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Tensor Force Effects in Real & Model Spaces

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On the Role of NN Tensor Force in Nuclei

July 24, 2000 Y. Akaishi

Realistic NN interaction

$$V = V_C(r) + V_T(r)S_{12} + V_{LS}(r)\vec{L}\vec{S} + V_W(r)W_{12} + V_{LL}(r)\vec{L}^2$$

R. Tamagaki, Prog. Theor. Phys. 39 (1968) 91.

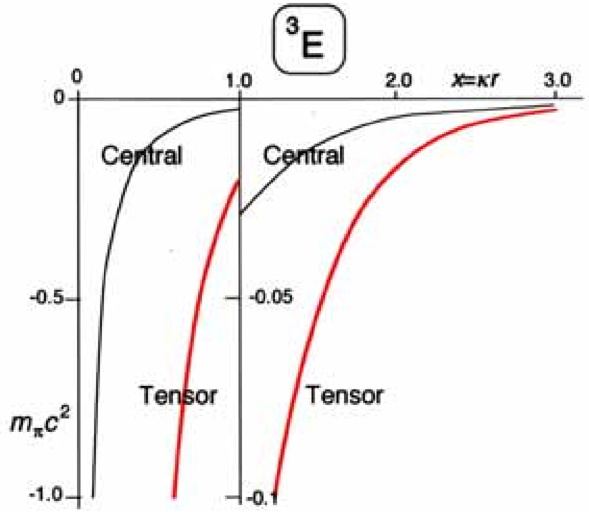
$$S_{12} = 3\frac{(\vec{\sigma}_1\vec{r})(\vec{\sigma}_2\vec{r})}{r^2} - \vec{\sigma}_1\vec{\sigma}_2$$

Y^M_{LSJ}

$$S_{12}\begin{bmatrix} Y_{J-1,1,J} \\ Y_{J,1,J} \\ Y_{J+1,1,J} \end{bmatrix} = \frac{1}{2J+1}\begin{bmatrix} -2(J-1) & 0 & 6\sqrt{J(J+1)} \\ 0 & 2(2J+1) & 0 \\ 6\sqrt{J(J+1)} & 0 & -2(J+2) \end{bmatrix}\begin{bmatrix} Y_{J-1,1,J} \\ Y_{J,1,J} \\ Y_{J+1,1,J} \end{bmatrix}$$

OPEP : Main origin of tensor force

Prog. Theor. Phys. Suppl. No.3 (1956)



Alpha Particle

$\hbar\omega = 21.6$ MeV

$KE = 3 \times \frac{3}{4}\hbar\omega = 48.6$ MeV

(MeV)

		H-J	RSC v8		Volkov
Energy		-20.6	-21.9		-29.0
Kin. E		131.1	103.6	are 2~2.5 times larger than Strong short-range correlation	48.6
Pot. E		-151.7	-125.4		-77.6
C	1E	-51.3	-37.2		-38.8
	3E	-26.2	-0.6		-38.8
	1O+3O	-0.4	0.5		0.0
T	3E	-69.7	-89.4	The largest contribution	0
	3O	-0.5	-0.7		0
LS+QLS		-3.6	1.9		0
P(D) %		12.8	11.0	D-state correlation due to tensor force	0

M. Sakai, I. Shimodaya, Y. Akaishi, J, Hiura & H. Tanaka, Prog. Theor. Phys. 56 (1974) 32.

Theory of Nuclear Matter

"Model space"

K.A. Brueckner & C.A. Levinson, Phys. Rev. 97 (1955) 1344
J. Goldstone, Proc. Roy. Soc. A239 (1957) 267
H.A. Bethe, Phys. Rev. 103 (1956) 1353

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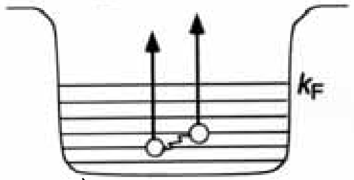
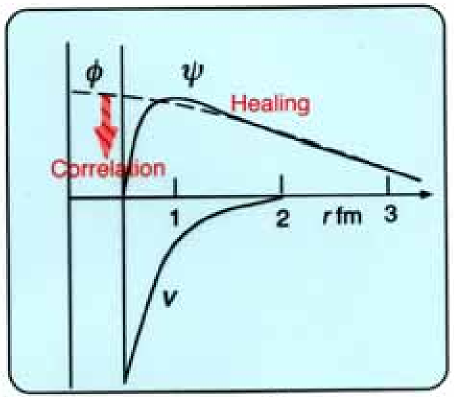
Foundation of shell model

Independent-pair scattering mode in nuclear matter

L.C. Gomes, J.D. Walecka & V.F. Weisskopf, Ann. Phys. 3 (1958) 241





$$g|\phi\rangle = v|\psi\rangle\ , \qquad |\psi\rangle = |\phi\rangle + \frac{Q}{\varepsilon_1 + \varepsilon_2 - t_1 - t_2} v|\psi\rangle$$

Hole-line expansion method

Y. Akaishi, H. Bando, A. Kuriyama & S. Nagata, Prog. Theor. Phys. 40 (1968) 288

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Independent-pair scattering mode in ^{4}He

Few-body
Tensor

Y. Akaishi & S. Nagata, Prog. Theor. Phys. 48 (1972) 133

Multiple Scattering Theory

K.M. Watson, Phys. Rev. 89 (1953) 575

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ATMS Method

"Real space"

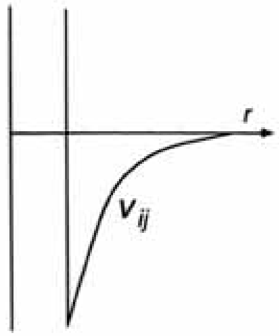
Y. Akaishi, H. Tanaka et al., Int. Rev. Nucl. Phys. Vol.4 (1986) 259

Real Space versus Model Space

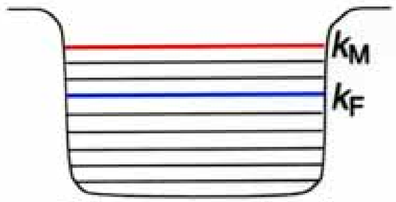
Real space

$$H|\Psi\rangle = E_0|\Psi\rangle$$

$$H = T + V, \qquad V = \sum_{(ij)} v_{ij}$$



Model space



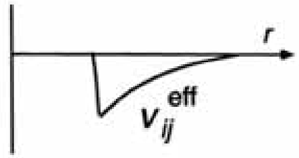
Plane wave basis

Truncated to

$$P = \sum_{\vec{k}}^{k \le k_M} |\vec{k}\rangle\langle\vec{k}|, \quad Q = 1 - P$$

$$H_M|\Phi\rangle = E_0|\Phi\rangle$$

$$H_M = P(T + V_M)P, \quad V_M = \sum_{(ij)} v_{ij}^{\text{eff}}$$



$$\langle\Phi|\Phi\rangle = 1$$

$$P|\Phi\rangle = |\Phi\rangle, \qquad Q|\Phi\rangle = 0,$$

Transformation

$$|\Psi\rangle = \hat{F}\,|\Phi\rangle$$

$$\hat{F} = 1 + \frac{Q}{e} V \hat{F}$$
$$e = E_0 - QTQ$$

↓

i) $P|\Psi\rangle = |\Phi\rangle$, ii) $\langle\Phi|\Psi\rangle = 1$

$$(E_0 - QTQ)|\Psi\rangle = (E_0 - QTQ)|\Phi\rangle + QV|\Psi\rangle$$
$$E_0|\Psi\rangle - TQ|\Psi\rangle = E_0|\Phi\rangle + QV|\Psi\rangle$$
$$E_0|\Psi\rangle - T(1-P)|\Psi\rangle = E_0|\Phi\rangle + (1-P)V|\Psi\rangle$$
$$E_0|\Psi\rangle - T|\Psi\rangle + T|\Phi\rangle = E_0|\Phi\rangle + V|\Psi\rangle - PV|\Psi\rangle$$
$$E_0|\Psi\rangle - H|\Psi\rangle = E_0|\Phi\rangle - T|\Phi\rangle - PV|\Psi\rangle$$

Now we define V_M so as to satisfy the relation;

$$PV_M|\Phi\rangle = PV|\Psi\rangle .$$

Then, $(E_0 - H)|\Psi\rangle = 0$

Reaction matrix

Def. of g

$$g_{ij} = v_{ij} + v_{ij}\frac{Q}{e}g_{ij}$$

Def. of $\hat{F}_{ij}$

$$g_{ij}\hat{F}_{ij} = v_{ij}\hat{F}$$

$$= v_{ij}\left(1+\frac{Q}{e}g_{ij}\right)\hat{F}_{ij}$$

$$= v_{ij}\left(1+\frac{Q}{e}\sum_{(kl)}v_{kl}\hat{F}\right)$$

$$= v_{ij}\left(1+\frac{Q}{e}\sum_{(kl)}g_{kl}\hat{F}_{kl}\right)$$

$$\hat{F} = 1+\sum_{(ij)}\frac{Q}{e}g_{ij}\hat{F}_{ij}$$

$$\hat{F}_{ij} = 1+\sum_{(kl)}{}'\frac{Q}{e}g_{kl}\hat{F}_{kl}$$

Multiple scattering process

$$PV|\Psi\rangle = P\sum_{(ij)}v_{ij}\hat{F}|\Phi\rangle = P\sum_{(ij)}g_{ij}\hat{F}_{ij}|\Phi\rangle$$

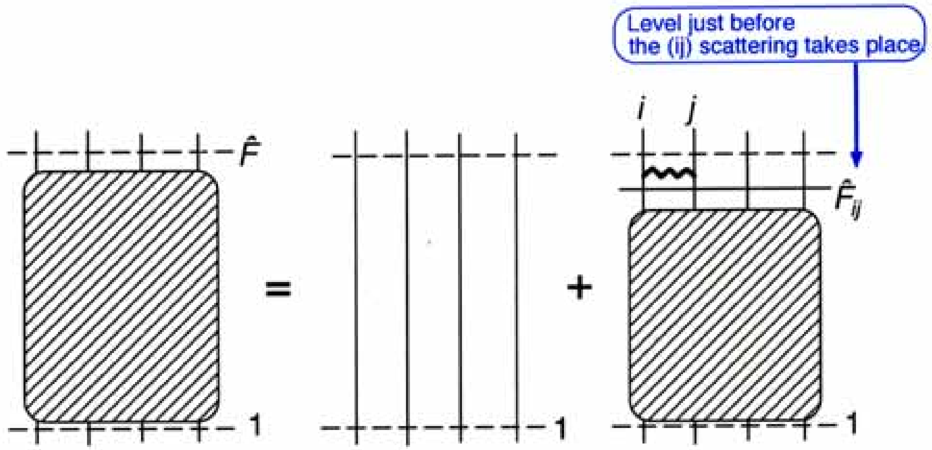
$$= P\sum_{(ij)}g_{ij}|\Phi\rangle + P\sum_{(ijk)}g_{ij}\frac{Q}{e}g_{jk}|\Phi\rangle + \ldots\ldots\ldots$$

$$PV_{\mathrm{M}}|\Phi\rangle = P\sum_{(ij)}v_{ij}^{\mathrm{eff}}|\Phi\rangle + \ldots\ldots\ldots$$

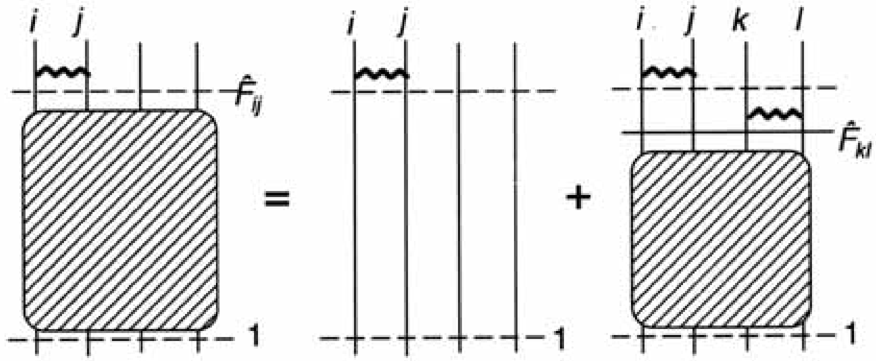
$$v_{ij}^{\mathrm{eff}} = g_{ij}\ ,\quad v_{ijk}^{\mathrm{eff}} = g_{ij}\frac{Q}{e}g_{jk}$$

Effective interaction

Multiple scattering process



$$\hat{F} = 1 + \sum_{(ij)} \frac{Q}{e} g_{ij} \hat{F}_{ij}$$



$$\hat{F}_{ij} = 1 + {\sum_{(kl)}}' \frac{Q}{e} g_{kl} \hat{F}_{kl}$$

Nuclear Matter

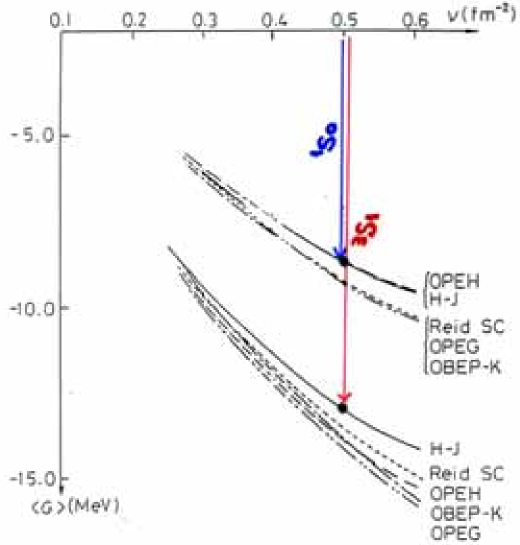
"Model space"

(MeV)

	H-J
E/A	-7.8
KE/A	23.9
PE/A	-31.7
^1S	-15.9
^3S	-15.8
^1P	3.2
^3P	0.3
^1D	-2.2
^3D	-1.3
P(D)	No D

Similar to "Volkov".

$v_T \frac{Q}{e} v_T$ is included.



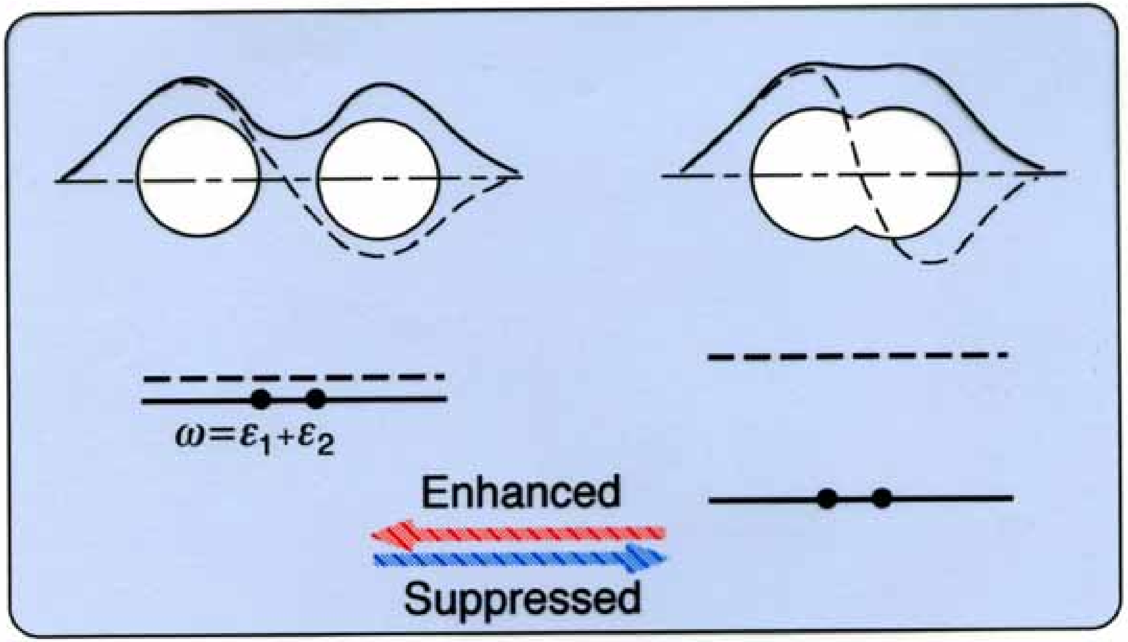
	N.M.	^4He
$^3S/^1S$ ratio	1.0	1.5

"Tensor enhancement"

Y. Akaishi & S. Nagata,
Prog. Theor. Phys. **48** (1972) 133.

Y. Akaishi, H. Bando & S. Nagata, Prog. Theor. Phys. Suppl. **52** (1972) 339.

Tensor force effects on clusterization

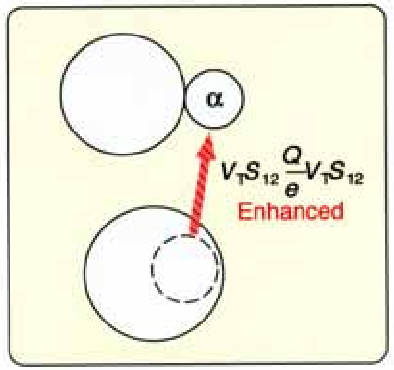


$$\boxed{V_T S_{12} \frac{Q}{e} V_T S_{12}} \approx -8\frac{V_T^{\,2}}{|\Delta|} + 2\frac{V_T^{\,2}}{|\Delta|} S_{12},$$

Central Tensor

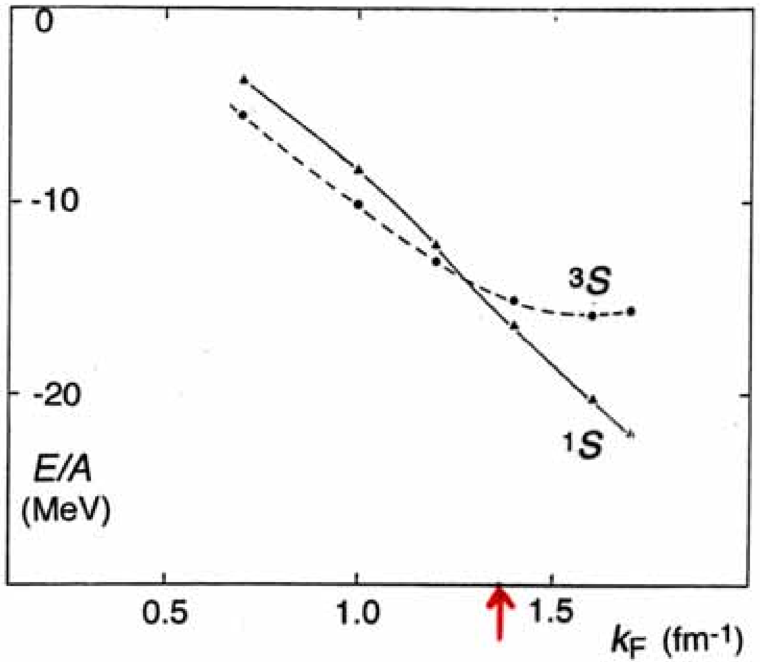
$$\Delta = (\varepsilon_1+\varepsilon_2) - (t_1+t_2)_{av} \approx 200\,\text{MeV}$$

Central ~ -100 MeV at 1 fm



Saturation of Nuclear Matter

H.A. Bethe, Ann. Rev. Nucl. Sci. **21** (1971) 93.



The saturation depends on three factors,
in decreasing order of importance:

(a) tensor force; (b) exchange force; (c) repulsive core.

Alpha Particle

Live fish

(MeV)		H-J
Energy		-20.6
Kin. E		131.1 (92.5(S), 38.6(D))
Pot. E		-151.7
C	1E	-51.3
	3E	-26.2
	1O+3O	-0.4
T	3E	-69.7
	3O	-0.5
LS+QLS		-3.6
P(D) %		12.8

H-J (D.tr.)
-20.6
106.1
-126.7
-58.8
-67.9

Tuncation of D-state

"Sashimi"

H-J (Sc.tr.)
-20.6
48.6
-69.1
-30.0
-39.1

Truncation of short-range correlation

Structure-dependent $^3S_1$ interaction

$\hbar\omega = 21.6$ MeV

$KE = 3 \times \frac{3}{4}\hbar\omega = 48.6$ MeV

Volkov
-29.0
48.6
-77.6
-38.8
-38.8
0.0
0
0
0
0

ATMS Representation of Multiple Scattering Operator

$$\hat{F} = \hat{F}_{ij} + \frac{Q}{e} g_{ij} \hat{F}_{ij}$$

$$n_{\text{pair}} \hat{F} = \sum_{(ij)} \hat{F}_{ij} + \sum_{(ij)} \frac{Q}{e} g_{ij} \hat{F}_{ij}$$

$$= \sum_{(ij)} \hat{F}_{ij} + \left(\hat{F} - 1\right)$$

$$\hat{F} = 1 + \frac{1}{n_{\text{pair}} - 1} \sum_{(ij)} \left(\hat{F}_{ij} - 1\right)$$

$$\left\{ \hat{F} = \left(1 + \frac{Q}{e} g_{ij}\right)\left(\hat{F}_{ij} - 1\right) + \left(1 + \frac{Q}{e} g_{ij}\right) \right\} |\Phi_0\rangle$$

off-shell $\bar{u}_{ij}$

on-shell corell. fn. u_{ij}

$$F = \bar{u}_{ij}\left(F_{ij} - 1\right) + u_{ij}$$

$$\left(F_{ij} - 1\right) = \bar{u}_{ij}^{-1}\left(F - u_{ij}\right)$$

$$F = 1 + \frac{1}{n_{\text{pair}} - 1} \sum_{(ij)} \bar{u}_{ij}^{-1}\left(F - u_{ij}\right)$$

$$F = \frac{1}{D}\left[\prod_{(kl)} \bar{u}_{kl}\right]\left[\sum_{(ij)} \frac{1}{\bar{u}_{ij}} u_{ij} - \left(n_{\text{pair}} - 1\right)\right]$$

$$D = \left[\prod_{(kl)} \bar{u}_{kl}\right]\left[\sum_{(ij)} \frac{1}{\bar{u}_{ij}} - \left(n_{\text{pair}} - 1\right)\right]$$

Amalgamation of Two-body correlations into Multiple Scattering process

Realistic Wave Function of ^{4}He

Spatial symmetric

$$\Psi = \Psi_S + \Psi_D = \left(F_S + F_D \right) \Phi^S \{0,0\}^A$$

S- , D- correlations

Spin-isospin antisymmetric

$$F_S = \sum_{(ij)} \left\{ w_{ij}^{1E} P_{ij}^{1E} + w_{ij}^{3E} P_{ij}^{3E} \right\}$$

$$F_D = \sum_{(ij)} w_{ij}^{TE} T_{ij}\left(\vec{r}_{ij}, \vec{r}_{ij} \right) P_{ij}^{3E}$$

$$w_{ij}^{1E} = \prod_{(kl)}{}' \bar{u}(r_{kl}) \left\{ {}^1u_S\left(r_{ij}\right) - \frac{5}{6}\bar{u}\left(r_{ij}\right) \right\}$$

$$w_{ij}^{3E} = \prod_{(kl)}{}' \bar{u}(r_{kl}) \left\{ {}^3u_S\left(r_{ij}\right) - \frac{5}{6}\bar{u}\left(r_{ij}\right) \right\}$$

$$w_{ij}^{TE} = \prod_{(kl)}{}' \bar{u}(r_{kl}) \frac{3}{r_{ij}^2} \, {}^3u_D\left(r_{ij}\right)$$

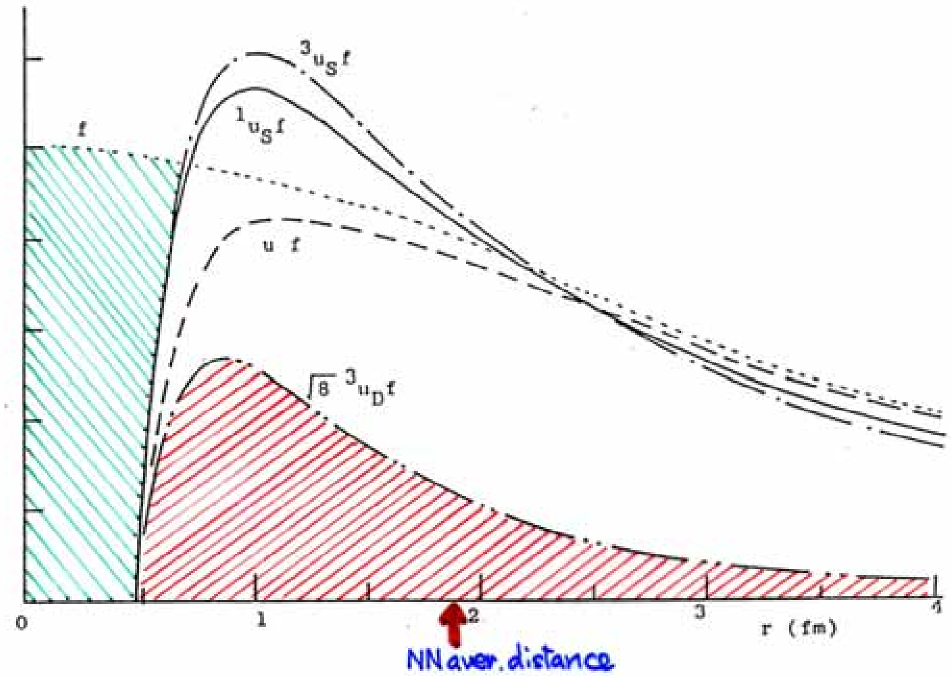
Two-body correlation

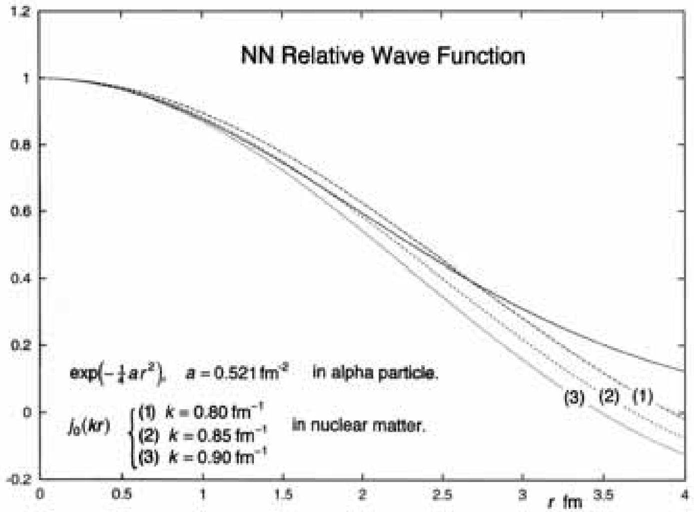
$$u_{ij} = {}^1u_S P_{ij}^{1E} + \left\{ {}^3u_S + {}^3u_D S_{ij} \right\} P_{ij}^{3E}$$

$$T_{ij}\left(\vec{a}, \vec{b} \right) = \frac{1}{2} \left\{ \left(\vec{\sigma}_i \vec{a}\right)\left(\vec{\sigma}_j \vec{b}\right) + \left(\vec{\sigma}_i \vec{b}\right)\left(\vec{\sigma}_j \vec{a}\right) \right\} - \frac{1}{3}\left(\vec{a}\vec{b}\right)\left(\vec{\sigma}_i \vec{\sigma}_j\right)$$

Traceless 2nd-rank tensor

ATMS





Y. Akaishi & S. Nagata, P. T. P. 48 (1972) 133

Akaishi-Bando-Nagata, P. T. P. Suppl. 52 (1972) 339

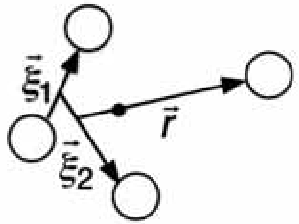
Tensor effects on clusterization

Momentum Distribution of N in ^{4}He

"Density correlation"

$$W(\vec{p}) = (2\pi)^{-3} \int d\vec{r} d\vec{r}' \exp(i\vec{p}(\vec{r} - \vec{r}'))\, \rho(\vec{r}, \vec{r}')$$

$$\rho(\vec{r}, \vec{r}') = \left(\frac{4}{3}\right)^3 \int\int d\vec{\xi}_1 d\vec{\xi}_2\, \Psi^* \left(\vec{\xi}_1, \vec{\xi}_2, \frac{4}{3}\vec{r}\right) \Psi\left(\vec{\xi}_1, \vec{\xi}_2, \frac{4}{3}\vec{r}'\right)$$



89 % 11%

$$W(\vec{p}) = C\left\{\exp\left(-B\vec{p}^2\right) + s \exp\left(-B\vec{p}^2 / t\right)\right\}$$

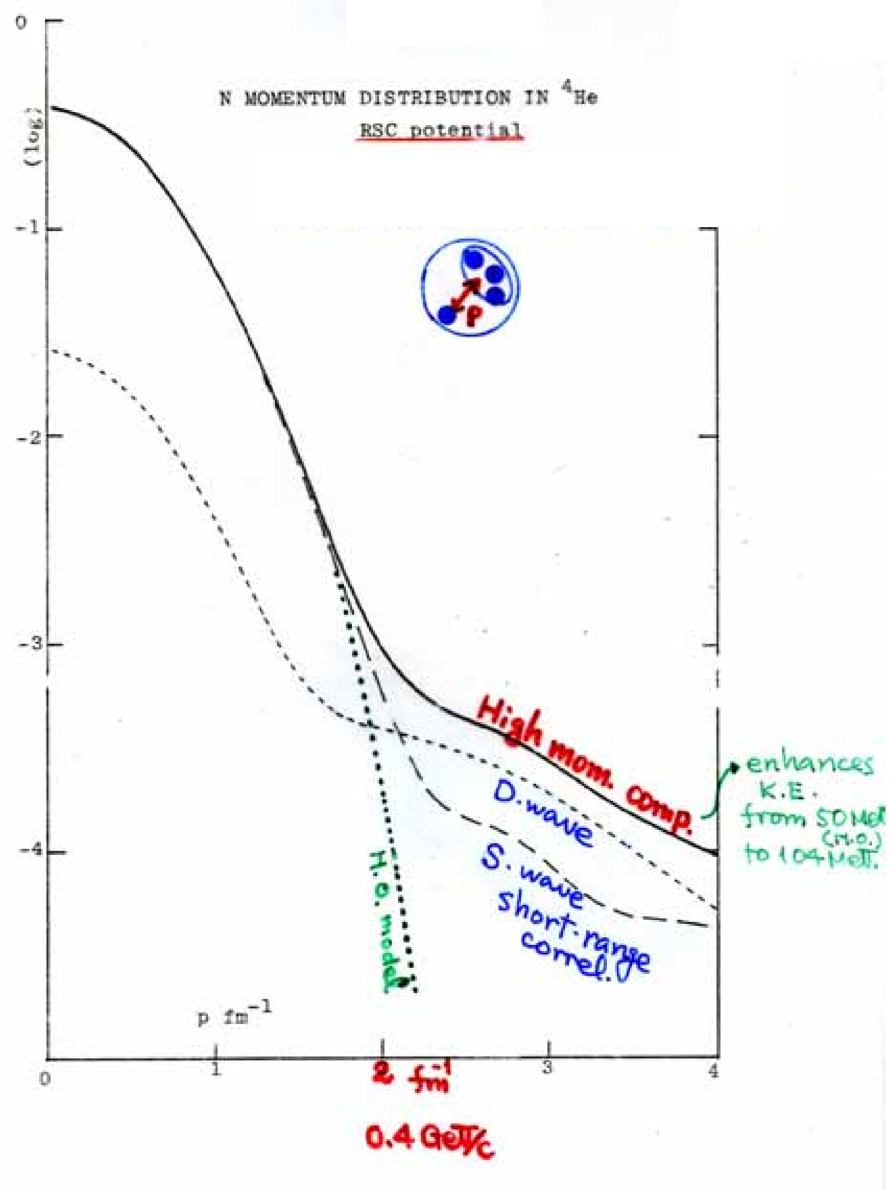
$B = 1.79\ \mathrm{fm}^2$, $t = 12$, $s = 0.00286$.

$$\text{K.E.} = (A-1)\frac{A-1}{A}\int d\vec{p}\, W(\vec{p})\, \frac{\hbar^2}{2M}\vec{p}^2$$

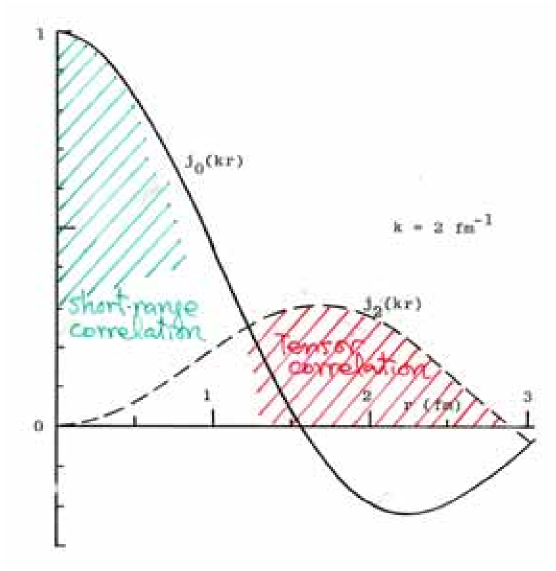
Form factor

"Density fluctuation"

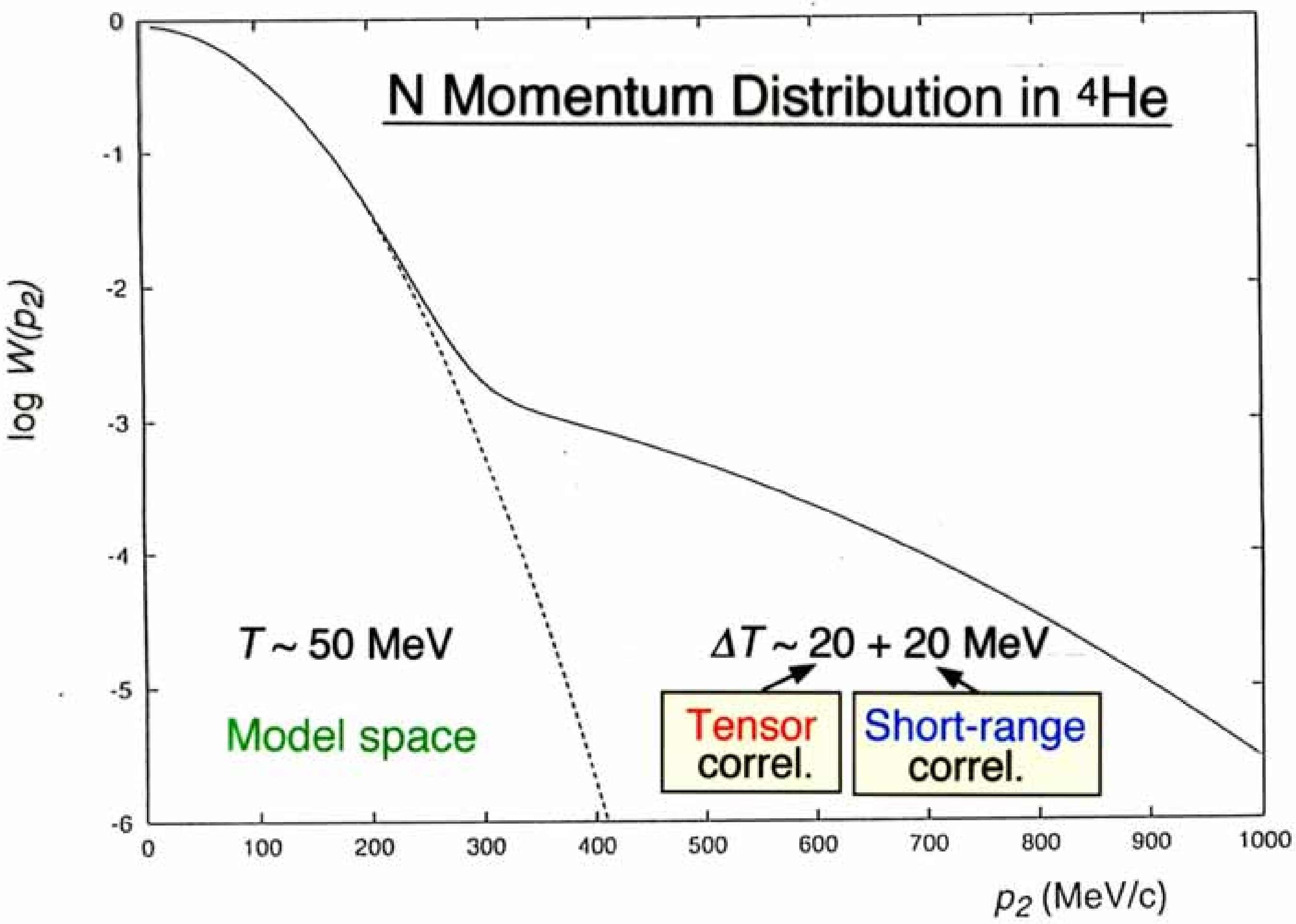
$$F(\vec{p}) = \int d\vec{r}\ \exp(i\vec{p}\vec{r})\, \rho(\vec{r}, \vec{r})$$

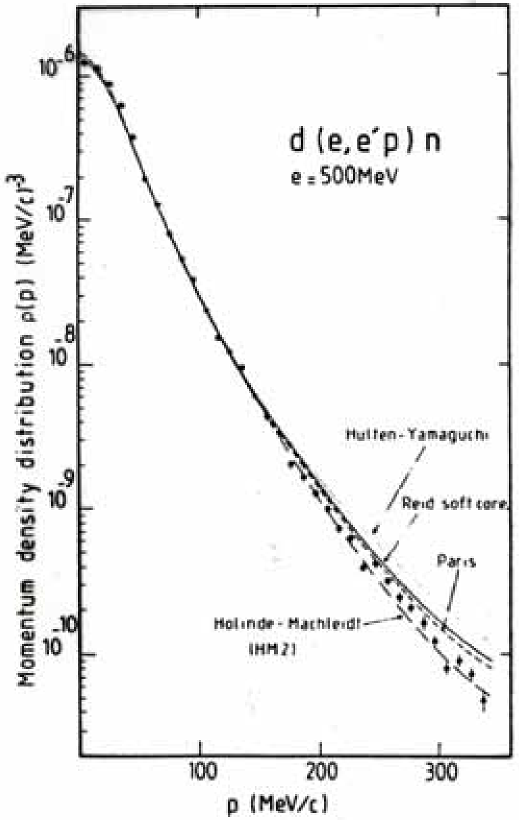


N MOMENTUM DISTRIBUTION IN 4He
RSC potential
(log)
p fm-1
High mom. comp.
enhances K.E. from 50MeV (H.O.) to 104MeV.
D. wave
S. wave short-range correl.
H.O. model
2 fm-1
0.4 GeV/c

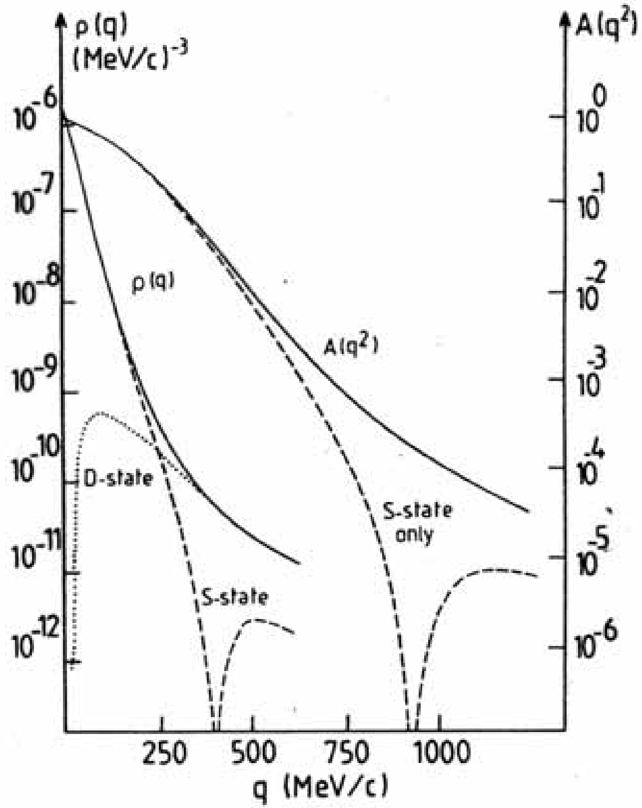


j0(kr)
k = 2 fm-1
j2(kr)
short-range correlation
Tensor correlation
r (fm)


N Momentum Distribution in 4He
log W(p2)
0
-1
-2
-3
-4
-5
-6
0
100
200
300
400
500
600
700
800
900
1000
p2 (MeV/c)
T ~ 50 MeV
Model space
ΔT ~ 20 + 20 MeV
Tensor correl.
Short-range correl.



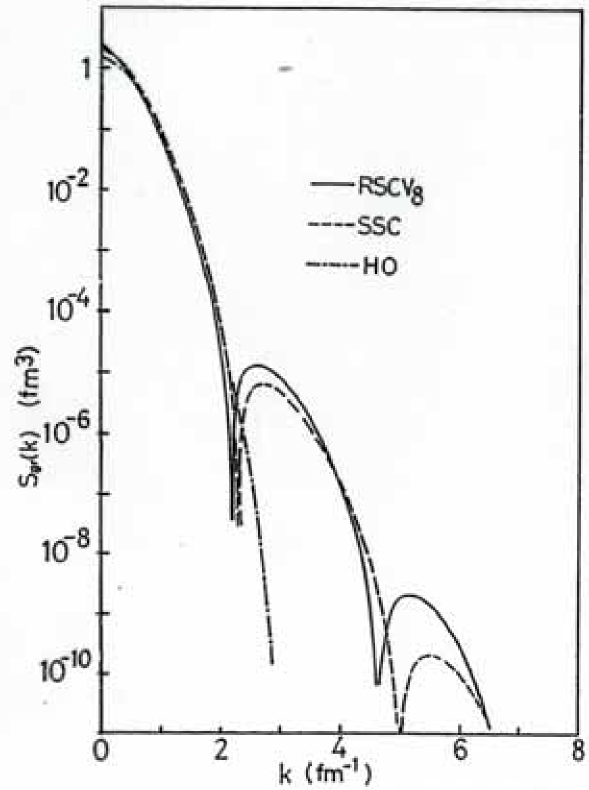
d (e,e'p) n
e = 500MeV
Momentum density distribution ρ(p) (MeV/c)^-3
Hulten-Yamaguchi
Reid soft core
Paris
Holinde-Machleidt (HM2)
p (MeV/c)



ρ(q) (MeV/c)^-3
A(q²)
ρ(q)
D-state
S-state
S-state only
q (MeV/c)

Detection of Short-Range Correlation

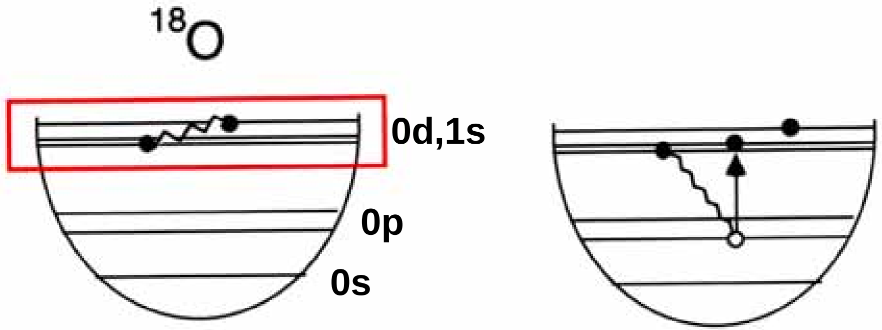
S. Tadokoro, T. Katayama, Y. Akaishi and H. Tanaka, Prog. Theor. Phys. **78** (1987) 732.

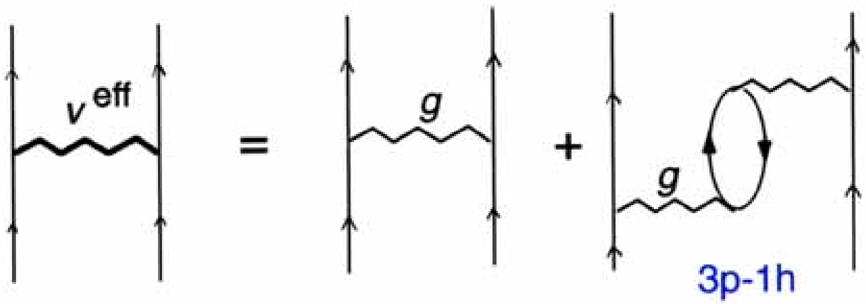


^{4}He ground spectral function

$^4He(e, e'p)\,^3H_{gr}$

Theory of Effective Interaction





T.T.S. Kuo & G.E. Brown,
Nucl. Phys. 85 (1966) 40

AMD
Y. Kanada-En'yo, H. Horiuchi, ...

K. Ando, H. Bando, S. Nagata et al., Prog. Theor. Phys. Suppl. No.65 (1979)

Recipe for Effective Interaction

Y. Akaishi & K. Takada, Prog. Theor. Phys. 37 (1967) 847

^{1}E central:

$$u^{JS}_{LL'} \quad \Psi/\Phi$$

$$g_C^{L=0}(r) = v_C u_{00}^{00} / j_0(kr) \qquad \to v_C \quad \text{at } r \ge r_{\text{healing}}$$

Short-range correl.

^{3}E central:

$$g_C^{L=0}(r) = v_C u_{00}^{11} / j_0(kr) + \sqrt{8} v_T u_{02}^{11} / j_0(kr) \quad \to v_C$$

$$v_T \frac{Q}{e} v_T$$ **Tensor renorm.**

^{3}E tensor:

$$g_T^{LL'=02}(r) = v_T u_{00}^{11} / j_0(kr) + \frac{1}{\sqrt{8}} \{v_C - 2v_T - 3v_{LS} - 3v_{QLS}\} u_{02}^{11} / j_0(kr) \quad \to v_T$$

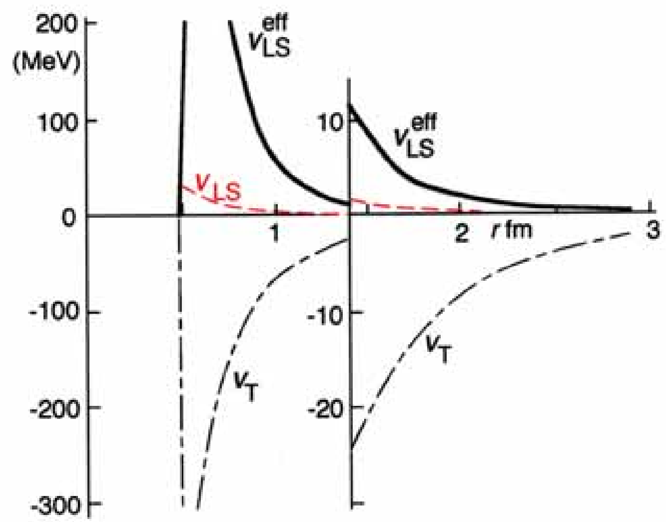
$$g_T^{LL'=22}(r) = \frac{7}{120}[-3\sqrt{8} v_T u_{20}^{11} / j_2(kr) - 3\{v_C - 2v_T - 3v_{LS} - 3v_{QLS}\} u_{22}^{11} / j_2(kr) + 5\{v_C + 2v_T - v_{LS} + 11v_{QLS}\} u_{22}^{21} / j_2(kr) - 2\{v_C - \tfrac{4}{7}v_T + 2v_{LS} + 2v_{QLS}\} u_{22}^{31} / j_2(kr)] \quad \to v_T + \tfrac{7}{2} v_{QLS}$$

^{3}O spin-orbit:

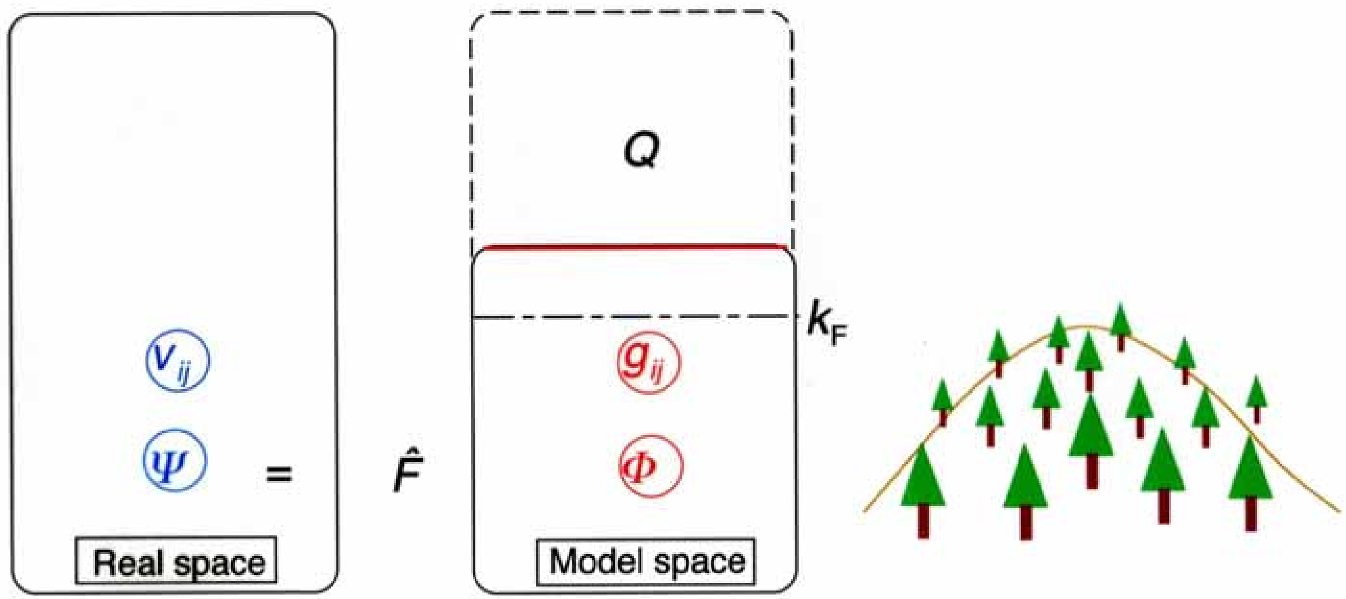
$$g_{LS}^{L=1}(r) = \frac{1}{12}[-2\{v_C - 4v_T - 2v_{LS} - 2v_{QLS}\} u_{11}^{01} / j_1(kr) - 3\{v_C + 2v_T - v_{LS} + 3v_{QLS}\} u_{11}^{11} / j_1(kr) + 5\{v_C - \tfrac{2}{5}v_T + v_{LS} + v_{QLS}\} u_{11}^{21} / j_1(kr) + 6\sqrt{6} v_T u_{13}^{21} / j_1(kr)] \quad \to v_{LS}$$

^{3}E spin-orbit

$$g_{LS}^{L=2}(r) = \frac{1}{60}[-9\sqrt{8} v_T u_{20}^{11} / j_2(kr) - 9\{v_C - 2v_T - 3v_{LS} - 3v_{LL}\} u_{22}^{11} / j_2(kr) - 5\{v_C + 2v_T - v_{LS} + 11v_{LL}\} u_{22}^{21} / j_2(kr) + 14\left\{v_C - \frac{4}{7}v_T + 2v_{LS} + 2v_{LL}\right\} u_{22}^{31} / j_2(kr) + 24\sqrt{3} v_T u_{24}^{31} / j_2(kr)] \quad \to v_{LS}$$

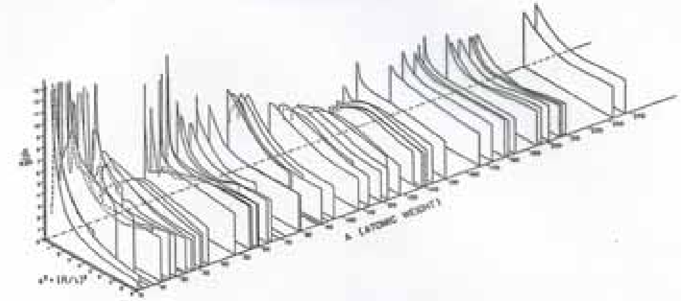


What is "Model"?



$$g_{ij} = v_{ij} + v_{ij}\frac{Q}{e}g_{ij}$$

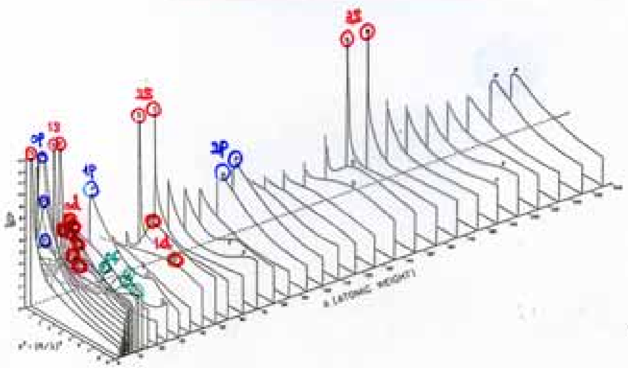
Optical Model



$E_n = 0 \sim 3\,\text{MeV}$

H.H. Barschall, Phys. Rev. **86** (1952) 431.

$I = 500\,\text{keV}$ → smearing → Gross structure

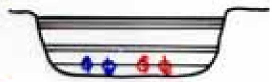


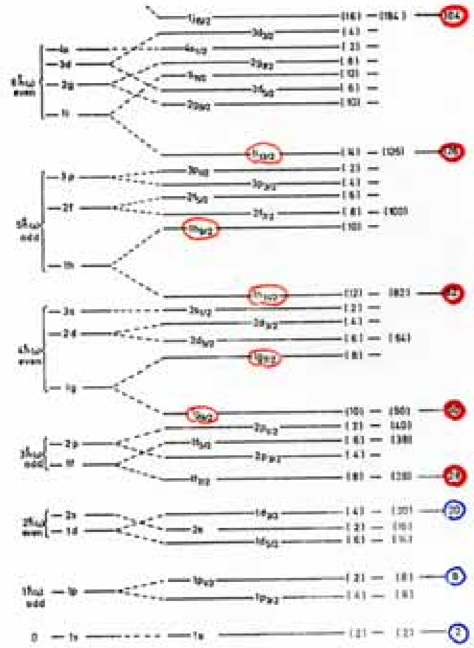
H. Feshbach, C.E. Porter & V.F. Weisskopf,
Phys. Rev. **96** (1954) 448.

$$U(r) = \begin{cases} -V_0 - iW_0, & r < R = r_0 A^{-1/3} \\ 0, & r > R \end{cases}$$

$V_0 = 42\,\text{MeV}, \quad W_0 = 1.26\,\text{MeV}, \quad r_0 = 1.45\,\text{fm}$

Shell Structure





due to LS splitting

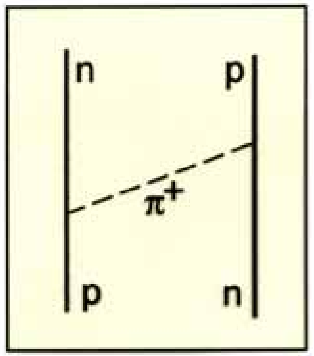
$\hbar\omega \approx 41\, A^{-1/3}\,\text{MeV}$

池田 2002年1月

K. Ikeda's idea
Tensor → Pion

パリティを破らない前提とパリティを破る要求
パイオンの働きを自由にするパリティ混合一粒子状態

S. Sugimoto et al.



殻模型の牙城で勝負！
A citadel of S.M.

Y. Akaishi

Tensor BHF calculation of ^{4}He

$$\Phi_{\text{intr}} = \prod_{k=1}^{4} F(\vec{r}_k; \vec{\sigma}_k, \vec{\tau}_k)\, \chi_{\text{spin-isospin}}$$

$$F(\vec{r}_k; \vec{\sigma}_k, \vec{\tau}_k) = \left\{ f_s(r_k) - i(\vec{\sigma}_k \vec{r}_k) f_p(r_k) g(\vec{\tau}_k) \right\}$$

$$(\vec{\sigma}\,\vec{r}\,)\alpha Y_{00} = -r \left| \left(\ell = 1, s = \tfrac{1}{2}\right) j = j_z = \tfrac{1}{2} \right\rangle$$

$$g(\vec{\tau})p = \frac{1}{2}(1-i)p - \sqrt{\frac{1}{2}}\,n$$

$$g(\vec{\tau})n = \frac{1}{2}(1+i)n + \sqrt{\frac{1}{2}}\,p$$

The AV8' Potential

R.B. Wiringa, V.G.J. Stoks & R. Schiavilla, Phys. Rev. C**51** (1995) 38.

$$V = v^{\pi} + v_{ST}^{\mathrm{R}}$$

OPEP

$$v^{\pi} = f^2 \left(\frac{m}{m_c} \right)^2 \frac{1}{3} mc^2 (\vec{\tau}_1 \vec{\tau}_2) \left\{ (\vec{\sigma}_1 \vec{\sigma}_2) Y_m(r) + T_m(r) S_{12} \right\}$$

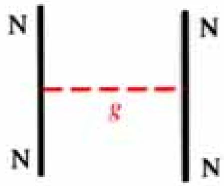
$$Y_m(r) = \frac{e^{-mr}}{mr} \left(1 - e^{-cr^2} \right)$$

$$T_m(r) = \left\{ 1 + \frac{3}{mr} + \frac{3}{(mr)^2} \right\} \frac{e^{-mr}}{mr} \left(1 - e^{-cr^2} \right)^2$$

$$f^2 = 0.075, \quad m = \frac{1}{3}(m_0 + 2m_c), \quad c = 2.1 \mathrm{fm}^{-2}$$

$$v_{ST}^{\mathrm{R}} = v_{ST}^{\mathrm{C}} + v_{ST}^{\mathrm{T}} S_{12} + v_{ST}^{\mathrm{LS}} \vec{L}\vec{S}$$

Brueckner-Hartree-Fock Calculation on Gaussian Basis



<u>Nucleon-nucleus potential</u>

$$U^{\mathrm{eq}}_{\mu}(\vec{r}_1)\varphi_{\mu}(\vec{r}_1) \equiv U_{\mathrm{H}}(\vec{r}_1)\varphi_{\mu}(\vec{r}_1) + \int d\vec{r}_2 U_{\mathrm{F}}(\vec{r}_1,\vec{r}_2)\varphi_{\mu}(\vec{r}_2)$$

g-matrix

$$U_{\mathrm{H}}(\vec{r}_1) = \int d\vec{r}_2 \sum_{\nu} \varphi^*_{\nu}(\vec{r}_2) g(\vec{r}_1,\vec{r}_2) \varphi_{\nu}(\vec{r}_2)$$

$$U_{\mathrm{F}}(\vec{r}_1,\vec{r}_2) = -\sum_{\nu} \varphi^*_{\nu}(\vec{r}_2) g(\vec{r}_1,\vec{r}_2) \varphi_{\nu}(\vec{r}_1)$$

$$g(\vec{r}_1-\vec{r}_2) = \sum_j \gamma^{\mu}_j \exp\left\{-\left((\vec{r}_1-\vec{r}_2)/c_j\right)^2\right\}, \quad c_1,\ldots,c_{20} \text{ fixed}$$

$$U^{\mathrm{eq}}_{\mu}(\vec{r}_1) = \sum_j \alpha^{\mu}_j \exp\left\{-\left(r_1/a_j\right)^2\right\}, \quad a_1,\ldots,a_{10} \text{ fixed}$$

$$\varphi_{\mu}(\vec{r}_1) = \sum_j \beta^{\mu}_j \exp\left\{-\left(r_1/b_j\right)^2\right\} r_1^{\ell} y_{\ell_1 s_1 j_1 m_1}(\hat{r}_1), \quad b_1,\ldots,b_{20} \text{ fi}$$

Self-consistently determined

Matrix elements for s-shell hyperon:

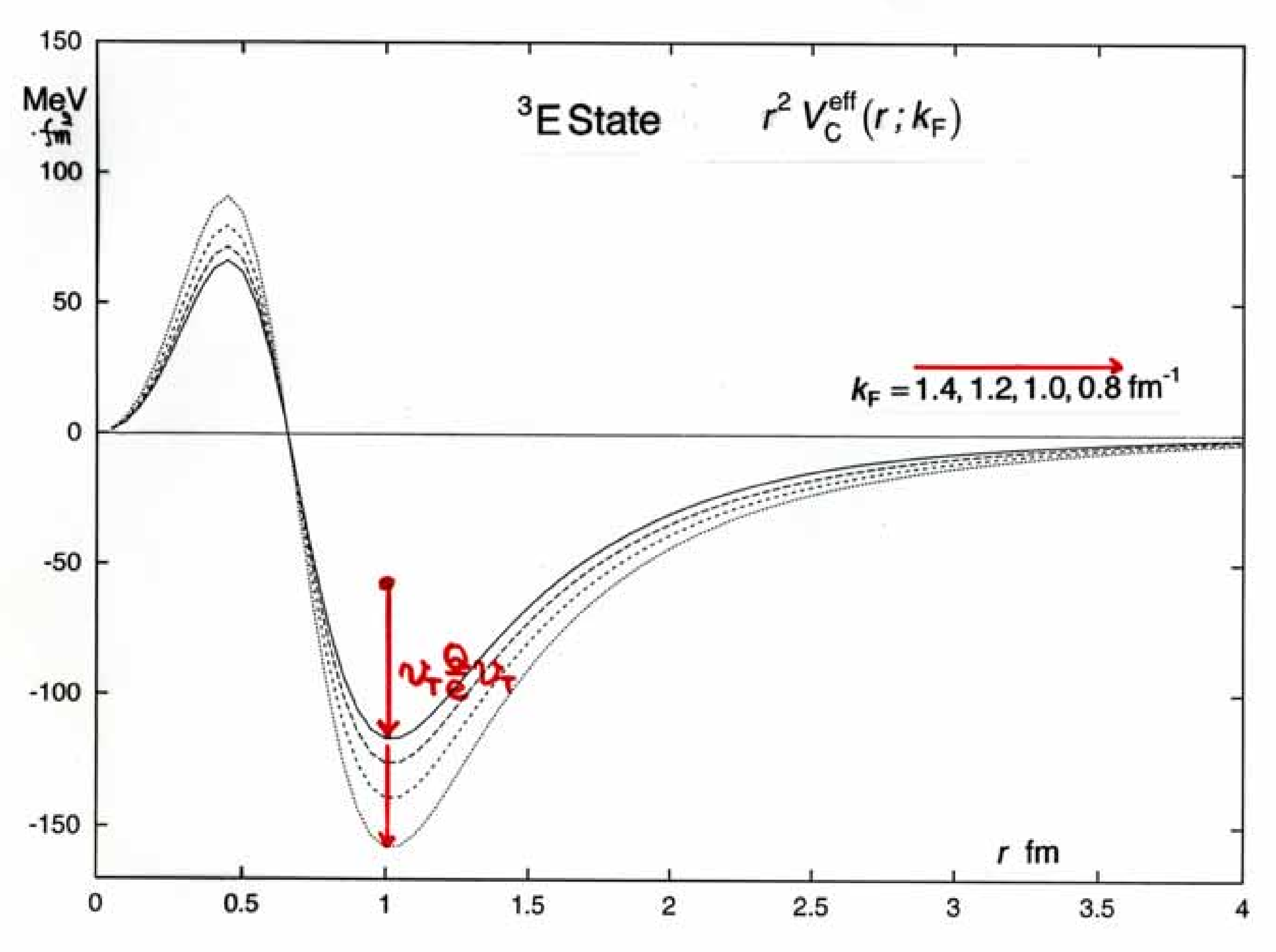
$$\sum_{\phi_1}\sum_{\mu_1} \left\langle (a_1'' j_1\mu_1\phi_1)(a_2'' j_2\mu_2\phi_2) \right| g \left| (a_1 j_1\mu_1\phi_1)(a_2 j_2\mu_2\phi_2) - \text{exch.} \right\rangle$$

$$= (2j_2+1)(2\ell_2+1)^2 \sum_{\ell'=0}^{\ell_2}\sum_{\ell''=0}^{\ell_2} \left(\frac{M_1}{M_1+M_2}\right)^{\ell'+\ell''} \sqrt{\binom{2\ell_2}{2\ell'}\binom{2\ell_2}{2\ell''}} \sum_n\sum_{n'}(2n+1)(2n'+1)$$

$$\times \sum_{\kappa} \int_0^{\infty} dr\, r^{\ell'+\ell''+2} \exp\left(-\left(A_{12}''+A_{12}+c_{\kappa}^2\right)r^2\right) \int_0^{\infty} dR\, R^{2\ell_2-\ell'-\ell''+2} \exp\left(-(a_1''+a_2''+a_1+a_2)R^2\right)$$
$$\times i^{n'} j_{n'}(ia_{12}'' rR) i^{n} j_{n}(ia_{12} rR)$$

$$\times \sum_{\bar{\ell}} (\ell' n 00|\bar{\ell}0)(\ell'' n' 00|\bar{\ell}0) \sum_{\bar{L}} (\ell_2-\ell' n 00|\bar{L}0)(\ell_2-\ell'' n' 00|\bar{L}0) \begin{Bmatrix} n & \ell' & \bar{\ell} \\ \ell_2 & \bar{L} & \ell_2-\ell' \end{Bmatrix} \begin{Bmatrix} n' & \ell'' & \bar{\ell} \\ \ell_2 & \bar{L} & \ell_2-\ell'' \end{Bmatrix}$$

$$\times \left[\sum_{K=\ell_2} \frac{2K+1}{2} \begin{Bmatrix} l_2 & K & \frac{1}{2} \\ 0 & \frac{1}{2} & \ell_2 \end{Bmatrix}^2 \sum_{J=\bar{\ell}} (2J+1) \begin{Bmatrix} \bar{\ell} & 0 & J \\ K & \bar{L} & \ell_2 \end{Bmatrix}^2 \frac{1-(-)^{n-\ell'}}{2} \left\{ V^{J,F=0}_{\bar{\ell},S=0}(\kappa) \right\}_{\text{Yn and Yp}} \right.$$
$$\left. + \cdots \left\{ V^{J,F=1}_{\bar{\ell},S=0}(\kappa) \right\}_{\text{Yn and Yp}} + \cdots \left\{ V^{J,F=0}_{\bar{\ell},S=1}(\kappa) \right\}_{\text{Yn and Yp}} + \cdots \left\{ V^{J,F=1}_{\bar{\ell},S=1}(\kappa) \right\}_{\text{Yn and Yp}} \right]$$



M. Serra et al.: *g*-matrix→RMF

Tensor BHF Calculation of ^{4}He

$$\Phi_{\text{intr}} = \prod_{k=1}^{4} F(\vec{r}_k\,;\,\vec{\sigma}_k,\vec{\tau}_k)\,\chi_{\text{spin-isospin}}$$

$$F(\vec{r}_k\,;\,\vec{\sigma}_k,\vec{\tau}_k) = \left\{ f_s(r_k) - i\,\underline{(\vec{\sigma}_k\vec{r}_k)}\,f_p(r_k)\,\underline{g(\vec{\tau}_k)} \right\}$$

$$\left(\vec{\sigma}\,\vec{r}\,\right)\alpha Y_{00} = -r\left|\left(\ell=1, s=\tfrac{1}{2}\right) j = j_z = \tfrac{1}{2}\right\rangle$$

$$g(\vec{\tau})\,p = \frac{1}{2}(1-i)\,p - \sqrt{\frac{1}{2}}\,n$$

$$g(\vec{\tau})\,n = \frac{1}{2}(1+i)\,n + \sqrt{\frac{1}{2}}\,p$$

Projection: $\Psi = P^{\tau}\,P^{\pi}\,\Phi_{\text{intr}}$

Complex

$$f_s(r) = \sum_{n=1}^{12} C_n \exp\left\{-\left(\frac{r}{b_n}\right)^2\right\}, \quad f_p(r) = \sum_{n=1}^{12} D_n \exp\left\{-\left(\frac{r}{b_n}\right)^2\right\}$$

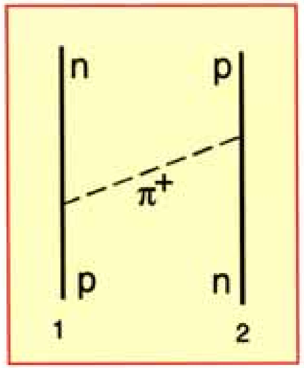
$b_n/b_{n-1} = c;\qquad b_1 = 0.1\,\text{fm},\quad b_{12} = 6.0\,\text{fm}$

Parameter search: Simplex method

OPEP

$$\left(\vec{\nabla}_2^{\,2}-\kappa^2\right)\phi^{(+)} = -4\pi \left[i\frac{f}{\kappa}\left(\vec{\sigma}_1\vec{\nabla}_1\right)\tau^{(-)}\right]\delta(\vec{r}_2-\vec{r}_1)$$

$$\tau^{(-)} = \tfrac{1}{\sqrt{2}}\left(\tau_x - i\tau_y\right), \quad \tau^{(-)}p = \sqrt{2}n$$



$$\vec{\nabla}_2^{\,2}\phi = -4\pi e_1\delta(\vec{r}_2-\vec{r}_1)$$

$$\phi = \frac{e_1}{|\vec{r}_2-\vec{r}_1|}$$

$$V_{Coul}(\vec{r}_2-\vec{r}_1) = e_2\phi = \frac{e_2 e_1}{|\vec{r}_2-\vec{r}_1|}$$

$$\phi^{(+)}(\vec{r}_2-\vec{r}_1) = \left[i\frac{f}{\kappa}\tau^{(-)}\left(\vec{\sigma}_1\vec{\nabla}_1\right)\right]\frac{\exp(-\kappa|\vec{r}_2-\vec{r}_1|)}{|\vec{r}_2-\vec{r}_1|}$$

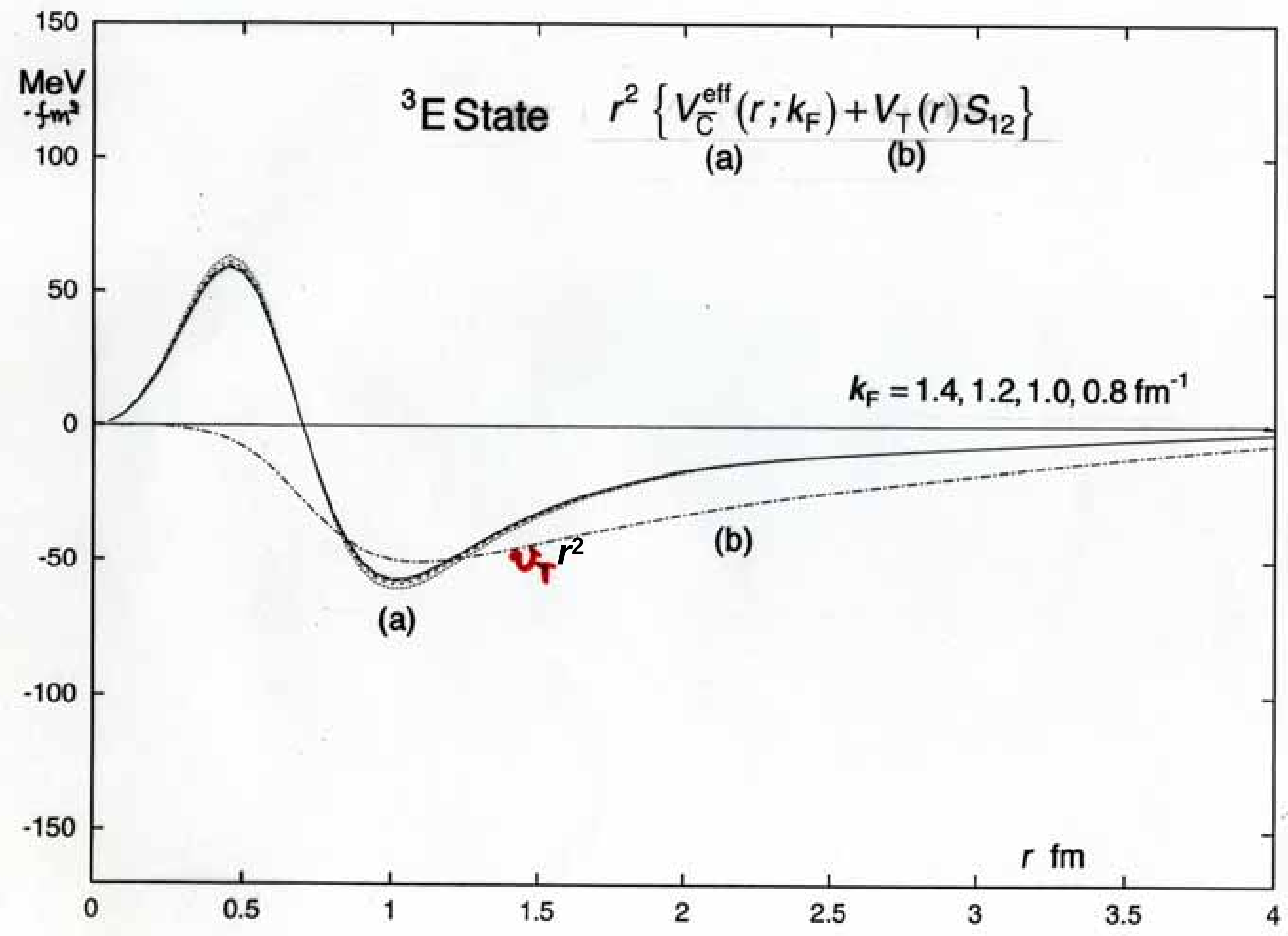
$$V_{OPEP}(\vec{r}_2-\vec{r}_1) = \left[i\frac{f}{\kappa}\left(\vec{\sigma}_2\vec{\nabla}_2\right)\right]\left\{\vec{\tau}_2^{(+)}\vec{\tau}_1^{(-)} + \vec{\tau}_2^{(0)}\vec{\tau}_1^{(0)} + \vec{\tau}_2^{(-)}\vec{\tau}_1^{(+)}\right\}\left[i\frac{f}{\kappa}\left(\vec{\sigma}_1\vec{\nabla}_1\right)\right]\frac{\exp(-\kappa|\vec{r}_2-\vec{r}_1|)}{|\vec{r}_2-\vec{r}_1|}$$

$$\vec{r} = \vec{r}_2-\vec{r}_1, \quad \vec{x} = \kappa\vec{r}, \quad \vec{\nabla}_2 = -\vec{\nabla}_1 = \kappa\vec{\nabla}_x, \quad \kappa = \frac{m_\pi c}{\hbar} \approx 0.7\ \mathrm{fm}^{-1}$$

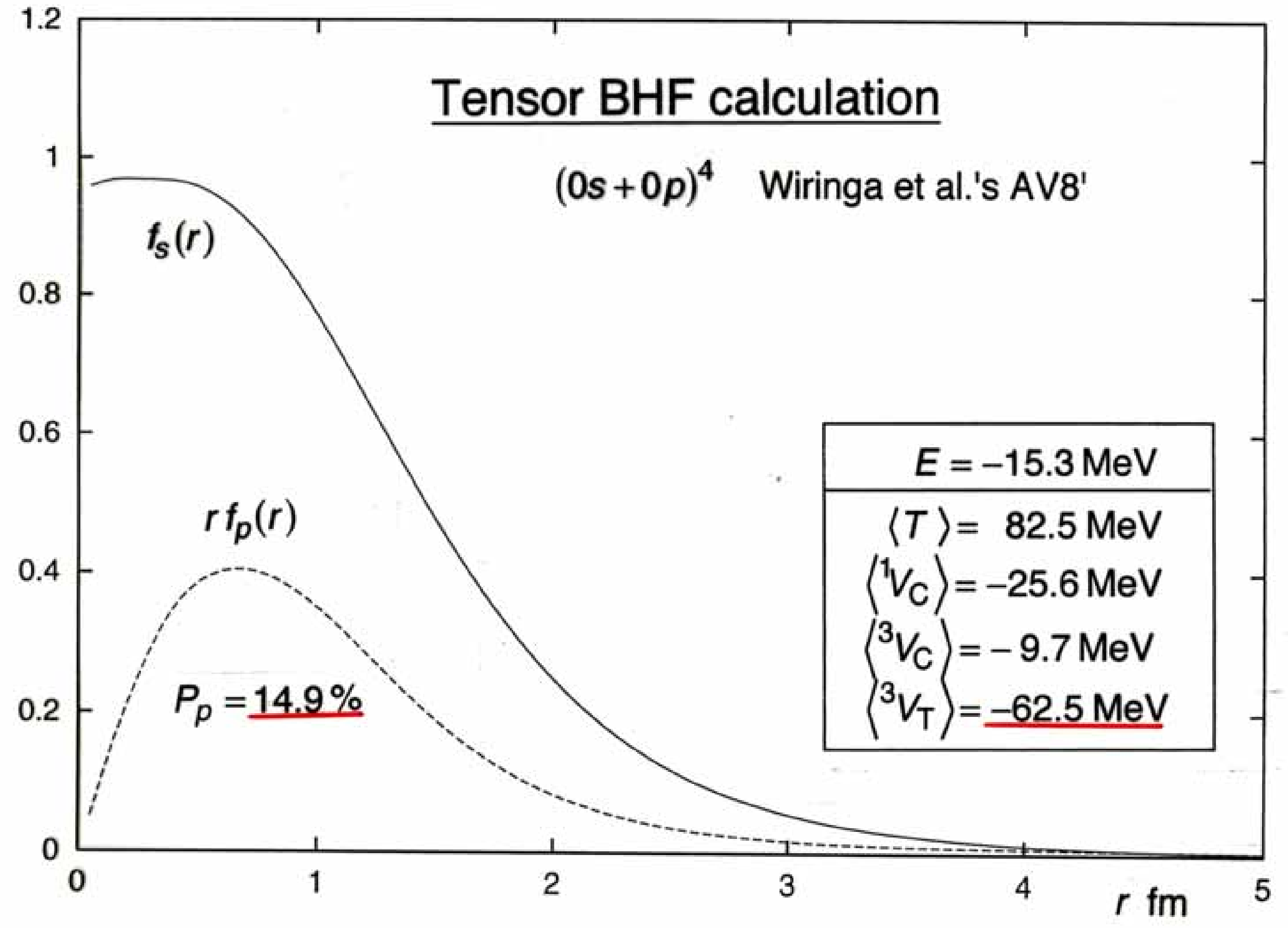
$$V_{OPEP}(\vec{r}) = f^2\kappa\left(\vec{\tau}_2\vec{\tau}_1\right)\left(\vec{\sigma}_2\vec{\nabla}_x\right)\left(\vec{\sigma}_1\vec{\nabla}_x\right)\frac{\exp(-x)}{x}$$

$$= \left(\frac{f^2}{\hbar c}\right)m_\pi c^2\,\frac{1}{3}\left(\vec{\tau}_1\vec{\tau}_2\right)\left\{\left(\vec{\sigma}_1\vec{\sigma}_2\right) + S_{12}\left(1+\frac{3}{x}+\frac{3}{x^2}\right)\right\}\frac{\exp(-x)}{x}$$

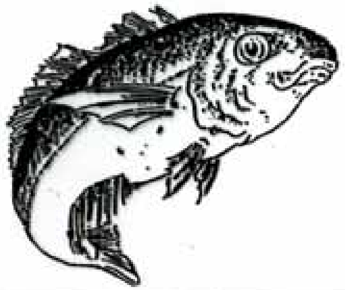
$$-\frac{1}{r^3}\left\{3\frac{(\vec{\mu}_1\vec{r})(\vec{\mu}_2\vec{r})}{r^2} - \vec{\mu}_1\vec{\mu}_2\right\}$$



^{3}E State
$r^2 \left\{ V_{\bar{C}}^{\text{eff}}(r; k_F) + V_T(r) S_{12} \right\}$
(a)
(b)
MeV
·fm²
150
100
50
0
-50
-100
-150
$k_F = 1.4, 1.2, 1.0, 0.8$ fm^{-1}
$v_T r^2$
(a)
(b)
r fm
0
0.5
1
1.5
2
2.5
3
3.5
4



Tensor BHF calculation
$(0s+0p)^4$ Wiringa et al.'s AV8'
$f_s(r)$
$r f_p(r)$
$P_p = 14.9\%$
$E = -15.3$ MeV
$\langle T \rangle = 82.5$ MeV
$\langle {}^1V_C \rangle = -25.6$ MeV
$\langle {}^3V_C \rangle = -9.7$ MeV
$\langle {}^3V_T \rangle = -62.5$ MeV
r fm
1.2
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5



Alpha Particle

(MeV)	AV8'
Energy	-25.9
Kin. E	102.4
Pot. E	-128.3
C [1E, 3E, 1O+3O]	-55.3
T [SD, DD]	-68.4
LS	-4.7
P(D) %	13.9

Phys. Rev. C**64** (2001) 044001.
Benchmark test calculation of 4N

" 兜煮"

AV8' (Sc.tr.)	
-15.3	
82.5	55.9(s) 26.6(p)
-97.8	
-25.6, -9.7	-35.3
-62.5	
P(p) = 14.9 %	

Charge & parity-mixed
single particle model

" 地動説"
Copernican

" 刺身"

AV8' (D.tr.)
-25.3
54.1
-79.4
-32.0
-47.4

Ordinary
single particle model

" 天動説"
Ptolemaic

Concluding Remarks

Ordinary single-particle model

Ptolemaic (geocentric)

State-dependent effective interactions
(Density, cluster, halo etc.)

Charge-parity nonconserving single-particle model

Copernican (heliocentric)

High-momentum phenomena
due to NN long-range tensor force

Pion (chiral) plays a leading role.