Reaction Theory (for exotic nuclei)

Jeff Tostevin, University of Surrey, UK.

- How do we learn about nuclear structure (single particle) from reactions? Can we?

$$\frac{d\sigma}{d\Omega}, \sigma_R, \sigma_{-n}, \frac{d\sigma}{dp_{//}} \Longleftrightarrow \Phi_A, \phi_{n\ell j} \quad ?$$

- Methods (currently) available for practical calculations of wavefunctions of reacting systems + observables; (approximations, reliability, availability)

- Overview of basic concepts, current adventures!

No reactions 'black box' → several boxes

need

Awareness of timescales, mechanisms, interactions

RIB physics — secondary beams — many traditional spectroscopic tools unavailable (e,e),(e,e′p), (currently) or much more difficult, transfer reactions (p,p′), (p,2p)....

Reaction Theory — new regimes of very weak binding — driplines + halos — inclusive measurements *

Structure Theory — reasonably sophisticated — now precise shell model predictions + few body models — correct?

- Not easy to treat reaction and structure aspects with equal rigour — depends on experimental choices
  — energy, target (Z), detection geometry

use approximations which make structure input and sensitivity more transparent

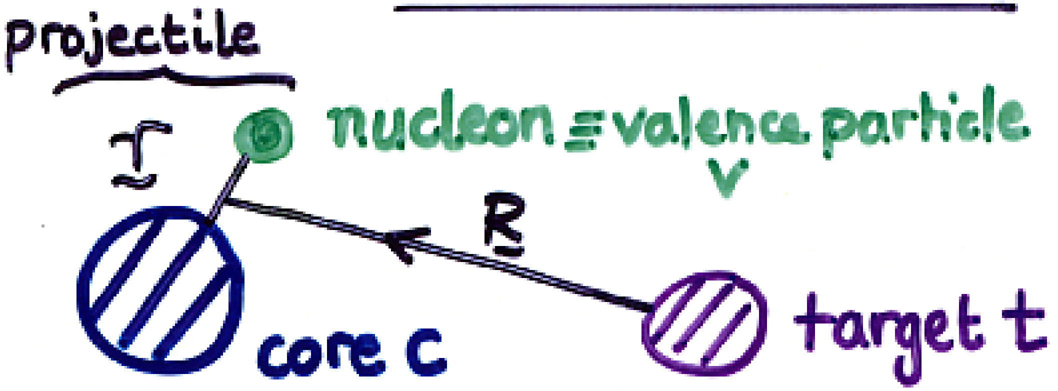
Single particle spectroscopy – reactions which

- minimally rearrange constituents (small number of degrees of freedom – excite a single nucleon if possible) – **DIRECT REACTIONS** (Austern, Satchler, Feshbach)

- elastic scattering
  - inelastic excitation
  - breakup – N removal without target excitation { diffraction / dissociation }
  - stripping – N removal + target excitation
  - transfer reactions

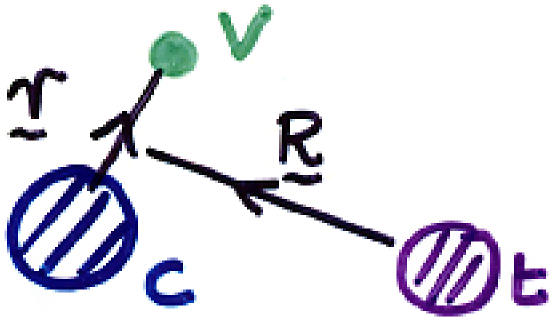
No practical many-body reaction theory →

FEW-BODY models ($n = 2, 3 \ldots$)



- Construct effective Hamiltonian H
- Solve, as best we can, $H\psi = E\psi$

Few-body model → $H\Psi = E\Psi$



$$H = \underbrace{T_r + V_{vc}}_{\text{projectile 'internal' motion}} + \underbrace{T_R + V_{ct} + V_{vt}}_{\text{projectile - target motion}} \equiv H_p + T_R + U(\underset{\sim}{R},\underset{\sim}{r})$$

$$\left.\begin{array}{l} \text{projectile g.s.} \quad \langle \underset{\sim}{r} | 0 \rangle = \phi_0(\underset{\sim}{r}) \\ \qquad\qquad \text{separation energy} \quad \varepsilon_0 \\ H_p\phi_0 = (T_r + V_{vc})\phi_0 = -\varepsilon_0\phi_0 \end{array}\right\} \text{weakly bound}$$

boundary conditions

$$\Psi(\underset{\sim}{r},\underset{\sim}{R}) = \underline{\phi_0(r)}\, e^{i\underset{\sim}{K}\cdot\underset{\sim}{R}} + \cdots\cdots \longrightarrow \text{structure enters here}$$

$\left.\begin{array}{l} V_{ct} \\ V_{vt} \end{array}\right\}$ effective (complex) interactions with target (nuclear + Coulomb)

— fitted to data – phenomenology

— theory, e.g.

$$V_{12}(\underset{\sim}{R}) = \int d\underset{\sim}{r_1} \int d\underset{\sim}{r_2}\, \rho_1(\underset{\sim}{r_1})\rho_2(\underset{\sim}{r_2})\, \overbrace{t_{NN}(\underset{\sim}{R} + \underset{\sim}{r_2} - \underset{\sim}{r_1})}^{\text{or } v_{NN}(\cdots)}$$

Results from scattering theory - point particles

$$\left.\begin{array}{l} E,\ k = (2\mu E/\hbar^2)^{1/2} \\ \left\{-\dfrac{\hbar^2}{2\mu}\nabla^2 + V\right\}\psi = E\psi \end{array}\right\} \text{scattering boundary conditions}$$

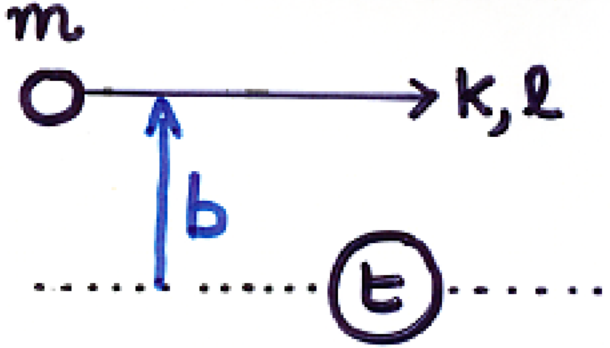
Partial wave expansion of ψ

$$u_\ell(r) \xrightarrow[r\to\infty]{} \frac{i}{2}\left\{\underbrace{H_\ell^{-}(kr)}_{\text{incoming wave}} - \underset{\substack{\uparrow \\ \text{partial wave S-matrix} \\ \text{(amplitude of outgoing wave)}}}{S_\ell}\, \overbrace{H_\ell^{(+)}(kr)}^{\text{outgoing wave}}\right\}$$

S_ℓ is probability amplitude that particle survives collision with angular momentum ℓ

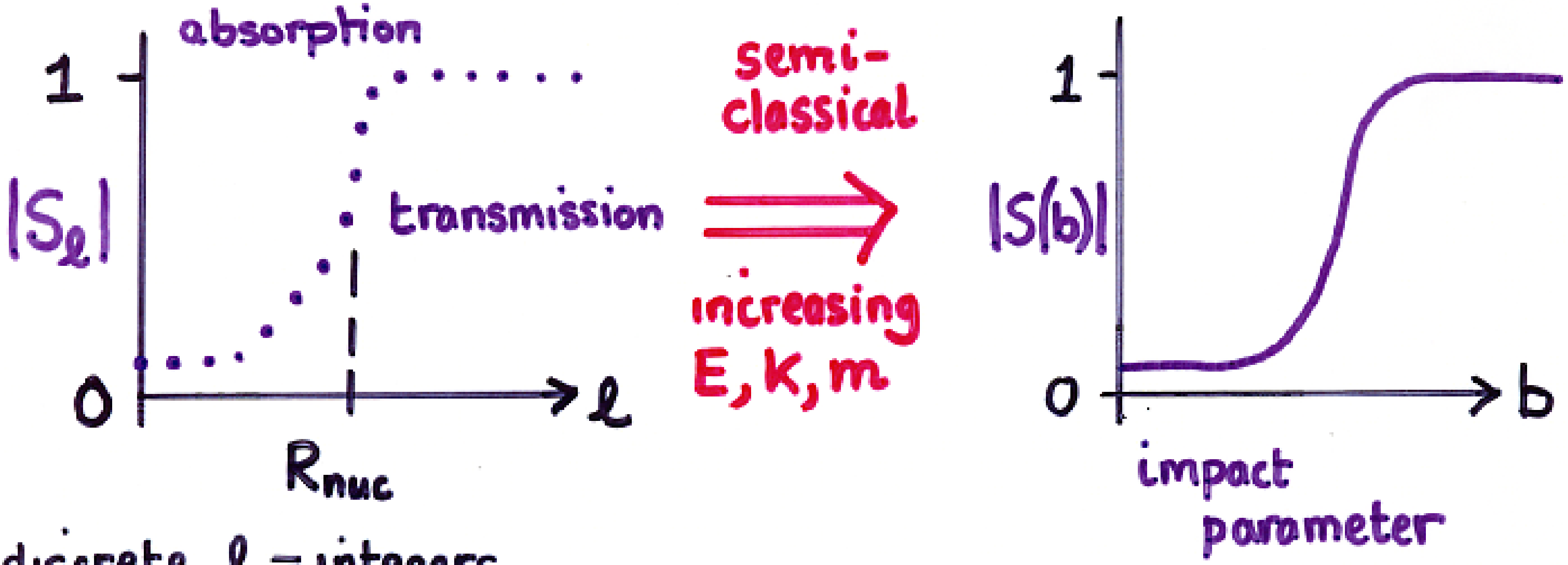
$|S_\ell|^2$ = survival probability (≤ 1 if V is complex)

$\{S_\ell = e^{2i\delta_\ell},\ \delta_\ell \equiv$ phase shift$\}$



$b \equiv$ impact parameter

$\ell \approx kb$ (actually $\ell + 1/2 \approx kb$)



discrete ℓ – integers

Observables – point particles

scattering from a complex potential can be only $\begin{cases} \text{elastic scattering} \\ \text{absorption/reaction} \end{cases}$

$$\sigma_{eL} = \frac{\pi}{k^2}\sum_{\ell}(2\ell+1)\,|1-S_\ell|^2 \longrightarrow \int d\underset{\sim}{b}\,|1-S(b)|^2$$

$$\sigma_R = \frac{\pi}{k^2}\sum_{\ell}(2\ell+1)\{1-|S_\ell|^2\} \longrightarrow \underbrace{\int d\underset{\sim}{b}}_{d\underset{\sim}{b}=2\pi b\,db}\{1-|S(b)|^2\}$$

$$\sigma_{tot} = \sigma_R + \sigma_{eL} = 2\int d\underset{\sim}{b}\,\{1-\mathrm{Re.}S(b)\}, \quad \text{etc.}$$

Eikonal approximation for point particles – approx. solution of Schrödinger equation

$$\left\{\nabla^2 + k^2 - \frac{2\mu}{\hbar^2}V(r)\right\}\psi = 0$$

assume $\psi(\underline{r}) = \underbrace{e^{i\underline{k}.\underline{r}}}_{\text{incident plane wave}}\ \underbrace{\omega(\underline{r})}_{\text{modifications due to V}}$

substitute in (still exact)

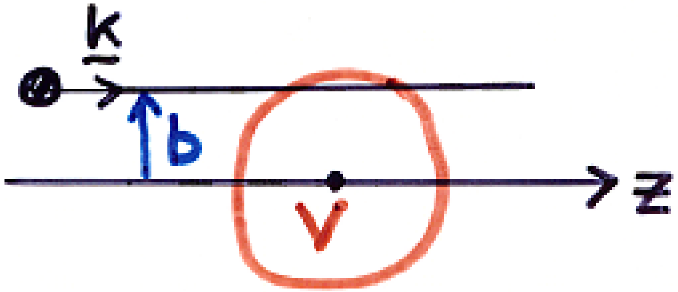
$$\left\{2\nabla\omega.\underset{\sim}{k}\, i - \frac{2\mu}{\hbar^2}V + \cancel{\nabla^2\omega}\right\}e^{i\underline{k}.\underline{r}} = 0$$

high energy (large k), smooth V(r), $\nabla^2\omega \ll 2\nabla\omega.\underline{k}$

$$\frac{d\omega}{dz} = -\frac{i\mu}{\hbar^2 k}V\omega$$

$$\omega(\underline{r}) = \exp\left\{-\frac{i\mu}{\underbrace{\hbar^2 k}}\int_{-\infty}^{z}dz'\,V(r')\right\}$$

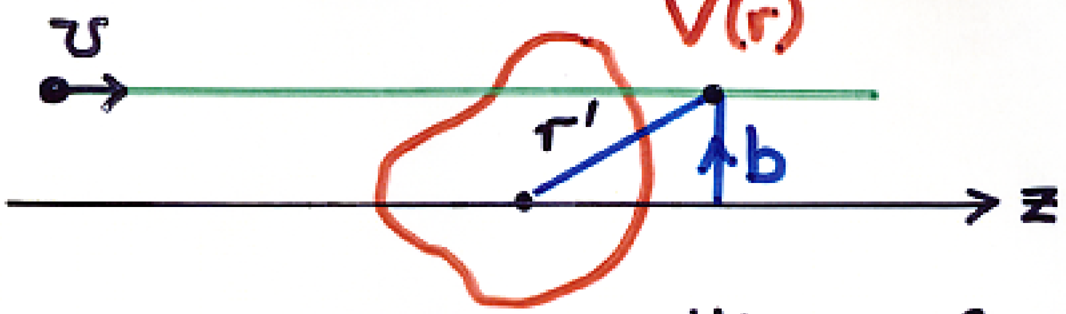
$\frac{\hbar k}{\mu} \equiv v$ = velocity



neglecting curvature ($\nabla^2\omega$) of ω

we have assumed effects of V(r) can be estimated by assuming straight line path — eikonal

Eikonal picture is:



$$\Psi(\underline{r}) = \omega(\underline{r})\, e^{i\underline{k}.\underline{r}} \equiv e^{i\underline{k}.\underline{r}} \exp\left\{\frac{-i}{\hbar v}\int_{-\infty}^{z} dz'\, V(r')\right\}$$

and as $z \to +\infty$, after collision

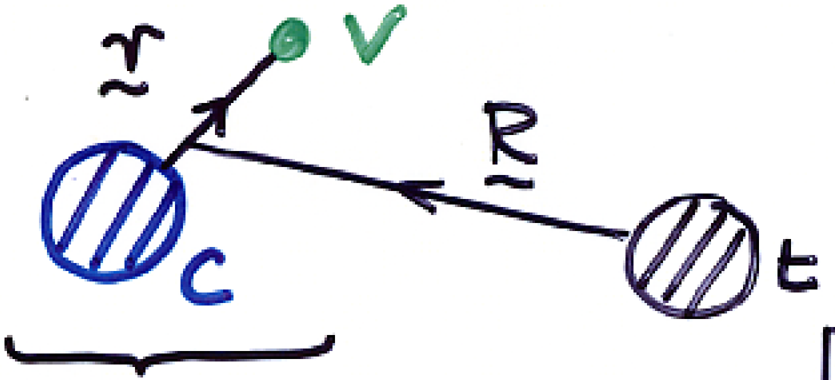
$$\Psi(\underline{r}) = e^{i\underline{k}.\underline{r}}\, \underbrace{\exp\left\{\frac{-i}{\hbar v}\int_{-\infty}^{\infty} dz'\, V(r')\right\}}$$

- function only of b –
- amplitude of outgoing plane wave after collision

⇒ eikonal approximation to S(b)

$$\Psi(\underline{r}) = S(b)\, e^{i\underline{k}.\underline{r}}$$ ⇒ generalises to few-body projectiles *

Return to few-body/composite systems



$$H_p = T_r + V_{cv}$$

$$U(\underset{\sim}{R},\underset{\sim}{r}) = V_{ct} + V_{vt}$$

no explicit internal structure to c and v constituents

$$\boxed{\mathcal{H} = H_p + T_R + U(\underset{\sim}{R},\underset{\sim}{r})}$$

- H_p has one bound state ϕ_0

  $$H_p \phi_0 = -\varepsilon_0 \phi_0$$

- All other states of H_p are unbound (continuum) scattering states

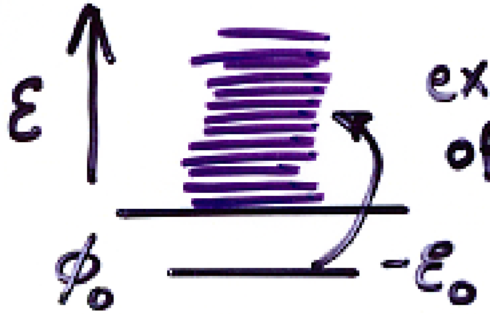
  $$H_p \phi_{\underset{\sim}{k}} = \varepsilon \phi_{\underset{\sim}{k}} \;;\; (\varepsilon, \underset{\sim}{k})$$

$$\left\{ \begin{array}{l} E_v \simeq \dfrac{m_v}{m_c + m_v} E \\[2ex] E_c \simeq \dfrac{m_c}{m_c + m_v} E \end{array} \right.$$

- $U(\underset{\sim}{R},\underset{\sim}{r})$ causes excitations of $\phi_0 \rightarrow \phi_{\underset{\sim}{k}}$

  $$\langle \phi_{\underset{\sim}{k}} | U(\underset{\sim}{R},\underset{\sim}{r}) | \phi_0 \rangle$$

Weakly bound systems – small ε_0 – breakup channel treatment



excitation to continuum of states with

$\varepsilon > 0, \underset{\sim}{k} \Rightarrow \phi_{\underset{\sim}{k}}(r)$

What are typical ε excited?

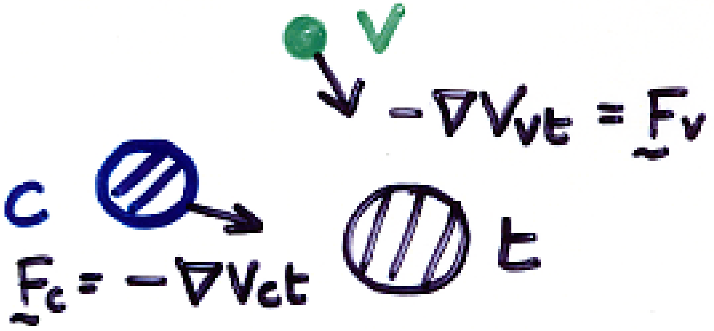
Does it matter which ε?

↓ yes

Big simplification if $E \gg \varepsilon$

(reaction 'fast' – internal motion 'slow')

dictated by geometry of tidal forces (V_{ct} and V_{vt}) experienced by components c and v



- sharp surfaces (nuclear forces) large $|\underset{\sim}{F}|$, larger $\varepsilon \lesssim 20\,\mathrm{MeV}$
- Coulomb forces – slow spatial variation, small $|\underset{\sim}{F}|$, small ε

- In both cases $\varepsilon, \phi_{\underset{\sim}{k}}$, such that $\langle H_p \rangle \ll E$

  Relatively low energies/velocities associated with relative motions

Nakamura et al. PRL 83 ('98) 1112

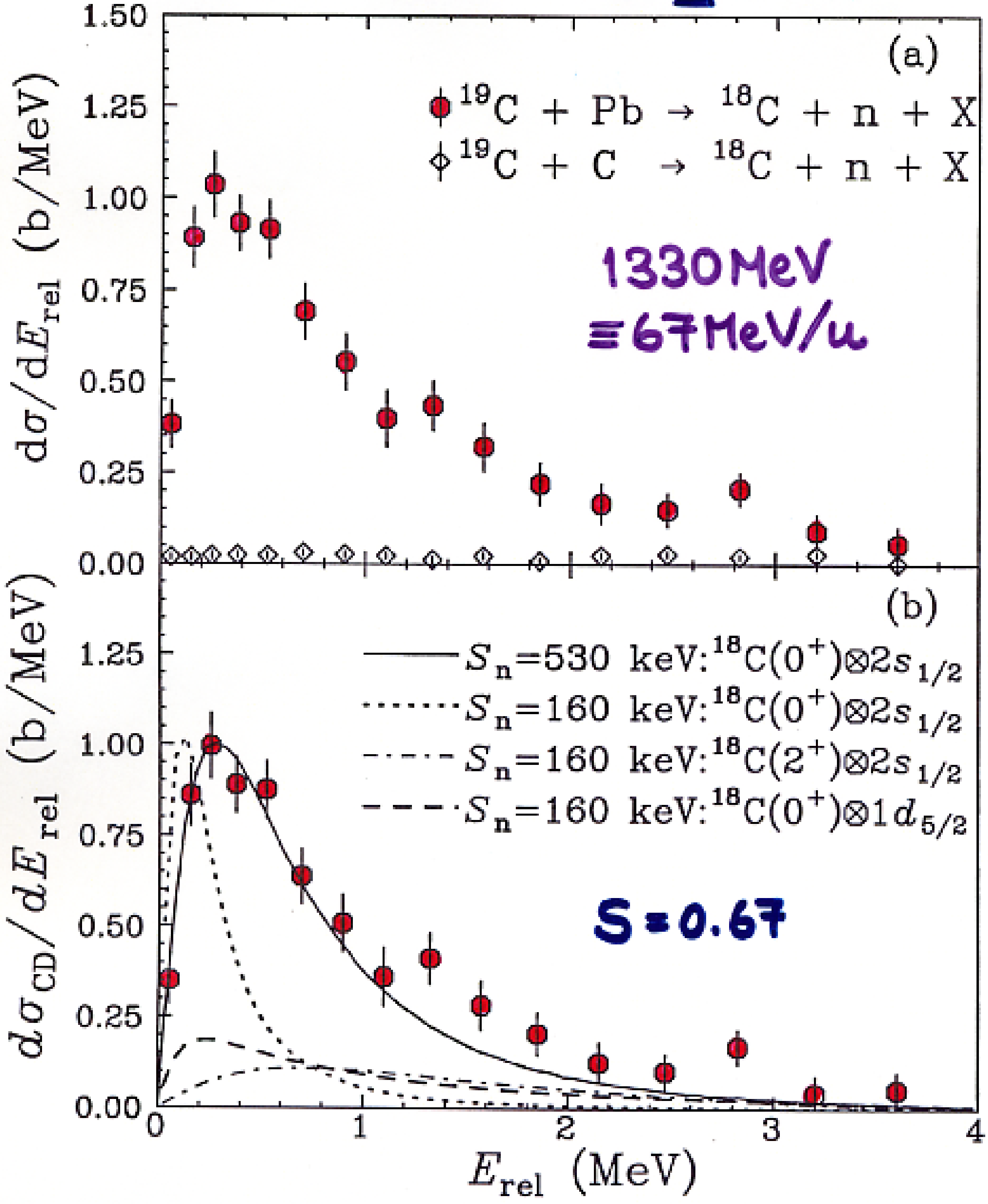
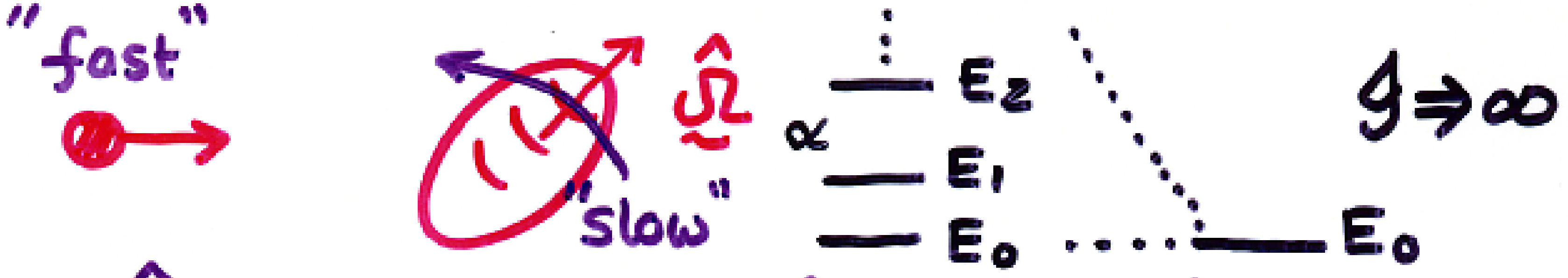


FIG. 1. (a) Dissociation cross sections as a function of relative energy E_{rel} for Pb (circles) and C (diamonds) targets. (b) Coulomb dissociation cross section for the Pb target, obtained by subtracting the nuclear contribution scaled from the C target spectrum in (a). The spectrum is compared with the calculations for the possible single-particle configurations described in the text.

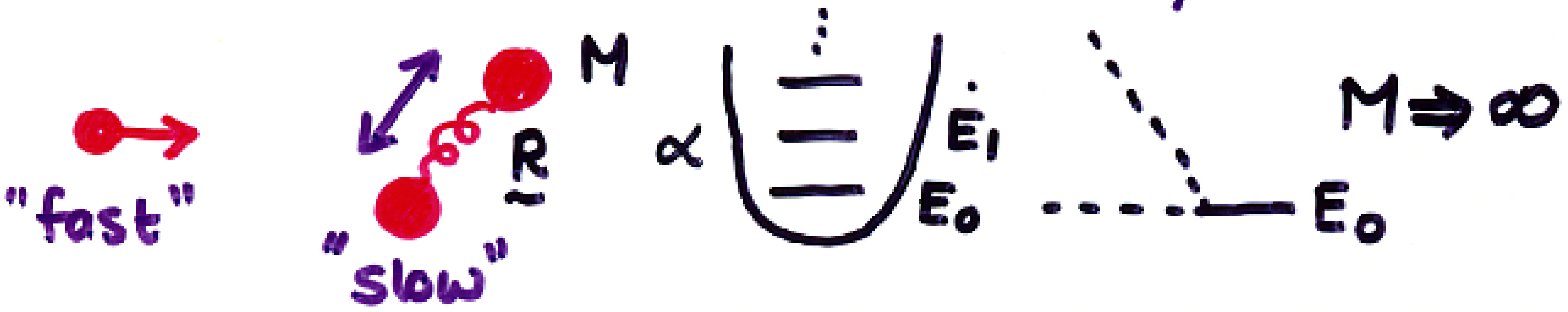
Adiabatic approximations: ($\equiv$ sudden appoximation)

- Identify $\left\{\begin{matrix}\text{Fast}\\ \text{Slow}\end{matrix}\right\}$ degrees of freedom



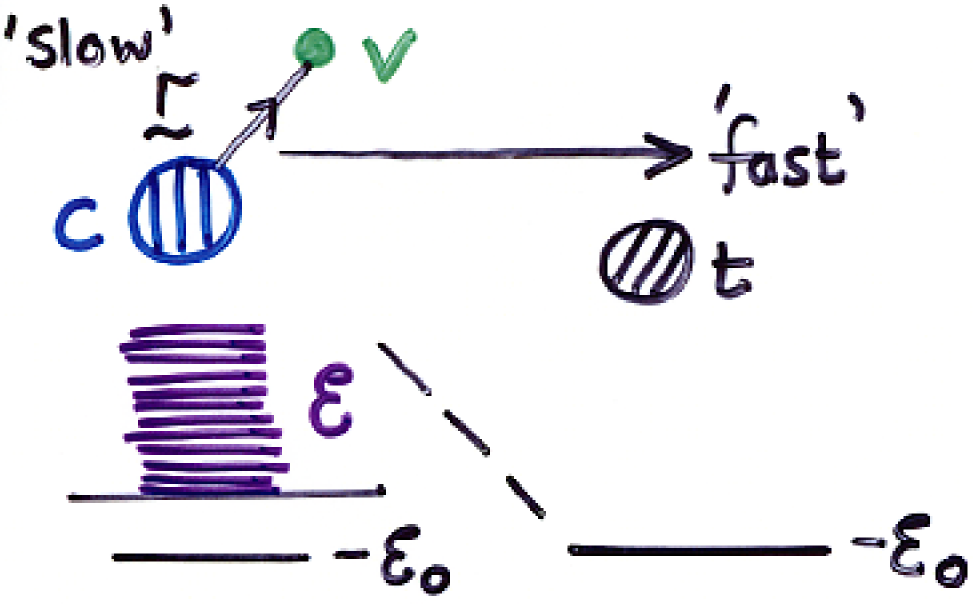
Fix $\hat{\underset{\sim}{\Omega}}$, calculate $f(\theta,\hat{\underset{\sim}{\Omega}})$ for all $\hat{\underset{\sim}{\Omega}}$

Amplitude for transition $\alpha \to \beta$ $\quad f_{\alpha\beta}(\theta) = \langle\beta|f(\theta,\hat{\underset{\sim}{\Omega}}|\alpha\rangle_{\hat{\underset{\sim}{\Omega}}}$



Calculate for all fixed $\underset{\sim}{R}$, $f_{\alpha\beta} = \langle\beta|f(\underset{\sim}{R})|\alpha\rangle_{\underset{\sim}{R}}$

In few-body (reaction) context



all excitations degenerate with ground state

freeze internal coordinate and then scatter c+v:

$f \equiv f(\theta, \underset{\sim}{r})$ for <u>all</u> fixed $\underset{\sim}{r}$

physical amplitude for breakup from $\phi_0 \rightarrow \phi_{\underset{\sim}{k}}$

$$f_{\underset{\sim}{k}}(\theta) = \langle \phi_{\underset{\sim}{k}} | f(\theta, \underset{\sim}{r}) | \phi_0 \rangle_{\underset{\sim}{r}}$$

- replaces $H_p \rightarrow -\varepsilon_0$, which should be good approx. if $E \gg \varepsilon$ → will be better with increasing E ($\varepsilon \approx$ fixed)

For practical purposes, replace $H_p \to -\mathcal{E}_0$

Adiabatic few-body model is: $\mathcal{H}^{AD} = T_R + U(\underset{\sim}{R},\underset{\sim}{r}) - \mathcal{E}_0$

$$\underline{\left\{ T_R + U(\underset{\sim}{R},\underset{\sim}{r}) - \mathcal{E}_0 \right\} \psi^{AD}(\underset{\sim}{R},\underset{\sim}{r}) = E\,\psi^{AD}(\underset{\sim}{R},\underset{\sim}{r})}$$

Eikonal model solution

as before $\psi^{AD}(\underset{\sim}{R},\underset{\sim}{r}) = \underbrace{e^{i\underset{\sim}{K}.\underset{\sim}{R}}\,\phi_0(\underset{\sim}{r})}_{\text{incident projectiles}}\,\omega(\underset{\sim}{R},\underset{\sim}{r})$

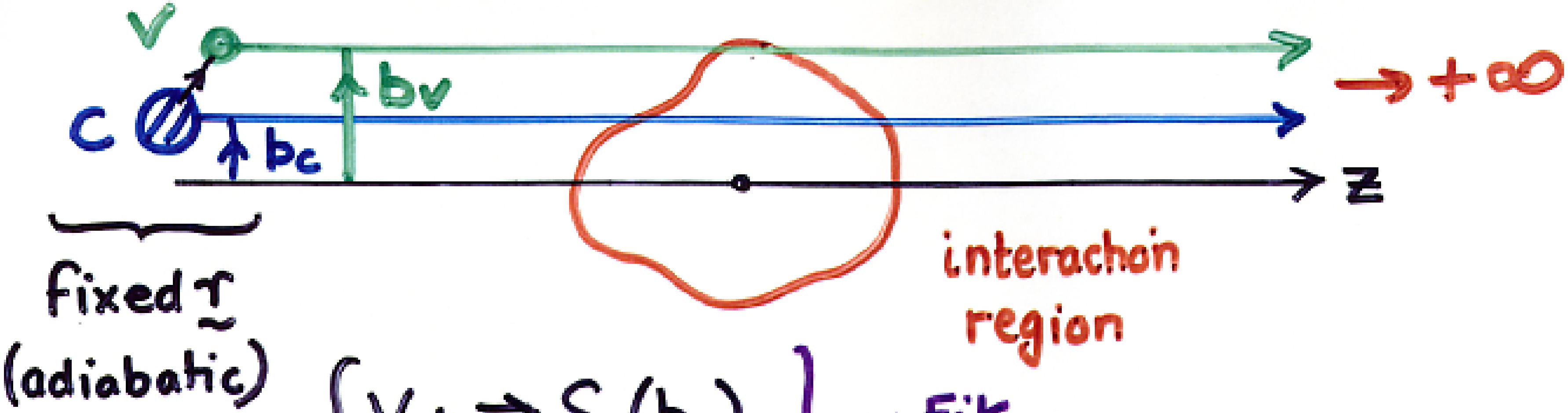
$$K = \left(2\mu\frac{[E+\mathcal{E}_0]}{\hbar^2}\right)^{1/2}$$

substitute and approximate $2\nabla_R\omega.\underline{K} \gg \nabla_R^2\omega$

$$\omega(\underset{\sim}{R},\underset{\sim}{r}) = \exp\left\{\frac{-i}{\hbar v}\int_{-\infty}^{Z} dZ'\,U(\underset{\sim}{R}',\underset{\sim}{r})\right\}$$

and as $U \equiv$ sum of 2-body potentials

as $Z \to +\infty$ $\quad\omega(\underset{\sim}{R},\underset{\sim}{r}) = S_c(b_c)S_v(b_v)$



$$\left.\begin{cases} V_{ct} \Rightarrow S_c(b_c) \\ V_{vt} \Rightarrow S_v(b_v) \end{cases}\right\} \psi^{Eik} \Rightarrow S_c(b_c)S_v(b_v) \times e^{i\underset{\sim}{K}\cdot\underset{\sim}{R}}\,\phi_0(\underset{\sim}{r})$$

So elastic S-matrix of projectile + target problem (including breakup effects – adiab + eikonal)

$$S_p(b) = \langle \phi_0 | \underbrace{\underbrace{S_c(b_c)S_v(b_v)}_{\text{survival amplitudes for } c,v \text{ at } b_c,\, b_v}}_{\text{probability } c,v \text{ in this configuration, + average over all configurations}} | \phi_0 \rangle \quad \Bigg\}$$

survival amplitude for proj. at c.m. impact parameter b.

FEW-BODY EIKONAL OR GLAUBER MODEL

For spectroscopy – beautiful + transparent formalism

$$S_p(b) = \langle \phi_0 \,|\, \underbrace{S_c(b_c)\, S_v(b_v)} \,|\, \phi_0 \rangle_{\underset{\sim}{r}}$$

- **dynamics**

  independent scatt. of c and v from target

- **Structure** (best possible wfns)

- If eikonal theory accurate (sufficiently) at energy of interest – spectroscopic tool.

- How accurate – how to use ?

More generally $S_p(b) = \langle \varphi | S_1 S_2 S_3 S_4 \cdots S_A | \varphi \rangle$

for any choice of 1, 2, 3 ... clusters.

- Tostevin et al. PRC <u>56</u> ('97) R2929
- Varga et al. ('02) 6-body - using QMC wfns. p+^{6}He, ^{6}Li

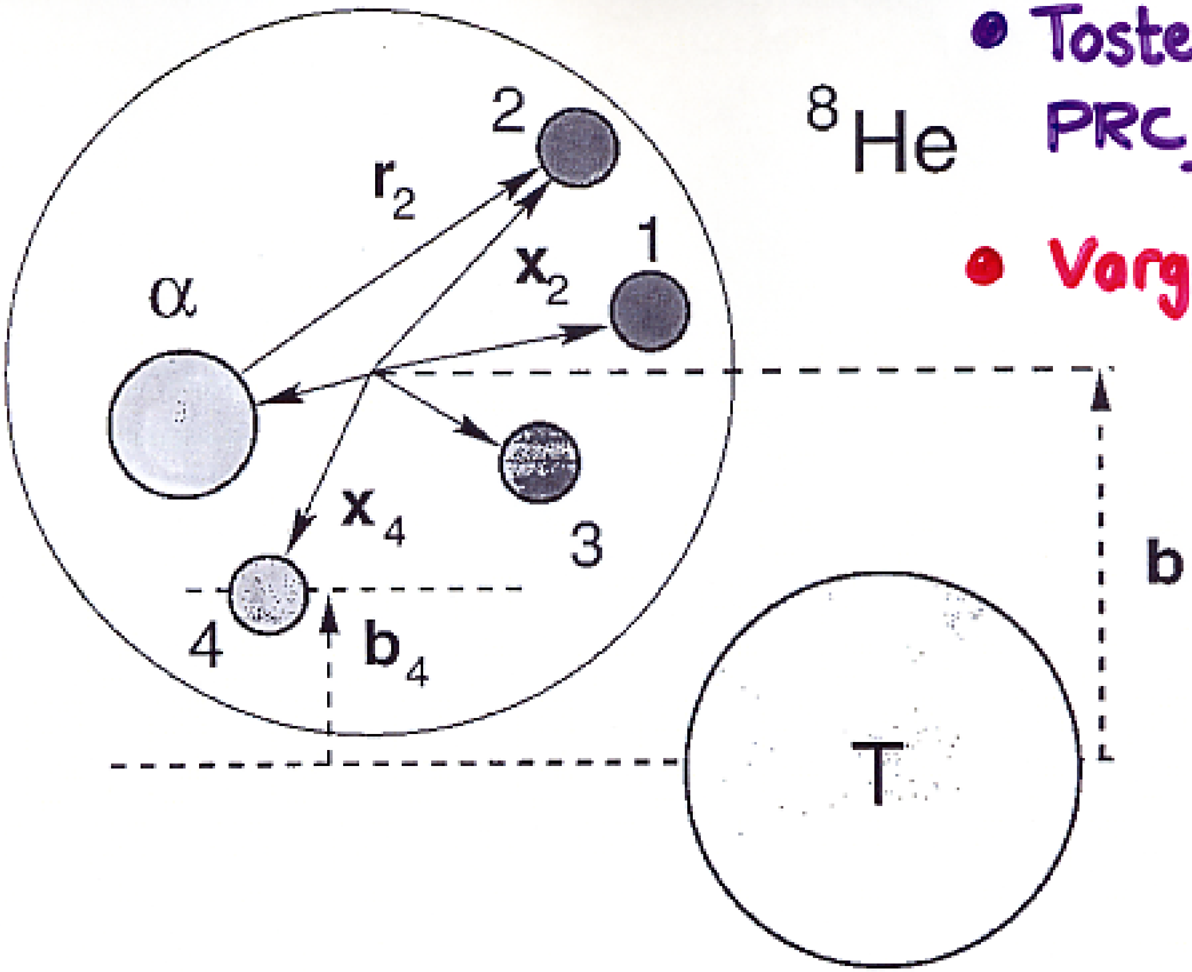


FIG. 2. Schematic representation of the coordinate system used for the effective six-body ^{8}He+target system.

$$S_8(b)=\langle\Phi_8|S_\alpha(b_\alpha)\prod_{i=1}^{4} S_i(b_i)|\Phi_8\rangle, \qquad (2)$$

Composite particle observables $\quad S_p = \langle \phi_0 | S_c S_v | \phi_0 \rangle$

$$\sigma_{el} = \int d\underset{\sim}{b}\, |1 - S_p|^2 = \int d\underset{\sim}{b}\, |1 - \langle \phi_0 | S_c S_v | \phi_0 \rangle|^2$$

$$\sigma_R = \int d\underset{\sim}{b}\, \{1 - |S_p|^2\} = \underbrace{\int d\underset{\sim}{b}\, \{1 - |\langle \phi_0 | S_c S_v | \phi_0 \rangle|^2\}}$$

used extensively: Ogawa, Al-Khalili + Tostevin,
Ozawa +....
to study structure effect (halo) on σ_R

Breakup cross sections: to state $\phi_{\underset{\sim}{k}}$

$$\sigma_{bu}(\underset{\sim}{k}) = \int d\underset{\sim}{b}\, |\langle \phi_{\underset{\sim}{k}} | S_c S_v | \phi_0 \rangle|^2$$

and for total breakup $\int d\underset{\sim}{k}$: $\sigma_{bu} = \int d\underset{\sim}{k} \int d\underset{\sim}{b}\, |\langle \phi_{\underset{\sim}{k}} | S_c S_v | \phi_0 \rangle|^2$

but, using closure relation (if one bound state)

$$\int d\underset{\sim}{k}\, |\phi_{\underset{\sim}{k}}\rangle\langle\phi_{\underset{\sim}{k}}| = 1 - |\phi_0\rangle\langle\phi_0| \underbrace{- |\phi_1\rangle\langle\phi_1| - \ldots}_{\text{if >1 bound state}}.$$

$$\underline{\sigma_{bu} = \int d\underset{\sim}{b}\, \{\langle \phi_0 | |S_c S_v|^2 | \phi_0 \rangle - |\langle \phi_0 | S_c S_v | \phi_0 \rangle|^2\}}$$

Moreover – simple formulas for absorptive (target excitation) σ's

$$\sigma_{abs} = \underbrace{\sigma_R - \sigma_{bu}}_{\text{target excitation}} = \int d\underset{\sim}{b}\, \langle \phi_o | \underbrace{1 - |S_c S_v|^2}_{\underbrace{|S_v|^2(1-|S_c|^2)}_{\substack{\text{v survives}\\ \text{c absorbed}}} + \underbrace{|S_c|^2(1-|S_v|^2)}_{\substack{\text{c-survives}\\ \text{v absorbed}}} + \underbrace{(1-|S_c|^2)(1-|S_v|^2)}_{\substack{\text{c absorbed}\\ \text{v absorbed}}}} | \phi_o \rangle$$

- $\sigma_{str} = \int d\underset{\sim}{b}\, \langle \phi_o |\, |S_c|^2 \{ 1 - |S_v|^2 \} | \phi_o \rangle$ etc. etc.

Cross section for stripping of v from projectile, being absorbed by (exciting) target, c surviving the collision.

NB: if $V_{vt} \equiv$ Real, $|S_v| = 1$, $\sigma_{str} = 0$

Related expressions for differential cross sections, etc. not shown here.

Reaction theory (for exotic nuclei)

Synopsis: Lecture I

- new challenge: weakly bound systems : continuum or breakup coupling
- direct reactions – excite minimally : single particle spect.
- few-body models – effective interactions – complex
- adiabatic approximation – slow/small $\mathcal{E}$ associated with internal motions
- eikonal few-body model for projectile p

$$\left. S_p(b) = \langle \underbrace{\phi_0 \,|\, \underbrace{S_c(b_c)\, S_v(b_v)}_{\text{dynamics}} \,|\, \phi_0}_{\text{structure}} \rangle \;\right\} \text{observables}$$

Use for nucleon knockout reactions

target

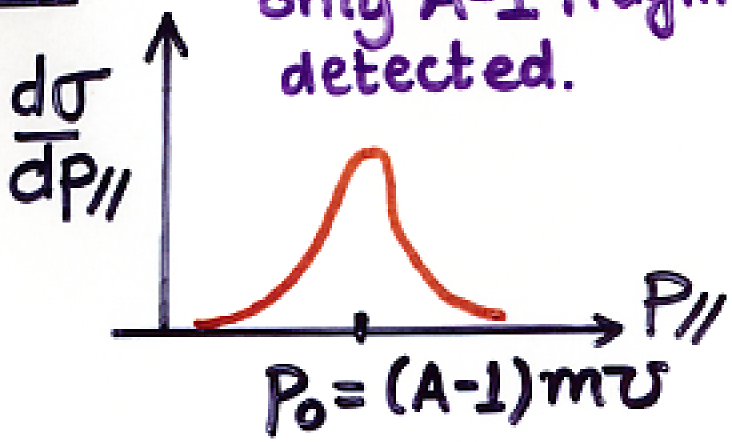
(^9Be, ^{12}C)

• Light

with or without γ-detection

if no γ-coincidences

$$\sigma = \sum_{\text{all final states } i} \sigma(i)$$



only A-1 fragment detected.

$$\sigma = \int dp_{\parallel} \frac{d\sigma}{dp_{\parallel}}.$$

A-1 fragment from (i) breakup ≡ <u>diffraction dissociation</u>

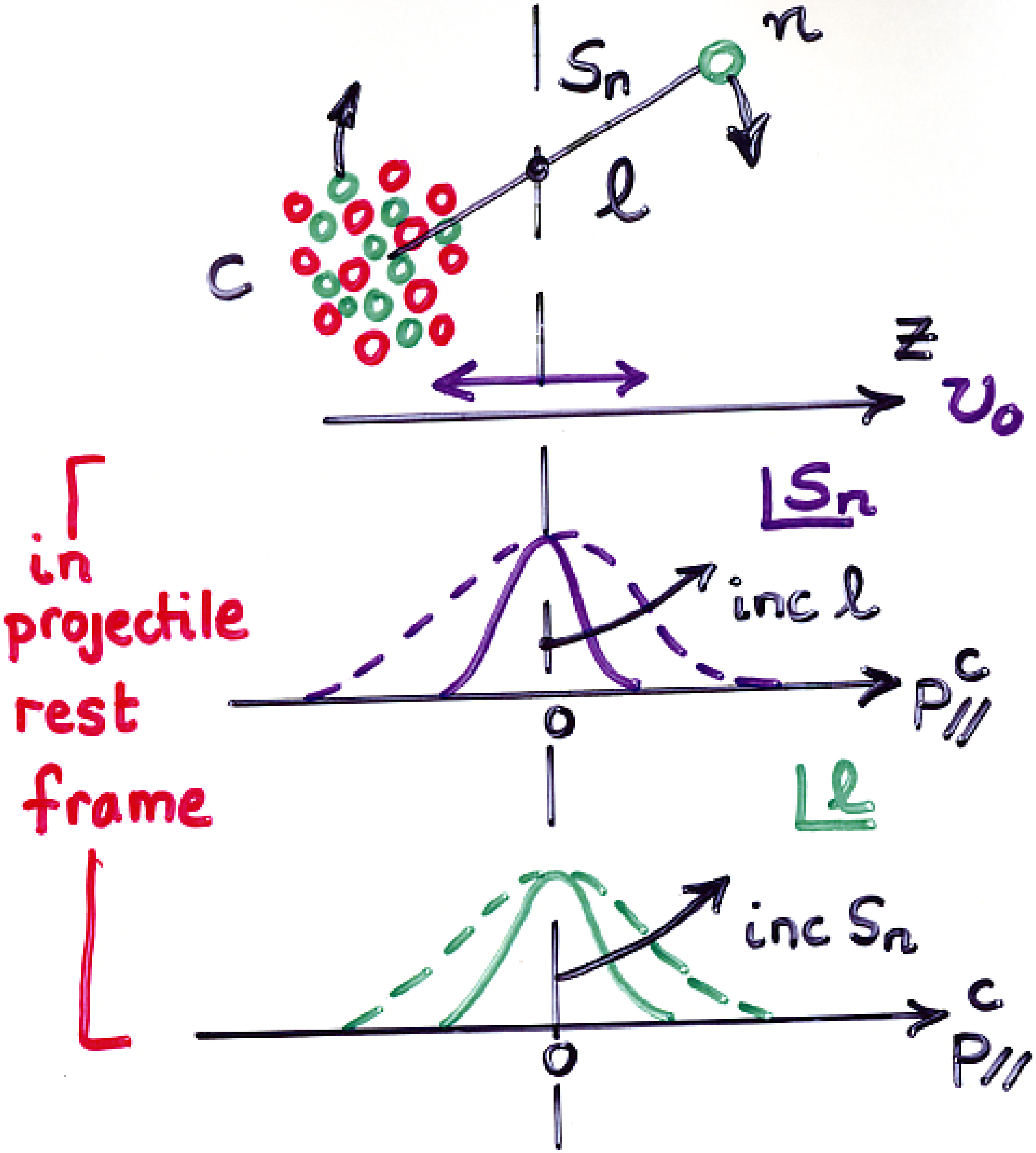
(target + (A-1) remain in g.s)

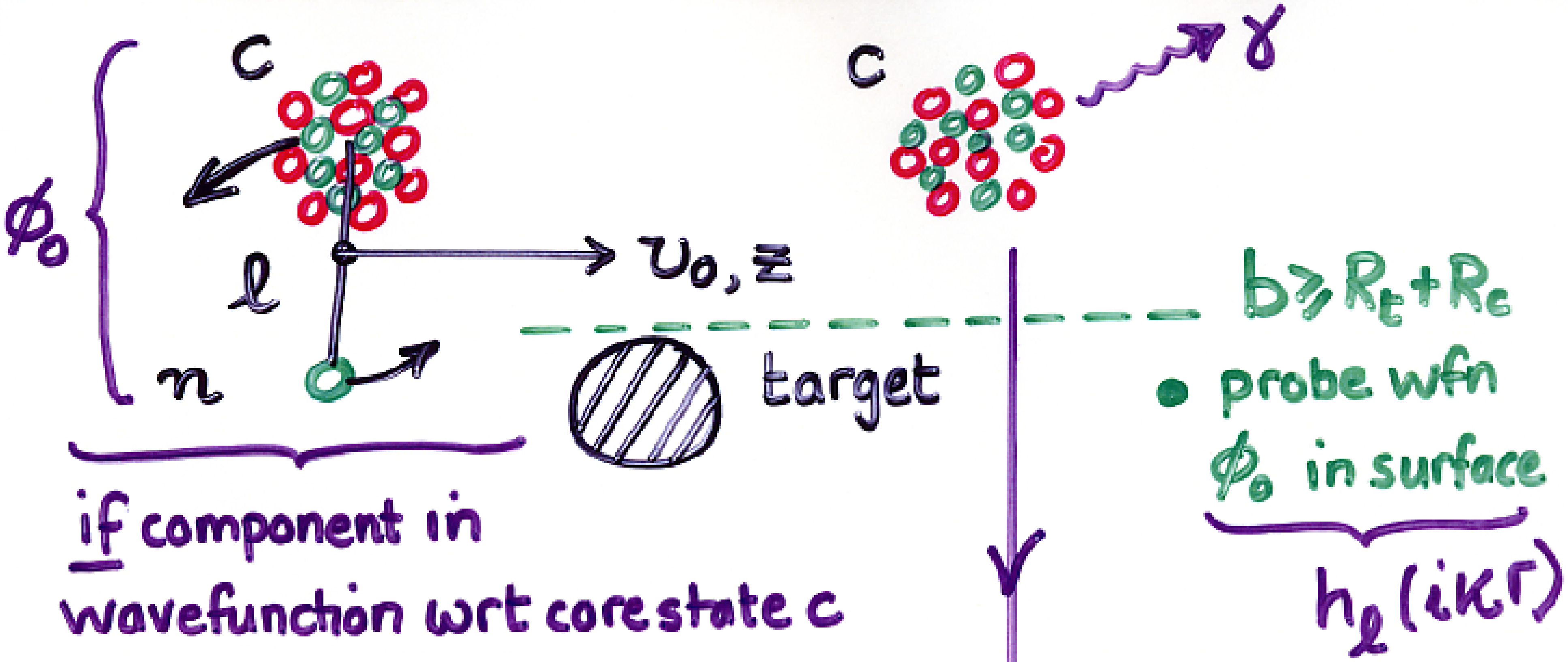
(ii) nucleon absorption by target

(target excitation) — <u>stripping</u>

different final states: (of target - incoherent)

$$\underline{\sigma = \sigma_{str} + \sigma_{diff}}$$


n
S_n
ℓ
c
z
v_0
S_n
inc ℓ
0
$P^c_{//}$
ℓ
inc S_n
0
$P^c_{//}$
in projectile rest frame



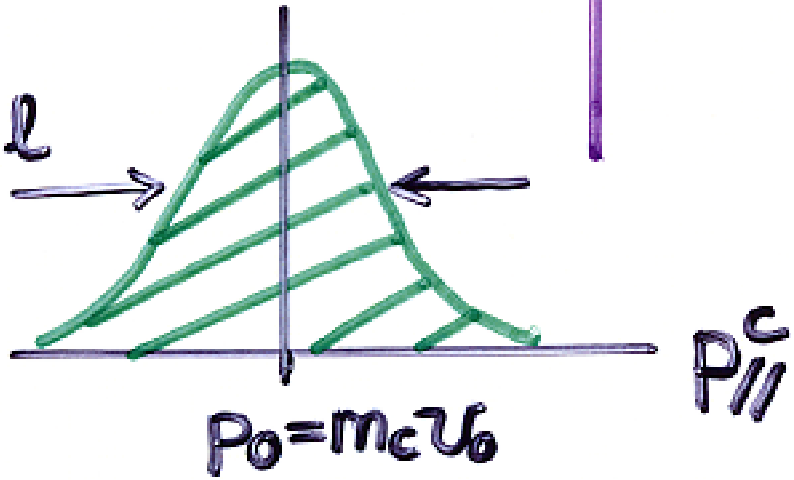
if component in wavefunction wrt core state c

- probe wfn ϕ_0 in surface

$h_\ell(i\kappa r)$

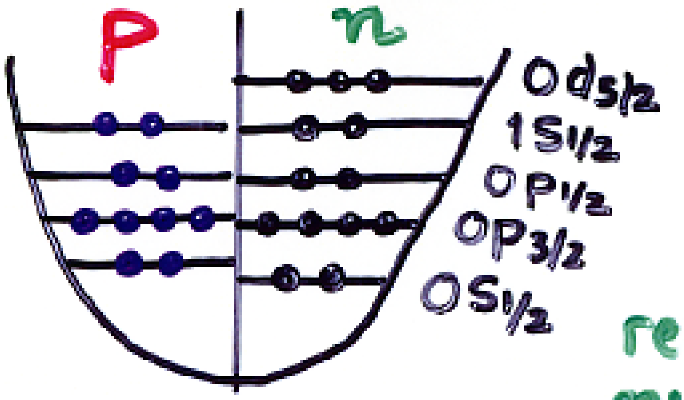
$\frac{d\sigma}{dp_{//}^c}$:

(i) $S_{n,\ell}$ width



(ii) integrated cross section σ_{-n} for core state c → parentage in ϕ_0

Structure information

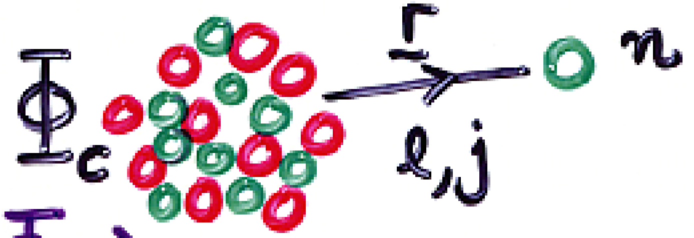


Removal of nucleon from Φ_A will leave remaining A−1 ≡ c = core in g.s or excited state of A−1, state c.

⇒ true even for extreme s.p. model

removed nucleon will have s.p. quantum numbers ℓ, j ; $\underset{\sim}{j} = \underset{\sim}{\ell} + \underset{\sim}{s}$

Formfactor



$F^c_{\ell j}(\underset{\sim}{r}) = \langle \Phi_c, \underset{\sim}{r} | \Phi_A \rangle$ ⇁ amplitude for $\Phi_c + \ell j$ in Φ_A

$$\int d\underset{\sim}{r}\, |F^c_{\ell j}(\underset{\sim}{r})|^2 = C^2 S(\ell j) \equiv \text{Spectroscopic Factor}$$

$$F^c_{\ell j}(\underset{\sim}{r}) = [C^2 S(\ell j)]^{1/2} \underbrace{\phi^c_{\ell j}(\underset{\sim}{r})}_{\text{model used, e.g. Woods-Saxon potential}} \; ; \; \int d\underset{\sim}{r}\, |\phi^c_{\ell j}(\underset{\sim}{r})|^2 = 1.$$

Calculate cross sections for removal of nucleon with unit single particle strength — the $\phi^c_{\ell j}(\underset{\sim}{r})$ — for n removal

- $$\sigma^{\rm diff}_{\rm sp} = \frac{1}{2J+1}\int d\underset{\sim}{\mathbf{b}} \left[\sum_M \langle \phi^c_{JM} | |(1-\mathcal{S}_c\mathcal{S}_n)|^2 | \phi^c_{JM}\rangle - \sum_{M,M'} |\langle \phi^c_{JM'}|(1-\mathcal{S}_c\mathcal{S}_n)|\phi^c_{JM}\rangle|^2 \right] \qquad (2)$$

and

- $$\sigma^{\rm str}_{\rm sp} = \frac{1}{2J+1}\int d\underset{\sim}{\mathbf{b}} \sum_M \langle \phi^c_{JM}|(1-|\mathcal{S}_n|^2)|\mathcal{S}_c|^2|\phi^c_{JM}\rangle. \qquad (3)$$

n-separation energy is $S_n = |E_A - E_c|$

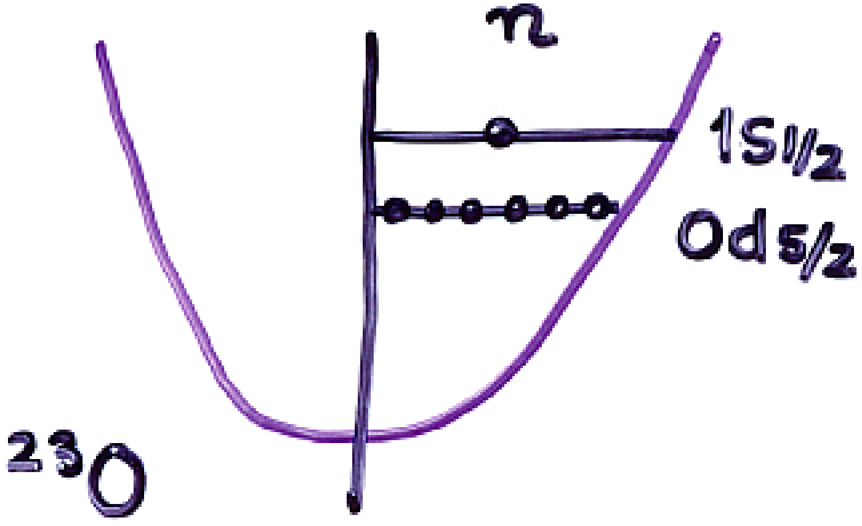
Calculation of Partial Cross Sections for Knockout to Individual Final States nI^{π} in a Direct Reaction Model

$$\sigma(nI^{\pi}) = \sum_{j} C^2 S(j, nI^{\pi})\, \sigma_{sp}(j, B_n)$$

$$\sigma_{sp}(j, B_n) = \sigma_{sp}^{strip}(j, B_n) + \sigma_{sp}^{diffr}(j, B_n)$$

Simplest viewpoint

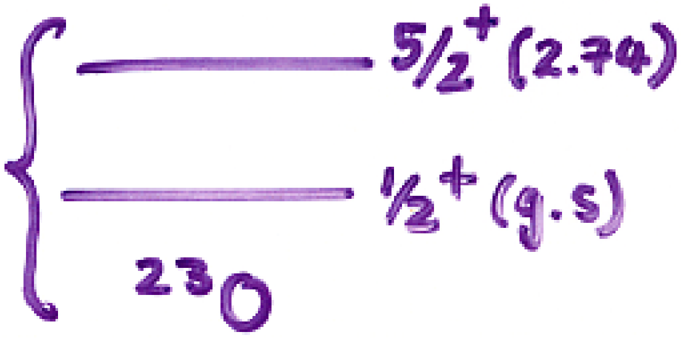
$^{23}O + ^{12}C$, 72MeV/u



$(S_n = 2.7\text{MeV})$

$(S_n = 5.5\text{MeV})$

Shell
model
(Brown)



$$\sigma_{sp}(1/2^+) = 64\text{mb}$$

$$\sigma_{sp}(5/2^+) = 23\text{mb}$$

$$\sigma_{-n} = \underbrace{6\sigma(5/2^+)} + \underbrace{\sigma(1/2^+)} = 202\text{mb} \Leftrightarrow 233(37)\text{mb}$$

RIKEN 72MeV/u

PRL <u>88</u> ('02) 142502

$O^{22}(2^+)$, $c^2S = 2.5$, $O^{22}(g.s)$

$O^{22}(3^+)$, $c^2S = 3.5$,

$\{d_{5/2}^{-1} \otimes s_{1/2}\}_J$

TABLE I: Calculated spectroscopic factors and nucleon removal cross sections in the reactions $^{12}\mathrm{C}(^{23}\mathrm{O},^{22}\mathrm{O}(I^\pi))\mathrm{X}$; 72MeV/nucleon

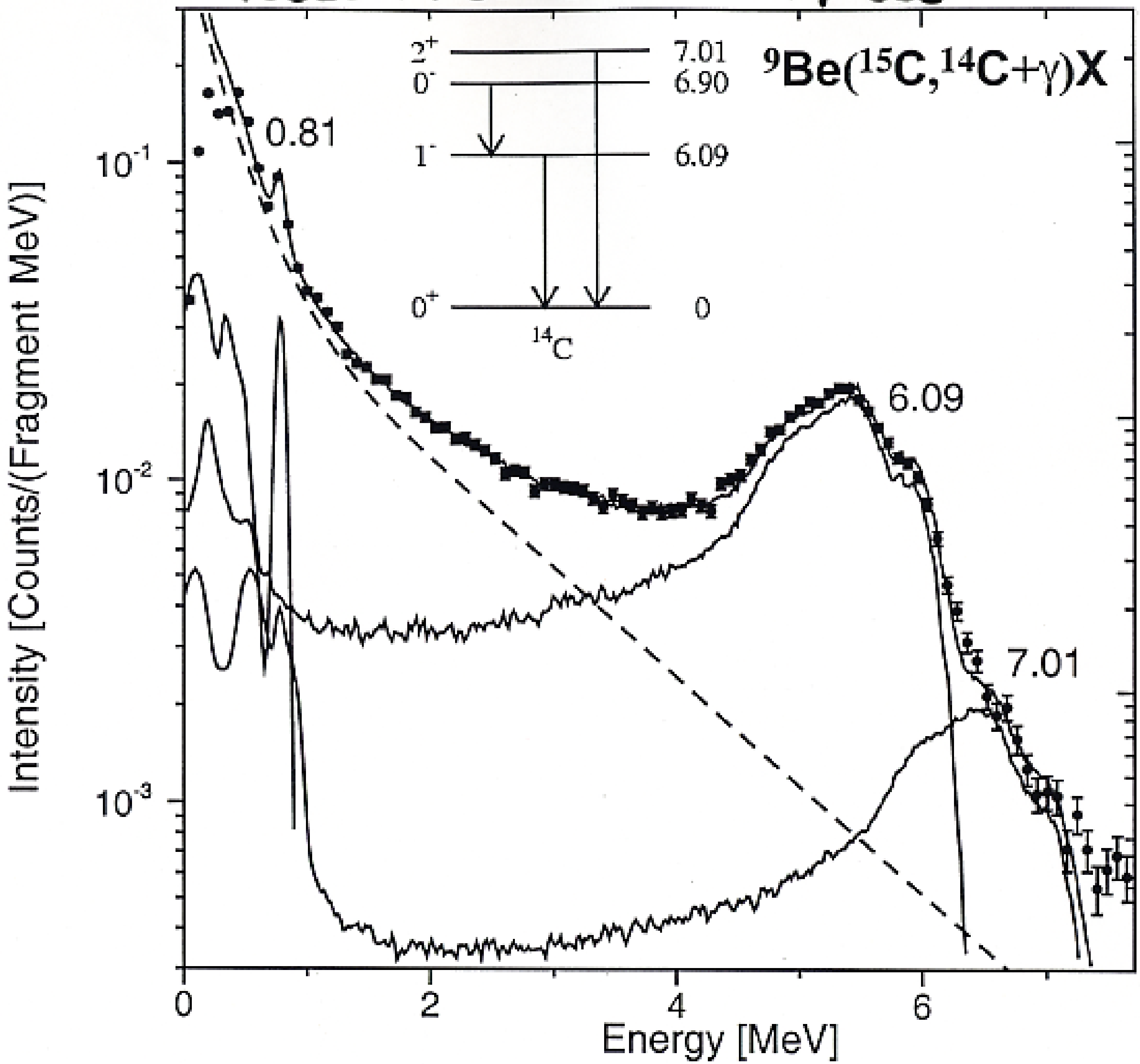
Shell model states below n threshold

Energy (MeV)	I^π	ℓ	C^2S	σ_{sp} (mb)	σ_{1n} (mb)	Expt
0	0^+	0	0.797	64.2	51.2	
3.38	2^+	2	2.130	22.8	48.6	⇒ 3.19Me
4.62	0^+	0	0.115	32.0	3.7	
4.83	3^+	2	3.079	20.4	62.9	⇒ 4.57Me
5.32	1^-	1	0.851	17.8	15.2	} $P_{1/2}^{-1}$
5.93	0^-	1	0.332	16.9	5.6	
6.50	2^+	2	0.242	18.0	4.4	
				Sum:	191	

• Brown •

• (233(37)mb)
RIKEN

Tostevin et al. PRC in press



Navin et al. PRL 81 ('98) 5089

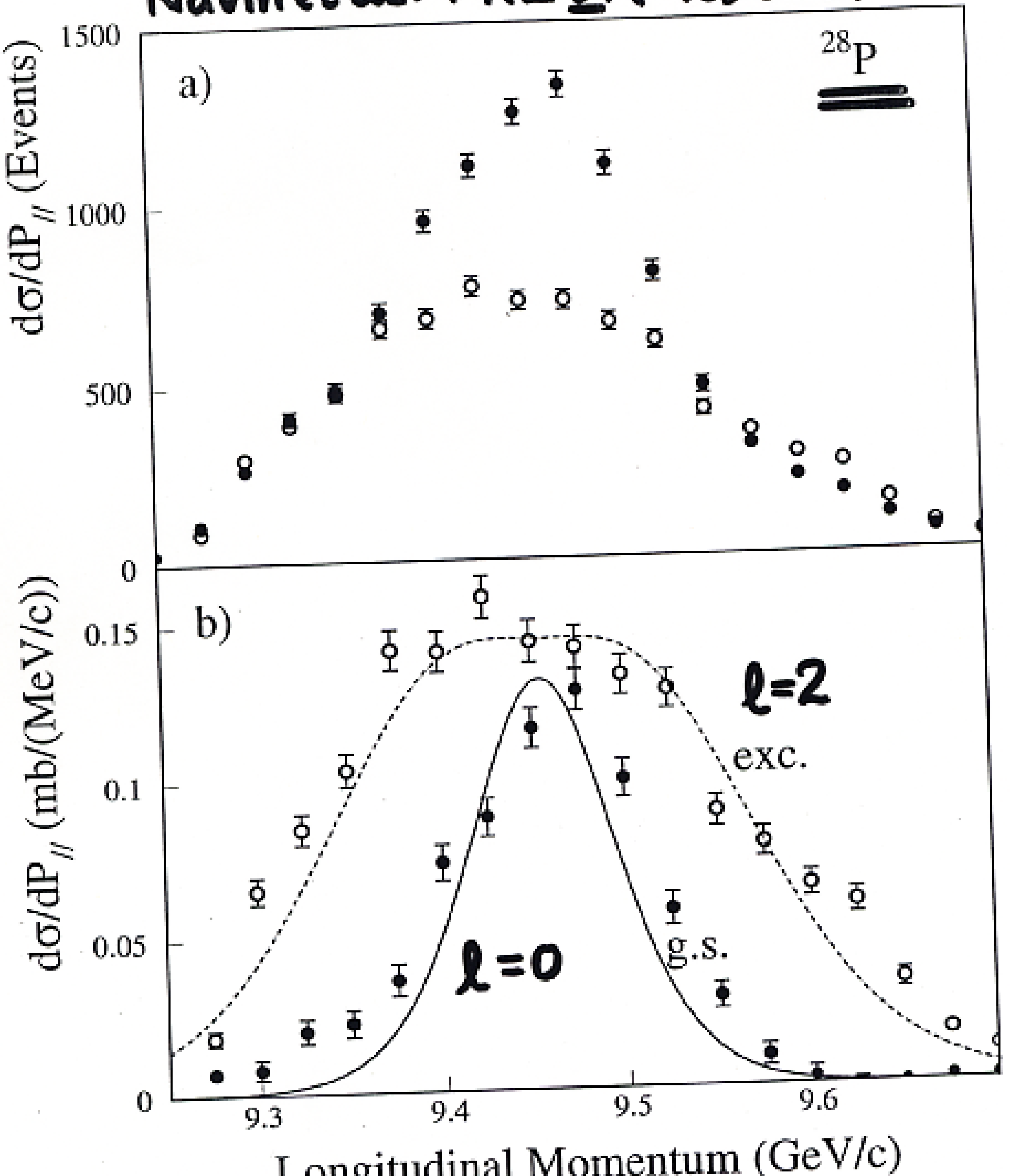
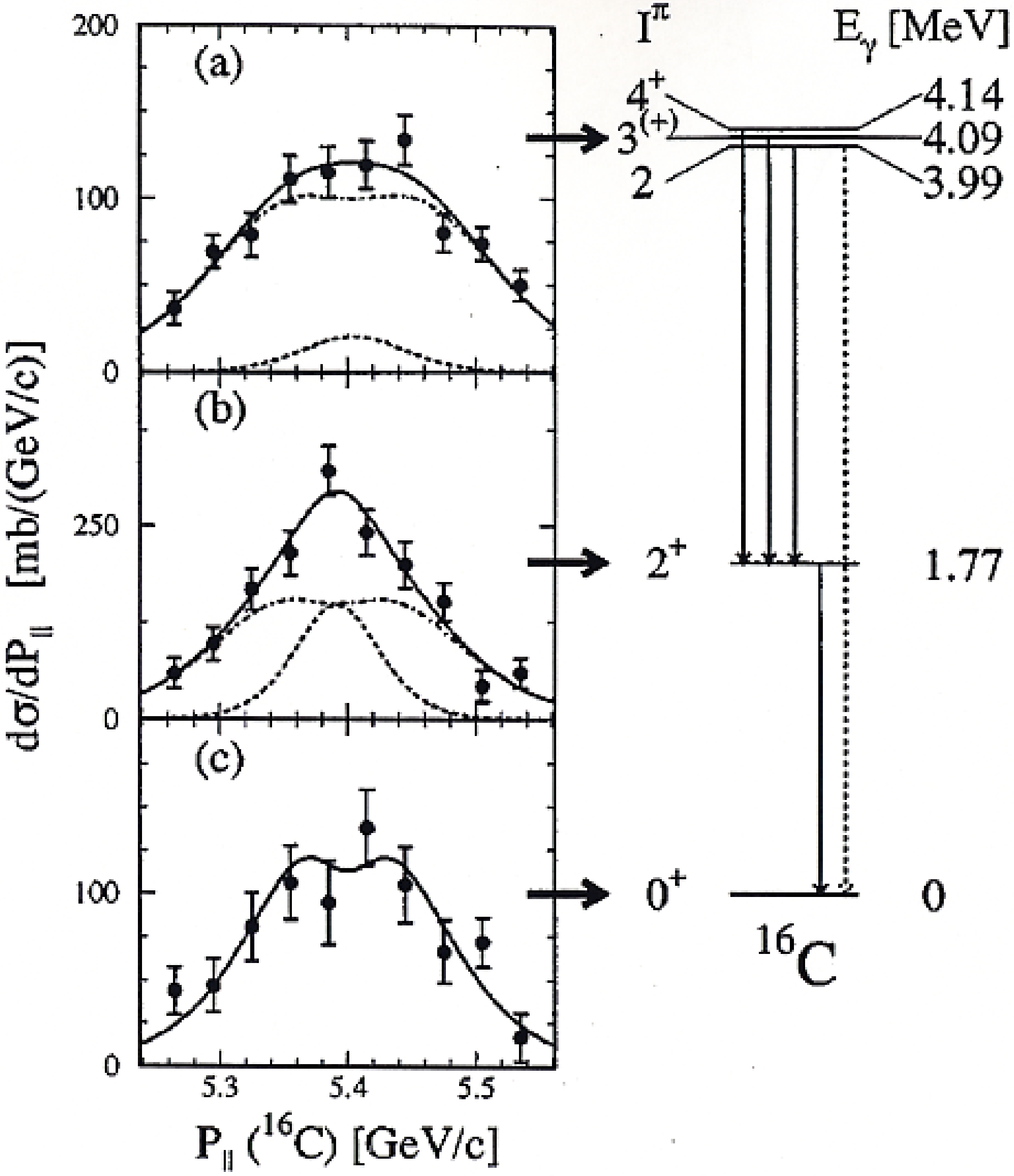


FIG. 4. Longitudinal momentum spectra for ^{27}Si projectile residues. (a) The (filled) open circles correspond to the absence (presence) of coincident γ rays in the NaI(Tl) array. (b) Derived longitudinal momentum spectrum corresponding to the ground (filled) and excited (open) states in the projectile residue ^{27}Si obtained from Fig. 4a. The continuous and dashed lines are the calculated longitudinal momentum distributions for the *s* and *d* states, respectively. The widths are 93 and 248 MeV/*c* in the laboratory frame.



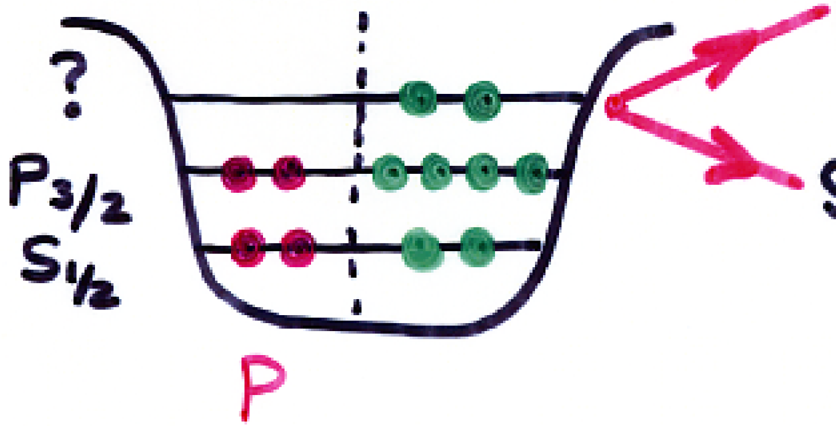
$\ell=0,2$

$\ell=0,2$

$\ell=2$

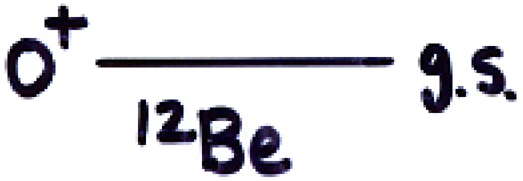
N = 8 shell closure

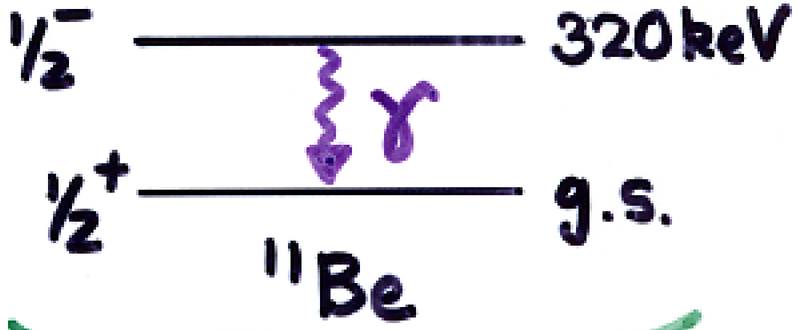
^{12}Be (Z = 4, N = 8)



$P_{1/2}$ — ^{11}Be ($1/2^-$), 320keV

$S_{1/2}$ — ^{11}Be ($1/2^+$), g.s.





level inversion

Navin et al. PRL 85 (2000), 266

Navin et al. PRL 85 (2000) 266

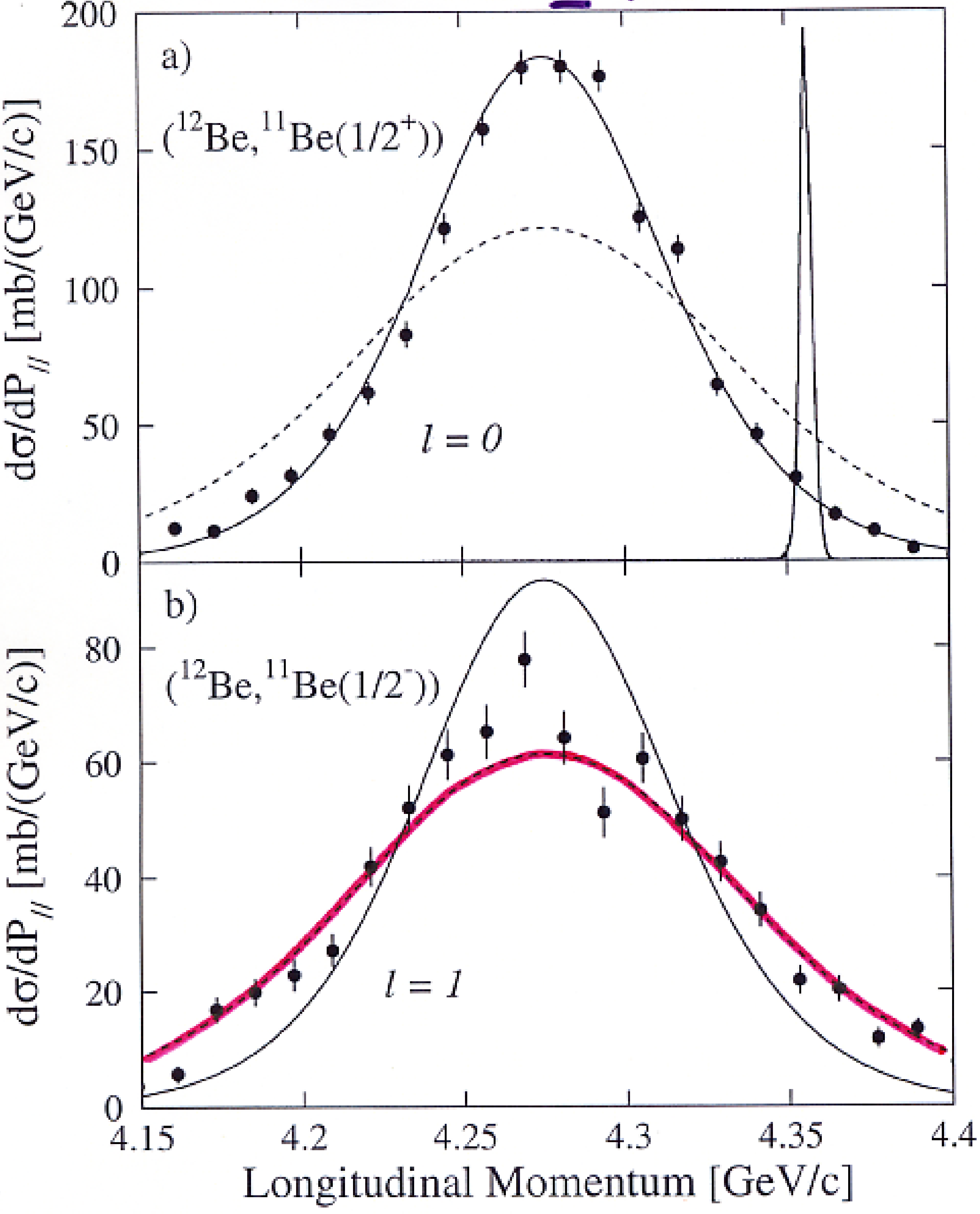


FIG. 2. Laboratory frame longitudinal momentum distributions for ^{11}Be residues in the ground (a) and excited (b) states. The solid (dashed) curves are calculated for $l = 0$ (1) neutron removal. The narrow line in (a) illustrates the line profile of the spectrograph.

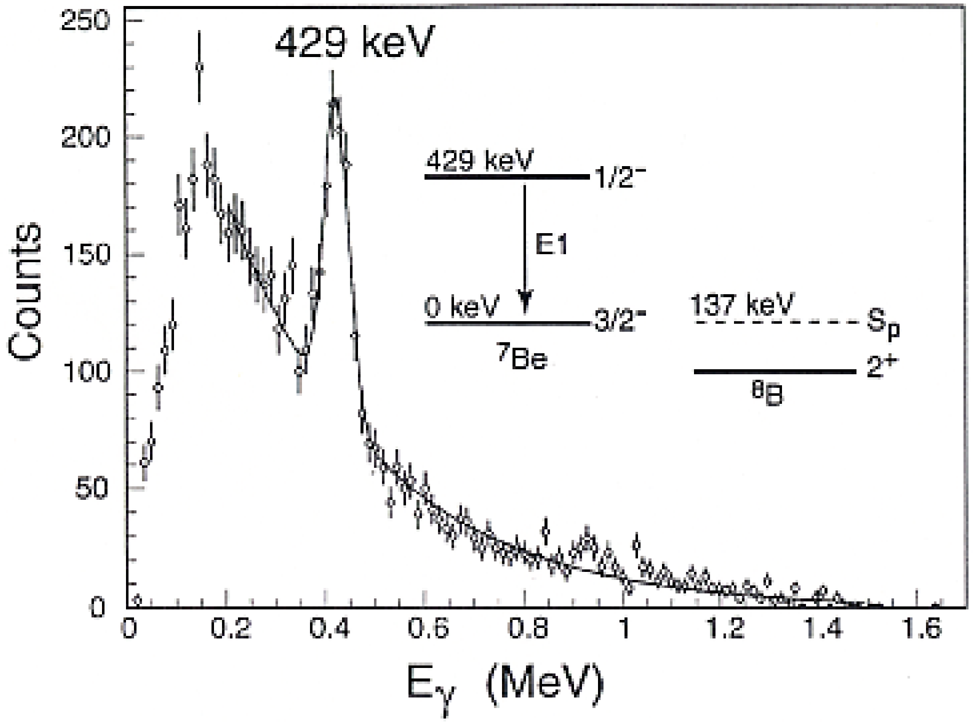


Fig. 2. Spectrum of γ rays in coincidence with ^{7}Be fragments after one-proton removal reactions of ^{8}B in a carbon target. The spectrum is obtained from the measured γ spectrum after Doppler correction. The solid line is a fit to the data with a Gaussian and a decaying exponential.

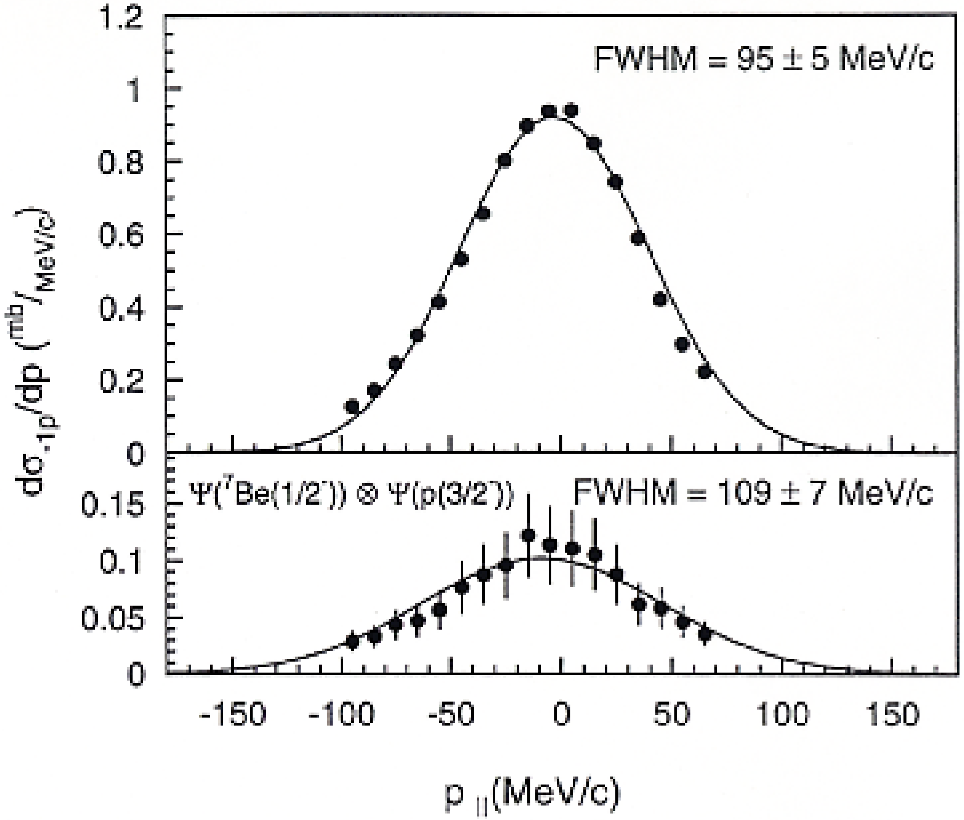
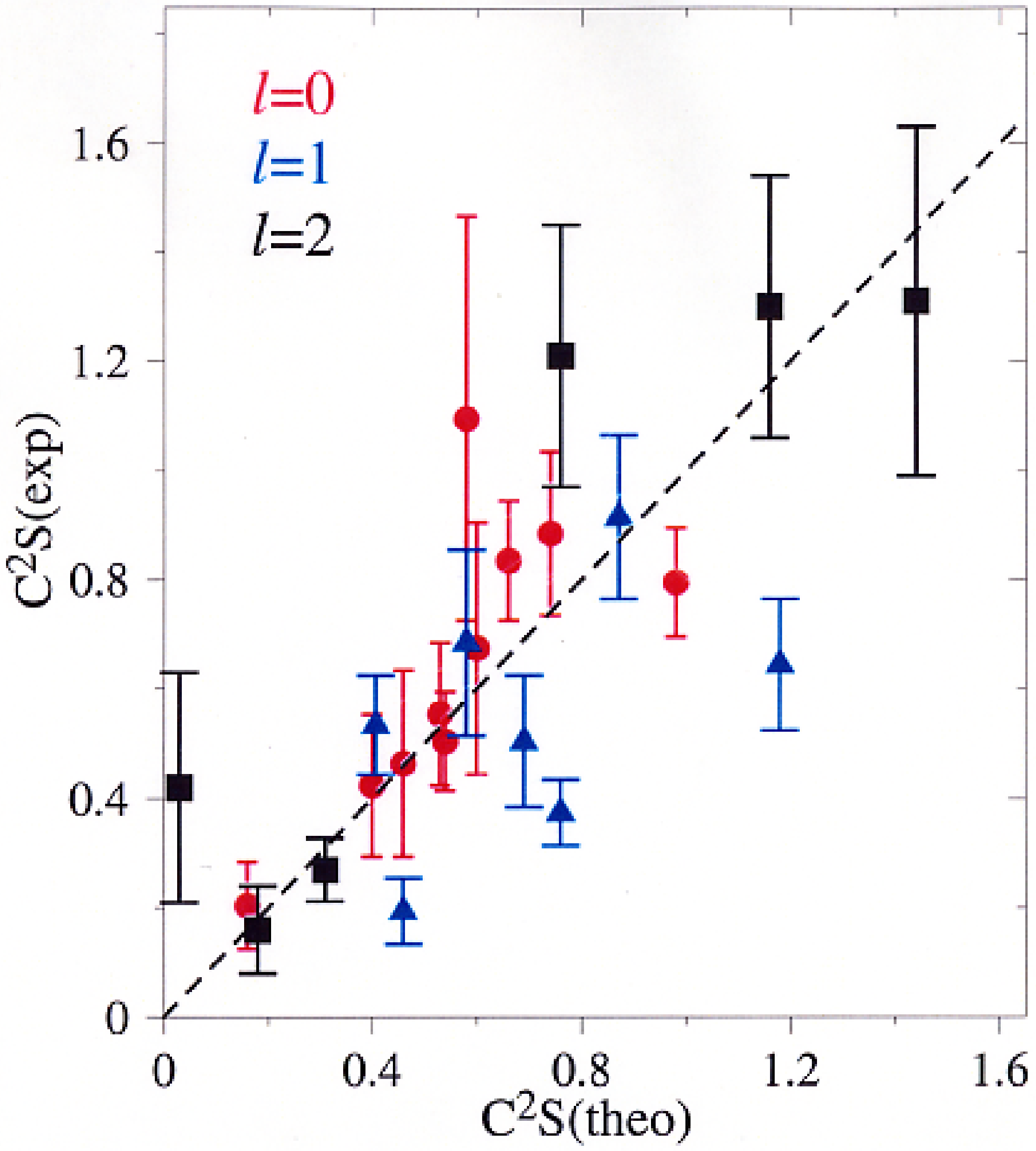


Fig. 3. The upper panel shows (points) our results for inclusive $p_{\|}^{\mathrm{total}}$ distribution yielding a width of 95 ± 5 MeV/c, in agreement with previous results [3]. The lower panel (points) shows our results for exclusive $p_{\|}^{\mathrm{exc.}}$ distribution, considering only the core excited con-

Comparison of experimental and calculated spectroscopic factors for reactions leading to individual final levels in the nuclei ^{11,12}Be, ^{14}B, ^{15,16,17,19}C and ^{26,27,28}P

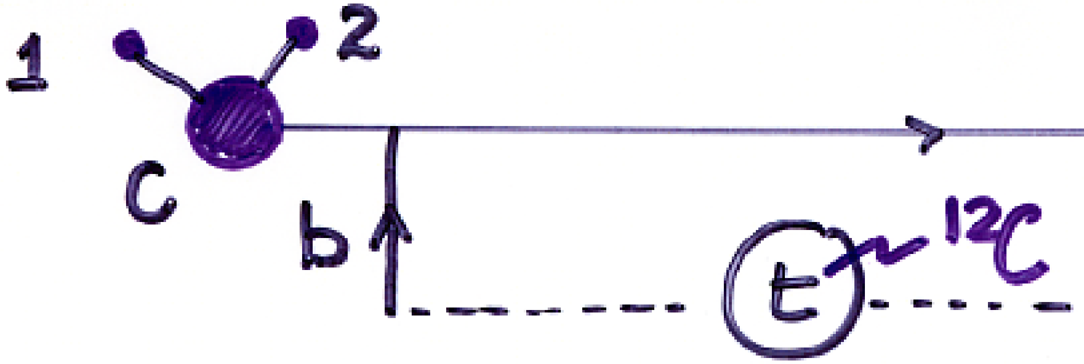


Two-neutron removal (2N removal more generally)

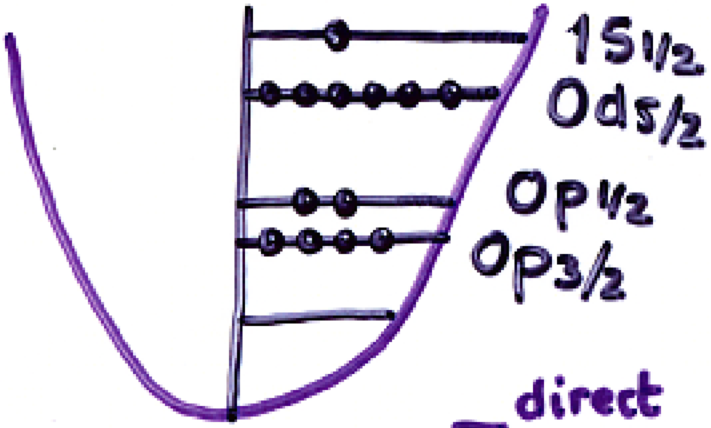
$^{23}O \rightarrow {}^{21}O$ (RIKEN, 72 MeV/u) $\Rightarrow$ 82(25) mb – large

Stripping component (dominant)

$$\sigma_{-2n} = \int d\underset{\sim}{b} \, \langle \phi | \, |S_c|^2 \{1-|S_1|^2\}\{1-|S_2|^2\} \, | \phi \rangle$$



includes simultan. + seqential 2n removal



$\sigma_{-2n}(sd) = 0.9$ mb

$\sigma_{-2n}(dd) = 0.6$ mb

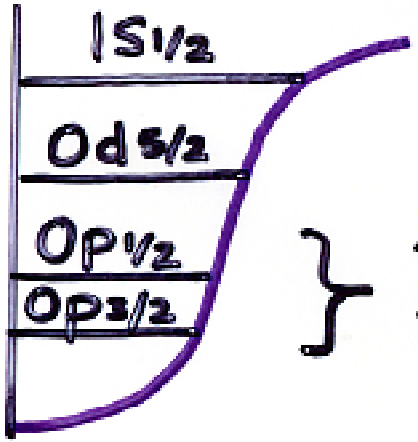
for all combinations

$$\sigma_{-2n}^{direct} \approx 15\sigma(dd) + 6\sigma(sd) \approx \underline{14 \text{mb}}\,!$$

Shell model ~ p strength near ^{22}O n-threshold

$\sigma_{-n}(p)$ leaves ^{22}O predominantly in continuum → ^{21}O

two body potential ($V_0 = 45$ MeV, $V_{LS} = 10$ MeV)



S_n(MeV)	σ_{sp} (mb)
2.74	~64
6 MeV	~23
12.0	~12
16.8	~11

$$\sigma_{-n}(p) \approx 2\sigma(p_{1/2}) + 4\sigma(p_{3/2})$$

$$\underline{\approx 68\text{mb}}$$

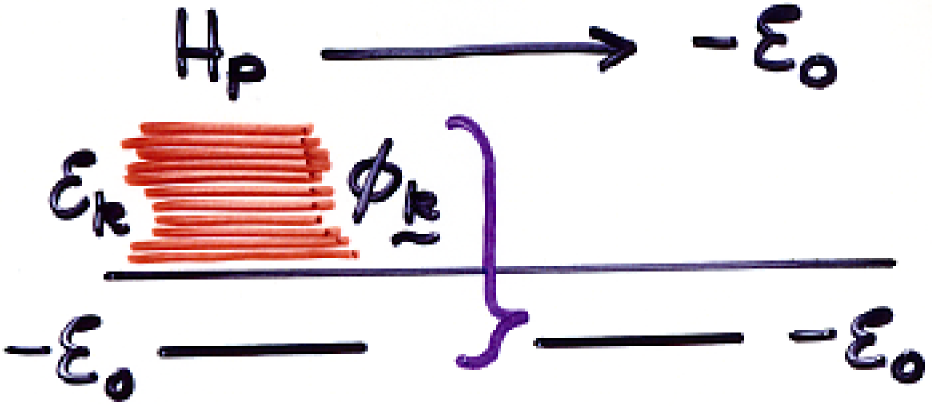
if 1 unit of p-strength to bound ^{22}O (shell model)?

$\underline{\sigma_{-n}(p) \simeq 57\text{mb}}$ to ^{22}O continuum

$(+\sigma_{-2n}^{\text{direct}} = 14\text{mb})$

$$\sigma(^{23}\text{O} \to {}^{21}\text{O}) \approx 71-82\text{mb}$$

Beyond adiabatic (high energy) approximations



approx treatment of
breakup configurations
but without explicit
reference to $\phi_{\underset{\sim}{k}}$

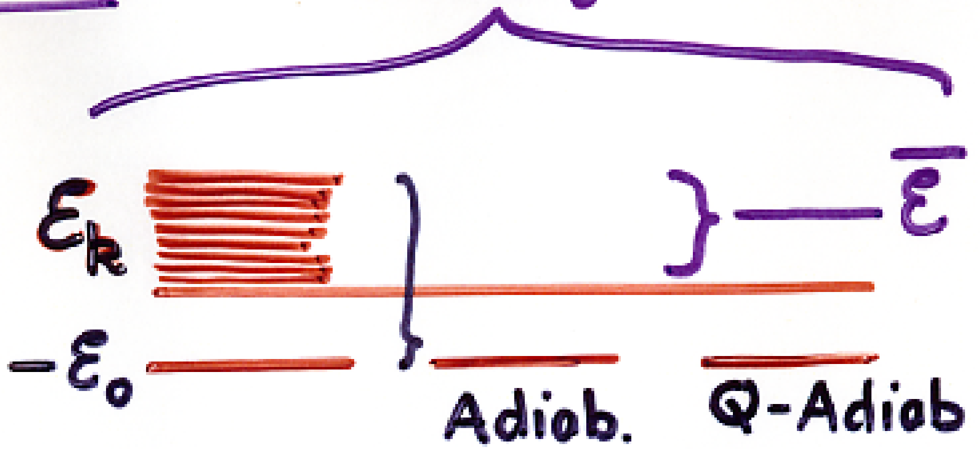
$$\{T_R + U(\underset{\sim}{R},\underset{\sim}{r}) - \varepsilon_0\}\,\underbrace{\Psi^{AD}(\underset{\sim}{R},\underset{\sim}{r}) = E\,\Psi^{AD}(\underset{\sim}{R},\underset{\sim}{r})}_{\Psi^{AD} = \Psi^{AD}_{el} + \Psi^{AD}_{bu}}$$

- (H_p) $\qquad \underline{\Psi^{AD}_{el} = |\phi_0\rangle\langle\phi_0|\Psi^{AD}\rangle} \qquad (-\varepsilon_0)$

$$\left\{\begin{array}{l} \underbrace{\Psi^{AD}_{bu} = \Psi^{AD} - \Psi^{AD}_{el}} \\ \langle\phi_{\underset{\sim}{k}}|\Psi^{AD}_{bu}\rangle \neq 0 \end{array}\right\} \text{less well treated}$$

Quasi-adiabatic approximations

$$\underbrace{\{T_R + U + \underline{H_p} - E\}\Psi(\underset{\sim}{R},\underset{\sim}{r})=0}_{\Psi_{el} + \Psi_{bu}}$$



$$\{T_R + U + H_p - E\}\Psi_{bu} = [E + \varepsilon_0 - T_R - U]\Psi_{el}$$

$$\approx \underbrace{[E + \varepsilon_0 - T_R - U]\,\Psi_{el}^{AD}}_{\text{source term}}$$

guess #1 $\bar{\varepsilon} \approx \langle \Psi_{bu}^{AD} | H_p | \Psi_{bu}^{AD} \rangle / \langle \Psi_{bu}^{AD} | \Psi_{bu}^{AD} \rangle$

compute Ψ_{bu}^{QAD} from inhomog. equation

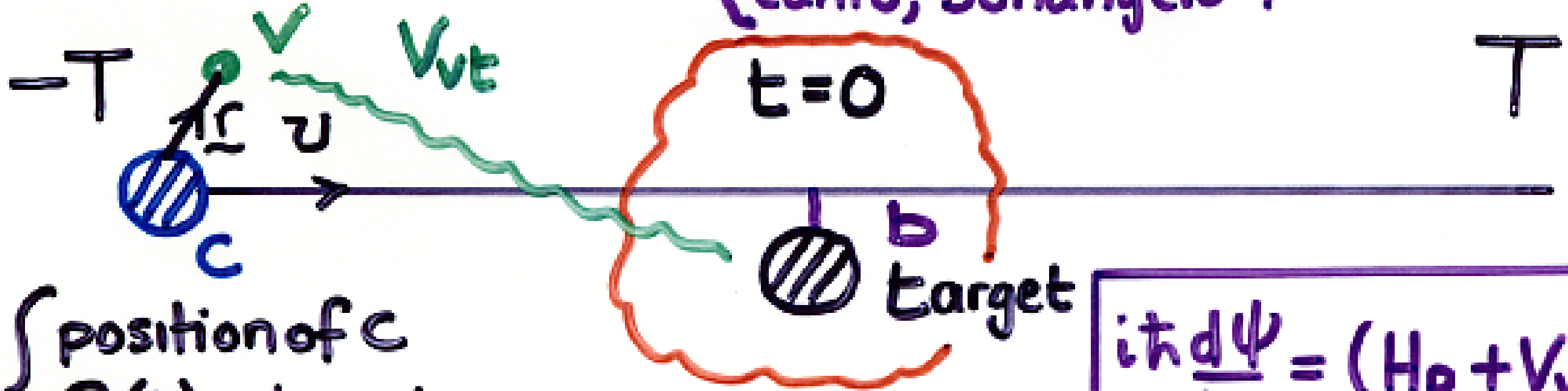
guess #2 $\bar{\varepsilon} \approx \langle \Psi_{bu}^{QAD} | H_p | \quad \rangle / \langle \quad | \quad \rangle$

+ iterate

Non-adiabatic (eikonal-like, trajectory based)

- time-dependent solution of Schrödinger equation for valence particle motion

- {Bertsch and Esbensen, Suzuki, Typel and Baur, Melezhik and Baye, Canto, Donangelo +} FURTHER READING



{position of c
$\underset{\sim}{R}(t)=\underset{\sim}{b}+\underset{\sim}{v}t$

$$\boxed{i\hbar\frac{d\Psi}{dt}=(H_p+V_{vt})\Psi(\underset{\sim}{r},t)}$$

as $t\to-\infty$, $\Psi(\underset{\sim}{r},t)=\phi_0$ as $t\to+\infty$ $\Psi_f(\underset{\sim}{r},T)$

- not exact: no explicit treatment of dynamics of V_{ct} and no energy transfer between core and internal ($\underset{\sim}{r}$) motion

if high Z target – use coulomb trajectory

$$\Psi(\mathbf{r},t)=\frac{1}{r}\sum_{lm} u_{lm}(r,t)Y_{lm}(\hat{r}). \tag{4}$$

This gives the following set of coupled equations,

$$i\hbar\frac{d}{dt}u_{lm}(r,t)=\left[\underbrace{\frac{\hbar^2}{2m_0}\left(-\frac{d^2}{dr^2}+\frac{l(l+1)}{r^2}\right)+U_{nc}(r)}_{Hp(\ell)}\right]u_{lm}(r,t)+C_{lm}(r,t), \tag{5}$$

where m_0 is the neutron-core reduced mass. The last term is the coupling generated by the neutron-target interaction,

$$C_{lm}(r,t)=\sum_{l'm'}\langle Y_{lm}|U_{nt}[|\mathbf{R}(t)-\alpha\mathbf{r}|]|Y_{l'm'}\rangle u_{l'm'}(r,t). \tag{6}$$

Solution using finite difference methods or $\underset{\sim}{r}$, t grids - care needed.

no core absorption.

$$P_{-1n} = 1 - |\langle 0|\Psi_f\rangle|^2. \tag{8}$$

The one-neutron stripping probability is calculated from the norm of the wave function after the collision according to

n-absorbed

$$P_{\text{str}}(b) = 1 - \langle \Psi_f|\Psi_f\rangle. \tag{9}$$

The diffraction dissociation probability is determined as the norm of the continuum part of the wave function after the collision. If the ground state is the only bound state, then the continuum part of the wave function is

n emerges, proj. not in g.s.

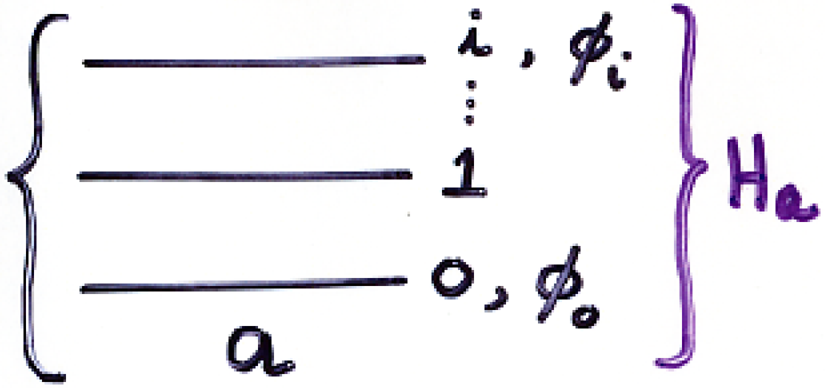
$$\Psi_f^{\text{cont}} = \Psi_f - |0\rangle\langle 0|\Psi_f\rangle. \tag{10}$$

The diffraction norm is

$$P_{\text{diff}}(b) = \langle \Psi_f|\Psi_f\rangle - |\langle 0|\Psi_f\rangle|^2. \tag{11}$$

Coupled channels methods

$$H_a \phi_a = \varepsilon_a \phi_a$$



General: $a + A$

$$\Psi_{aA} = \chi_0(\underset{\sim}{R})\phi_0 + \chi_1(\underset{\sim}{R})\phi_1 + \cdots\cdots$$

$$\left.\begin{aligned} &\{T_R + H_a + V_{aA}\}\Psi_{aA} = E\,\Psi_{aA} \\ &\Psi_{aA} = e^{i\underset{\sim}{k}\cdot\underset{\sim}{r}}\phi_0 + \cdots\cdots\cdots \end{aligned}\right\}$$

Ψ_{aA} in Schrödinger equation, and overlapping with each ϕ_i:

$$\{(E-\varepsilon_a) - T_R\}\chi_i(\underset{\sim}{R}) = \sum_j \langle\phi_i|V_{aA}|\phi_j\rangle\chi_j(\underset{\sim}{R})$$

- coupled differential equations for $\chi_i(\underset{\sim}{R})$

$$\left.\begin{aligned} &\bullet\ \chi_0(\underset{\sim}{R}) = e^{i\underset{\sim}{K}\cdot\underset{\sim}{R}} + \text{outgoing waves} \\ &\bullet\ \chi_i(R) = \text{outgoing waves}\ (i \neq 0) \end{aligned}\right\} \begin{aligned} &R\rightarrow\infty \text{ boundary} \\ &\text{conditions} \end{aligned}$$

Basis can be any complete set

$$\Psi(\underline{R},\underline{r}) = \sum_i \hat{\chi}_i(\underline{R})\,\hat{\phi}_i(\underline{r}) \quad \left\{ \begin{array}{l} \text{reduces to finite} \\ \text{c.channels problem} \end{array} \right.$$

continuum may have resonances of H_p, useful to base $\hat{\phi}_i(\underline{r})$ on H_p → CDCC method

For k in Δk_i , $\Delta k_i = k_i - k_{i-1}$

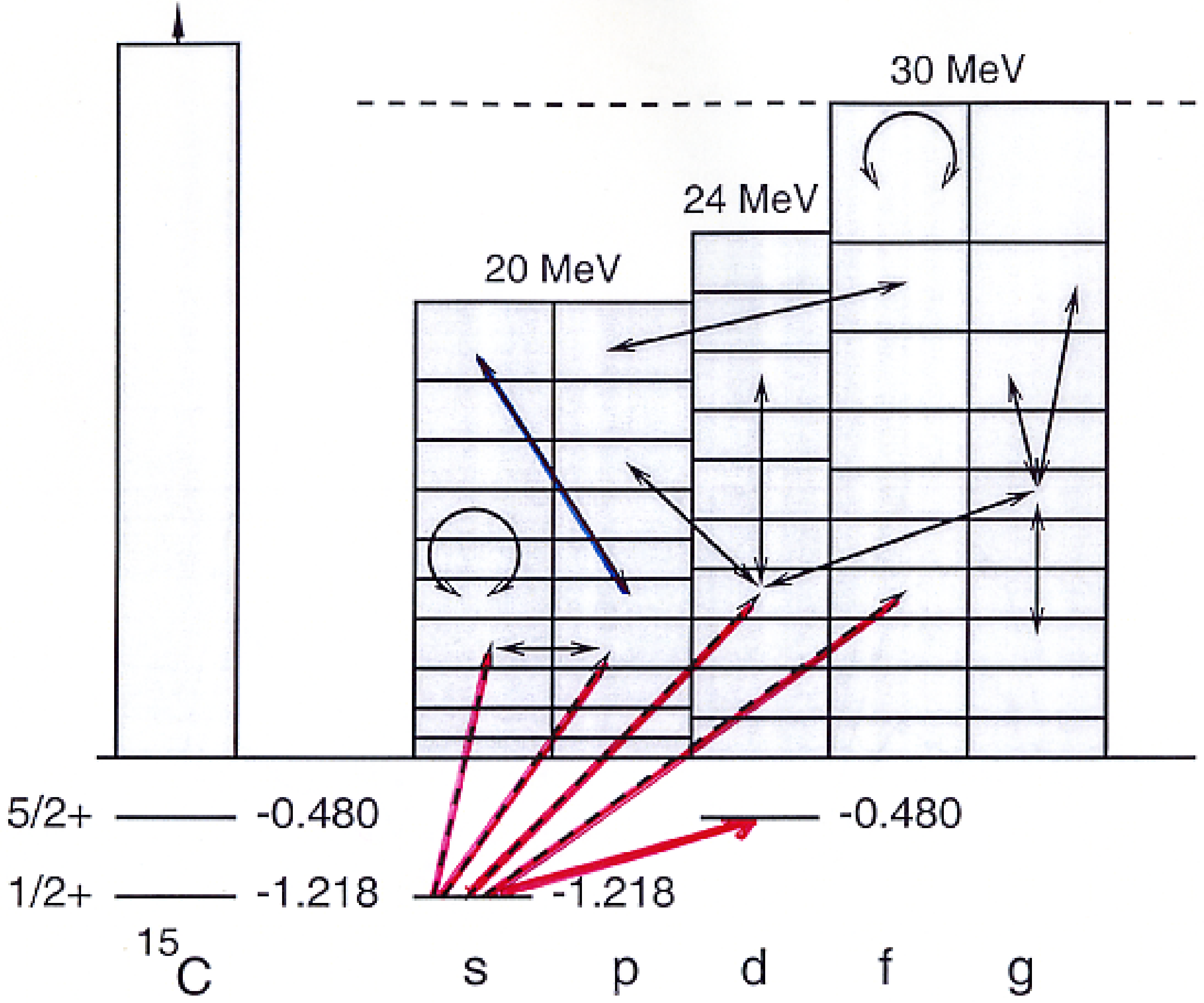
$$\hat{\phi}_i(\underline{r}) = \sqrt{\frac{2}{\pi N_i}} \int_{k_{i-1}}^{k_i} \underbrace{f_i(k)}_{} \, \underbrace{\phi_{\ell j}(k,\underline{r})}_{\text{continuum states}} \, dk$$

$$\text{in } \Delta k_i : \quad \begin{array}{l} H_p \\ \left\{ \begin{array}{l} \text{unresonant} \\ \text{resonant} \end{array} \right. \end{array} \begin{array}{c} \text{weights} \\ \left\{ \begin{array}{l} = 1 \\ = \sin\delta_\ell(k) \end{array} \right\} \end{array} \qquad N_i = \int_{\Delta k_i} dk \, |f_i(k)|^2$$

$$\phi_{\ell j}(\underline{r}) \rightarrow \{\cos\delta_\ell \, F_\ell(kr) + \sin\delta_\ell \, G_\ell(kr)\} \, Y_\ell(\hat{r}) \ldots$$

- Key is:

$$\langle \hat{\phi}_i | \hat{\phi}_j \rangle_{\underline{r}} = \delta_{ij} \, , \quad \hat{\varepsilon}_i = \langle \hat{\phi}_i | H_p | \hat{\phi}_i \rangle$$


30 MeV
24 MeV
20 MeV
5/2+
-0.480
1/2+
-1.218
-0.480
-1.218
15C
s
p
d
f
g

Coupled continuum channels $H_a \rightarrow H_p\,,\ V_{aA} \rightarrow U(\underset{\sim}{R},\underline{r})$

but $\phi_a \longrightarrow \phi_{\underset{\sim}{k}}(\underline{r})$, infinite in number, infinite in range

$$\left\{(E-\varepsilon_k)-T_R\right\}\chi_i(\underset{\sim}{R}) = \sum\!\!\!\!\!\!\int d\underset{\sim}{k}_j \, \langle \phi_i | U(\underset{\sim}{R},\underline{r}) | \phi_j \rangle_{\underline{r}} \, \chi_j(\underset{\sim}{R})$$

- infinite number of coupled systems
- continuum-continuum couplings $\langle \phi_{\underset{\sim}{k}'} | U | \phi_{\underset{\sim}{k}} \rangle$ a problem.
- $\phi_0 \Rightarrow (\ell_0 j_0)$ – (e.g. ^{15}C, $\tfrac{1}{2}^+$, $\ell_0=0$, $j_0=\tfrac{1}{2}$)

  $\phi_{\underset{\sim}{k}} \Rightarrow (k,\ell,j)$ – $\Big(\underbrace{\tfrac{1}{2}^+}_{\ell=0},\ \underbrace{\tfrac{1}{2}^-,\ \tfrac{3}{2}^-}_{\ell=1},\ \underbrace{\tfrac{3}{2}^+,\ \tfrac{5}{2}^+}_{\ell=2},\ \ldots\ldots\Big)$

  $\longrightarrow \phi_{\ell j}(k,r)$

Must go back to Hamiltonian Hp and use some 'discrete' basis of states $\hat{\phi}_i(\underline{r})$
(put in 'box')

$$\sum\!\!\!\!\!\!\int d\underset{\sim}{k}\, |\phi_{\underset{\sim}{k}}\rangle\langle\phi_{\underset{\sim}{k}}| \approx \sum_i |\hat{\phi}_i\rangle\langle\hat{\phi}_i|$$

Higher order effects — time spent within range of tidal forces $U(\underset{\sim}{R},\underset{\sim}{r})$.

Depends on both energy and interaction range

if $U(\underset{\sim}{R},\underset{\sim}{r})$ act once – "Born Approximation" $\phi_0 \xrightarrow{u} \phi_{\underset{\sim}{k}}$

(favoured by high energy, short range interactions, well bound ϕ_0) $\langle \phi_{\underset{\sim}{k}} | U(\underset{\sim}{R},\underset{\sim}{r}) | \phi_0 \rangle$

higher order effects of $U(\underset{\sim}{R},\underset{\sim}{r})$ $\phi_0 \xrightarrow{u} \phi_{\underset{\sim}{k}} \xrightarrow{u} \phi_{\underset{\sim}{k}'} \xrightarrow{u} \cdots\cdots$

(low energy, long range U, extended and weakly bound ϕ_0)

$\langle \phi_{\underset{\sim}{k}} | U(\underset{\sim}{R},\underset{\sim}{r}) | \phi_0 \rangle$

'continuum-continuum couplings' $\longleftarrow$ $\langle \phi_{\underset{\sim}{k}'} | U(\underset{\sim}{R},\underset{\sim}{r}) | \phi_{\underset{\sim}{k}} \rangle$

$\vdots$

problem for higher order calculations

$\phi_{\underset{\sim}{k}}$ of infinite range — non-convergent couplings.

- similarly, excitations of c (and v) and t *

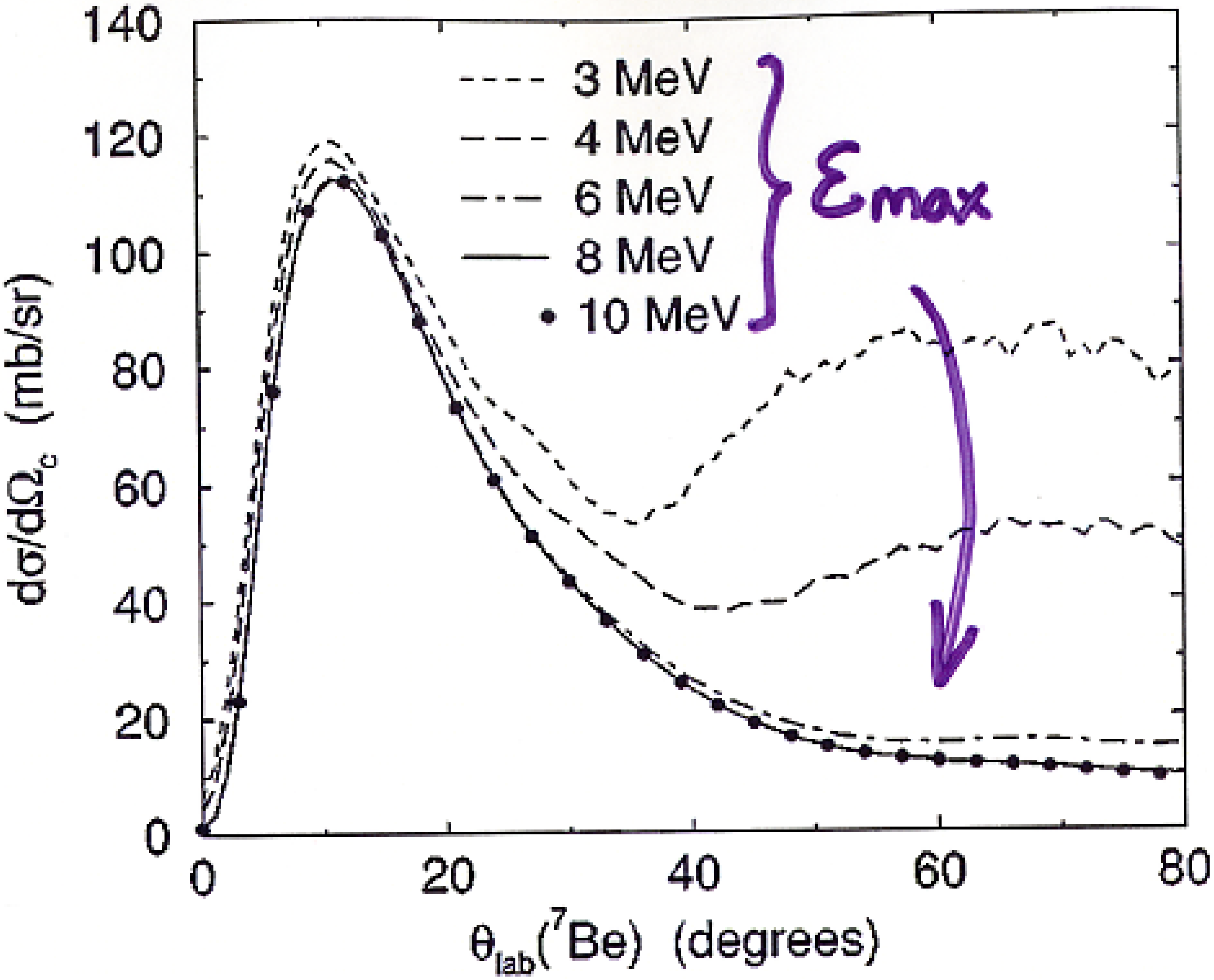


FIG. 1. Convergence of the calculated laboratory-frame ^{7}Be cross section angular distribution following the breakup of ^{8}B on ^{58}Ni at 25.8 MeV as a function of the maximum proton-^{7}Be relative energy included in the calculation.

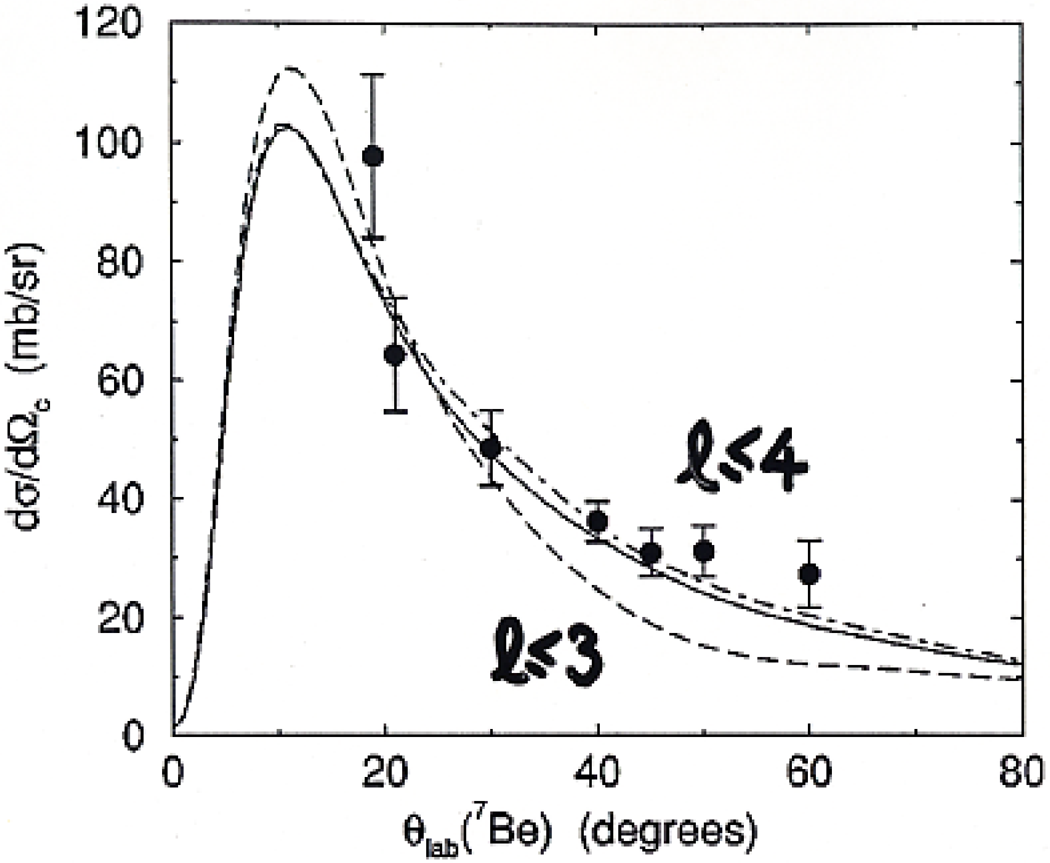
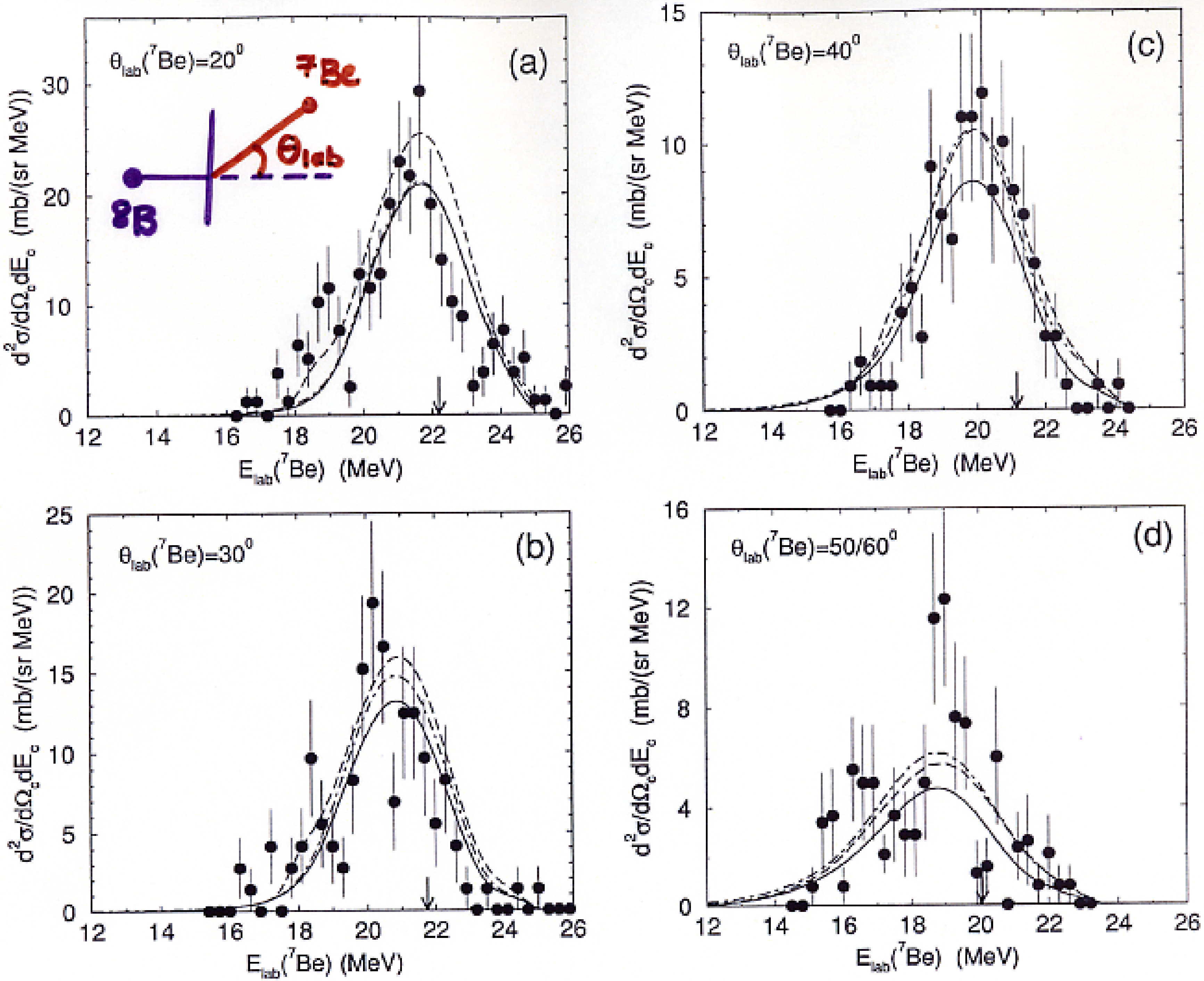
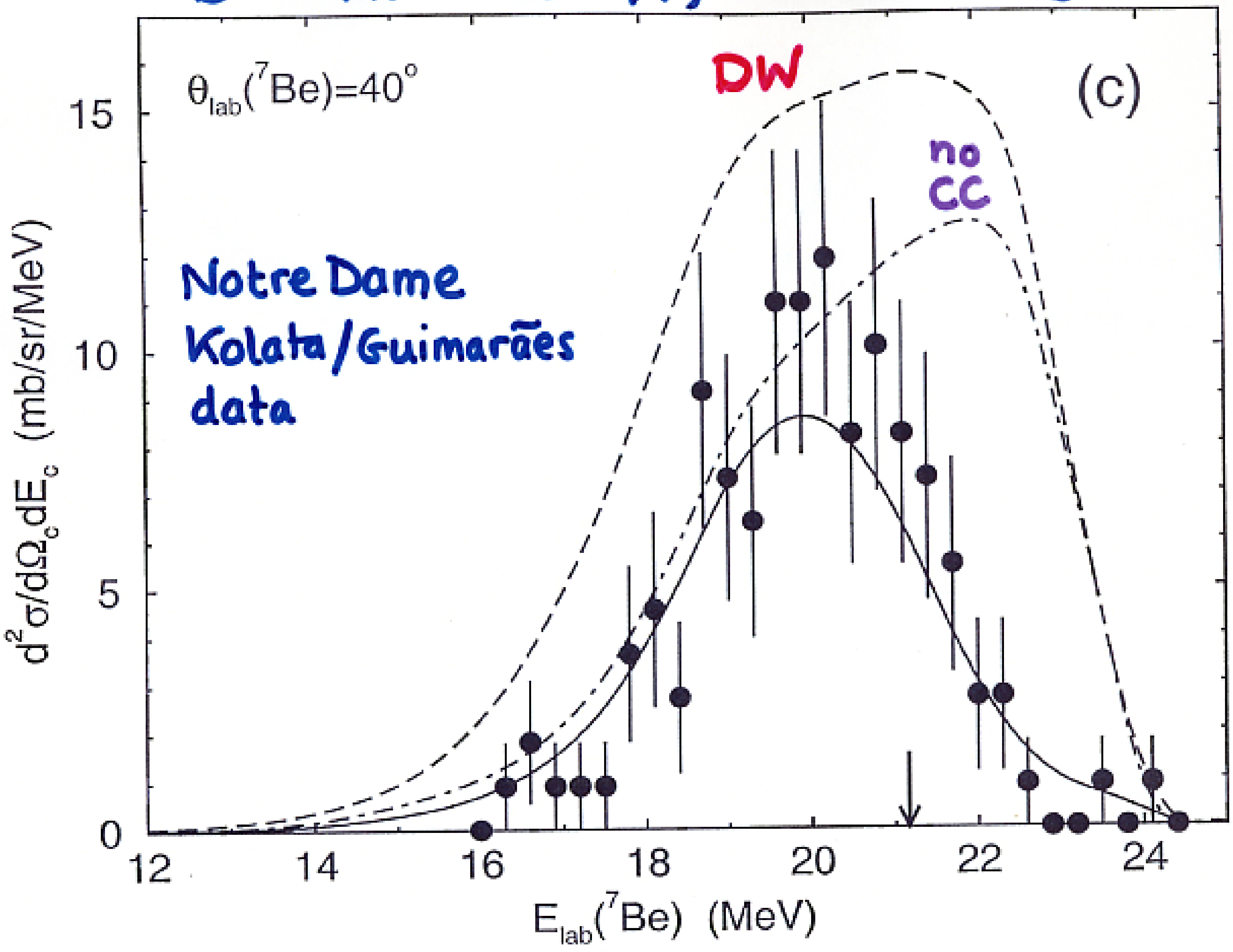


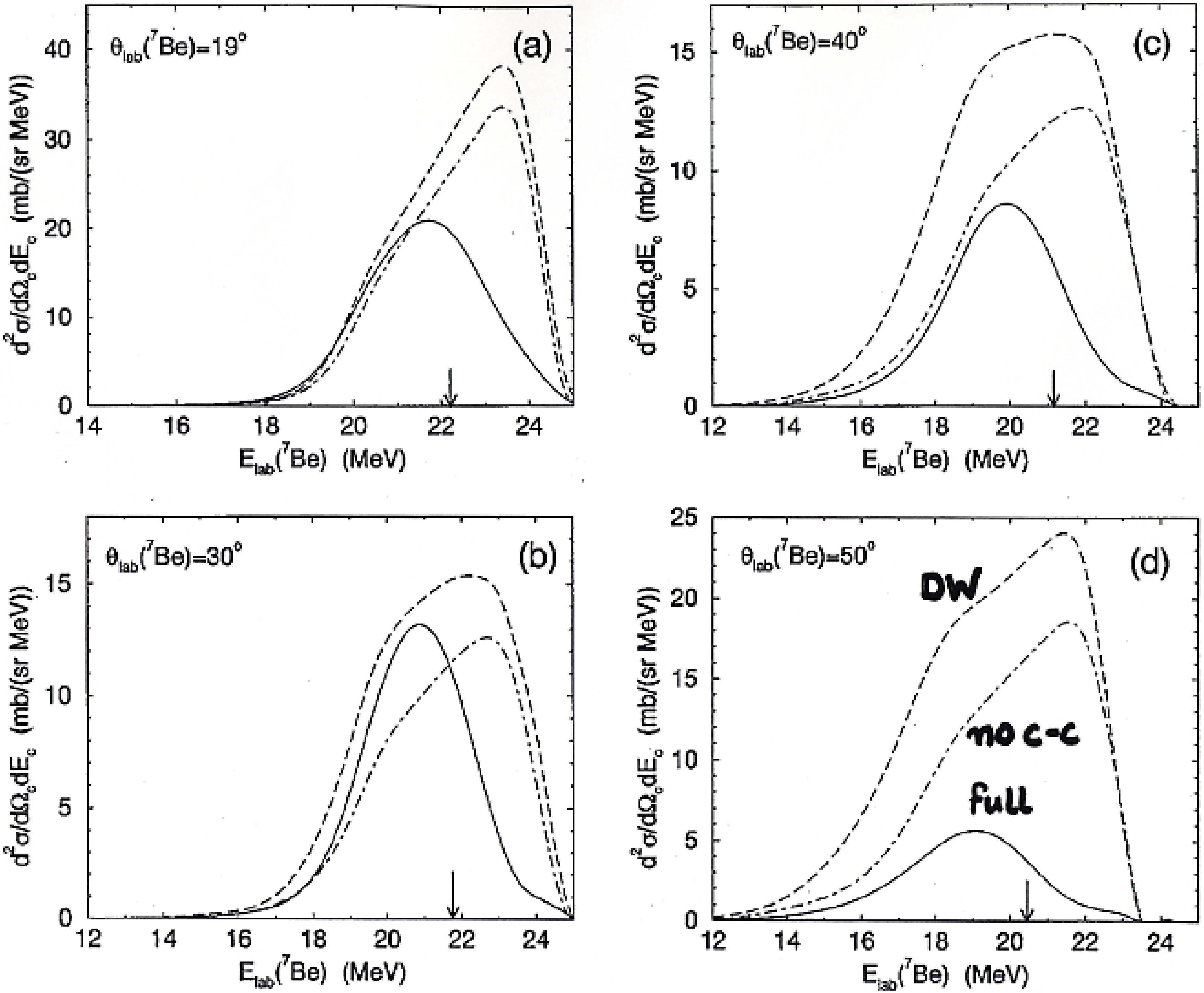
FIG. 2. The calculated laboratory-frame ^{7}Be cross section angular distribution following the breakup of ^{8}B on ^{58}Ni at 25.8 MeV. The long-dashed curve is the $\mathcal{E}_{\max}=10$ MeV, $l\leqslant 3$, $q\leqslant 2$, calculation from Fig. 1. The solid curve includes $q=3$ multipole terms while the dot-dashed curve includes both $q=4$ and $l=4$ effects.



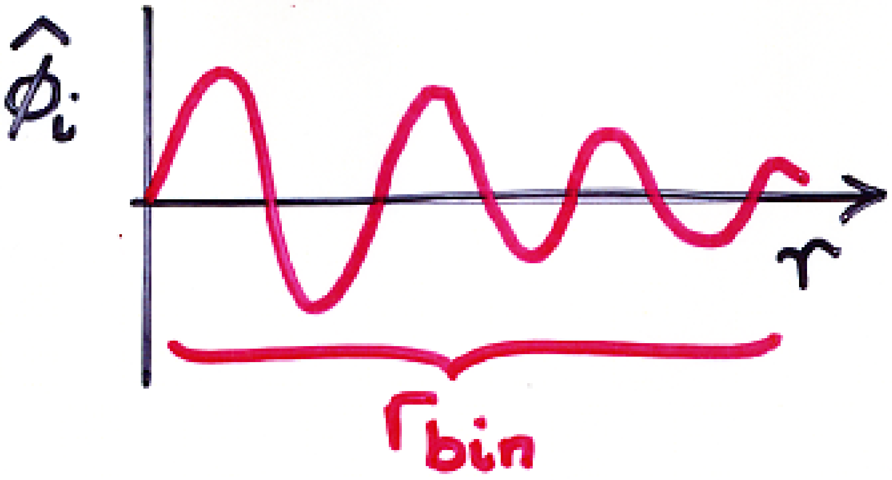
Data. Kolata et al. PRC 63 ('01) 024616

$^8B + ^{58}Ni \rightarrow ^7Be + X$, 26 MeV = $E_{^8B}$




$\theta_{lab}(^7Be)=19°$
(a)
$d^2\sigma/d\Omega_c dE_c$ (mb/(sr MeV))
0
10
20
30
40
14
16
18
20
22
24
$E_{lab}(^7Be)$ (MeV)
$\theta_{lab}(^7Be)=40°$
(c)
0
5
10
15
12
14
16
18
20
22
24
$\theta_{lab}(^7Be)=30°$
(b)
0
5
10
15
12
14
16
18
20
22
24
$\theta_{lab}(^7Be)=50°$
(d)
DW
no c-c
full
0
5
10
15
20
25
12
14
16
18
20
22
24

CDCC $\hat{\phi}_i$ are called 'bin' states – normalisable discrete excited states of H_p in continuum



Uncertainty principle

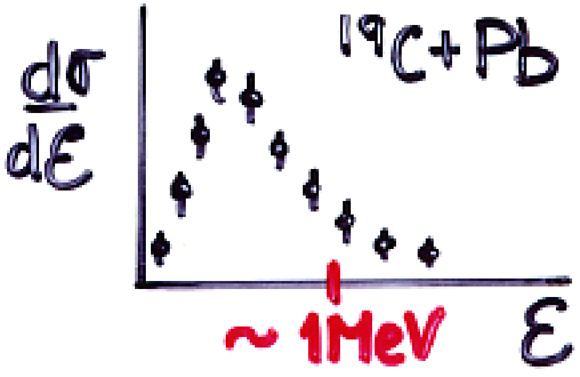
$$\hat{\phi}_i(r) \Longleftrightarrow \int_{\Delta k_i} dk \ldots$$

$$r_{bin} \Longleftrightarrow \Delta k_i$$

↳ small (particularly for Coulomb forces)

↓

long range couplings $\langle \hat{\phi}_i | U(\underline{R},\underline{r}) | \hat{\phi}_j \rangle$ etc.



r_{bin}, Δk_i must be chosen carefully

→ convergence

$\mathcal{E}_{max}$, ℓ_{max}, Δk_i ?

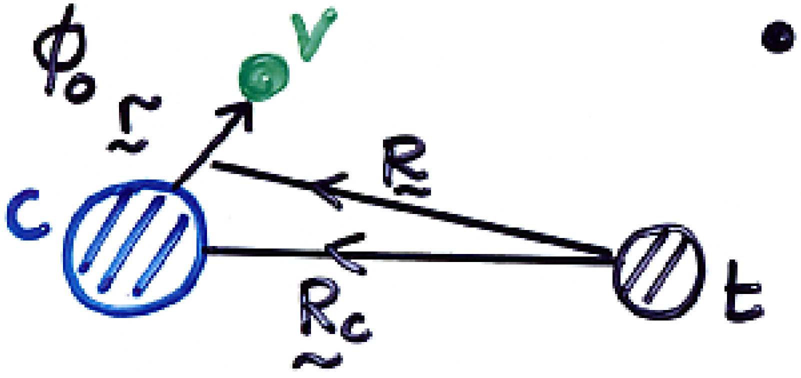
Adiabatic – non-eikonal

$$\{T_R + U(\underset{\sim}{R},\underset{\sim}{r}) - \varepsilon_0\}\,\psi^{AD}(\underset{\sim}{R},\underset{\sim}{r}) = E\,\psi^{AD}(\underset{\sim}{R},\underset{\sim}{r})$$

for each fixed $\underset{\sim}{r}$ can be solved exactly — $\underbrace{U(\underset{\sim}{R},\underset{\sim}{r})\ \text{non-central}}_{\text{coupled differential equations (notes)}}$

<u>but</u> if in $U(\underset{\sim}{R},\underset{\sim}{r})$, $\underline{V_{vt} \equiv 0}$ (e.g. heavy core, Coulomb case)

$$\{T_R + V_{ct}(\underset{\sim}{R_c})\}\,\tilde{\Psi}(\underset{\sim}{R},\underset{\sim}{r}) = E_0\,\tilde{\Psi}(\underset{\sim}{R},\underset{\sim}{r})\ ,\quad E_0 = E + \varepsilon_0$$



Recoil adiabatic
Limit

- <u>exact solution</u>

$$\tilde{\Psi}(\underset{\sim}{R},\underset{\sim}{r}) = e^{i\alpha \underset{\sim}{K}\cdot \underset{\sim}{r}}\,\chi^{(+)}_{\underset{\sim}{K}}(\underset{\sim}{R_c})\,\phi_0(\underset{\sim}{r})$$

$$\begin{cases} \alpha = \dfrac{m_v}{m_v + m_c} \\ (T_{R_c} + V_{ct}(\underset{\sim}{R_c}))\,\chi^{(+)}_{\underset{\sim}{K}}(\underset{\sim}{R_c}) = E_0\,\chi^{(+)}_{\underset{\sim}{K}}(\underset{\sim}{R_c}) \end{cases}$$

So $f_{el}(\theta)$ from this $\tilde{\Psi}(\underset{\sim}{R},\underset{\sim}{r})$?

$$f_{el}(\theta) = \underbrace{\langle \underset{\sim}{K}'\phi_0(\underset{\sim}{r})}_{e^{-i\underset{\sim}{K}'\cdot\underset{\sim}{R}} = e^{-i\underset{\sim}{K}'\cdot(\underset{\sim}{R}_c-\alpha\underset{\sim}{r})}} |\, V_{ct}(R_c) \,| \underbrace{\tilde{\Psi}(\underset{\sim}{R},\underset{\sim}{r})\rangle}_{e^{i\alpha\underset{\sim}{K}\cdot\underset{\sim}{r}}\,\chi^{(+)}_{\underset{\sim}{K}}(\underset{\sim}{R}_c)\,\phi_0(\underset{\sim}{r})}$$

$$= \underbrace{\left\{\int d\underset{\sim}{r}\,|\phi_0(\underset{\sim}{r})|^2 e^{i\alpha\underset{\sim}{q}\cdot\underset{\sim}{r}}\right\}}_{F(\alpha\underset{\sim}{q})} \underbrace{\langle \underset{\sim}{K}'|V_{ct}|\chi^{(+)}_{\underset{\sim}{K}}\rangle}_{f_{pt}(\theta)}$$

$$\boxed{f_{el}(\theta) = F(\alpha\underset{\sim}{q})\,f_{pt}(\theta)} \qquad \underset{\sim}{q} = \underset{\sim}{K}-\underset{\sim}{K}'$$

extended c+v system

scattering of composite by Vct if pointlike.

Recoil adiabatic solution ($V_{vt} \equiv 0$)

$$f_{el}(\theta) = F(\alpha\underset{\sim}{q})\, f_{pt}(\theta) - \boxed{\text{EXACT}}$$

↳ calc from $V_{ct}(R_c)$ } includes breakup ●

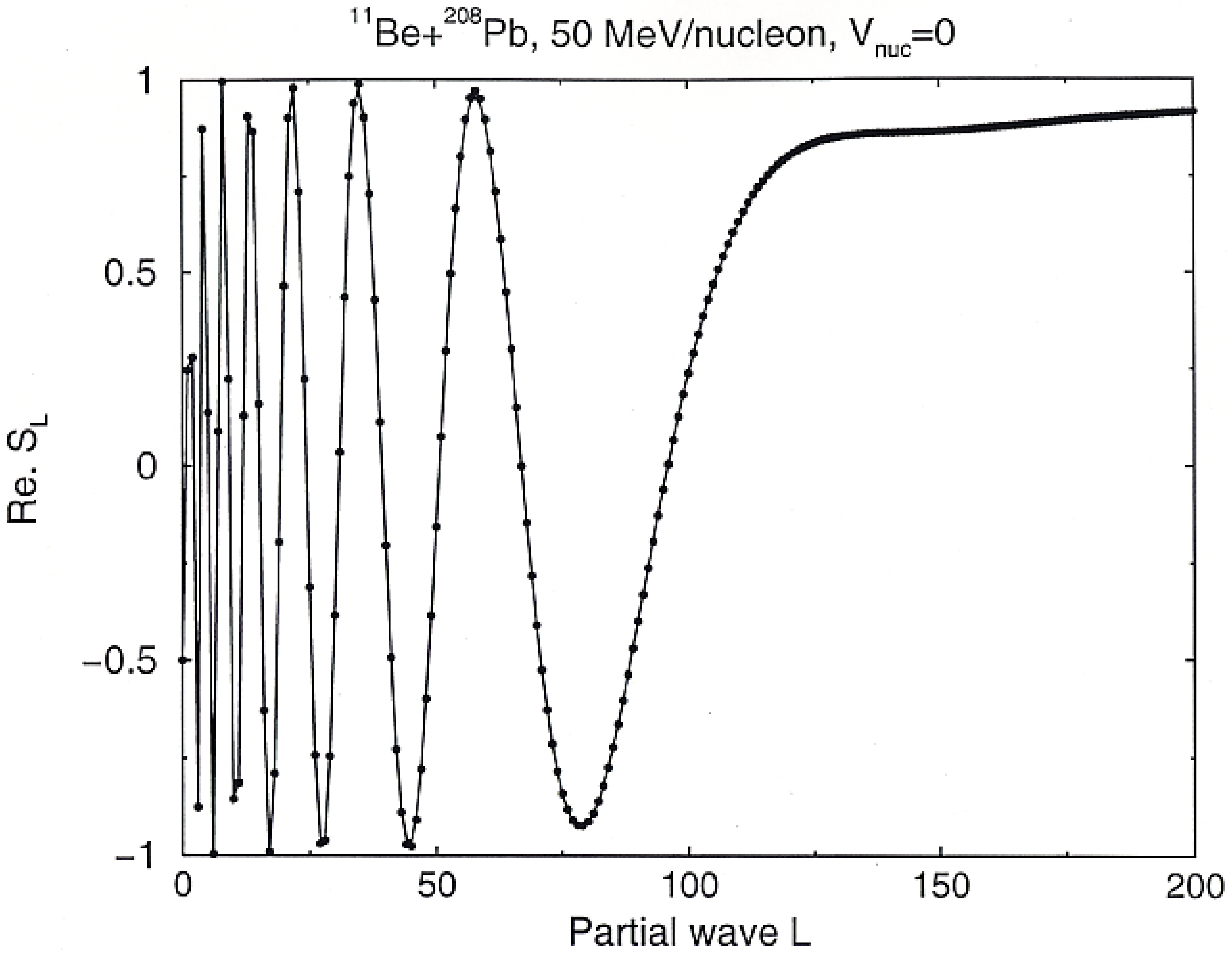
deviations from Coulomb scattering $f_{E\ell} = f_{el}(\theta) - f_c(\theta)$

$\int_0^\pi \sin\theta\, d\theta\, f_{E\ell} \rightarrow f_L \rightarrow S_L$: partial wave S-matrix element. ⇒ exact solution of (adiabatic) 3-body problem with Coulomb V_{ct} component

- no bin decomposition, Δk_i, kmax
- no ℓ-truncation
- no radial truncation, etc, etc.

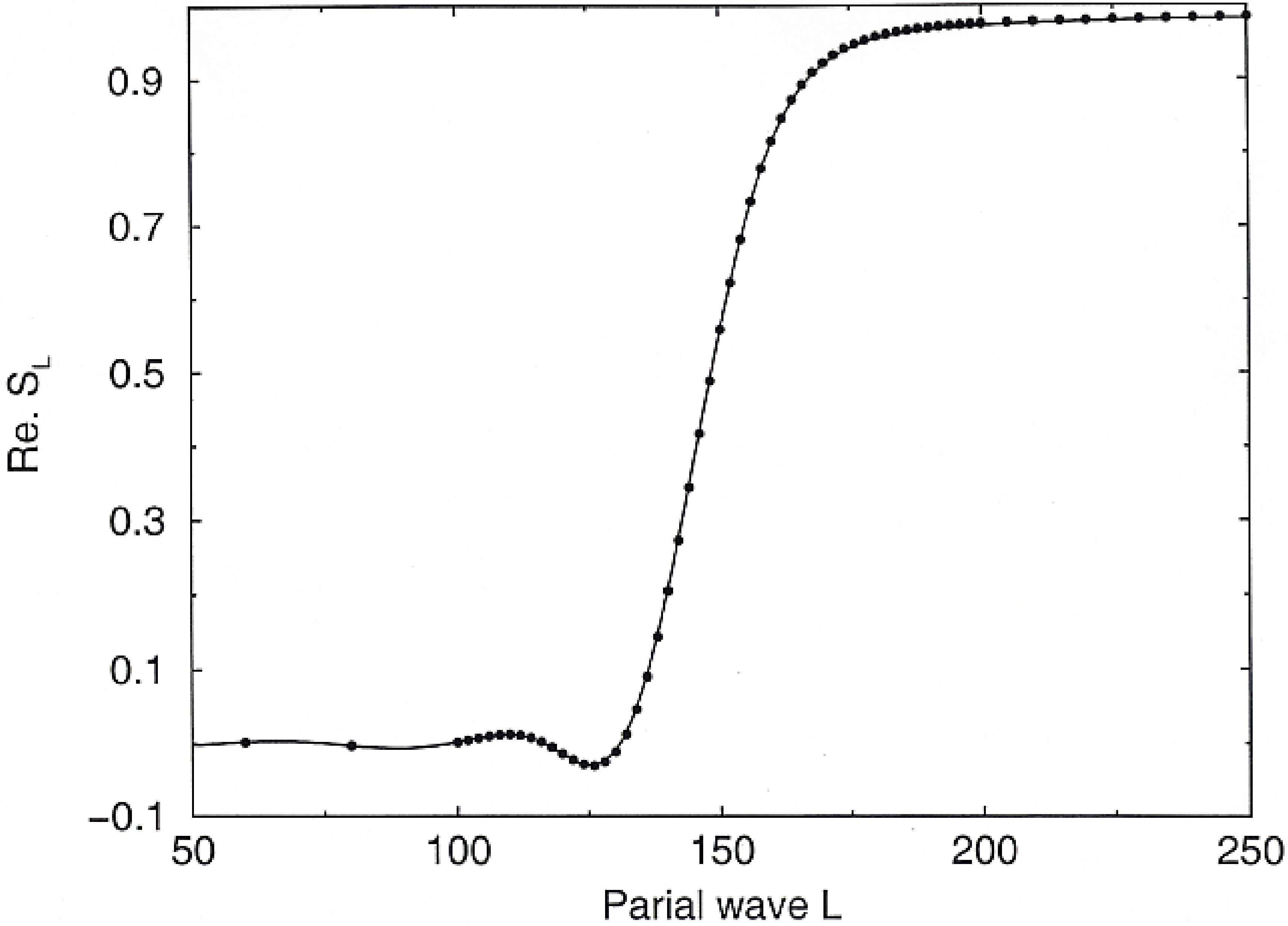
} of CDCC

● Should agree with CDCC calculation if replace all $\hat{\mathcal{E}}_i \rightarrow -\mathcal{E}_0$ in coupled equations



^{11}Be+^{208}Pb, 50 MeV/nucleon, V_{nuc}=0

$V_{nuc} \neq 0$



FURTHER READING
REACTION THEORY (for exotic nuclei)

Time-dependent and semi-classical methods

Nonperturbative time-dependent approach to breakup of halo nuclei, V. S. Melezhik and D. Baye, Physical Review C 59, 3232-3239

Time-dependent analysis of the Coulomb breakup method for determining the astrophysical S factor, V. S. Melezhik and D. Baye, Physical Review C 64, 054612 (11 pages)

Validity of the semiclassical approximation for the breakup of weakly bound nuclei, H. D. Marta, L. F. Canto, R. Donangelo et al., Physical Review C 66, 024605 (5 pages)

Elastic scattering and breakup of 17F at 10 MeV/nucleon, J. F. Liang, J. R. Beene, H. Esbensen et al., Physical Review C 65, 051603 (5 pages)

Eikonal approximation in heavy-ion fragmentation reactions, H. Esbensen and G. F. Bertsch, Physical Review C 64, 014608 (6 pages)

Dynamical description of the breakup of one-neutron halo nuclei 11Be and 19C, S. Typel and R. Shyam, Physical Review C 64, 024605 (8 pages)

Higher order effects in electromagnetic dissociation of neutron halo nuclei, S. Typel and G. Baur, Physical Review C 64, 024601 (7 pages)

Higher order effects in electromagnetic dissociation of fast particles, a soluble model and application to 11Li, S. Typel and G. Baur, Nuclear Physics A 573 (1994) 486-500

Higher-order effects in the Coulomb dissociation of 8B into 7Be + p, S. Typel, H.H. Wolter and G. Baur, Nuclear Physics A 613 (1997) 147-164

Electromagnetic dissociation of 8B and the rate of the 7Be(p,γ)8B reaction in the Sun, B. Davids, Sam M. Austin, D. Bazin et al., Physical Review C 63, 065806 (14 pages)

Eikonal theory and momentum distributions

Evaluation of an eikonal model for 11Li-nucleus elastic scattering, J.S. Al-Khalili, I.J. Thompson and J.A. Tostevin, Nuclear Physics A 581 (1995) 331-355

Momentum Content of Single-Nucleon Halos, P. G. Hansen, Phys. Rev. Lett. 77, 1016-1019 (1996)

Momentum distributions in stripping reactions of single-nucleon halo nuclei, H. Esbensen, Phys. Rev. C 53, 2007-2010 (1996)

Breakup reactions of the halo nuclei 11Be and 8B, K. Hencken, G. Bertsch, and H. Esbensen, Phys. Rev. C 54, 3043-3050 (1996)

How large are the halos of light nuclei?, J.A. Tostevin and J.S. Al-Khalili, Nuclear Physics A 616 (1997) 418-425

Elastic and quasielastic scattering of 8He from 12C, J. A. Tostevin, J. S. Al-Khalili, M. Zahar, M. Belbot, J. J. Kolata, K. Lamkin, D. J. Morrissey, B. M. Sherrill, M. Lewitowicz, and A. H. Wuosmaa, Phys. Rev. C 56, R2929-R2933 (1997)

Few-body calculations of proton-6,8He scattering, J. S. Al-Khalili and J. A. Tostevin, Phys. Rev. C 57, 1846-1852 (1998)

Nuclear breakup of Borromean nuclei, G. F. Bertsch, K. Hencken, and H. Esbensen, Phys. Rev. C 57, 1366-1377 (1998)

Nuclear induced breakup of halo nuclei, H. Esbensen and G. F. Bertsch, Physical Review C 59, 3240-3245

Systematic study of 8B breakup cross sections, Henning Esbensen and Kai Hencken, Physical Review C 61, 054606 (8 pages)

Noneikonal calculations for few-body projectiles, J. M. Brooke, J. S. Al-Khalili, and J. A. Tostevin, Physical Review C 59, 1560-1566

Calculations of reaction cross sections for 19C at relativistic energies, J. A. Tostevin and J. S. Al-Khalili, Physical Review C 59, R5-R8

Manifestation of halo size in scattering and reactions, J.A. Tostevin, R.C. Johnson and J.S. Al-Khalili, Nuclear Physics A 630 (1998) 340-351

Eikonal models for spectroscopic studies

Core excitation in halo nucleus breakup, J.A. Tostevin, J. Phys. G: Nucl. Part. Phys. **25** (1999) 735-739.

Direct Evidence for the Breakdown of the $N = 8$ Shell Closure in 12Be, A. Navin, et al., Phys. Rev. Lett. 85, 266-269 (2000)

One-Neutron Knockout from Individual Single-Particle States of 11Be, T. Aumann, et al., Phys. Rev. Lett. 84, 35-38 (2000)

Spectroscopy of Radioactive Beams from Single-Nucleon Knockout Reactions: Application to the sd Shell Nuclei 25Al and 26,27,28P, A. Navin, et al., Phys. Rev. Lett. 81, 5089-5092 (1998)

One-neutron removal reactions on neutron-rich psd-shell nuclei, E. Sauvan et al., Physics Letters B 491 (2000) 1-7

Cross sections, momentum distributions, and neutron angular distributions for Be induced reactions on silicon, F. Negoita et al., Phys. Rev. C 59, 2082 (1999)

Single-neutron knockout reactions: Application to the spectroscopy of 16,17,19C, V. Maddalena, T. Aumann, D. Bazin et al., Physical Review C, 63, 024613 (17 pages)

Absolute spectroscopic factors from nuclear knockout reactions, B. A. Brown, P.

G. Hansen, B. M. Sherrill et al., Physical Review C 65, 061601 (4 pages)

Single-neutron knockout from 34,35Si and 37S, J. Enders, A. Bauer, D. Bazin et al., Physical Review C 65, 034318 (10 pages)

Single-nucleon knockout reactions at fragmentation beam energies, J.A. Tostevin, Nuclear Physics A 682 (2001) 320-331

Spectroscopy of 13,14B via the one-neutron knockout reaction, V. Guimares, J. J. Kolata, D. Bazin, B. Blank, B. A. Brown, T. Glasmacher, P. G. Hansen, R. W. Ibbotson, D. Karnes, V. Maddalena, A. Navin, B. Pritychenko, B. M. Sherrill, D. P. Balamuth, and J. E. Bush, Phys. Rev. C 61, 064609 (2000)

Coupled channels treatment of breakup

Coupled-channels theory of breakup processes in nuclear reactions, Kamimura M, Yahiro M, Iseri Y, Sakuragi Y, Kameyama H and Kawai M, (1986) Progress in Theoretical Physics Supplement 89, 1.

Continuum discretized coupled-channels calculations for three-body models of deuteron-nucleus reactions, Austern N, Iseri Y, Kamimura M, Kawai M, Rawitscher G and Yahiro M, (1987) Physics Reports 154, 125.

Higher-order and E2 effects in medium energy 8B breakup, J. Mortimer, I. J. Thompson, and J. A. Tostevin, Physical Review C 65, 064619 (8 pages)

Electromagnetic dissociation of 8B and the rate of the 7Be(p,γ)8B reaction in the Sun, B. Davids, Sam M. Austin, D. Bazin et al., Physical Review C 63, 065806 (14 pages)

Coupling to breakup channels using a transformed harmonic oscillator basis, A. M. Moro, J. M. Arias, J. Gomez-Camacho et al., Physical Review C 65, 011602 (5 pages)

Multistep effects in sub-Coulomb breakup, F. M. Nunes and I. J. Thompson, Physical Review C 59, 2652-2659

Calculations of three-body observables in 8B breakup, J. A. Tostevin, F. M. Nunes, and I. J. Thompson, Physical Review C 63, 024617 (10 pages)

Multistep Coulomb and nuclear breakup of one-nucleon halo nuclei, I.J. Thompson, J.A. Tostevin and F.M. Nunes, Nuclear Physics A 690 (2001) 294-297

Single-neutron removal reactions from 15C and 11Be: Deviations from the eikonal approximation, J. A. Tostevin, D. Bazin, B. A. Brown, T. Glasmacher, P. G. Hansen, V. Maddalena, A. Navin, and B. M. Sherrill, Physical Review C 66, August 2002.

Adiabatic and recoil adiabatic models

Four-body adiabatic model applied to elastic scattering, J.A. Christley, J.S. Al-Khalili, J.A. Tostevin and R.C. Johnson, Nuclear Physics A 624 (1997) 275-292

Elastic Scattering of Halo Nuclei, R. C. Johnson, J. S. Al-Khalili, and J.A.

Tostevin Phys. Rev. Lett. 79, 2771-2774 (1997)

Manifestation of halo size in scattering and reactions, J.A. Tostevin, R.C. Johnson and J.S. Al-Khalili, Nuclear Physics A 630 (1998) 340-351

Coulomb breakup of light composite nuclei, J.A. Tostevin et al., Physics Letters B 424 (1998) 219-225

Coulomb dissociation of light nuclei, J. A. Tostevin, S. Rugmai, and R. C. Johnson, Phys. Rev. C 57, 3225-3236 (1998)

Coulomb breakup of two-neutron halo nuclei, P. Banerjee, J. A. Tostevin, and I. J. Thompson, Phys. Rev. C 58, 1337-1340 (1998)

Scattering and reactions of halo nuclei, R.C. Johnson, Progress in Theoretical Physics Supplement 140, (2000) 33.

Nonadiabatic corrections to elastic scattering of halo nuclei, N. C. Summers, J. S. Al-Khalili, and R. C. Johnson Phys. Rev. C 66, 014614 (2002)

Applications to transfer reactions

One- and two-nucleon transfer reactions, N.K. Glendenning, Chapter IX.E, in Nuclear Spectroscopy and Reactions, Vol. D, ed J. Cerny, Academic Press, 319.

Single-neutron transfer from 11Be(gs) via the (p,d) reaction with a radioactive beam, Nuclear Physics A 683 (2001) 48-78

Deuteron stripping and pick-up on halo nuclei, N. K. Timofeyuk and R. C. Johnson Phys. Rev. C 59, 1545 (1999)

Are coupled channel effects important for the asymptotic normalization coefficient method?, F. M. Nunes and A. M. Mukhamedzhanov, Phys. Rev. C 64, 062801 (2001)

Dynamics of two-neutron transfer reactions with the Borromean nucleus 6He, Yu. Ts. Oganessian, V. I. Zagrebaev, and J. S. Vaagen, Phys. Rev. C 60, 044605 (1999)