call the experimental value<sup>[16]</sup> is known to be  $\sim 0.285 \text{ fm}^{-2}$ . The si's in  $^{12}\text{C}$  is inspected by the variational calculation versus  $\nu_0$  in these choices are found to be good.

*Energy spectra and structure of  $^{12}\text{C}$*

## Samples of P.T.P. No. 68 (1980)



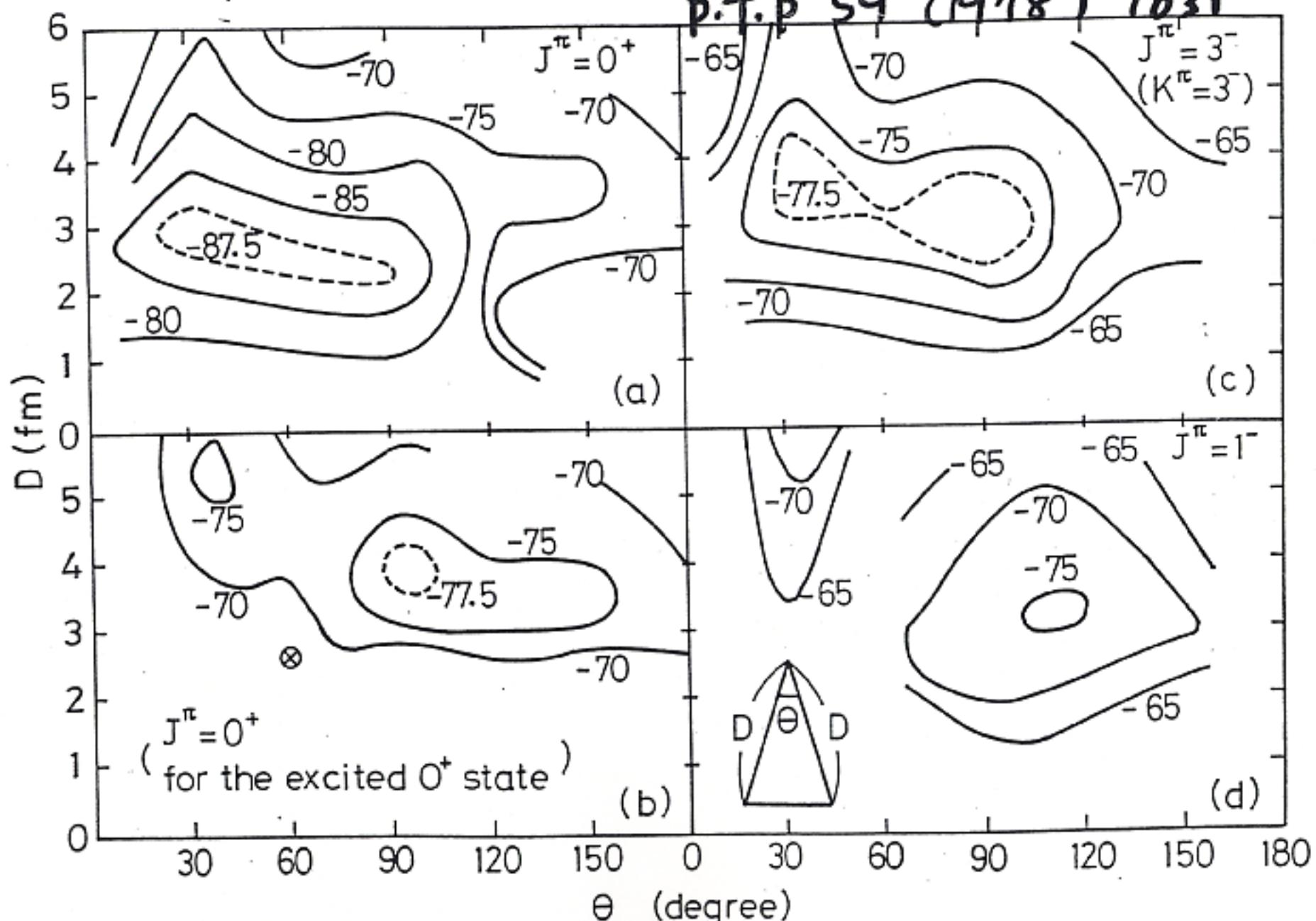


Fig. 2-3. Energy surfaces in the isosceles configuration (for  $D$  and  $\theta$ , see the illustration in (d)). The Volkov No. 1 ( $m=0.60$ ) force is used without the Coulomb force. (a)

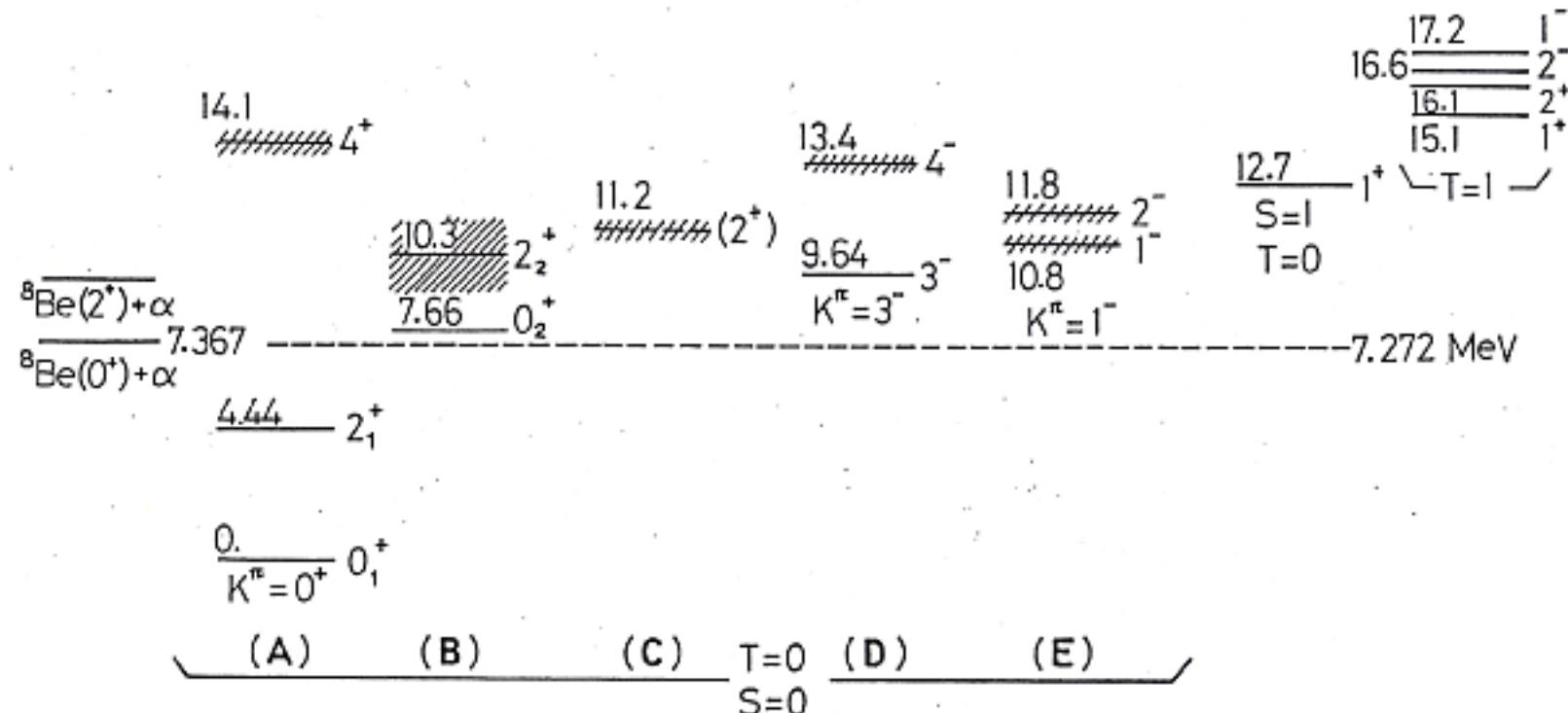
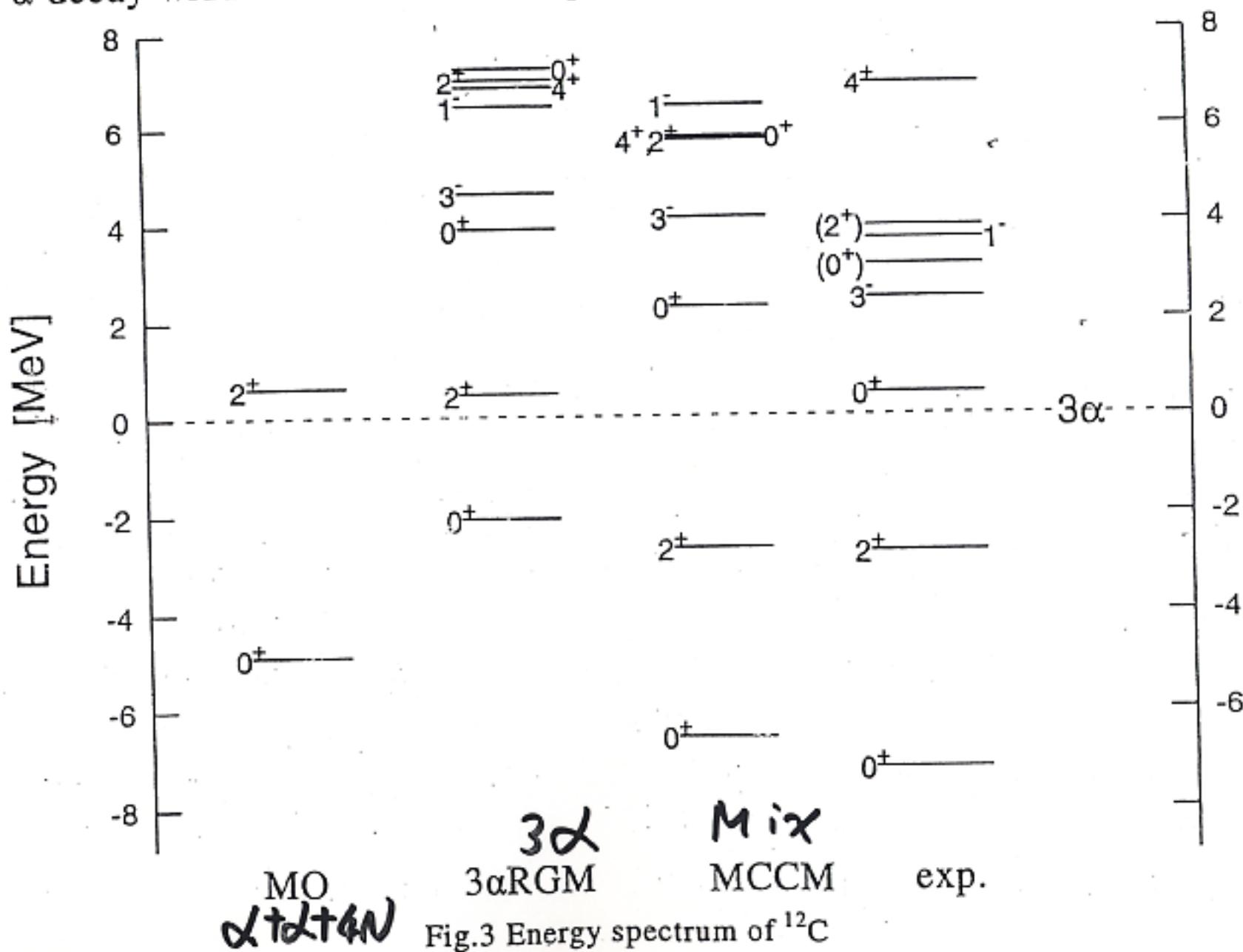


Fig. 1-1. Experimental energy levels of  $^{12}\text{C}$  are classified into several bands according to the dynamical calculations in § 2. (A)  $K^\pi=0^+$  ground rotational band, (B) the first family of the excited positive-parity states including the  $0_2^+$  and  $2_2^+$  states, (C) the second family of excited positive-parity states. (D) shows  $K^\pi=3^-$  band, (E)  $K^\pi=1^-$  band. The data are taken from Ref. 16). Here, the  $(0^+)$  level at 10.3 MeV and the  $(2^-)$  level at 13.35 MeV are assigned to  $J^\pi=2^+$  and  $4^-$  respectively, according to the dynamical calculations in § 2.

same problem by the hybrid model where the broken symmetry ( $[4^231]$ ) configuration of  $0p$ -shell shell model were introduced besides the  $3\alpha$ -clustering

and an  $\alpha$ -decay width from the  $v_2$  state. The present model has a transition probability of  $11.5 e^2 fm^4$  to the experimental value of  $13 \pm 4 e^2 fm^4$  and the  $\alpha$ -decay width of  $7.3 eV$  to the experimental value of  $8.5 \pm 1.0 eV$ .



# History of the cluster calculation

phenomenological  $\alpha$ -particle model

binding energy of  $4N$  nuclei are  
just the number of bonds of  $\alpha-\alpha$ ?

Brink model ( $\sim 1960$ )

microscopic calculation for light  $4N$  nuclei

$\sim 1970-1980$

microscopic calculation for nuclei  
consisting of cluster(s) and valence particles

$\sim 1980$

AMD ( $\sim 1990$ )

Green Function Monte Carlo

# Brink Model

$$\Psi = P [ A(\phi_1 \phi_2 \dots) ]$$

$$\phi_i(r) = \exp[-\nu(\vec{r} - \vec{R}_i)^2] \chi_i$$

P: Angular momentum and parity projection

A: Antisymmetrizer

$$\exp[-\nu(\vec{x} - \vec{X})^2] = (\sum_{n=0}^{\infty} X^n H_n(x) / n!) e^{-\nu x^2}$$

Local Gauss is equal to the coherent state  
of many ph states in the shell model picture

## Two nucleons case (with the same spin-isospin)

$$\phi_1(r) = \exp[-\nu(\vec{r} - \vec{x})^2] \chi_i$$

$$\phi_2(r) = \exp[-\nu(\vec{r} + \vec{x})^2] \chi_i$$

$$\psi_1(r) = \phi_1(r) + \phi_2(r)$$

$$\psi_2(r) = [\phi_1(r) - \phi_2(r)] / x$$

$$A(\phi_1(r) \phi_2(r)) = A(\psi_1(r) \psi_2(r))$$

$$x \rightarrow 0 \quad \cancel{\phi}_1(r) = \exp[-\nu r^2] \chi_i$$

$$\cancel{\phi}_2(r) = x \exp[-\nu r^2] \chi_i$$

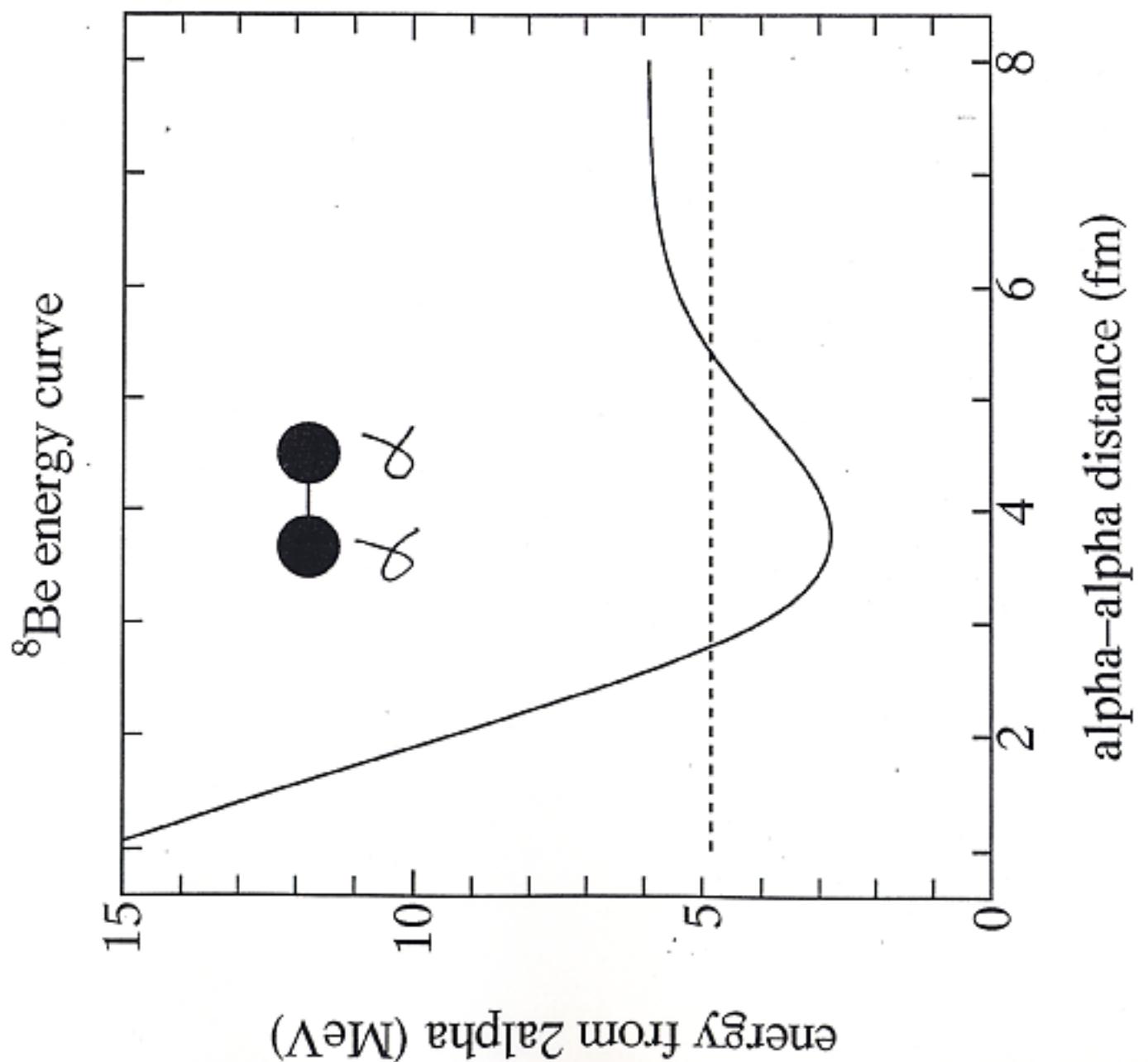
Cluster model space coincides with  
the lowest shell model at  $x \rightarrow 0$

**~ 1985 revolution of RNB experiment**

**Cluster structure survives in n-rich isotopes?**

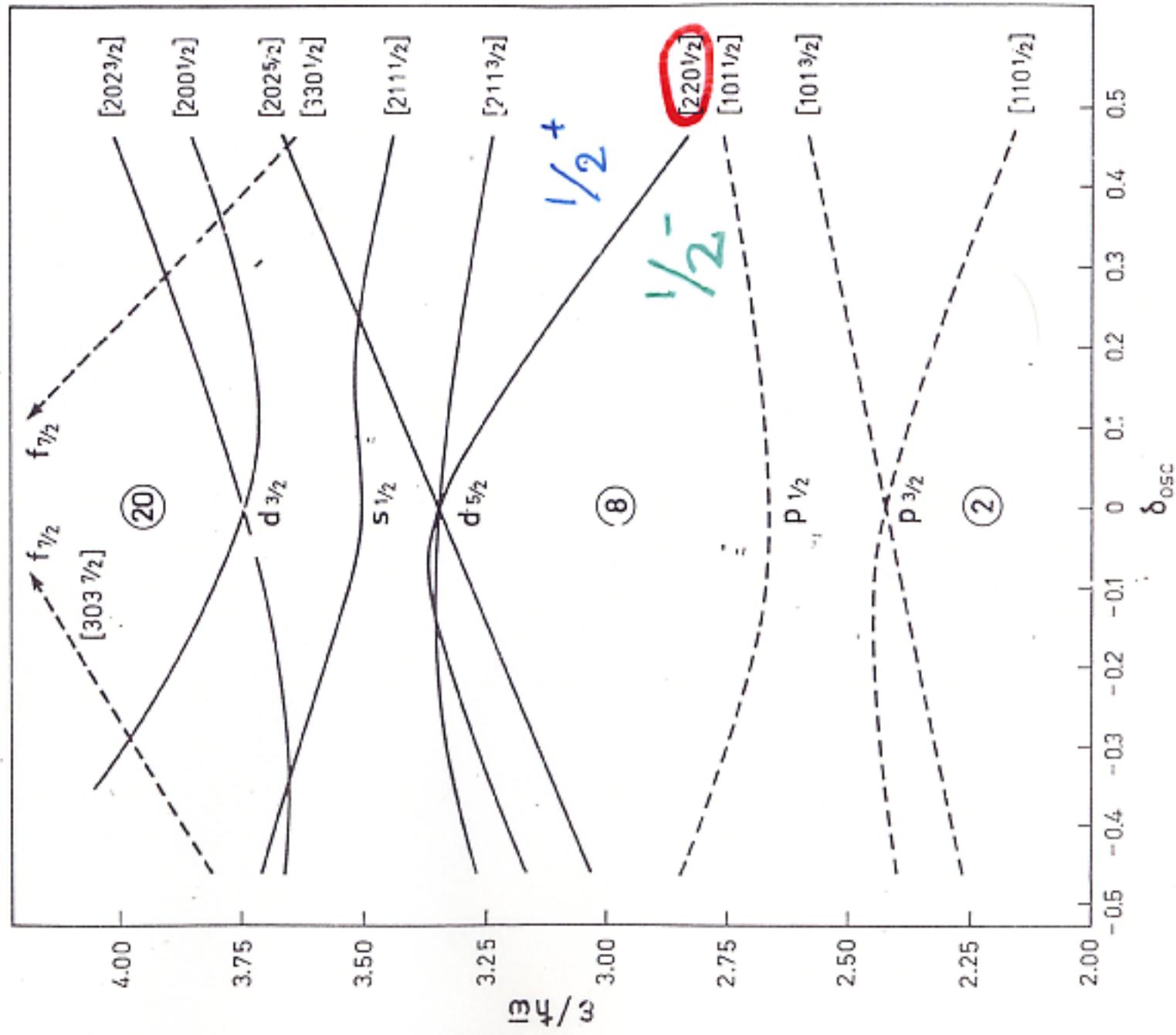
**Findings of the halo structure and  
disappearance of the magic number  
are related to the s-wave problem**

**The possibility that the cluster structure  
becomes more important due to  
the valence neutrons around it**



## § 5-1 STATIONARY STATES

221



**Figure 5-1** Spectrum of single-particle orbits in spheroidal potential ( $N$  and  $Z < 20$ ). The spectrum is taken from B. R. Mottelson and S. G. Nilsson, *Mat. Fys. Skr. Dan. Vid. Selsk.*, 1, no. 8 (1959). The orbits are labeled by the asymptotic quantum numbers  $[Nn_3\Lambda\Omega]$  referring to large prolate deformations. Levels with even and odd parity are drawn with solid and dashed lines, respectively.

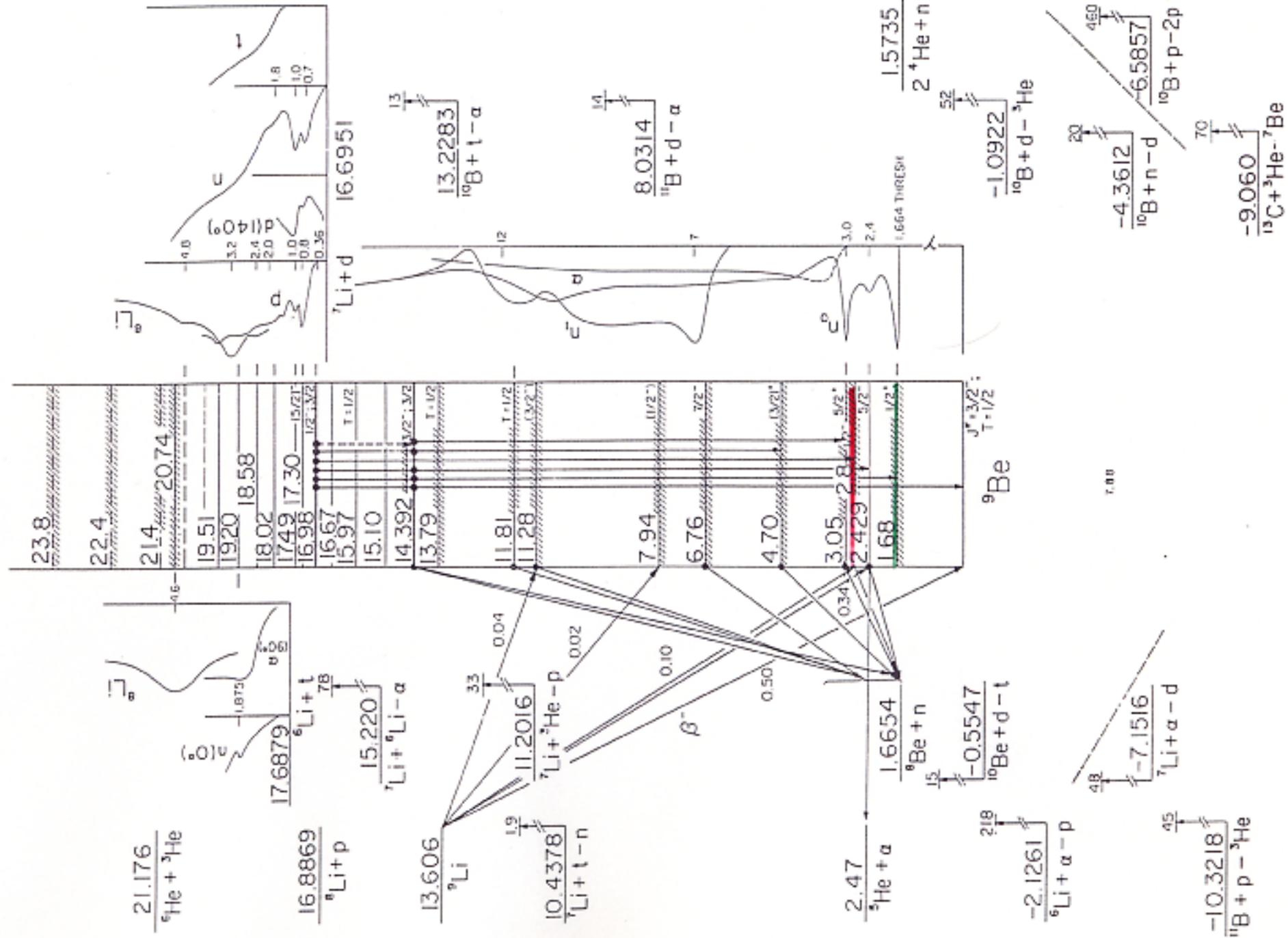


Fig. 16. Energy levels of  $^{9}\text{Be}$ : for notation, see fig. 1.

$$^9\text{Be} = \alpha + \alpha + n$$

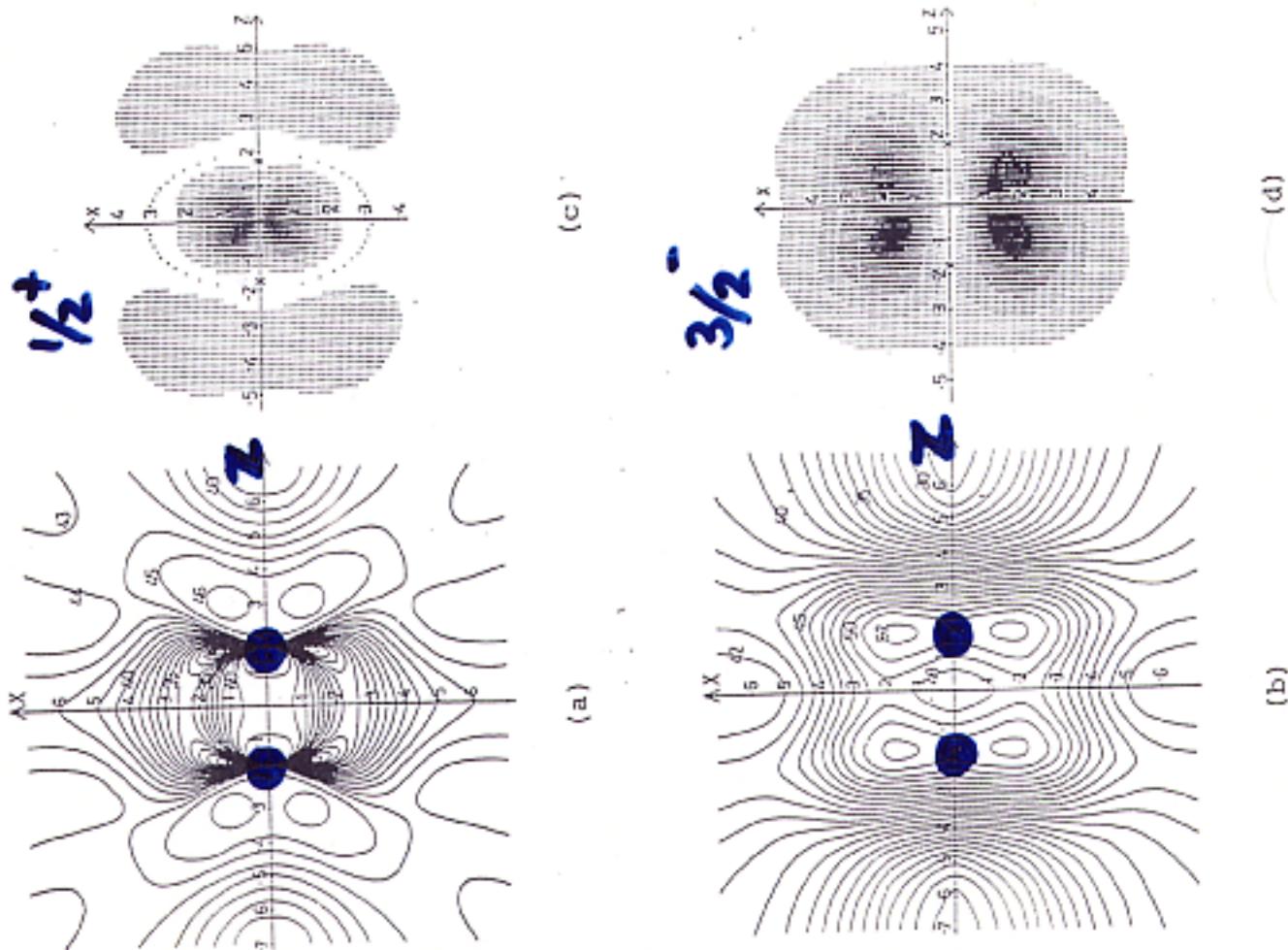
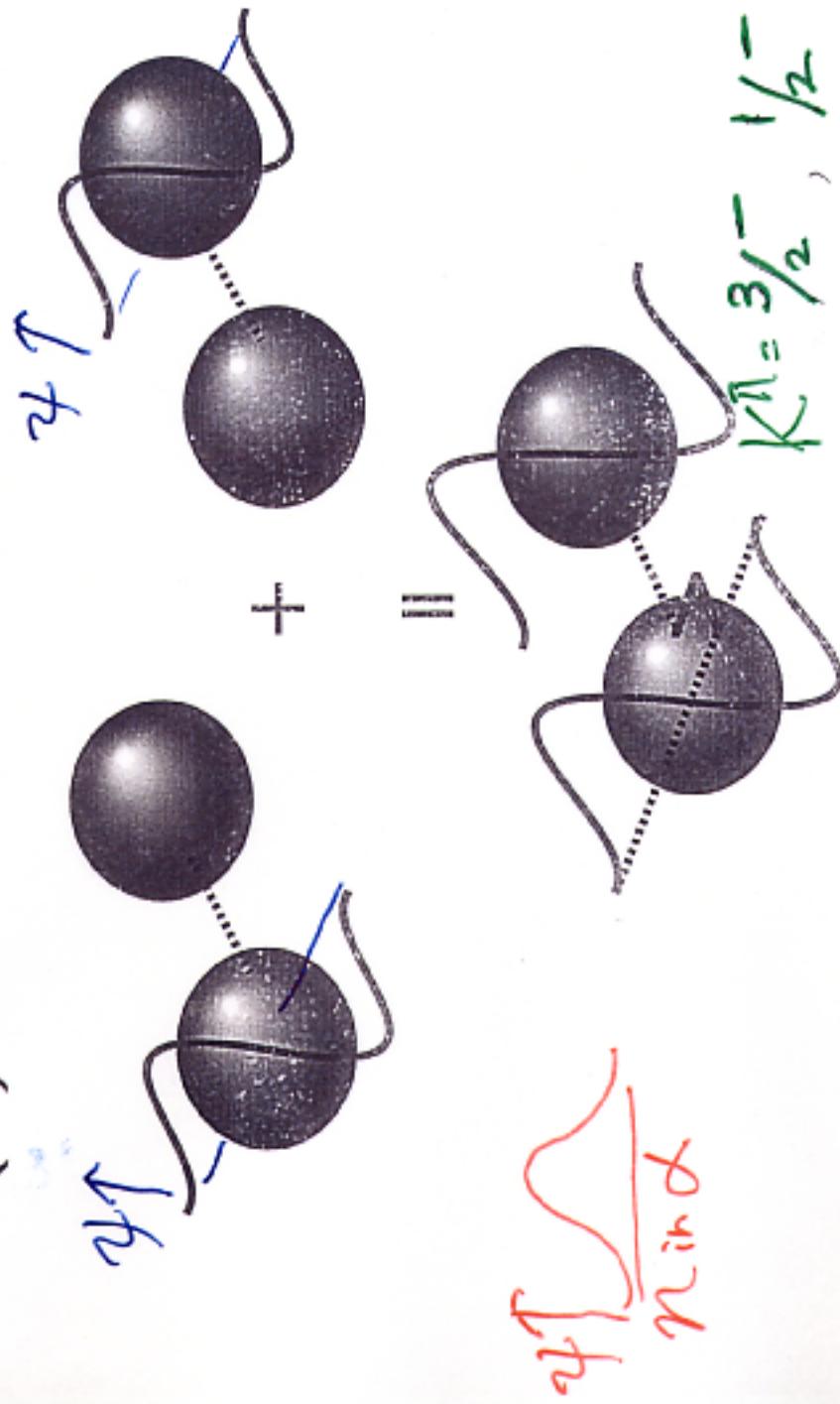


Fig. 3-2. Motion of valence neutron. The upper figures are for  $1/2^+$  state and the lowers for  $3/2^+$  state. (a) and (b): Contour maps of the total binding energy in MeV. Cross mark ( $\times$ ) denotes a position of an  $\alpha$ -particle. (c) and (d): Two-dimensional reduced width amplitudes  $|Z_L^J|$  for  $1/2^+$  ( $K=1/2, \mu=1/2$ ) and  $3/2^+$  ( $K=3/2, \mu=1/2$ ) configurations. Dotted line in (c) denotes nodal points. (e) and (f): The total energy curves vs  $R$  in the weak-coupling scheme  $[L(^9\text{Be}) \otimes l_J(n)]$ . (g) and (h): Neutron reduced width amplitude  $q_{LL}^{J*}$  (a). (Solid line:  $2\alpha+n$  method. Broken line: LCAO approximation.)

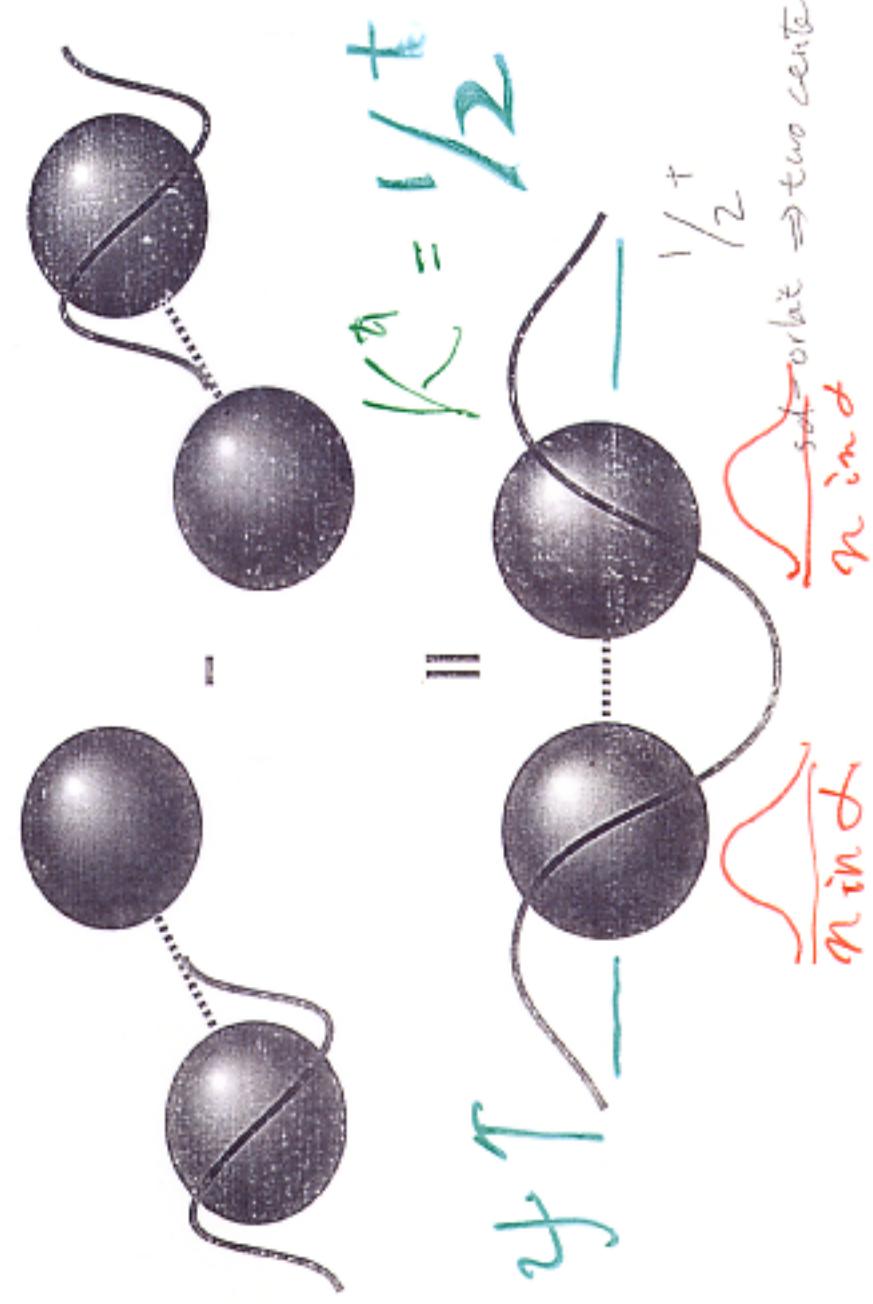
S. Okabe and Y. Abe  
P.T.P. 61, 1049 (1979)

(a)  $\pi$ -orbit One node



p-orbit  $\Rightarrow$  two center

(b)  $\sigma$ -Orbit Two nodes



$n$  node  $\Rightarrow$  two center

$n$  node

*Y.Kanada-En'yo H.Horiuchi and A.Ono*  
*Phys. Rev. C52 628 (1995)*

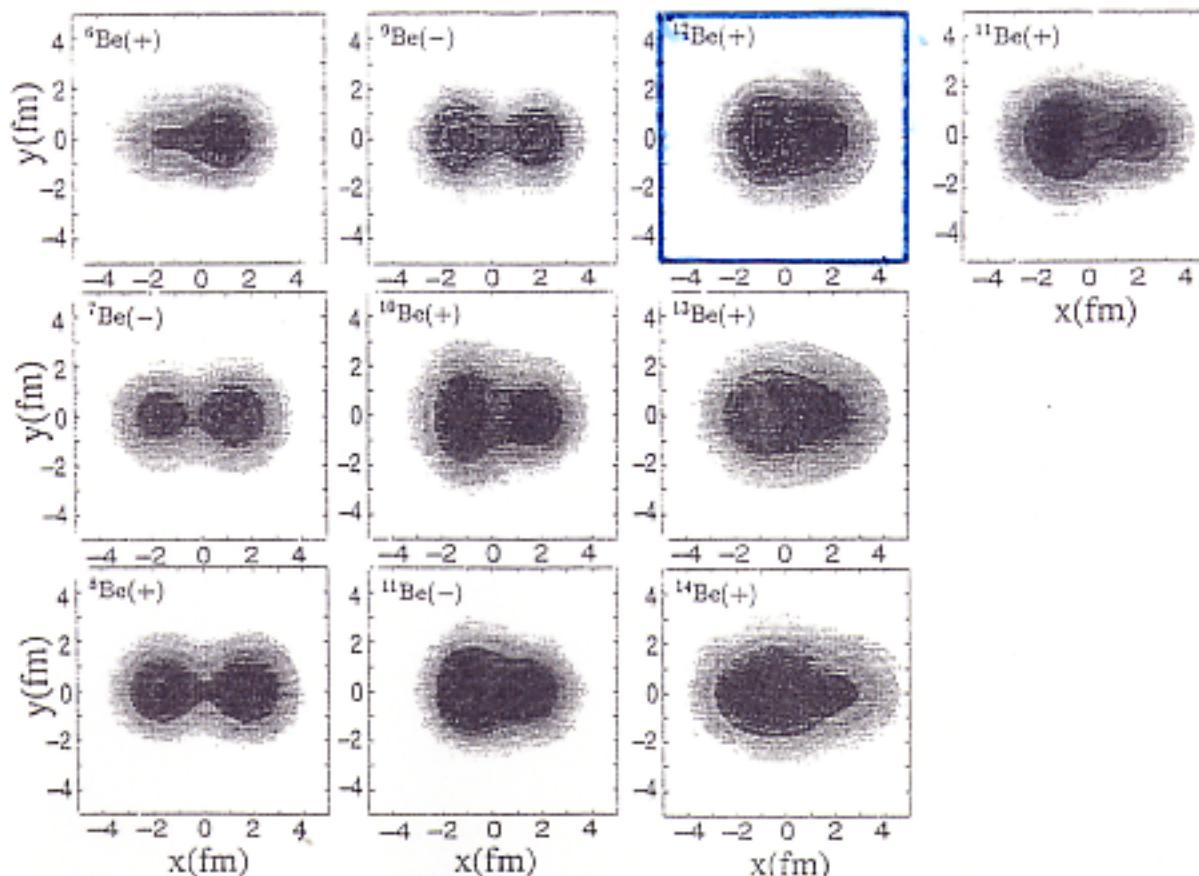


FIG. 9. Matter density distribution of AMD states of Be isotopes. The intrinsic density before parity projection is shown. Density is projected to a  $x$ - $y$  plane and integrated along the  $z$  axis perpendicular to the plane. Units of  $x$  and  $y$  axes are in fm.

Europhys. Lett., 34(1)  
(1996) 7,

$^7\text{Li}(^7\text{Li}, \alpha ^6\text{He})\alpha$   
 $^7\text{Li}(^7\text{Li}, \alpha \alpha)^6\text{He}$

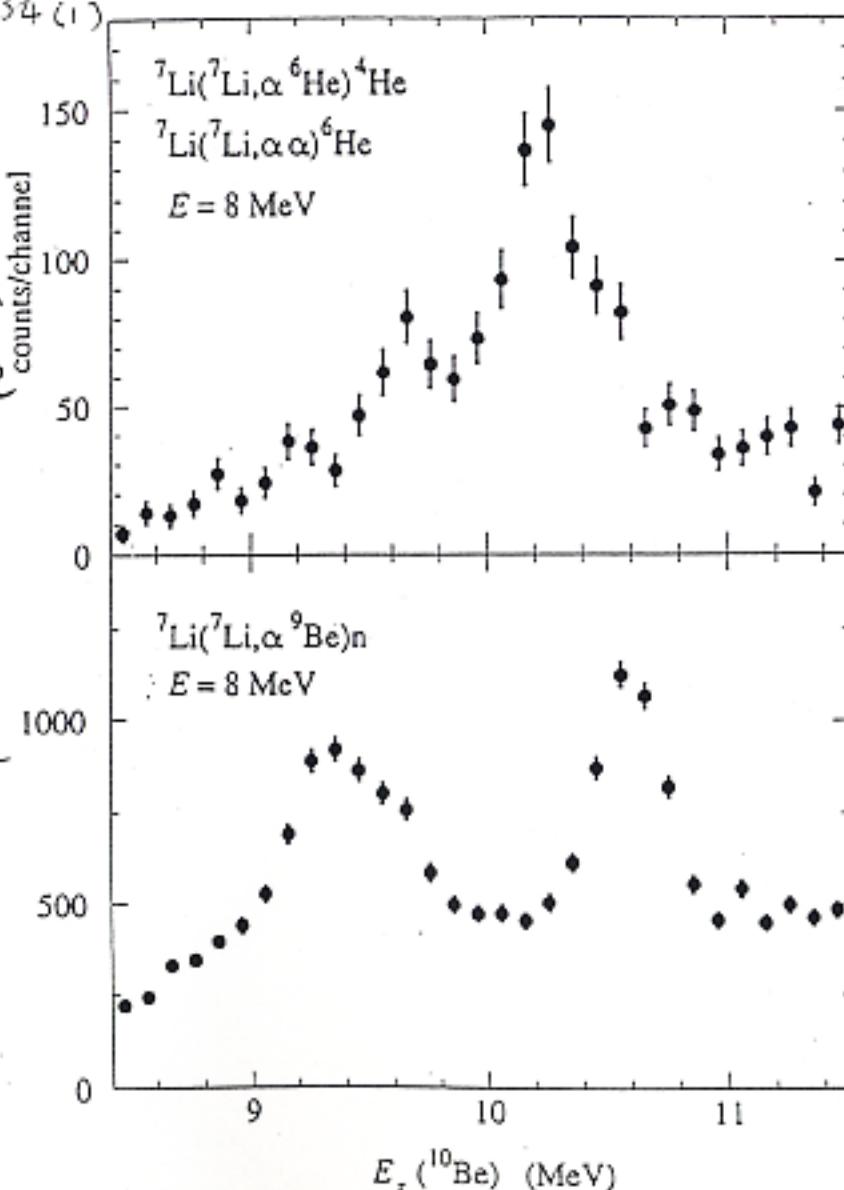
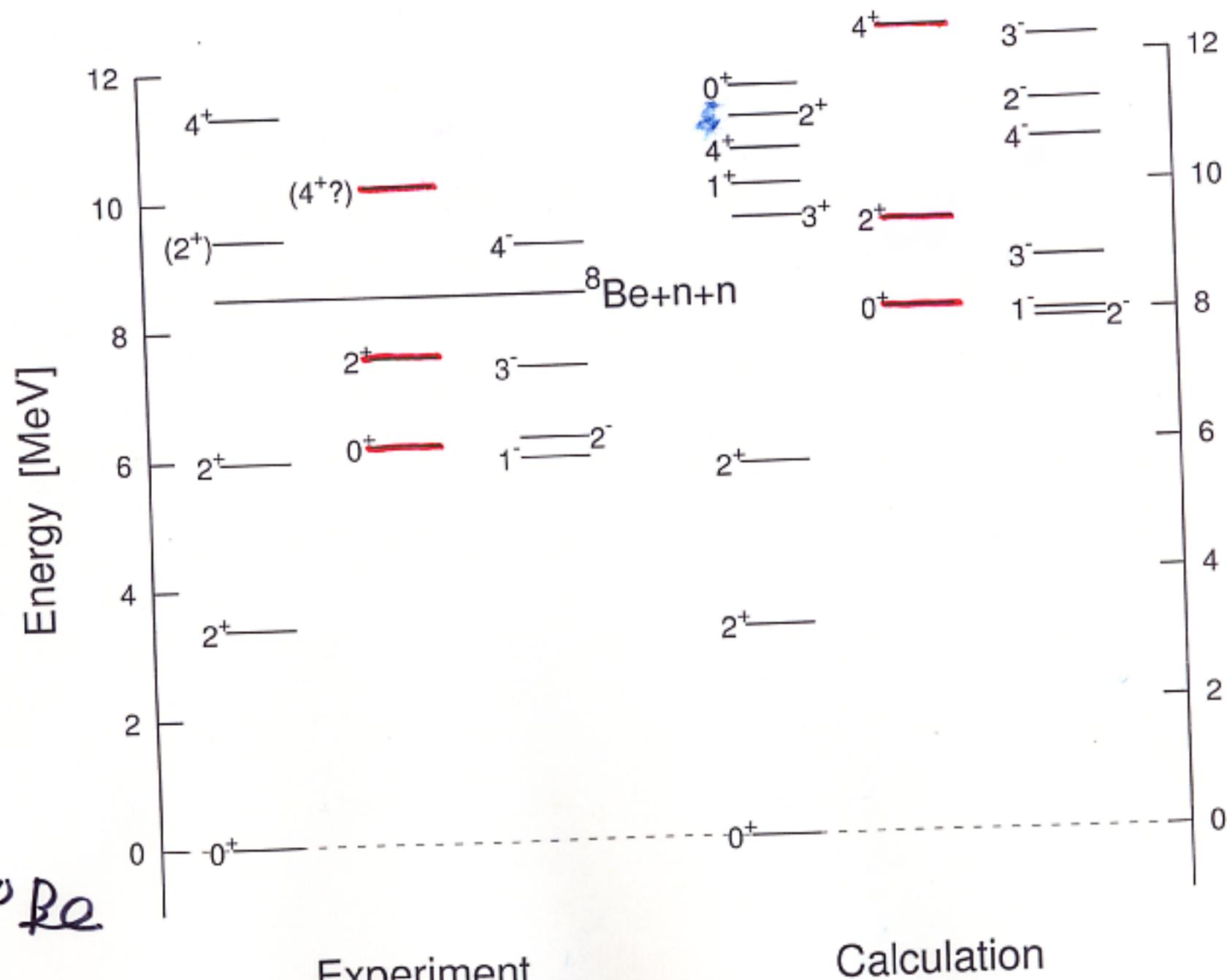


Fig. 3. –  $^{10}\text{Be}$  excitation spectra obtained from the  $^7\text{Li}(^7\text{Li}, \alpha ^6\text{He})^4\text{He}$  and  $^7\text{Li}(^7\text{Li}, \alpha ^9\text{Be})n$  reactions at  $E_i = 8$  MeV (the large difference in the number of events for these two processes is mainly due to the larger  $^9\text{Be}$  detector solid angle).

29/Jun/99

$^{10}\text{Be}$



$d + d + h + h$

N. Itagaki and S. Okabe, PRC 61(2000)  
 $\frac{nn\pi\pi}{nn\pi\pi}$

Freer et al.  
Phys. Rev. Lett. 82 (1999)  
Phys. Rev C63 (2001)

Cluster structure  
around 15 MeV region

Theoretical analysis:  
Ito, Phy. Rev. C62 (2000)

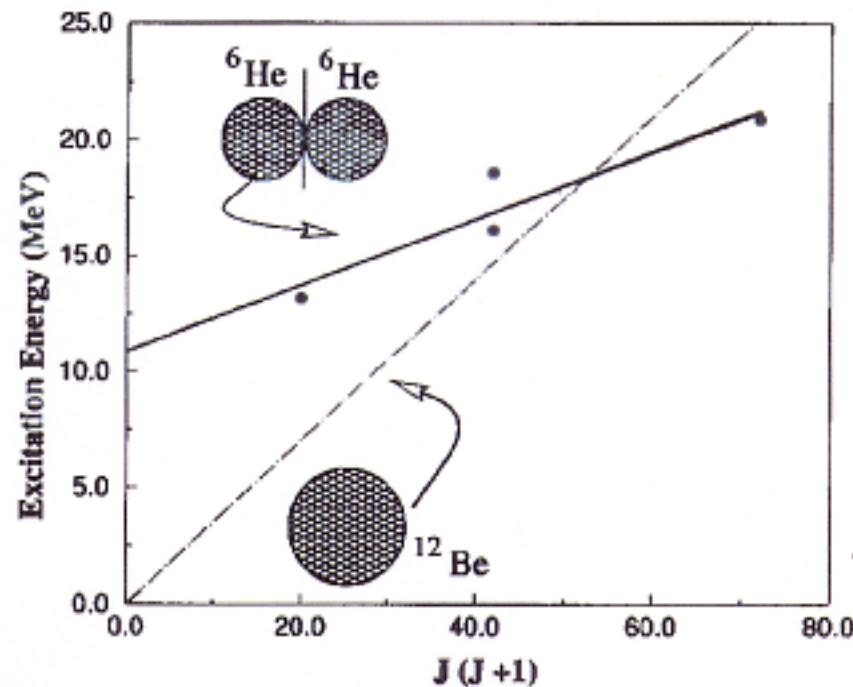
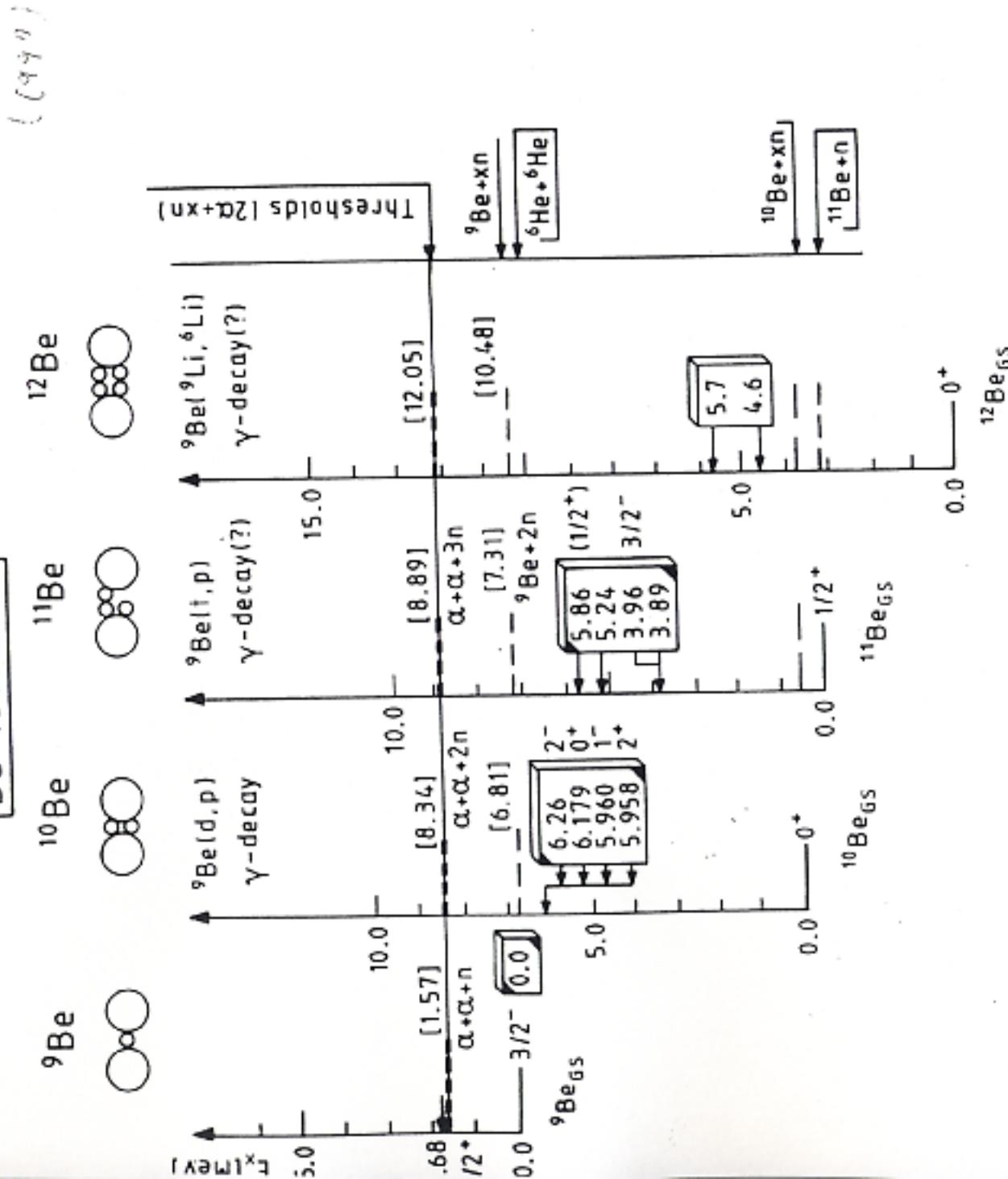


FIG. 4. The energy-spin systematics of the  ${}^6\text{He} + {}^6\text{He}$  breakup states (black dots). The solid line is a linear fit to the four points, and the dot-dashed line shows the extrapolated trajectory of a ground state band with a rotational energy of 350 keV.

## DIMERS BASED ON $(\alpha + \alpha) + x$ NEUTRONS

### Be-ISOTOPES



2. - Energy diagrams for dimers of the beryllium isotopes. The excitation energies (and thresholds) are shown relative to the energy of the  $(\alpha + \alpha + xn)$  threshold. Only the first possible generic two-center states are indicated (without rotational excitations) in boxes.

W. von Oertzen  
(1991)

# The puzzle of $^{12}\text{Be}$

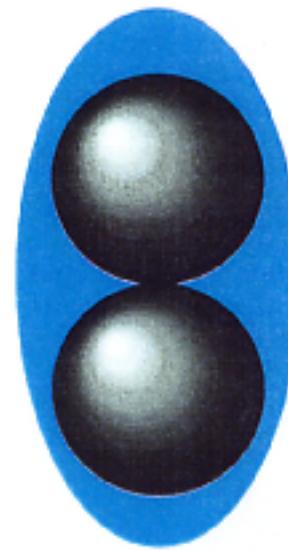
## Two Intrinsic Configurations

1. spherical configuration



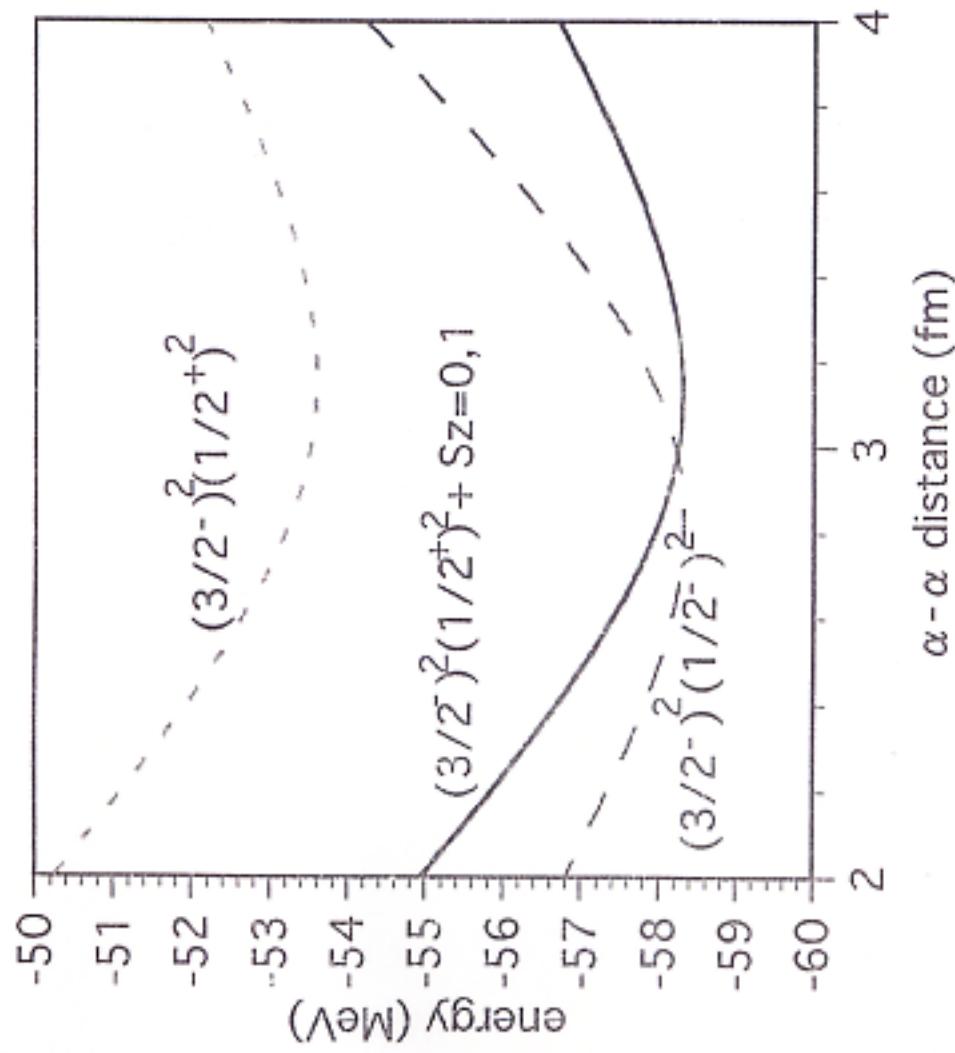
N=8 magic number

2. deformed (clusterized)  
configuration



Small O<sup>+</sup> - 2<sup>+</sup> level spacing  
slow  $\beta$  decay to  $^{12}\text{B}$

**F.C. Barker, J. Phys. C 2 L45 (1976)**  
T.Suzuki and T.Otsuka Phys. Rev. C56 (1997)



N. Itagaki, S. Okabe, and K. Ikeda  
 PRC 62 (2000), Fig. 3, 4, 301

$$^{12}\text{Be} = \alpha + \alpha + 4n$$

Iwasaki et al.  
Phys. Lett. B 481 (2000),  
Phys. Lett. B 491 (2000)

$\beta_2$  was deduced

$1^-$  state has been observed

$B(E1) = 0.051 \text{ e}^2 \text{fm}^2$

Theoretical analysis:  
Sagawa et al.  
Phys. Rev C63 (2001)

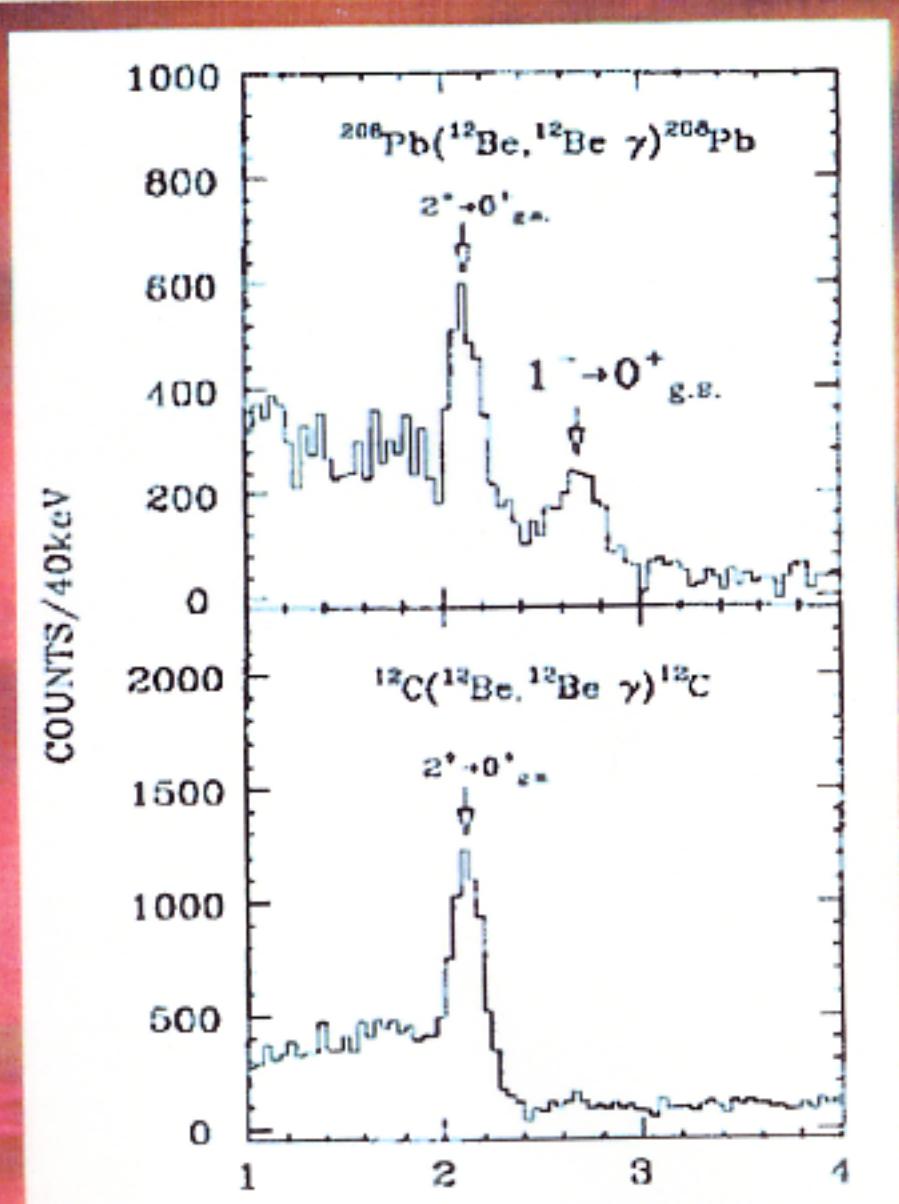
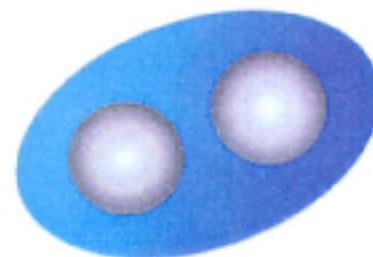


Fig. 1. Doppler-corrected  $\gamma$  ray energy spectra measured in the inelastic scattering of  $^{12}\text{Be}$  on the lead (top) and carbon (bottom) targets.

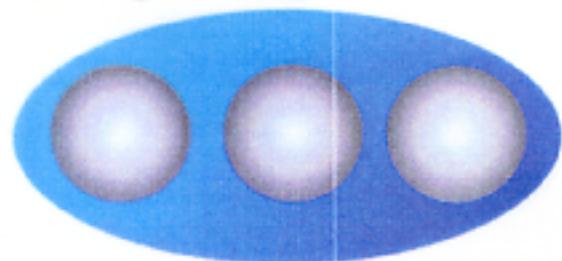
He halo



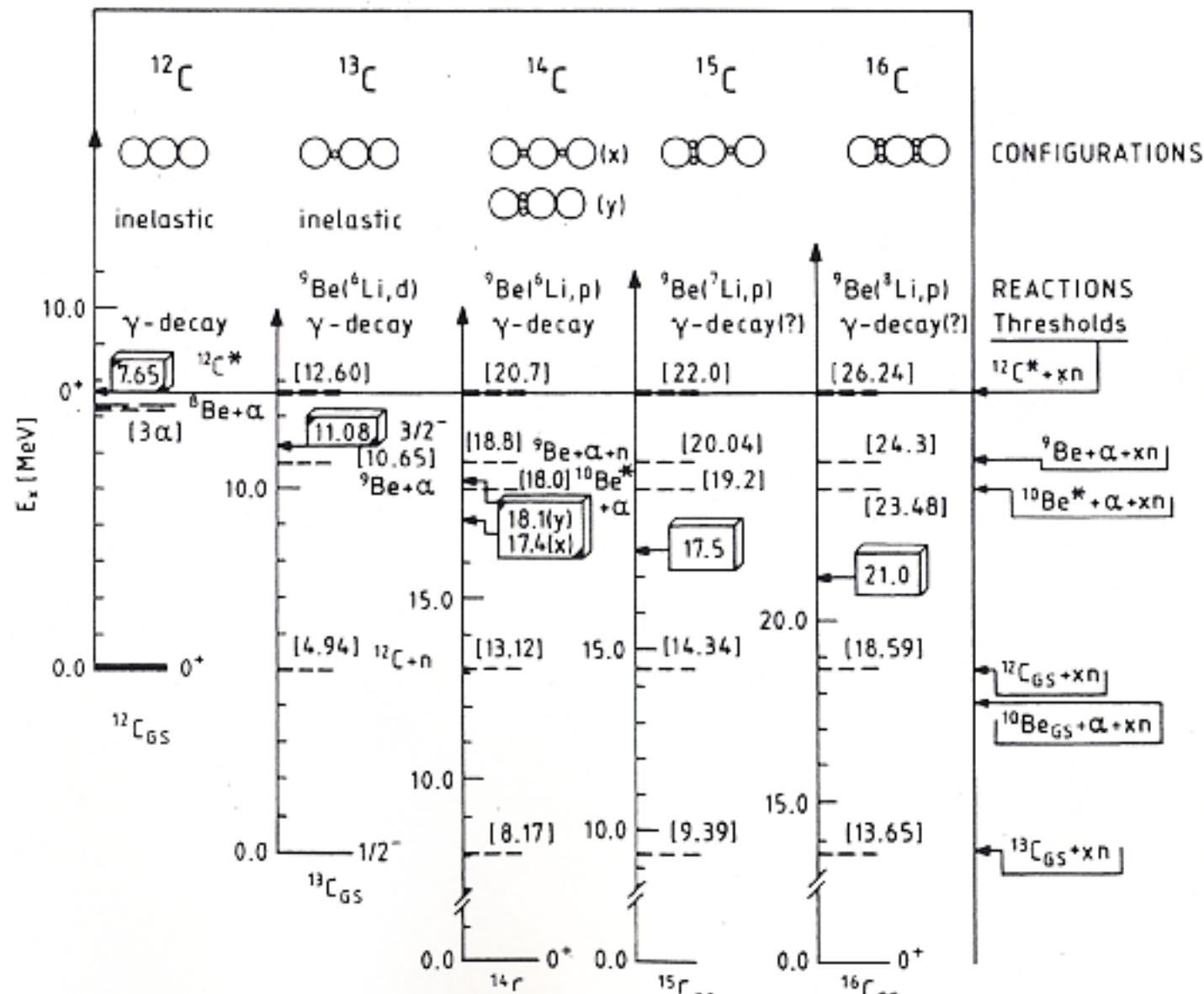
Be cluster

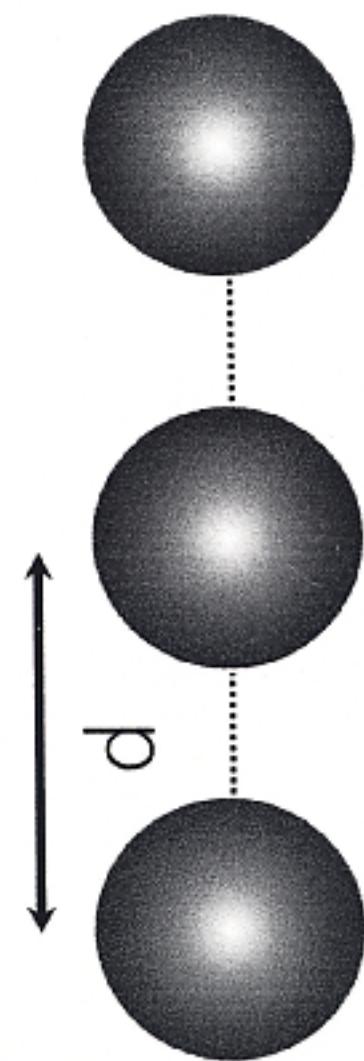


C polymer

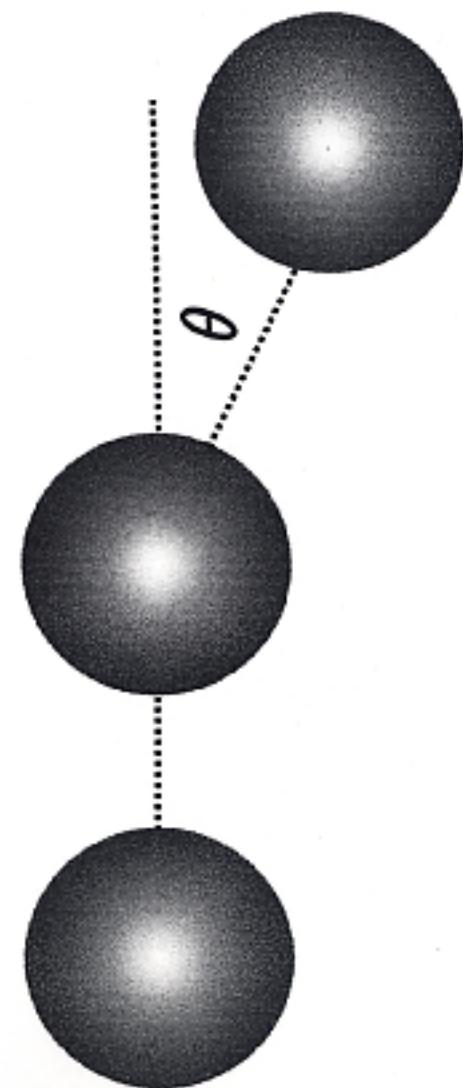


## CHAIN STATES OF CARBON ISOTOPES





(a)



(b)

Fig. 3

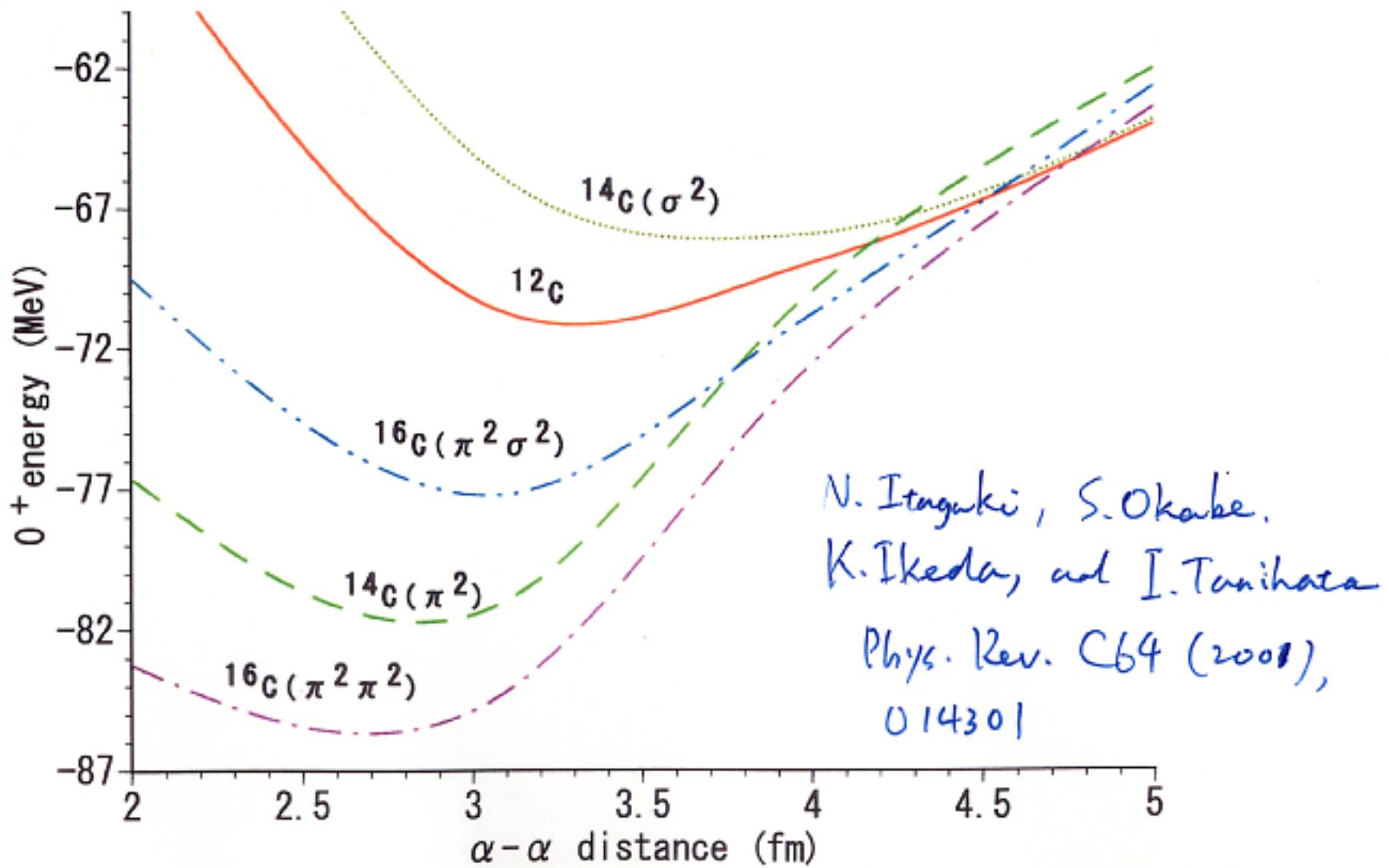
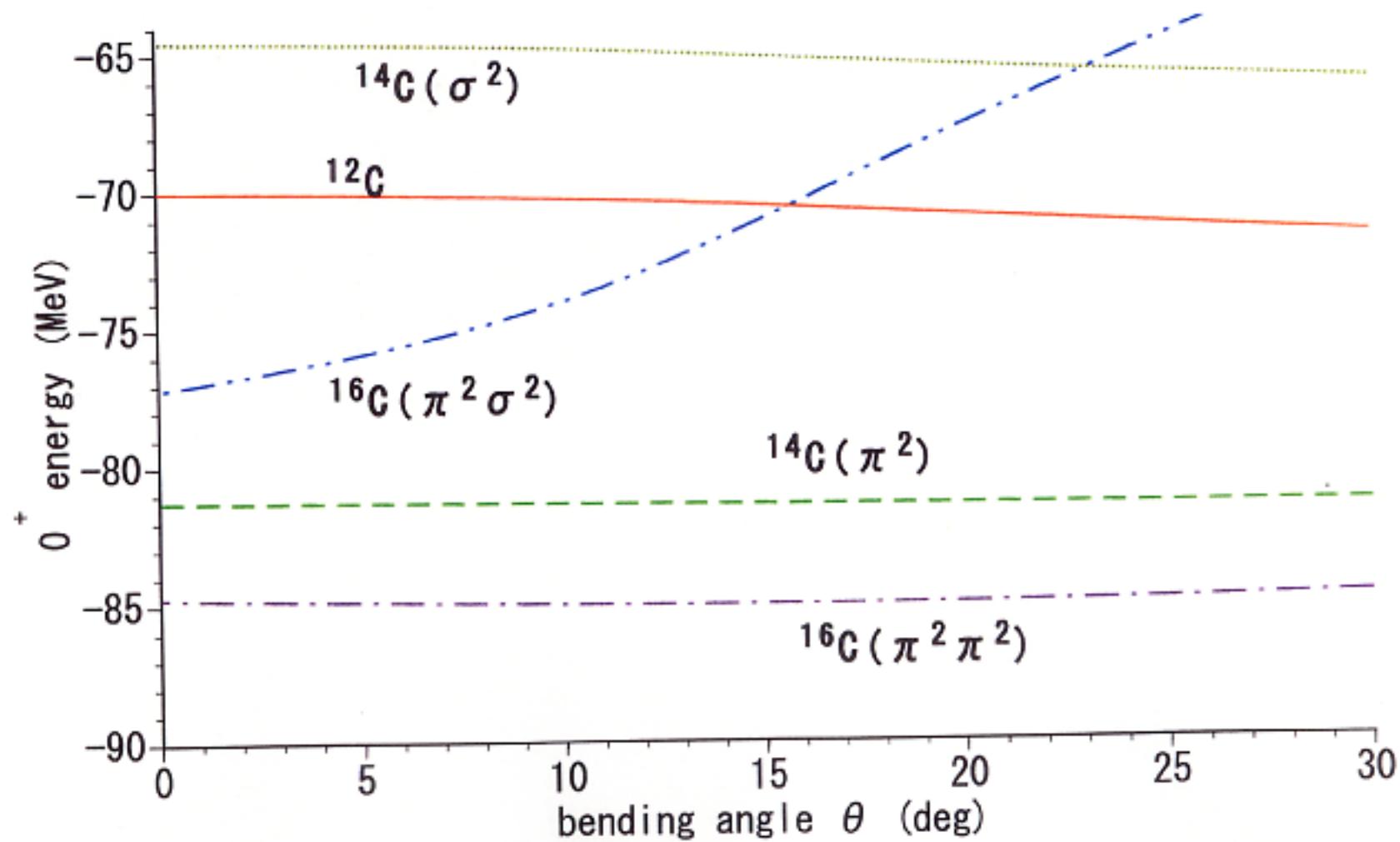


Fig. 4



N. Itagaki, S. Okabe, K. Ikeda, and I. Tanihata  
Phys. Rev. C64 (2001), 014301

## Superdeformation in the $N = Z$ Nucleus $^{36}\text{Ar}$ : Experimental, Deformed Mean Field, and Spherical Shell Model Descriptions

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(Received 8 May 2000)

A superdeformed rotational band has been identified in  $^{36}\text{Ar}$ , linked to known low-spin states, and observed to its high-spin termination at  $J'' = 16^+$ . Cranked Nilsson-Schulinsky and spherical shell model calculations assign the band to a configuration in which four  $p\ell$ -shell orbitals are occupied, leading to a low-spin deformation  $\beta_2 \approx 0.45$ . Two major shells are active for both protons and neutrons, yet the valence space remains small enough to be confronted with the shell model. This band thus provides an ideal case to study the microscopic structure of collective rotational motion.

PACS numbers: 21.10.Re, 21.60.Cs, 23.20.I.W, 27.30.+t

shown in Fig. 1(a), and a partial decay scheme for  $^{36}\text{Ar}$  is presented in Fig. 2. Transitions denoted by diamonds in Fig. 1(a) firmly link the band to known low-spin states. For example, Fig. 1(b) shows a spectrum of the band obtained with a single gate set on the 4950-keV linking transition. The spin and purity assignments in Fig. 2 are based on angular distribution measurements which establish stretched-E2 character for all of the in-band and high-energy linking transitions. Two examples are shown in Figs. 1(c) and 1(d). Legendre polynomial fits to these data (solid curves) yield ( $a_2, a_4$ ) coefficients of  $(0.31 \pm 0.02, -0.09 \pm 0.03)$  and  $(0.31 \pm 0.02, -0.08 \pm 0.02)$  for the 3352 and 4166-keV  $\gamma$  rays and establish the  $6^+$  and  $4^+$  states of the band. The 4951-keV  $2^+$  state was known from earlier  $^{35}\text{Cl}(p, \gamma)^{36}\text{Ar}$  studies [16]. These studies also identified a  $(0^+)$  state at 4329 keV which, based on the regular rotational spacing, is presumed to be the SD bandhead. The  $\gamma$ -ray branchings from the  $4^+$  and  $6^+$  states of the band yield in-band to decay-out  $B(E2)$  ratios of  $148 \pm 6$  and  $86 \pm 4$ , consistent with strongly enhanced in-band transitions. Assuming a similar ratio of  $B(E2)$ 's for the  $2^+$  state, a 622-keV  $2^+ \rightarrow 0^+$  in-band transition would be expected to carry  $\sim 0.3\%$  of the decay intensity. The present experiment yields an upper limit of 1.0% for this unobserved branch.

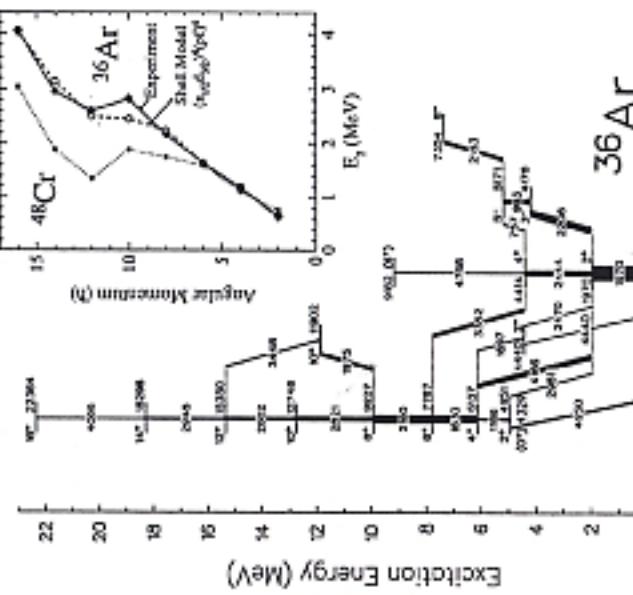


FIG. 2. Partial decay scheme for  $^{36}\text{Ar}$  showing the superdeformed band (left). Transition and level energies are given to the nearest keV and arrow widths are proportional to transition intensities. The inset compares the experimental and shell model “backbending” plots for the SD band in  $^{36}\text{Ar}$ , as well as experimental values for the ground band of  $^{45}\text{Cr}$  [2–4].

The single particle wave packet  $\psi_i$  consists of a spatial part,  $\phi_i$ , spin part,  $\chi_{si}$  and isospin part,  $\chi_{\tau i}$ . In the old version of the AMD,<sup>7)</sup> the spatial part  $\phi_i(r_j)$  is represented in the spherical Gaussian form

$$\phi_i(r_j) = (2\nu/\pi)^{3/4} \exp\left\{-\nu(r_j - Z_i)^2\right\}. \quad (2)$$

In the new version of AMD developed in this study, we employ the deformed Gaussian instead of the spherical one as the spatial part. This is given in Eq. (3). In the following, for simplicity, we call the old version of AMD “spherical AMD” and the new version of AMD “deformed AMD”. The deformed Gaussian has an independent width parameter for each direction,  $x$ ,  $y$  and  $z$ ;

$$\phi_i(r_j) = (8\nu_x\nu_y\nu_z/\pi^3)^{1/4} \exp\left\{-\sum_{\sigma=x,y,z} \nu_\sigma (r_j - Z_i)_\sigma^2\right\}. \quad (3)$$

Here, in Eqs. (2) and (3), complex variables  $Z_i$ , which represent the positions of the wave packets in the phase space, are the variational parameters and depend on the individual nucleons. The width parameters  $\nu$  [Eq. (2)] and  $\nu_x, \nu_y$  and  $\nu_z$  [Eq. (3)] are also variational parameters, but are common to all nucleons. Using the deformed spatial part of the single particle wave packet, we expect the following effects. 1) When the mean field is deformed, AMD single particle wave packets will also be deformed. Therefore, with deformed AMD single particle wave packets, we

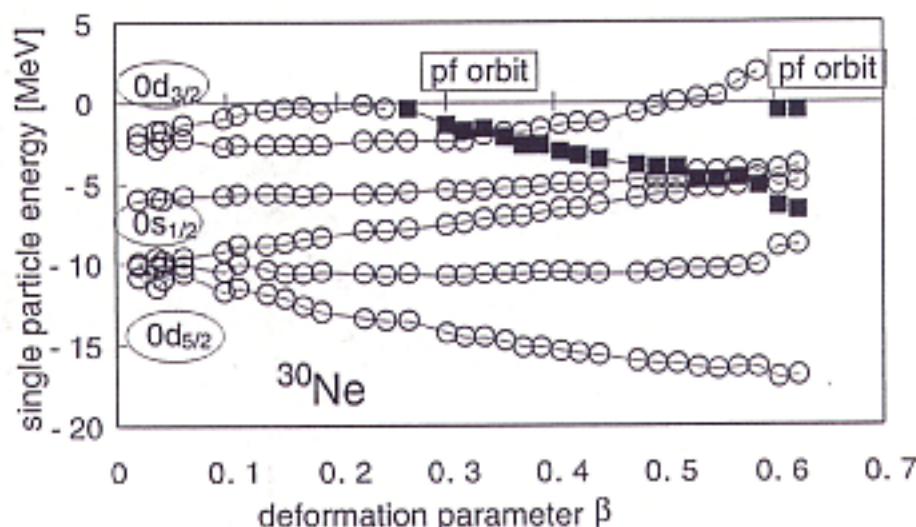
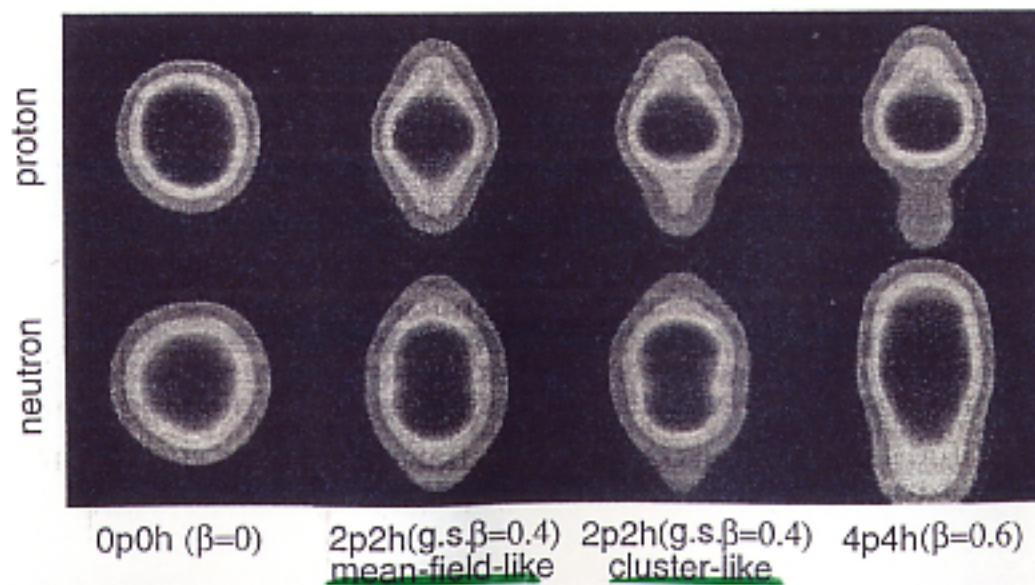


Fig. 5. Hartree-Fock single particle energies of neutrons in  $^{30}\text{Ne}$  as functions of the deformation parameter  $\beta$ . Open circles (filled boxes) correspond to the plus (minus) parity orbits.



## Quantum Monte Carlo calculations of $A=8$ nuclei

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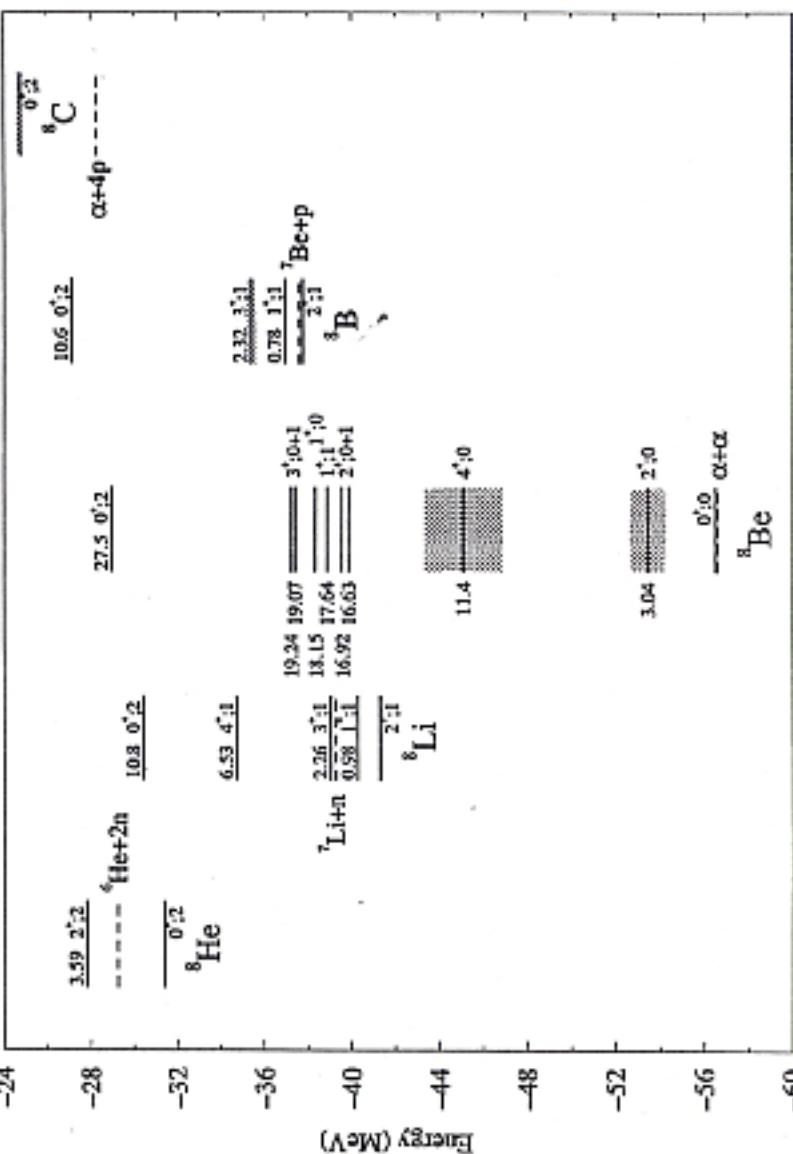
(Received 7 February 2000; published 1 June 2000)

We report quantum Monte Carlo calculations of ground and low-lying excited states for  $A=8$  nuclei using a realistic Hamiltonian containing the Argonne  $a_18$  two-nucleon and Urbana IX three-nucleon potentials. The calculations begin with correlated eight-body wave functions that have a filled  $\alpha$ -like core and four  $p$ -shell nucleons  $LS$  coupled to the appropriate ( $J^\pi, \Gamma$ ) quantum numbers for the state of interest. After optimization, these variational wave functions are used as input to a Green's function Monte Carlo calculation made with a new constrained path algorithm. We find that the Hamiltonian produces a  $^8\text{Be}$  ground state that is within 2 MeV of the experimental resonance, but the other eight-body energies are progressively worse as the neutron-proton asymmetry increases. The  $^6\text{Li}$  ground state is stable against breakup into subclusters, but the  $^8\text{He}$  ground state is not. The excited state spectra are in fair agreement with experiment, with both the single-particle behavior of  $^8\text{He}$  and  $^8\text{Li}$  and the collective rotational behavior of  $^8\text{Be}$  being reproduced. We also examine energy differences in the  $\Gamma=1,2$  isomultiplets and isospin-mixing matrix elements in the excited states of  $^8\text{Be}$ . Finally, we present densities, momentum distributions, and studies of the intrinsic shapes of these nuclei, with  $^8\text{Be}$  exhibiting a definite  $2\alpha$  cluster structure.

PACS number(s): 21.10.-k, 21.45.+v, 21.60.Ka, 27.20.+n

## QUANTUM MONTE CARLO CALCULATIONS OF $A=8$ NUCLEI

PHYSICAL REVIEW C 62 014001



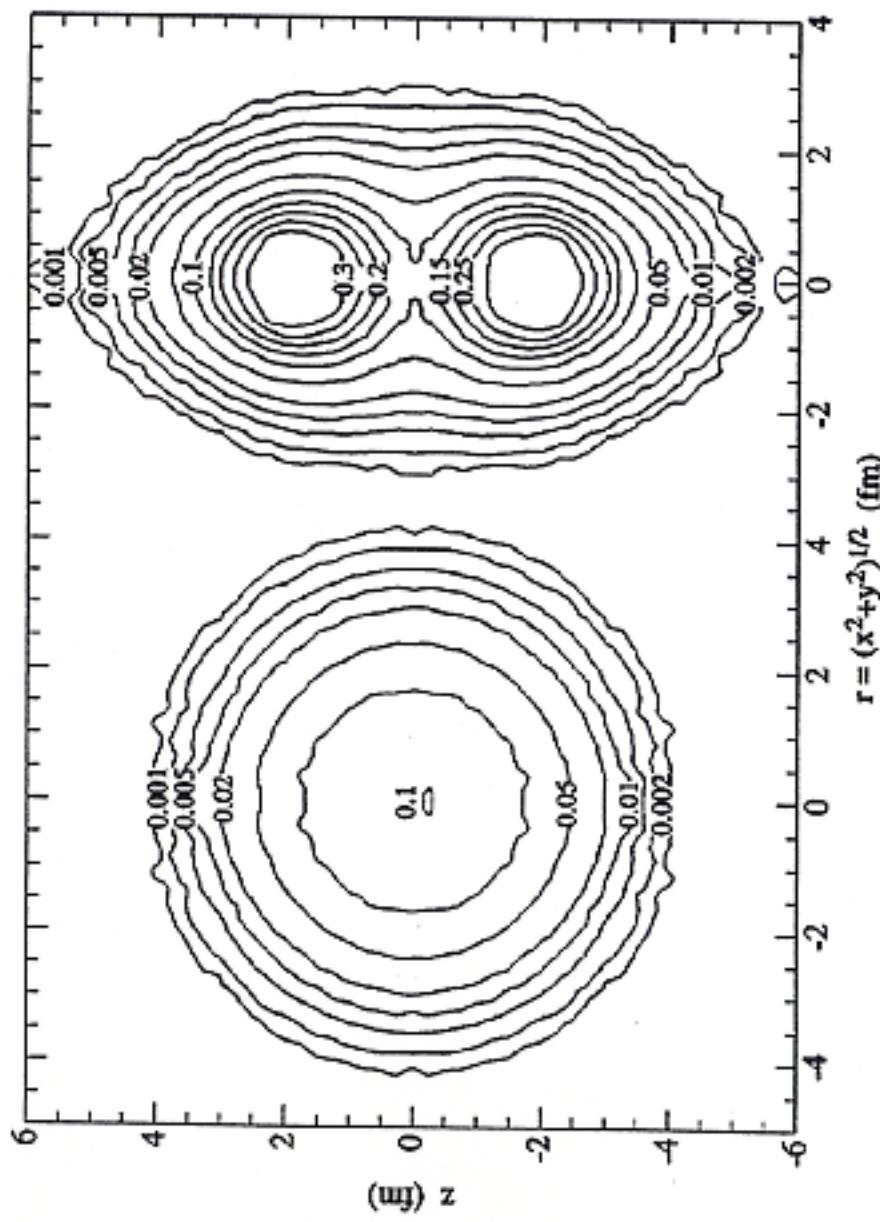


FIG. 15. Contours of constant density, plotted in cylindrical coordinates, for  ${}^8\text{Be}(0^+)$ . The left side is in the "laboratory" frame while the right side is in the intrinsic frame.

in Tables III and IV, for the  $(2^+; 0, 1)$  and  $(3^+; 0, 1)$  pairs are indeed fairly close to each other, since they have similar sizes and signs for the largest components. However, the  $(1^+; 0, 1)$  states are not so similar, particularly due to the large  $T=1$  [1P<sub>3</sub>1] component that is not available to the  $T=0$  state. This may be why the  $E_{01}(1)$  is noticeably smaller, which shows up through the change in sign of the magnetic moment contribution.

# Recent development and future perspective

Cluster study has long history  $\sim 40$  years

**It does not mean that everything is solved**

Now is the time to utilize knowlage and  
technic developed until now

We can discuss new phenomena related to  
the physics of RIB, Gamma sphere, Euro ball . . .

Theoretical iterest:

Is cluster deformation really defferent from  
mean-field deformation?

**unified model is necessary**