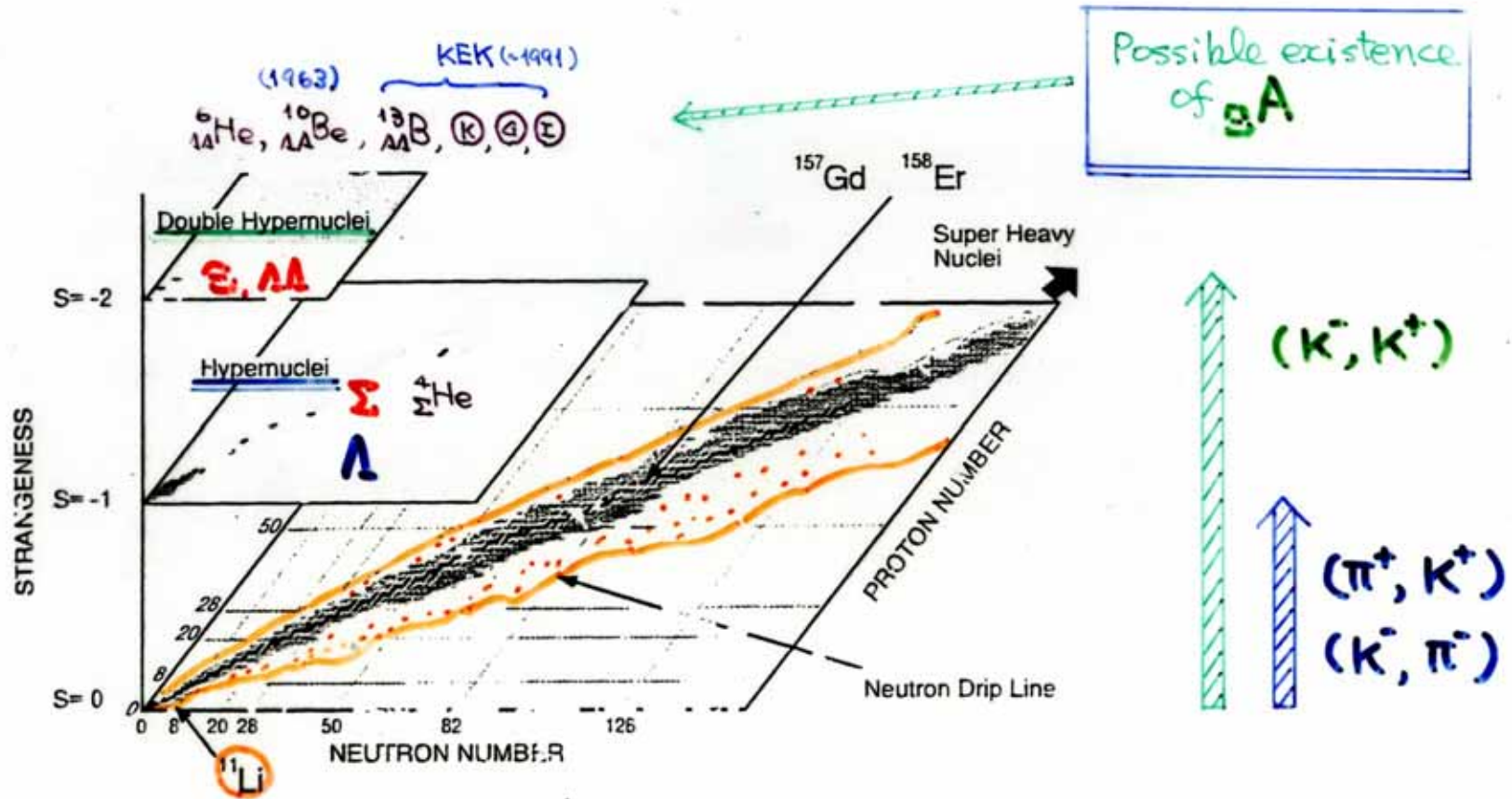


**CISS03**  
**Sept.16-20, 2003**

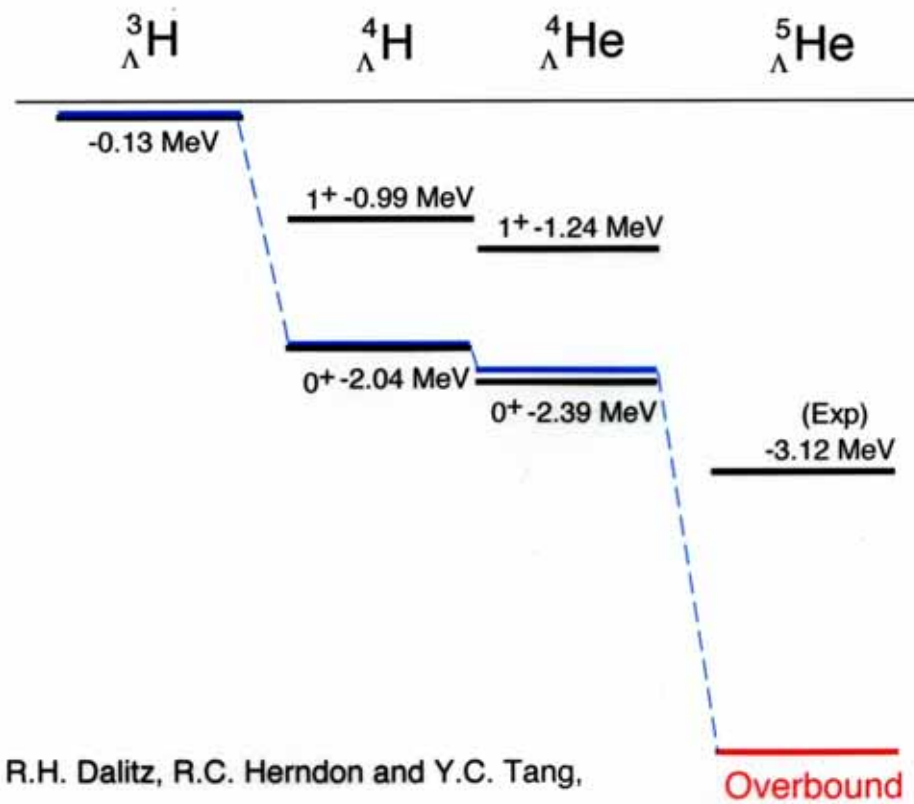
# **Coherent $\Lambda$ - $\Sigma$ Coupling in Asymmetric Hypernuclei**

Yoshinori AKAISHI

Institute of Particle and Nuclear Studies, KEK



# The Overbinding Problem

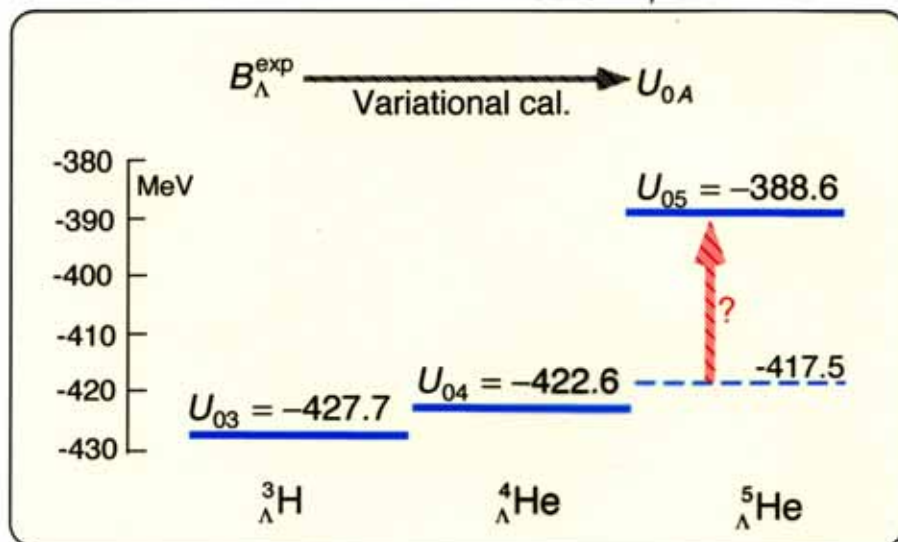


R.H. Dalitz, R.C. Herndon and Y.C. Tang,  
Nucl. Phys. **B47** (1972) 109

## Dalitz et al's analysis

$$V_{\Lambda N}(r) = \begin{cases} \infty, & r < d \\ U_{0A} \exp[-\lambda(r-d)], & r > d \end{cases}$$

$\lambda = 3.219 \text{ fm}^{-1}, d = 0.45 \text{ fm}$



$$\begin{aligned} U_{03} &= \frac{1}{4}U_0^t + \frac{3}{4}U_0^s \\ U_{04} &= \frac{1}{2}U_0^t + \frac{1}{2}U_0^s \\ U_{05} &= \frac{3}{4}U_0^t + \frac{1}{4}U_0^s \end{aligned}$$

$$\frac{1}{2}(U_{03} + U_{05}) - U_{04} = 0 \quad .03W_3$$

$W_3 = 480 \text{ MeV}$

$W_3 = 1.43 \text{ MeV}$

$$V_{\Lambda NN} = -\frac{1}{3}W_3(\vec{\sigma}_\tau \vec{\sigma}_2)(\vec{\tau}_1 \vec{\tau}_2) \frac{\exp(-\mu r_{1\Lambda})}{\mu r_{1\Lambda}} \frac{\exp(-\mu r_{2\Lambda})}{\mu r_{2\Lambda}}$$

Singlet int. is more attractive than triplet int.

Central YN interaction

	$\Lambda N$	$\Sigma N$
D0	○	
D2	○	○

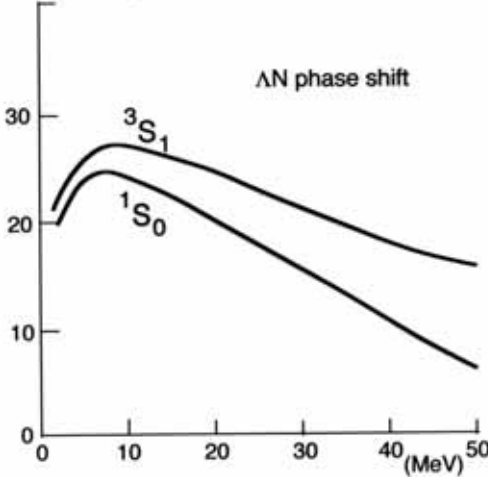
←  $\Lambda N$ - $\Sigma N$  coupling →

1-channel picture

2-channel picture

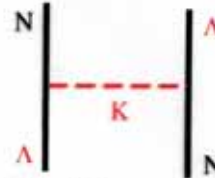
A.R. Bodmer  
(1966)

Phase-equivalent to Nijmegen D



# Brueckner-Hartree-Fock Calculation on Gaussian Basis

## Hyperon-nucleus potential



$$U_{\mu}^{\text{eq}}(\vec{r}_1)\varphi_{\mu}(\vec{r}_1) \equiv U_{\text{H}}(\vec{r}_1)\varphi_{\mu}(\vec{r}_1) + \int d\vec{r}_2 U_{\text{F}}(\vec{r}_1, \vec{r}_2)\varphi_{\mu}(\vec{r}_2)$$

$$U_{\text{H}}(\vec{r}_1) = \int d\vec{r}_2 \sum_{\nu} \varphi_{\nu}^*(\vec{r}_2) \overset{\text{g-matrix}}{g}(\vec{r}_1, \vec{r}_2) \varphi_{\nu}(\vec{r}_2)$$

$$U_{\text{F}}(\vec{r}_1, \vec{r}_2) = -\sum_{\nu} \varphi_{\nu}^*(\vec{r}_2) g(\vec{r}_1, \vec{r}_2) \varphi_{\nu}(\vec{r}_1)$$

$$g(\vec{r}_1 - \vec{r}_2) = \sum_j \gamma_j^{\mu} \exp\left\{-\left(\frac{|\vec{r}_1 - \vec{r}_2|}{c_j}\right)^2\right\}, \quad c_1, \dots, c_{20} \text{ fixed}$$

$$U_{\mu}^{\text{eq}}(\vec{r}_1) = \sum_j \alpha_j^{\mu} \exp\left\{-\left(r_1/a_j\right)^2\right\}, \quad a_1, \dots, a_{10} \text{ fixed}$$

$$\varphi_{\mu}(\vec{r}_1) = \sum_j \beta_j^{\mu} \exp\left\{-\left(r_1/b_j\right)^2\right\} r_1^{\ell} y_{\ell_1 s_1 j m_1}(\vec{r}_1), \quad b_1, \dots, b_{20} \text{ f}$$

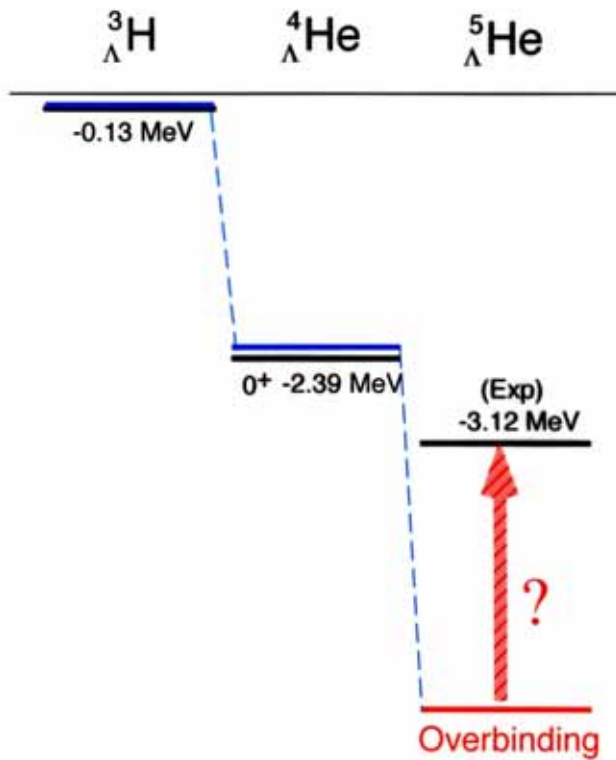
Self-consistently determined

Matrix elements for s-shell hyperon:

$$\begin{aligned} & \sum_{\phi_1 \mu_1} \sum_{\phi_2 \mu_2} \langle (a_1^{n_1} j_1 \mu_1 \phi_1) (a_2^{n_2} j_2 \mu_2 \phi_2) | g (a_1 j_1 \mu_1 \phi_1) (a_2 j_2 \mu_2 \phi_2) - \text{exch.} \rangle \\ &= (2j_2 + 1)(2\ell_2 + 1)^2 \sum_{\ell=0}^{\ell_2} \sum_{\ell'=0}^{\ell_2} \left( \frac{M_1}{M_1 + M_2} \right)^{\ell + \ell'} \sqrt{\frac{2\ell_2}{2\ell} \frac{2\ell_2}{2\ell'}} \sum_n \sum_{n'} (2n+1)(2n'+1) \\ & \times \int_0^{\infty} dr r^{\ell + \ell' + 2} \exp\left(-A_{12} r^2 + A_{12} + c_k^2\right) \int_0^{\infty} dR R^{2\ell_2 - \ell' - \ell + 2} \exp\left(-a_1 r^2 - a_2 R^2 + a_1 + a_2\right) R^2 \\ & \times i^{n'} j_{n'}(ia_{12} r R) j_n^{\ell}(ia_{12} r R) \\ & \times \sum_j \begin{pmatrix} \ell & n & 0 & 0 \\ \ell' & n' & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell_2 - \ell' & n & 0 & 0 \\ \ell_2 - \ell' & n' & 0 & 0 \end{pmatrix} \begin{Bmatrix} n & \ell & \ell \\ \ell_2 & \ell_2 - \ell & \ell \end{Bmatrix} \begin{Bmatrix} n' & \ell' & \ell' \\ \ell_2 & \ell_2 - \ell' & \ell' \end{Bmatrix} \\ & \times \left[ \sum_{K=\ell_2} \frac{2K+1}{2} \begin{Bmatrix} j_2 & K & \frac{1}{2} \\ 0 & \frac{1}{2} & \ell_2 \end{Bmatrix}^2 \sum_{J=\ell} (2J+1) \begin{Bmatrix} \ell & 0 & J \\ K & \ell & \ell_2 \end{Bmatrix}^2 \frac{1 - (-1)^{n-\ell}}{2} V_{\ell, S=0}^{J, F=0}(K) \right. \\ & \left. + \dots + V_{\ell, S=0}^{J, F=1}(K) \right]_{Y_n \text{ and } Y_p} + \dots + V_{\ell, S=1}^{J, F=0}(K) \left. + \dots + V_{\ell, S=1}^{J, F=1}(K) \right]_{Y_n \text{ and } Y_p} \end{aligned}$$

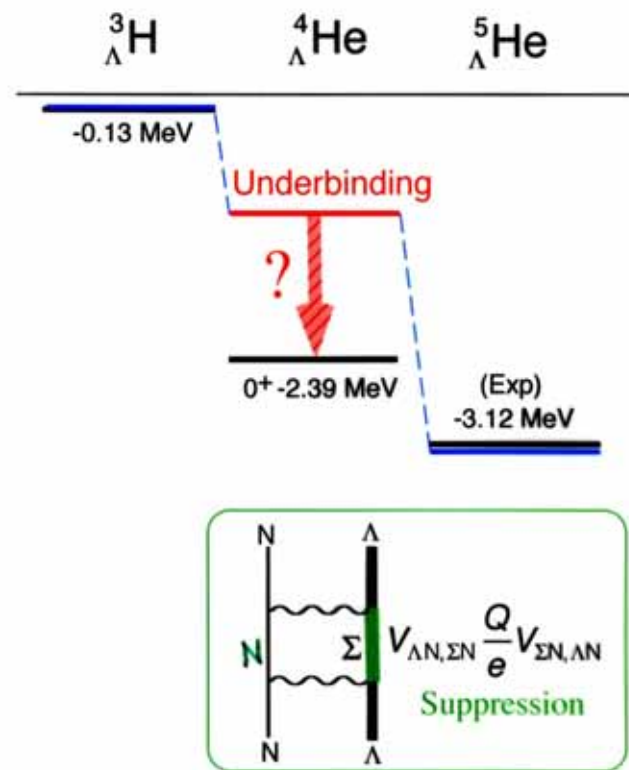
D0

The Overbinding Problem



D2

The Underbinding Problem



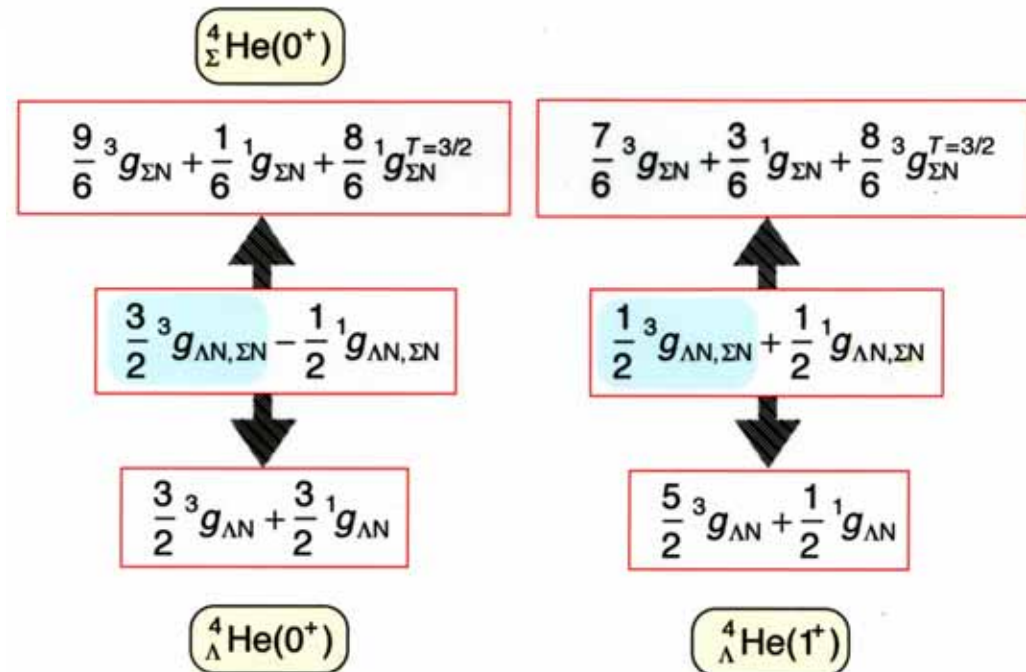
## $\Lambda$ - $\Sigma$ Coupling

B.F. Gibson, A. Goldberg and M.S. Weiss, Phys. Rev. C6 (1972) 741;  
J. Dabrowski, Phys. Rev. C8 (1973) 835.

$$|{}^5_\Lambda\text{He}\rangle = \Phi_\Lambda(\vec{r})|{}^4\text{He}\rangle \quad T=0$$

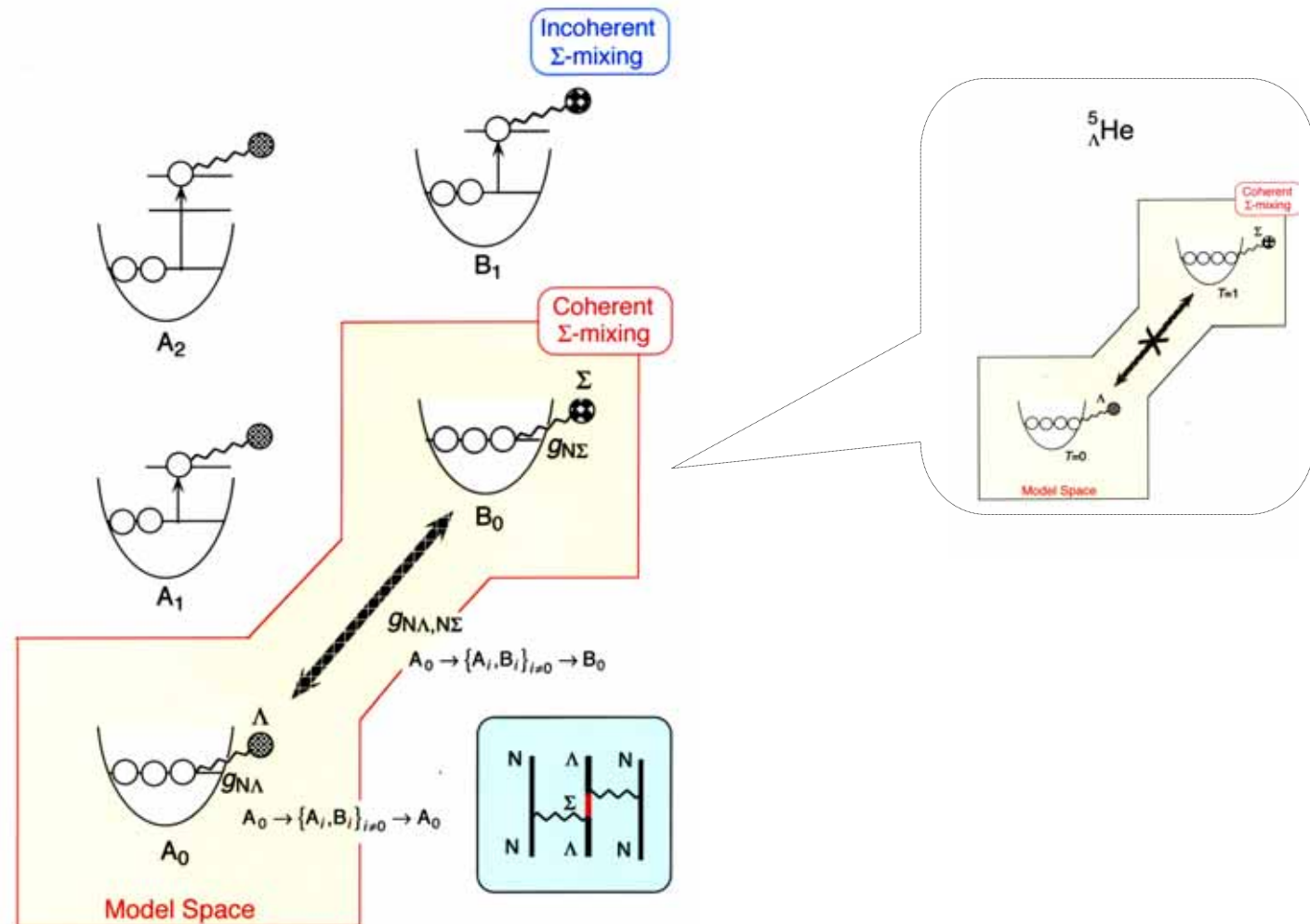
$$|{}^4_\Lambda\text{He}\rangle = \Phi_\Lambda(\vec{r})|{}^3\text{He}\rangle + \sqrt{\frac{2}{3}}\Phi_{\Sigma^+}(\vec{r})|{}^3\text{H}\rangle - \sqrt{\frac{1}{3}}\Phi_{\Sigma^0}(\vec{r})|{}^3\text{He}\rangle$$

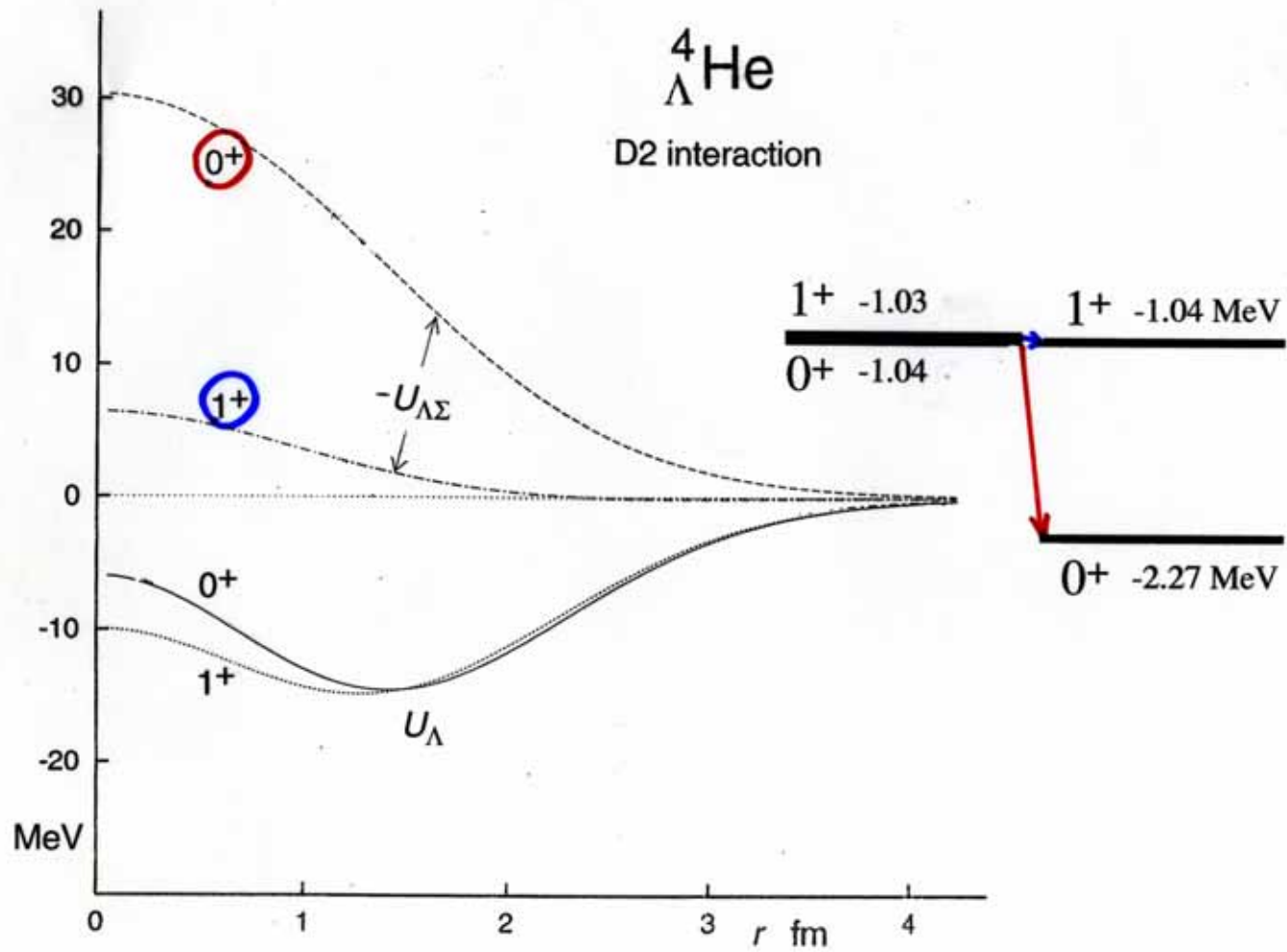
$T=1/2$





# Coherent $\Lambda$ - $\Sigma$ Coupling

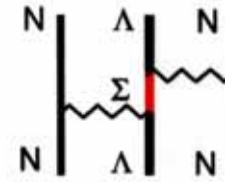




"The  $0^+-1^+$  difference is not a measure of  $\Lambda N$  spin-spin interaction."

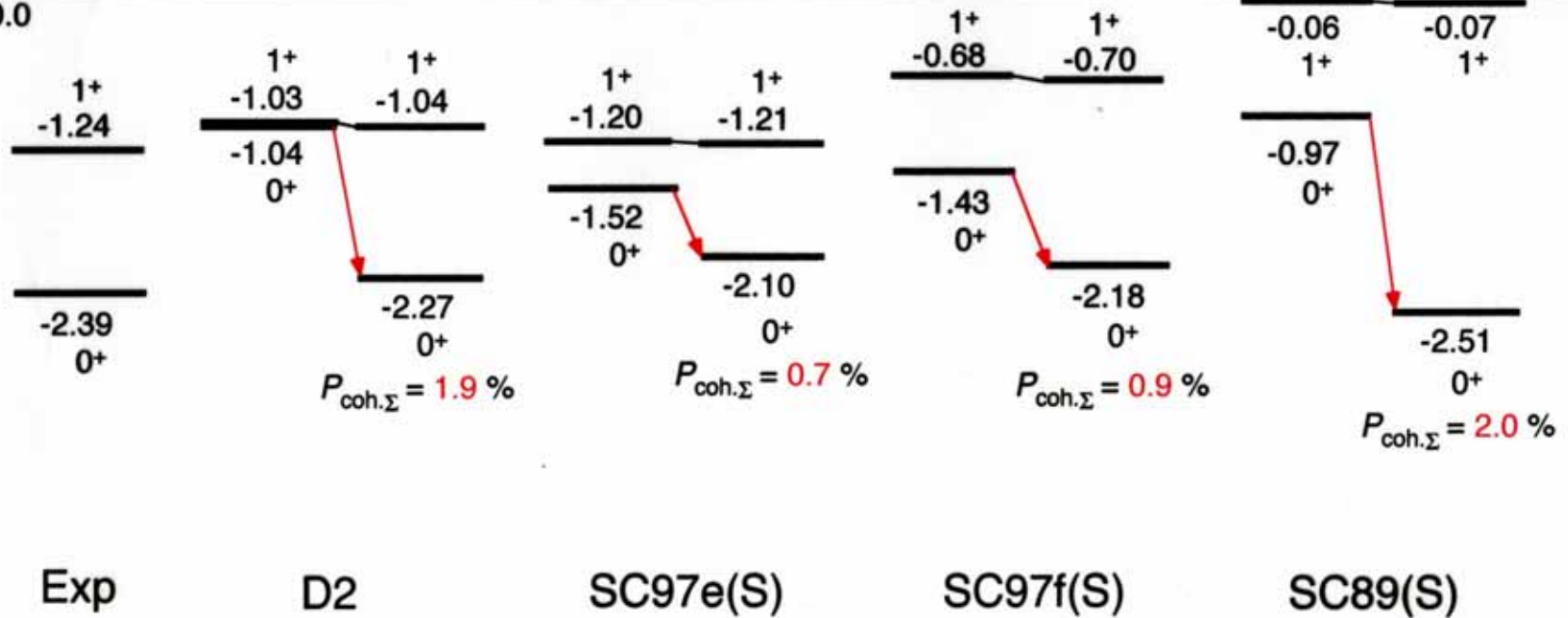
B. Gibson

${}^4_{\Lambda}\text{He}$



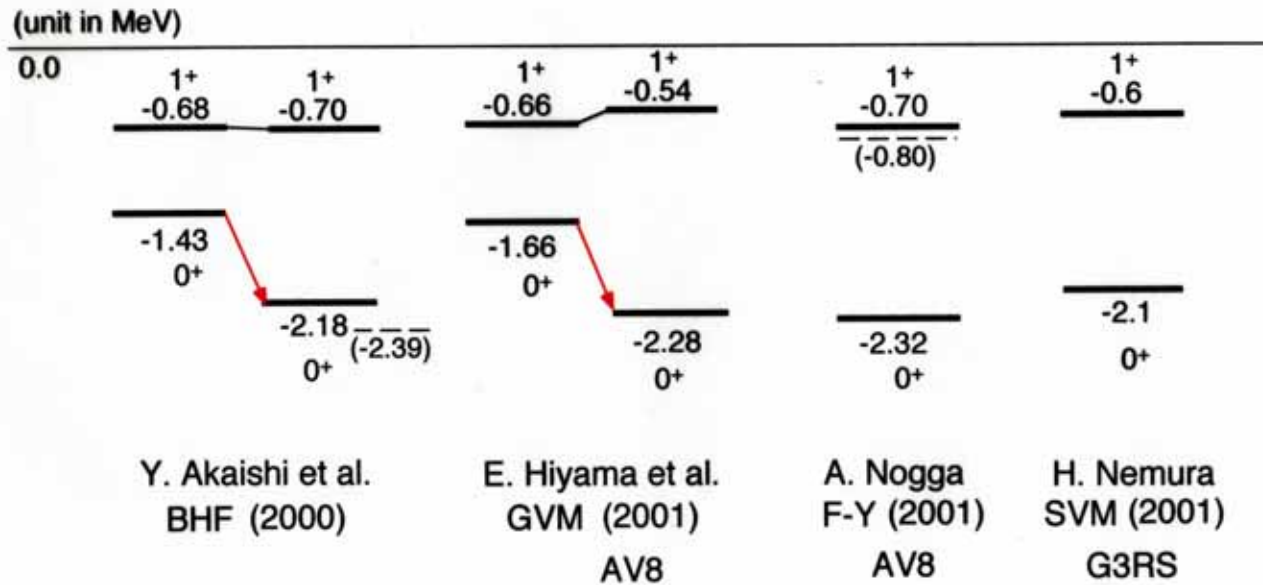
(unit in MeV)

0.0



T. Rijken et al., Phys. Rev. C59 (1999) 21.

${}^4_{\Lambda}\text{He}$  SC97f(S)



E. Hiyama, M. Kamimura, T. Motoba, T. Yamada & Y. Yamamoto, Phys. Rev. C65 (2001) 011301(R).  
 A. Nogga, Doctoral dissertation.

## Faddeev-Yakubovsky calculations for ${}^4_{\Lambda}\text{He}$

A. Nogga, H. Kamada and W. Glöckle, Phys. Rev. Lett. **88** (2002) 172501

<i>YNF</i>	$E_{\text{sep}}^{\Lambda}(0^+)$	$E_{\text{sep}}^{\Lambda}(1^+)$	$\Delta$
SC89	2.14	0.02	2.06
SC97 <i>f</i>	1.72	0.53	1.16
SC97 <i>e</i>	1.54	0.72	0.79
SC97 <i>d</i>	1.29	0.80	0.47
Expt.	2.39(3)	1.24(5)	1.15

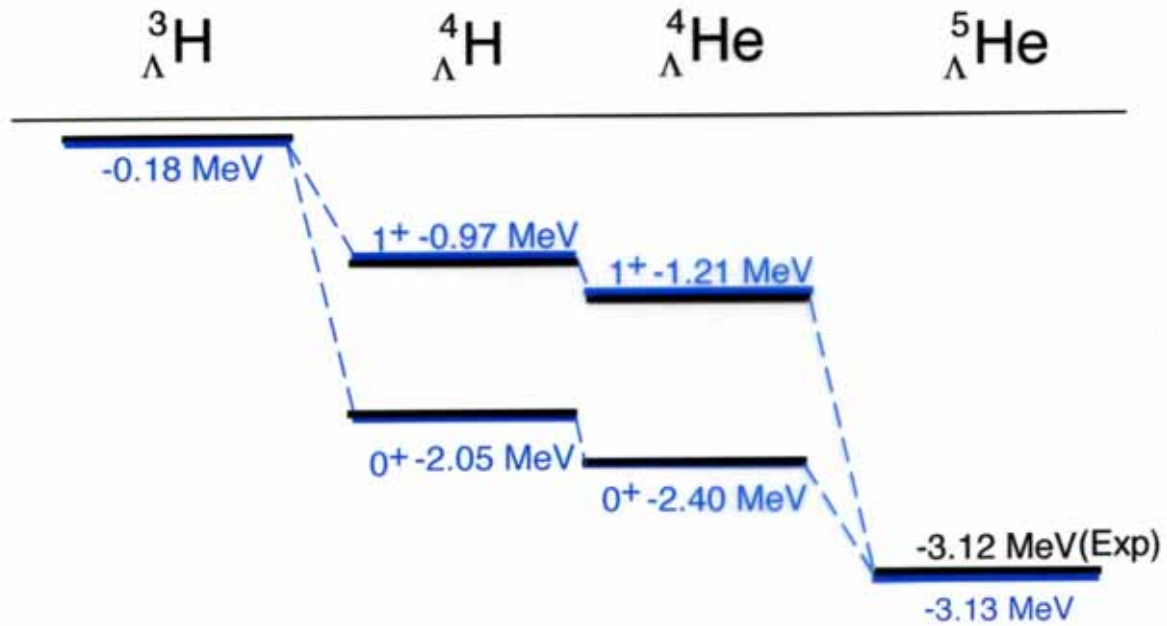
	${}^4_{\Lambda}\text{H}$	
$S=1$ pairs	$1^+$	$0^+$
$\Lambda p \Leftrightarrow -\sqrt{\frac{1}{3}}\Sigma^0 p + \sqrt{\frac{2}{3}}\Sigma^+ n$ $\left\{ \begin{array}{l} S_3=1 \\ S_3=0 \\ S_3=-1 \end{array} \right.$	$\begin{array}{c} -1/3 \\ +1/3 \\ +1/2 \end{array}$	$\begin{array}{c} +1/2 \\ +1/2 \\ +1/2 \end{array}$
$\Lambda n \Leftrightarrow \sqrt{\frac{1}{3}}\Sigma^0 n - \sqrt{\frac{2}{3}}\Sigma^- p$ $\left\{ \begin{array}{l} S_3=1 \\ S_3=0 \\ S_3=-1 \end{array} \right.$	$\begin{array}{c} -1/3 \\ +1/3 \\ +1/2 \end{array}$	$\begin{array}{c} +1/2 \\ +1/2 \\ +1/2 \end{array}$
Contribution to $U_{\Sigma\Lambda}$	$1/2$	$3/2$
$\Lambda$ - $\Sigma$ coupling effect ( $\sim\Lambda NN$ force)	<b>1</b>	<b>9</b>

cancel each other

coherently added

# The Overbinding Problem

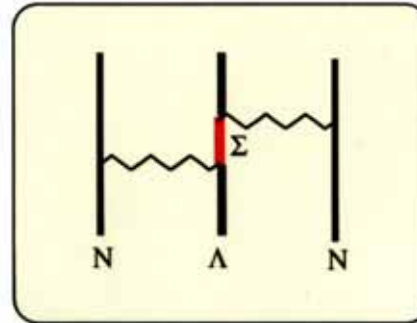
has been solved.



H. Nemura: D2'+Minnesota NN

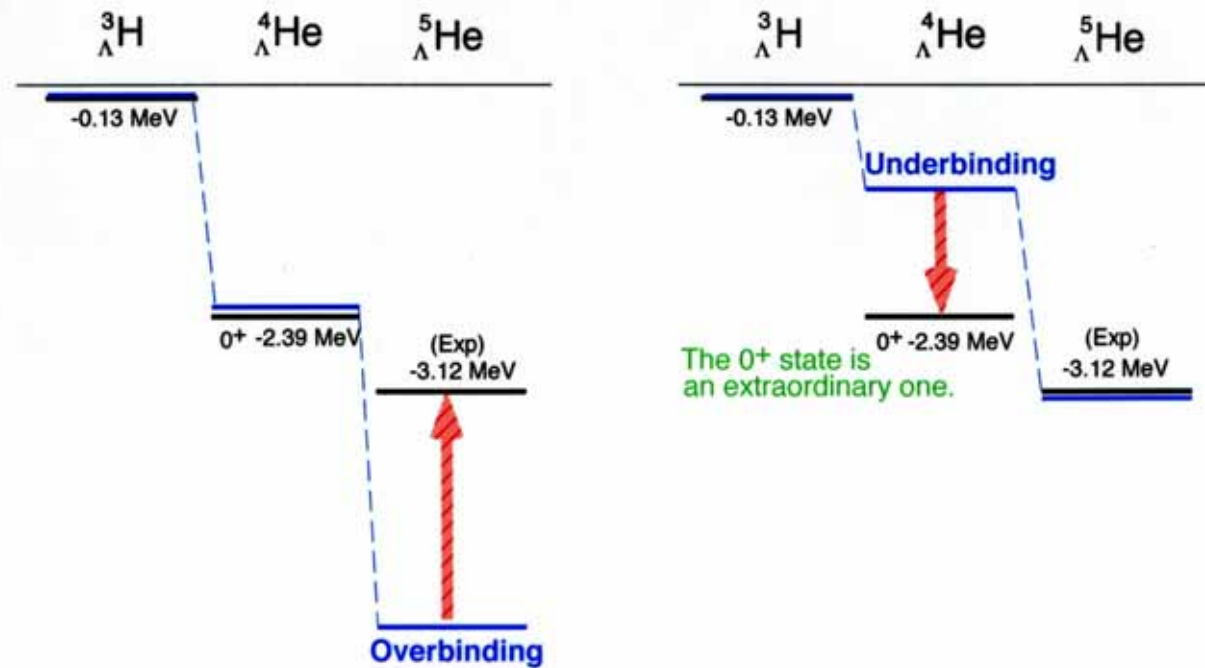
**Repulsion !**

Y. Nogami et al.  
Nucl. Phys. B19 (1970) 93



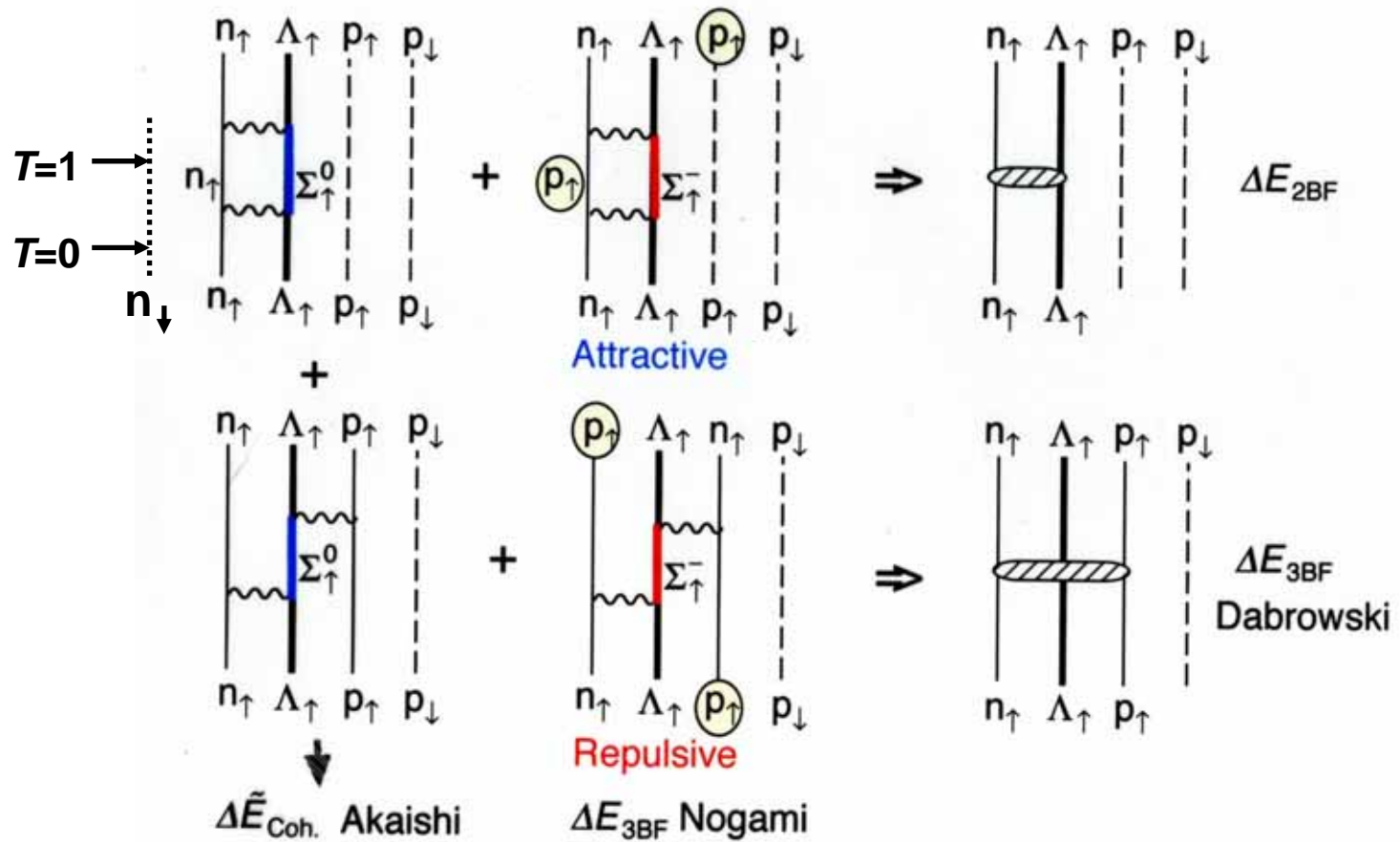
**Attraction !**

Y. Akaishi et al.  
Phys. Rev. Lett. 84 (2000) 3539

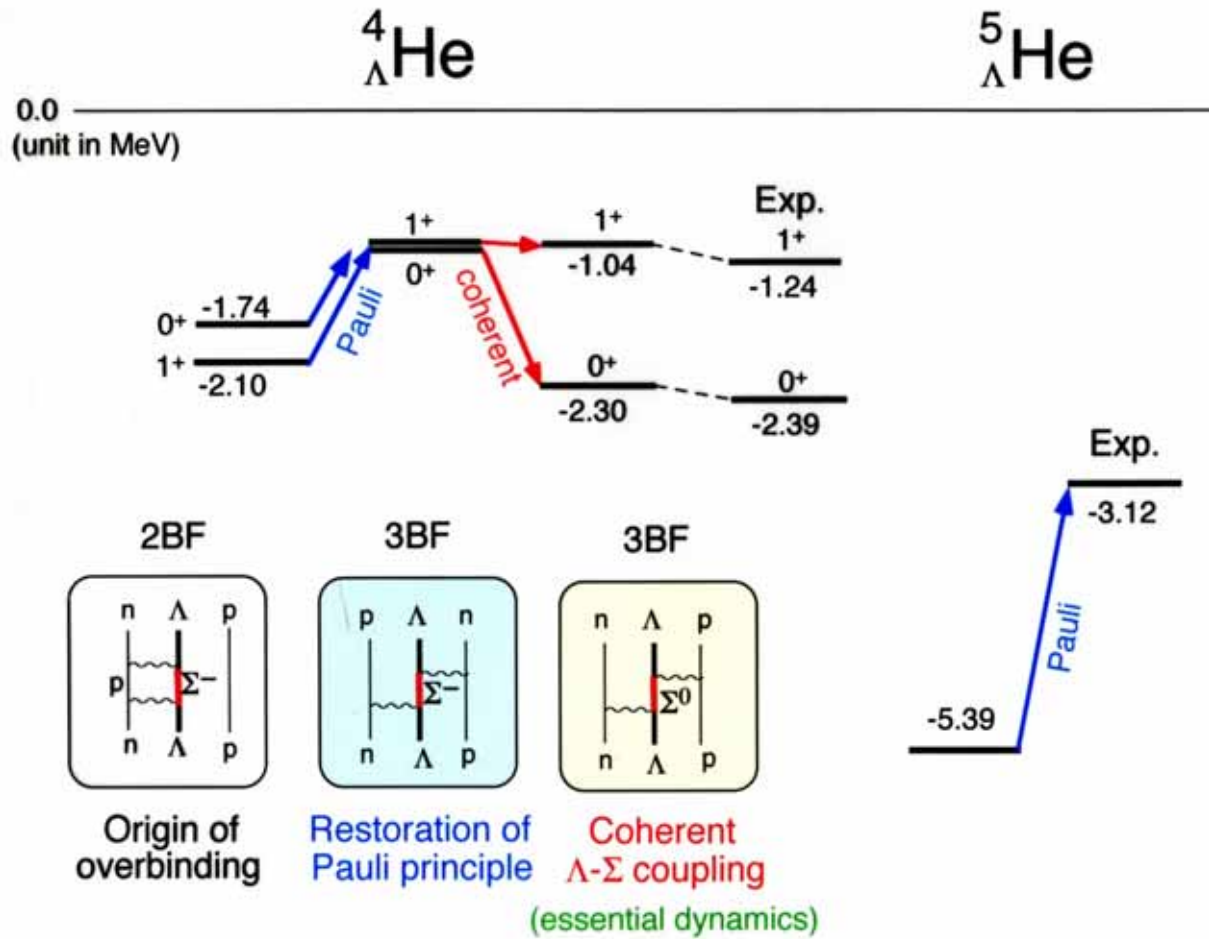




${}^4_{\Lambda}\text{He}$  NNN in  $(0s)^3$



## Effects of $\Lambda$ NN Three-Body Force

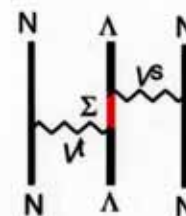


### Three-Body Force due to Coherent $\Lambda$ - $\Sigma$ Coupling : [for D0]

$$U_{\Lambda NN} = \sum_{\alpha=tt,ts,ss} W_3^\alpha(r_{1\Lambda}, r_{\Lambda 2}) \left[ a_\alpha + b_\alpha (\vec{\sigma}_1 \vec{\sigma}_2) + c_\alpha \frac{1}{2} \vec{\sigma}_\Lambda (\vec{\sigma}_1 + \vec{\sigma}_2) \right]$$

$\Lambda N$  spin-spin

$$\begin{pmatrix} a_{tt} & b_{tt} & c_{tt} \\ a_{ts} & b_{ts} & c_{ts} \\ a_{ss} & b_{ss} & c_{ss} \end{pmatrix} = \begin{pmatrix} \frac{7}{16} & \frac{3}{16} & \frac{3}{8} \\ \frac{1}{8} & \frac{1}{8} & -\frac{1}{4} \\ \frac{5}{48} & \frac{1}{48} & -\frac{1}{8} \end{pmatrix}$$



$$W_3^{ts}(r_{1\Lambda}, r_{\Lambda 2}) = V_{\Lambda N, \Sigma N}^t(r_{1\Lambda}) \frac{1}{\Delta M^*} V_{\Sigma N, \Lambda N}^s(r_{\Lambda 2}) + V_{\Lambda N, \Sigma N}^s(r_{1\Lambda}) \frac{1}{\Delta M^*} V_{\Sigma N, \Lambda N}^t(r_{\Lambda 2})$$

$$\langle V_{\Sigma N, \Lambda N}^s \rangle = -\beta \langle V_{\Sigma N, \Lambda N}^t \rangle \quad \beta = 0.67, \quad \langle W_3^t \rangle_4 = 0.74 \text{ MeV}$$

${}^5_\Lambda\text{He}$	$\frac{1}{2}(3 + \beta^2) \langle W_3^t \rangle_5$	$\approx 3 \text{ MeV}$
${}^4_\Lambda\text{H}^*(1^+)$	$\frac{1}{8}(9 + 2\beta + \beta^2) \langle W_3^t \rangle_4$	$\approx 1.0 \text{ MeV}$
${}^4_\Lambda\text{H}(0^+)$	$\frac{1}{8}(-3 - 6\beta + 5\beta^2) \langle W_3^t \rangle_4$	$\approx -0.44 \text{ MeV}$
${}^3_\Lambda\text{H}$	$\frac{1}{8}(-1 - 6\beta + 3\beta^2) \langle W_3^t \rangle_3$	$\approx -0.05 \text{ MeV}$

# Light Hypernuclei

A.R. Bodmer and Q.N. Usmani, Nucl. Phys. **A477** (1988) 621.

R. Sinha and Q.N. Usmani, Nucl. Phys. **A684** (2001) 586c.

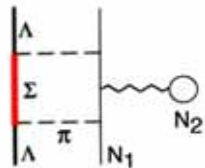
$$V_{\Lambda N} = \left[ (V_{\text{core}}(r) - \bar{V})(1 - \varepsilon + \varepsilon P_x) + \frac{1}{4} V_{\sigma} \bar{\sigma}_{\Lambda} \bar{\sigma}_N \right] T_{\pi}^2(r)$$

spin-spin

$$V_{\Lambda NN} = W_{\rho} \{\text{Nogami}\} + V_{\Lambda NN}^{\text{DS}} \{\text{Dispersive}\}$$

$$V_{\Lambda NN}^{\text{DS}}(r_{ij\Lambda}) = W_0 T_{\pi}^2(r_{i\Lambda}) T_{\pi}^2(r_{j\Lambda}) \left[ 1 + \frac{1}{6} \bar{\sigma}_{\Lambda} (\bar{\sigma}_i + \bar{\sigma}_j) \right]$$

spin-spin



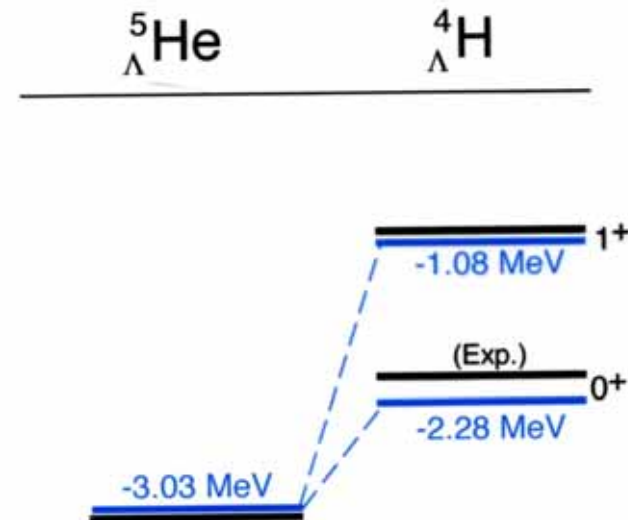
The  $0^+ - 1^+$  splitting

$$0.38 + 0.86 \text{ MeV} \quad (70\%)$$

The major part is attributed to  $\Lambda NN$  and not to  $\Lambda N$ .

$$0.56 + 0.92 \text{ MeV for SC97f} \quad (62\%)$$

$$-0.17 + 1.40 \text{ MeV for D2} \quad (114\%)$$

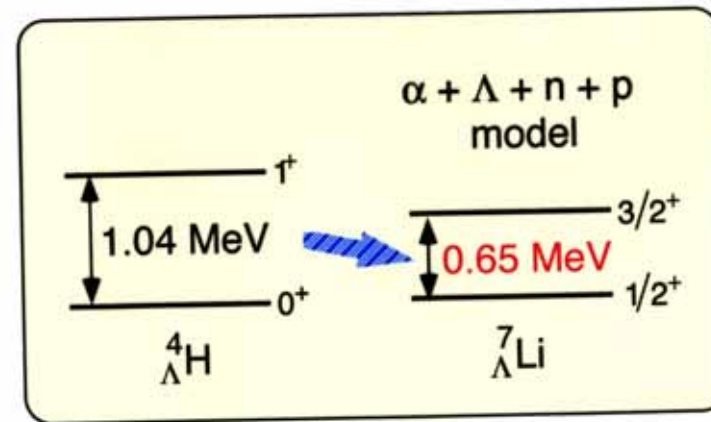
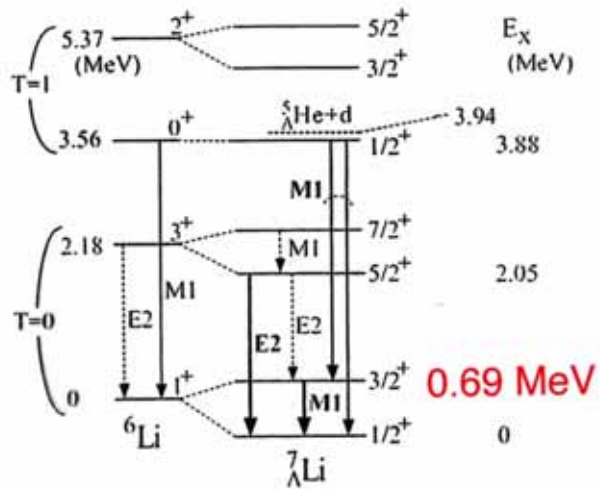
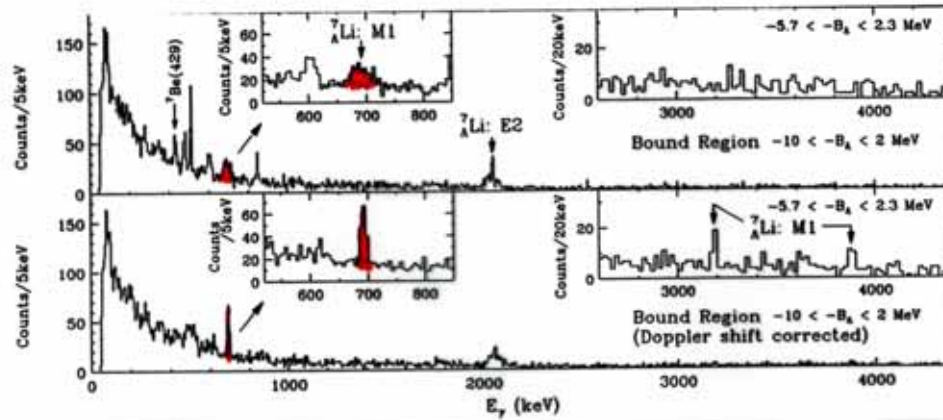


Variational Monte-Carlo

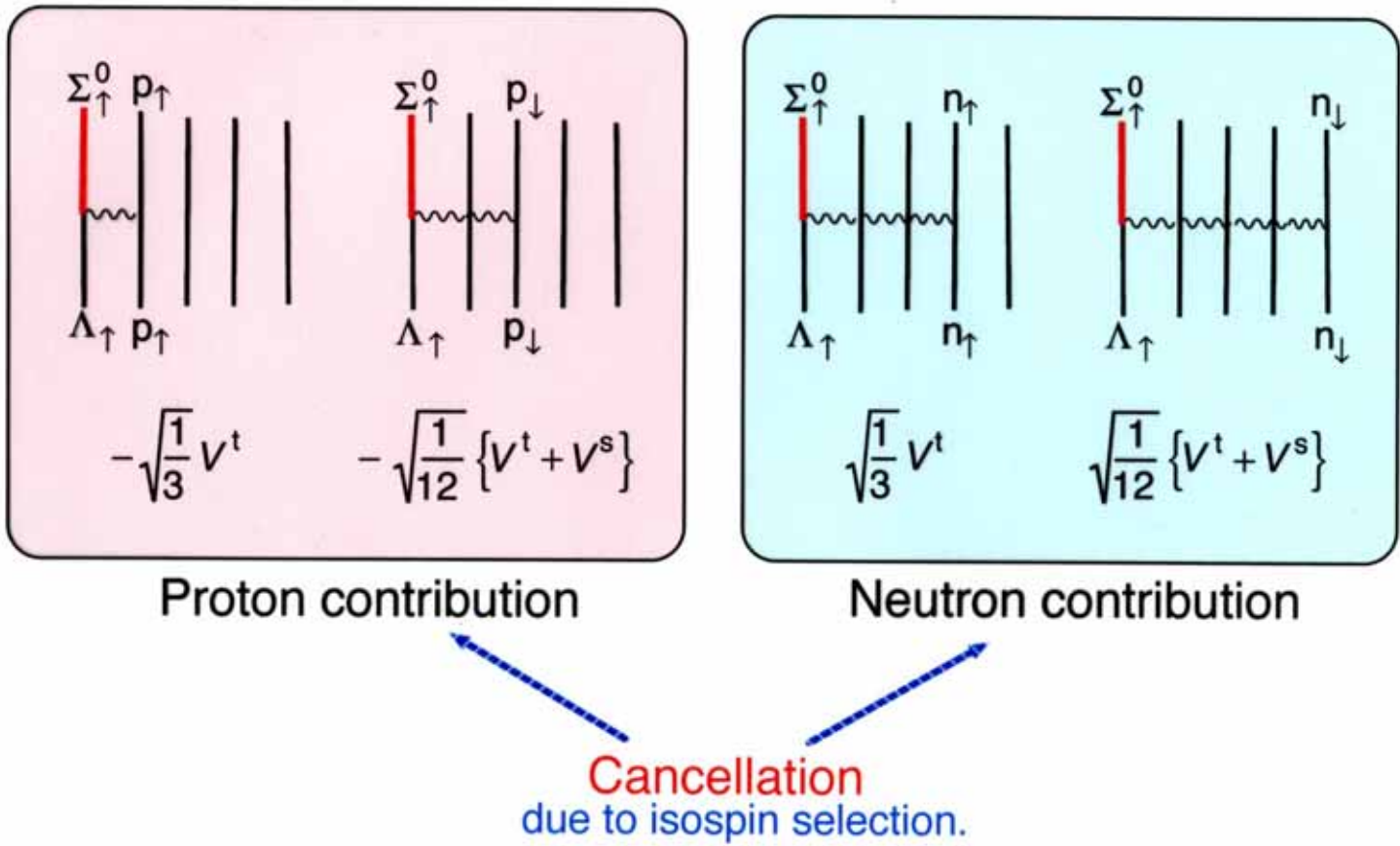
J. Lomnitz-Adler, V.R. Pandharipande & R.A. Smith, Nucl. Phys. **A361** (1981) 399.

# $\Lambda$ N spin-spin interaction

H. Tamura et al., Phys. Rev. Lett. **84** (2000) 5963

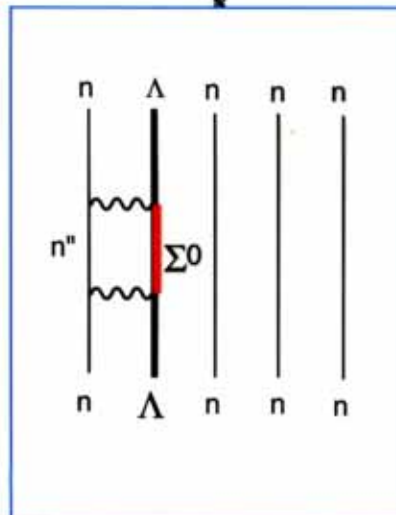


E. Hiyama et al., Nucl. Phys. **A639** (1998) 173c



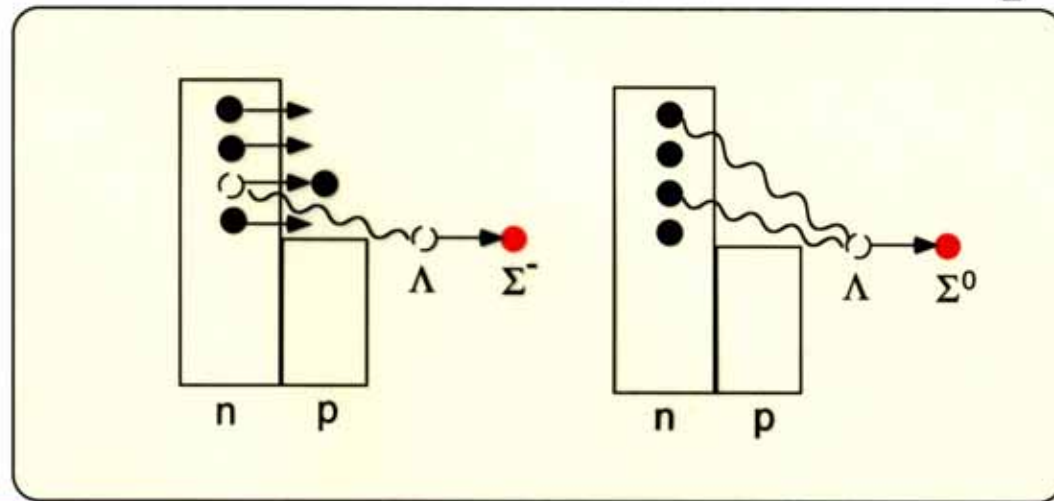
$$\{P_{\Sigma N}^+(^3S_1)V^tP_{\Lambda N}(^3S_1) + P_{\Sigma N}^+(^1S_0)V^sP_{\Lambda N}(^1S_0)\} \Lambda_{\uparrow}^{\uparrow} p_{\uparrow}^{\uparrow} p_{\downarrow}^{\downarrow} n_{\uparrow}^{\uparrow} n_{\downarrow}^{\downarrow} |0\rangle$$

*g*-matrix

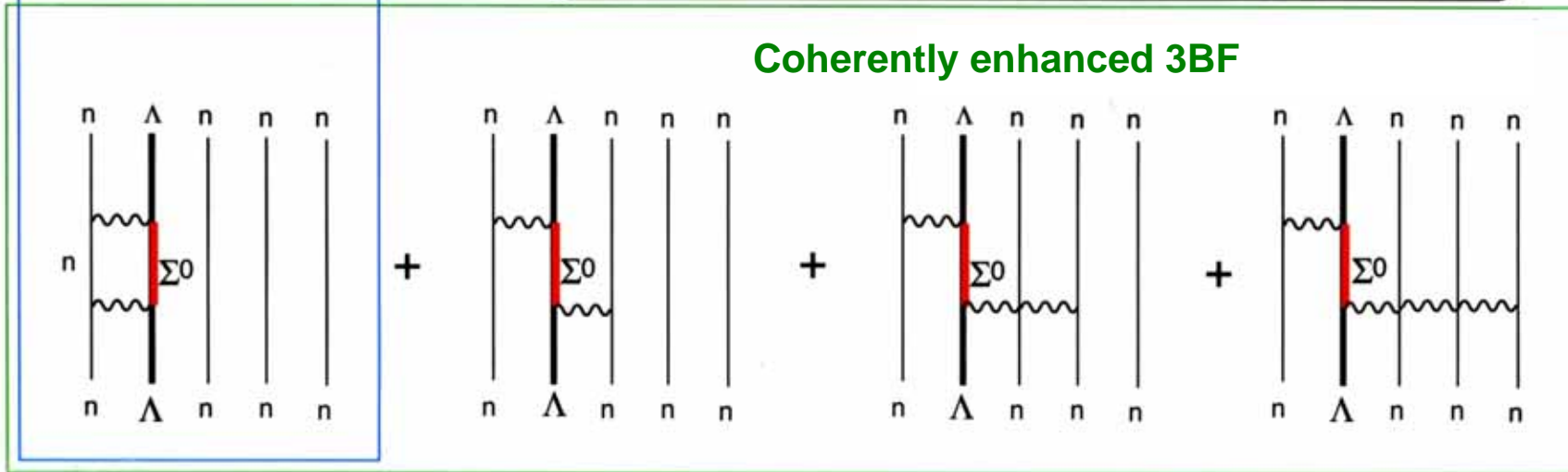


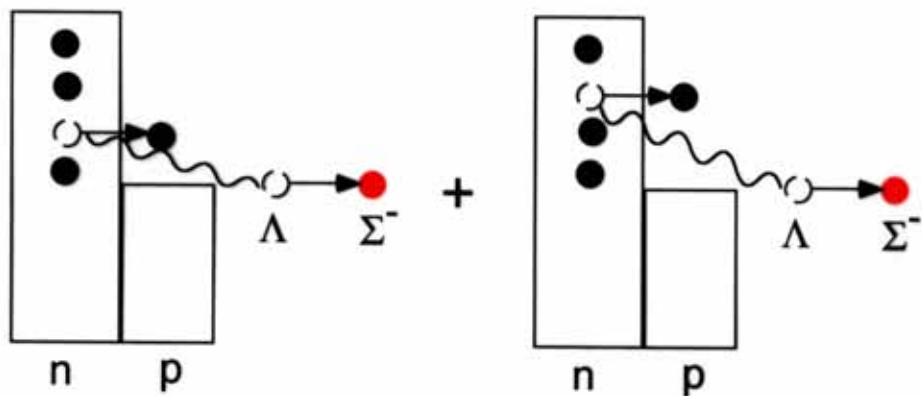
$$\sqrt{\frac{1}{T+1}}$$

$$\sqrt{\frac{T}{T+1}} \quad \text{for } T = T_z = \frac{N}{2}$$

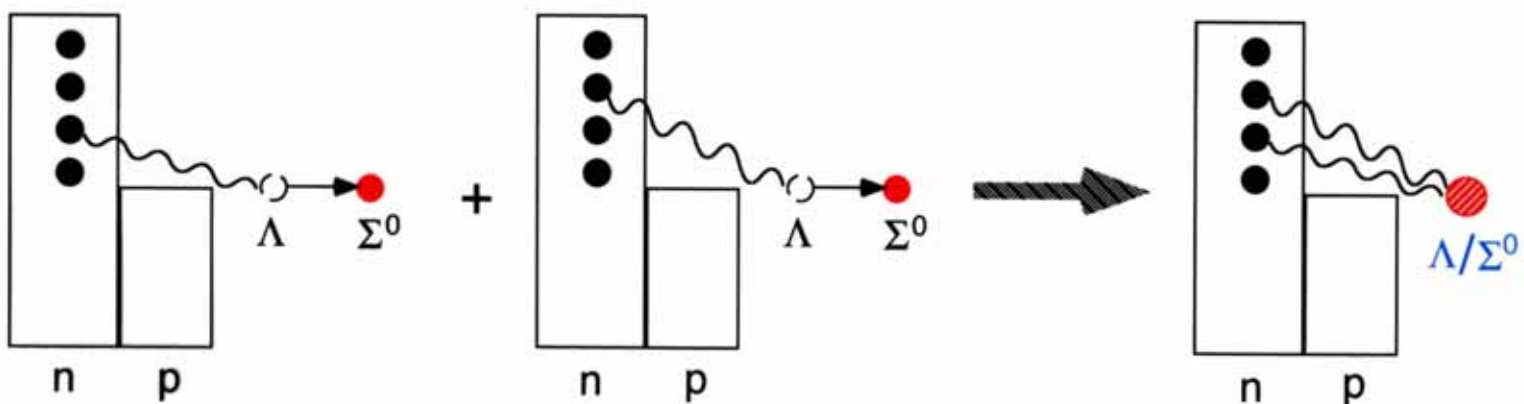


Coherently enhanced 3BF



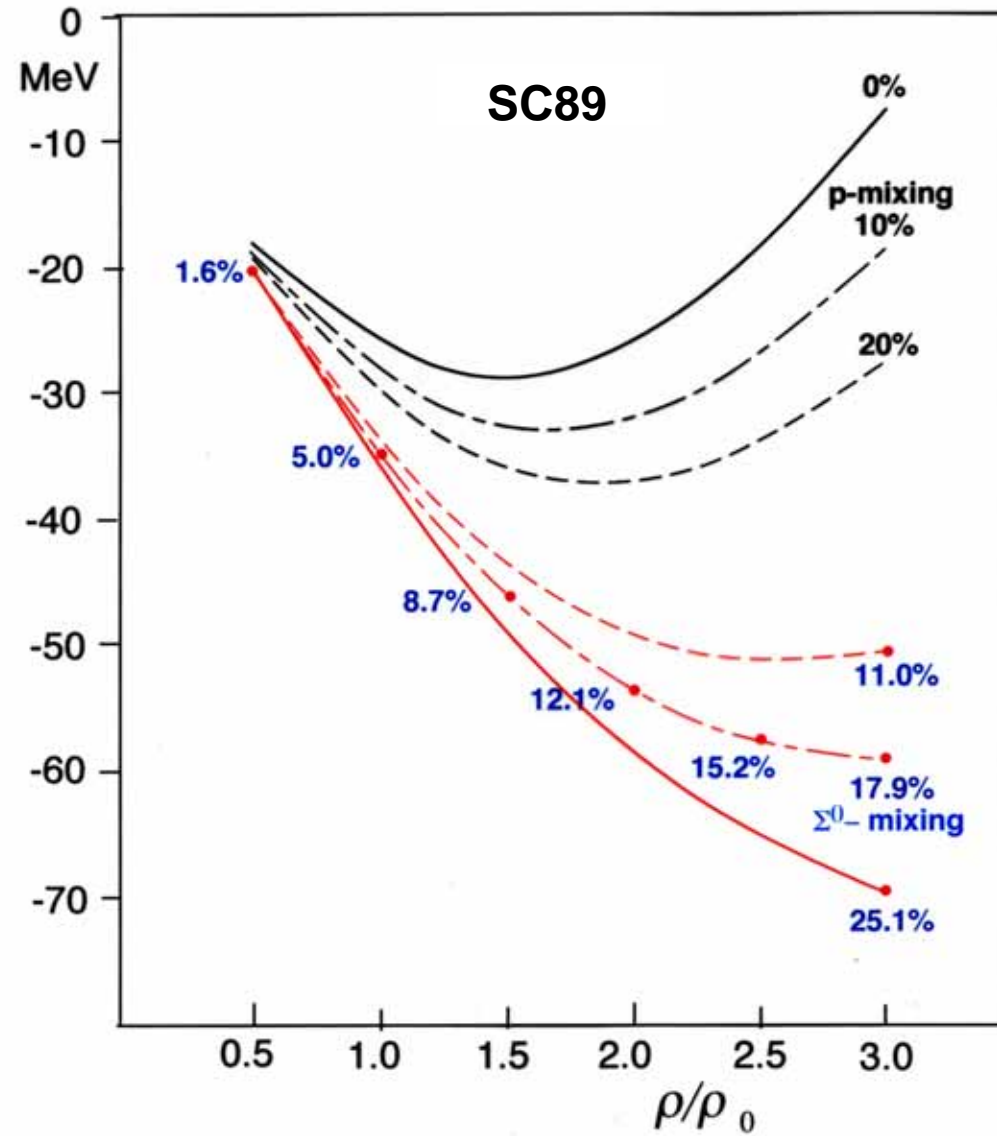


$$\sqrt{\frac{T}{T+1}} \text{ for } T = T_z = \frac{N}{2}$$



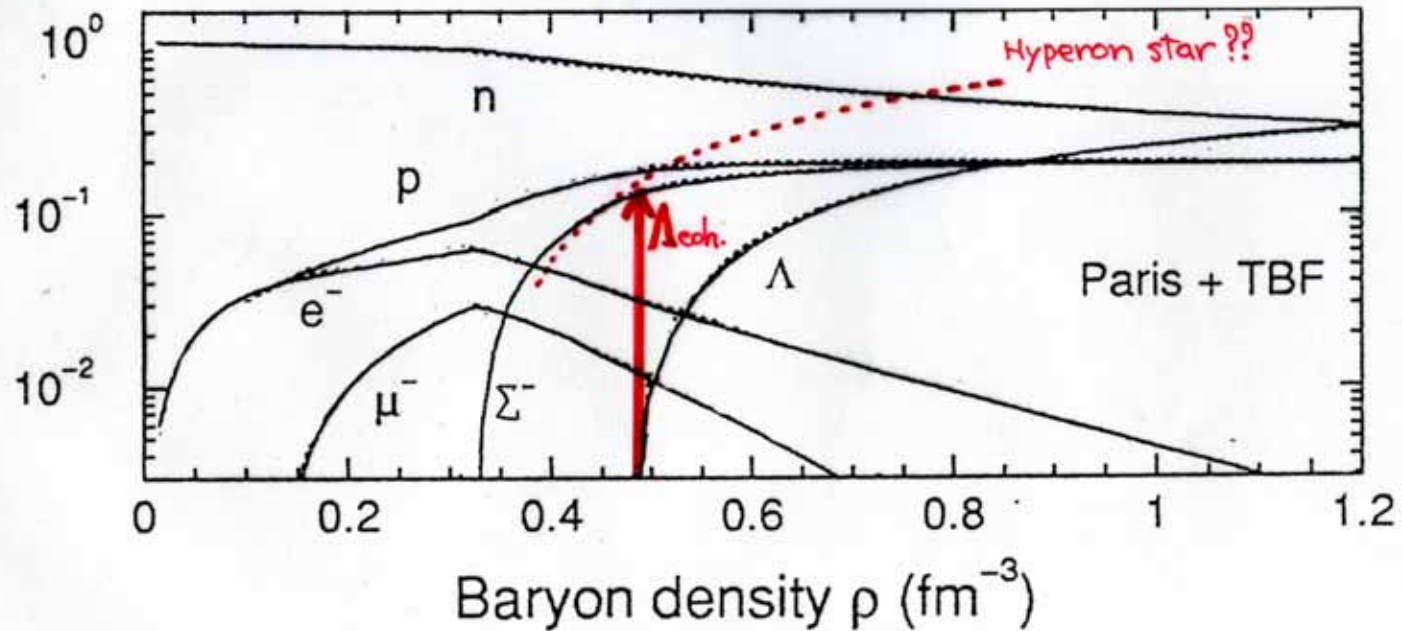


# $U_\Lambda$ in neutron matter



## Composition of neutron star matter

M. Baldo and G.F. Burgio, Phys. Rev. C61 (2000) 055801.  
( Brueckner-Bethe-Goldstone theory )



## Relativistic Mean-Field Model

Lagrangian density

$$L = L_B^0 + L_M^0 + L_{\text{int}}$$

Baryons : n, p,  $\Lambda$ ,  $\Sigma^0$ ,  $\Sigma^-$

Mesons :  $\sigma$ ,  $\rho$ ,  $\omega$

For  $\Lambda$  and  $\Sigma^0$

$$(\not{p} - \gamma^0 g_{\Lambda\Lambda\omega}\omega_0 - M_\Lambda + g_{\Lambda\Lambda\sigma}\sigma)\Lambda - \gamma^0 g_{\Lambda\Sigma\rho}\rho_0 \Sigma^0 = 0$$

$$(\not{p} - \gamma^0 g_{\Sigma\Sigma\omega}\omega_0 - M_\Sigma + g_{\Sigma\Sigma\sigma}\sigma)\Sigma^0 - \gamma^0 g_{\Sigma\Lambda\rho}\rho_0 \Lambda = 0$$

For mesons

$$m_\sigma^2 \sigma = \Sigma g_{BB\sigma} \langle \bar{B} B \rangle$$

$$m_\omega^2 \omega^0 = \Sigma g_{BB\omega} \langle \bar{B} \gamma^0 B \rangle$$

$$m_\rho^2 \rho^0 = \Sigma g_{BB\rho} \langle \bar{B} \gamma^0 B \rangle + g_{\Lambda\Sigma\rho} (\langle \bar{\Lambda} \gamma^0 \Sigma \rangle + \langle \bar{\Sigma} \gamma^0 \Lambda \rangle)$$

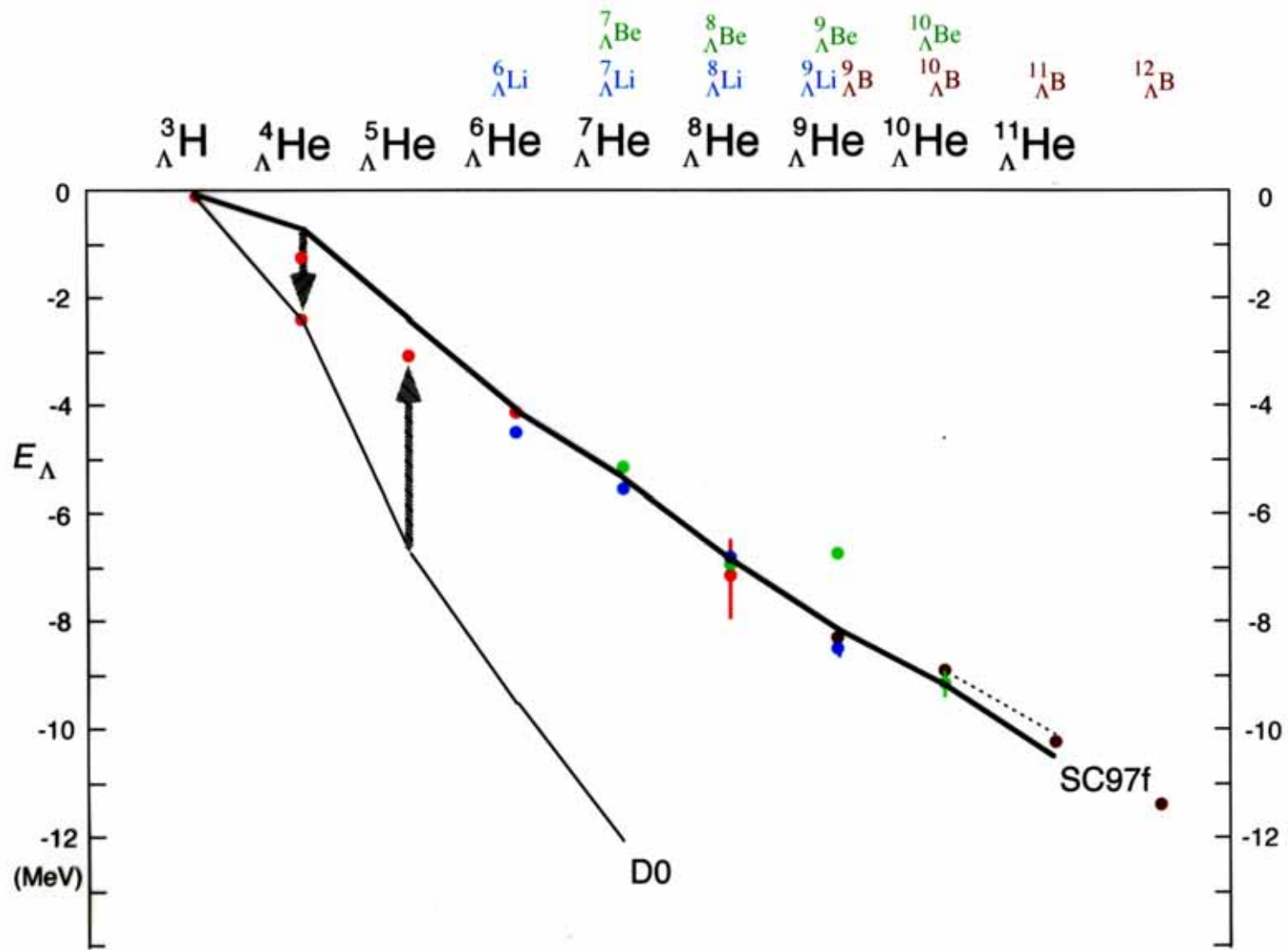
The normal state of infinite matter :

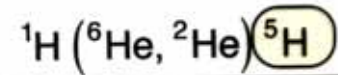
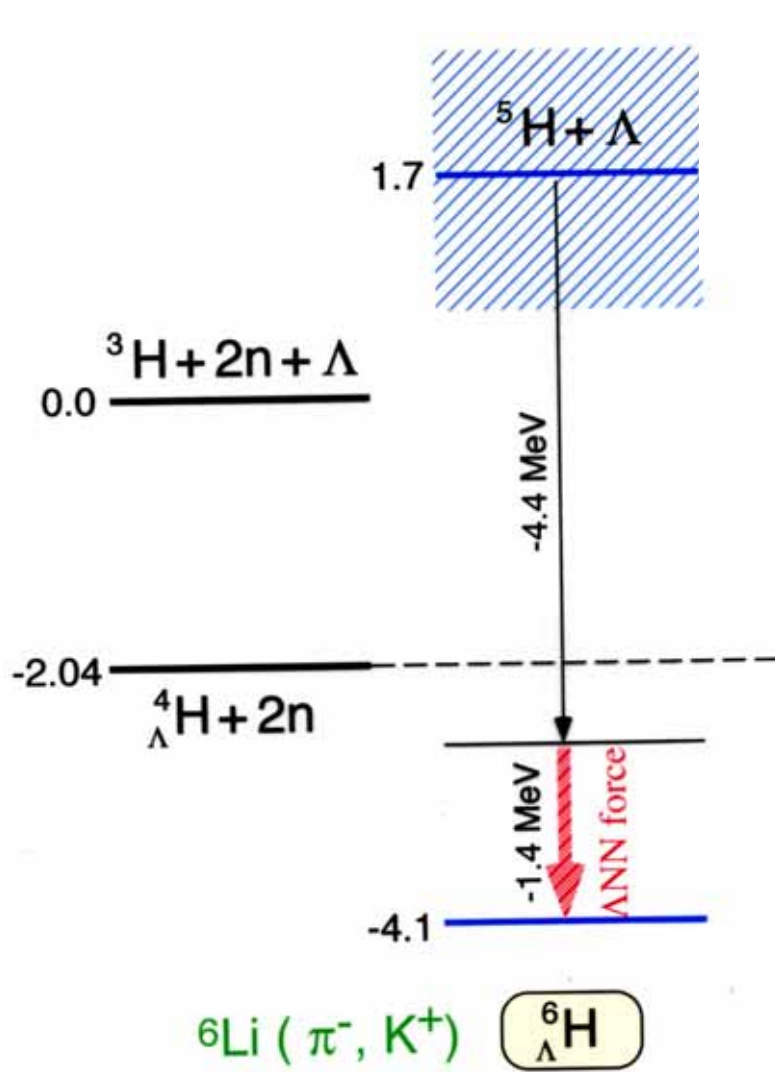
N.K. Glendenning, *Astrophys. J.* **293** (1985) 470.

Baryons in the medium carry the same Q.N.'s as in vacuum.

$$\text{[Yellow oval]} = 0$$

M. Serra





Superheavy hydrogen  
 A.A. Korshennikov et al.,  
 Phys. Rev. Lett. 87 (2001) 092501

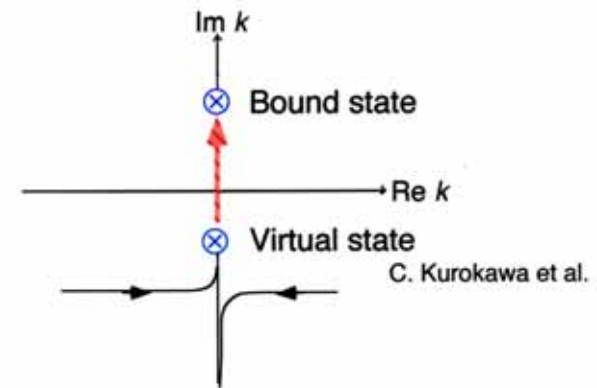
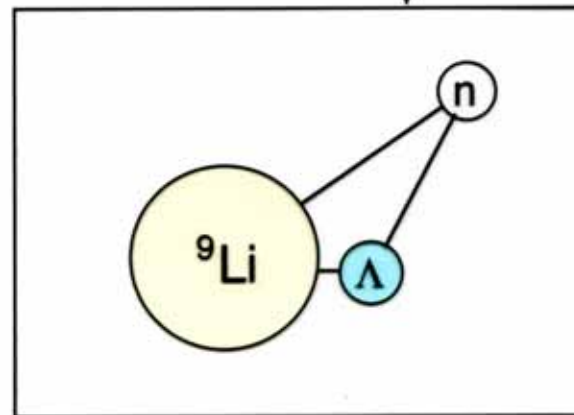
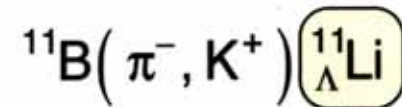
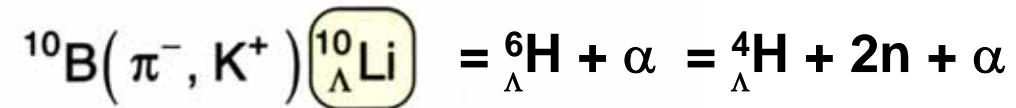
K. Arai, H. Nemura  
 ${}^7_{\Lambda}\text{H}$      ${}^8_{\Lambda}\text{H}$



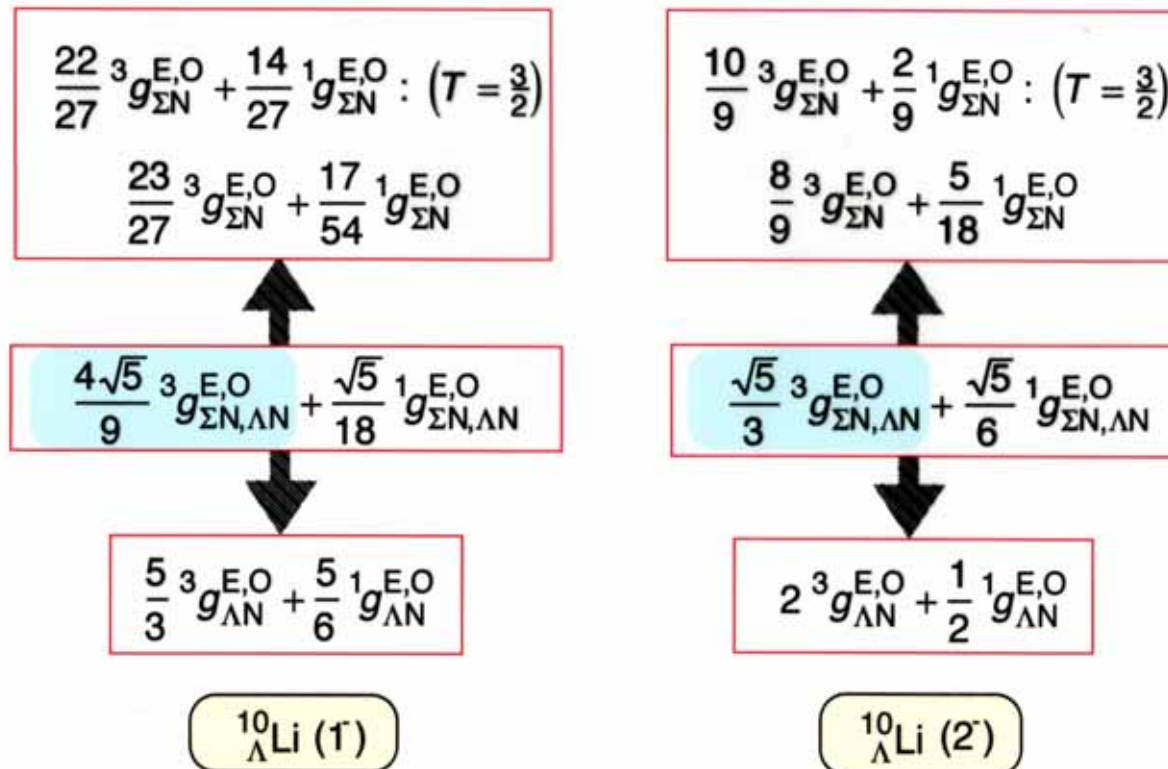
"Hyperheavy hydrogen"

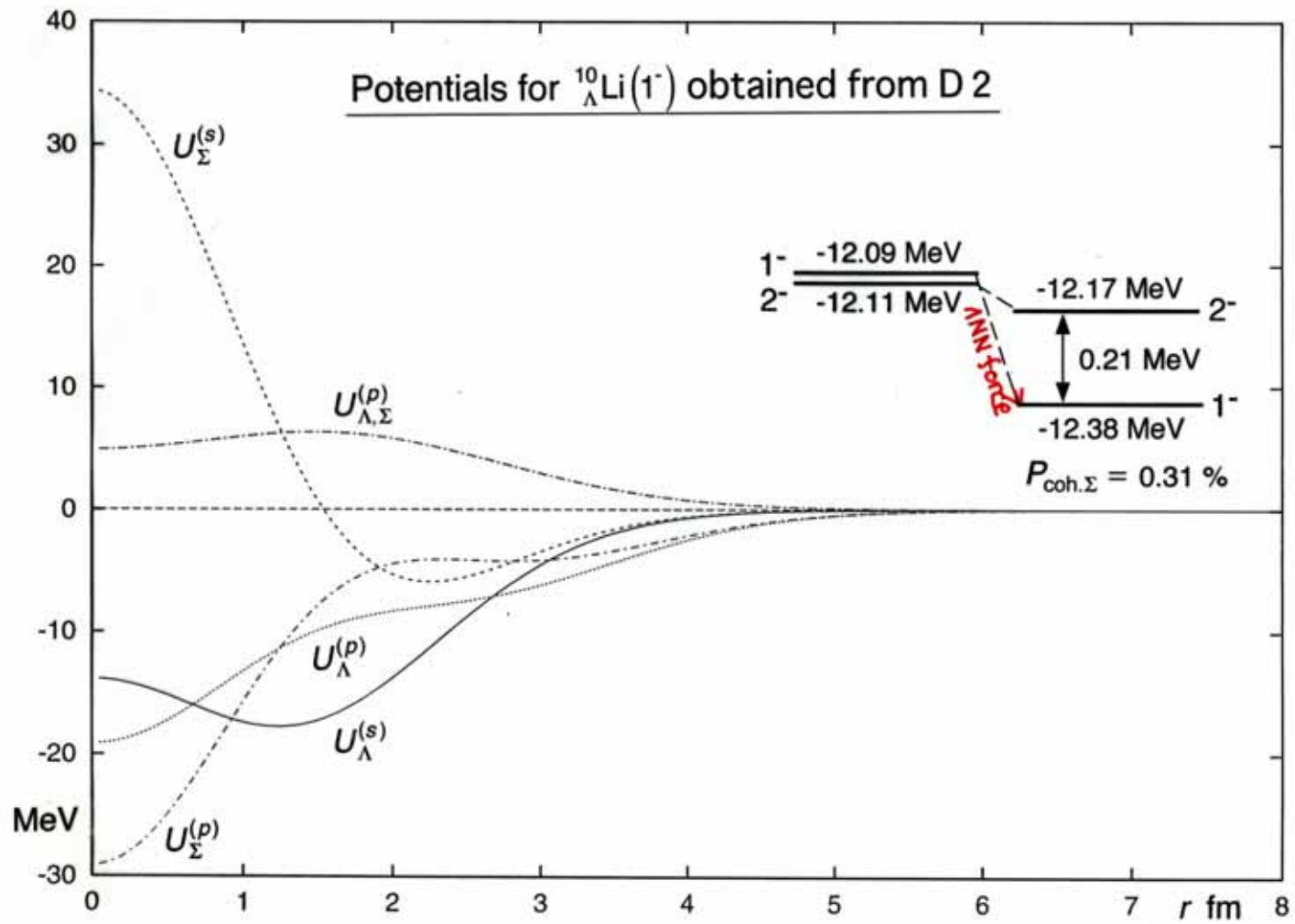
Khin Swe Myint & Y. Akaishi,  
 Prog. Theor. Phys. Suppl. 146 (2002) 599

# Double-charge & Strangeness Exchange Reaction



## YN interaction from p-shell nucleons





AMD cal.; Y. Enyo, A. Dote et al.

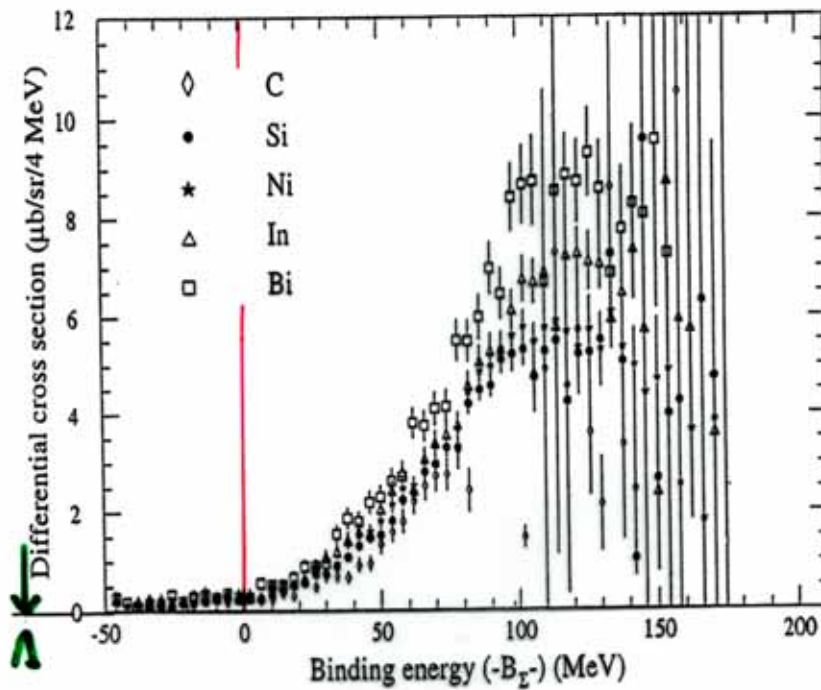


# Double-Charge Exchange Experiment

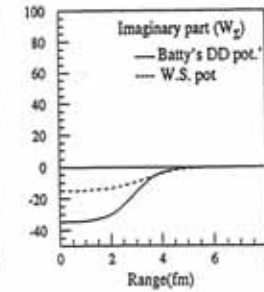
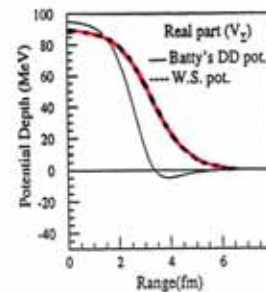
$(\pi^-, K^+)$  at KEK

H. Noumi, P.K. Saha et al., Phys. Rev. Lett. **89** (2002) 072301

$\Sigma^-$

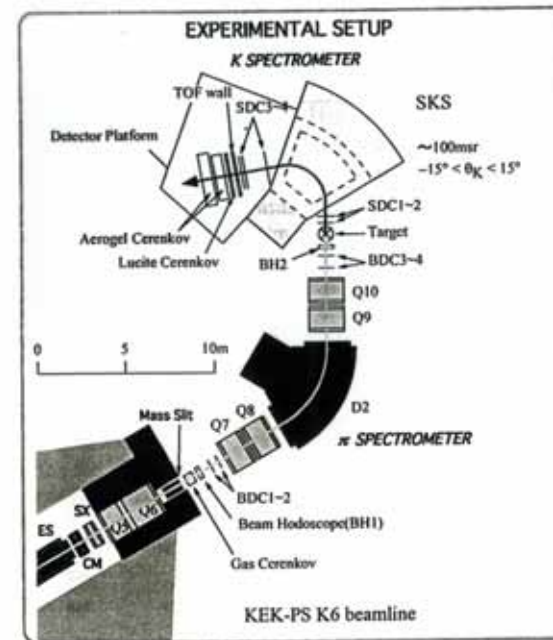
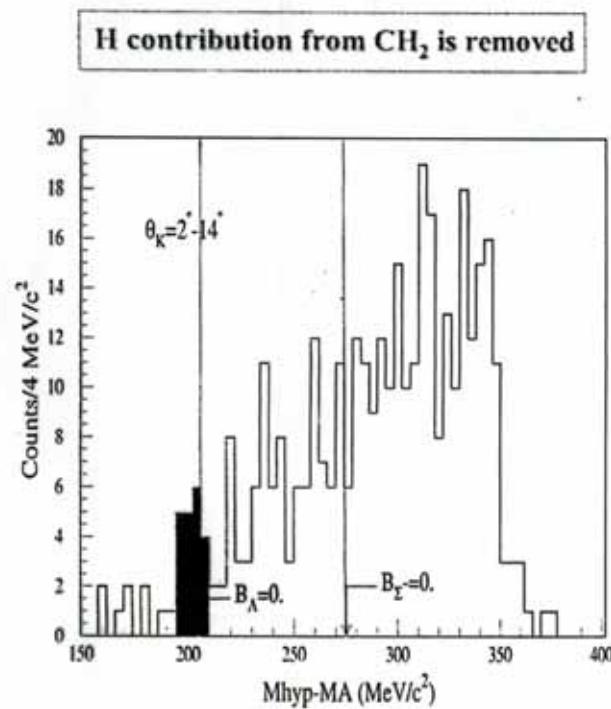


Strongly repulsive !?

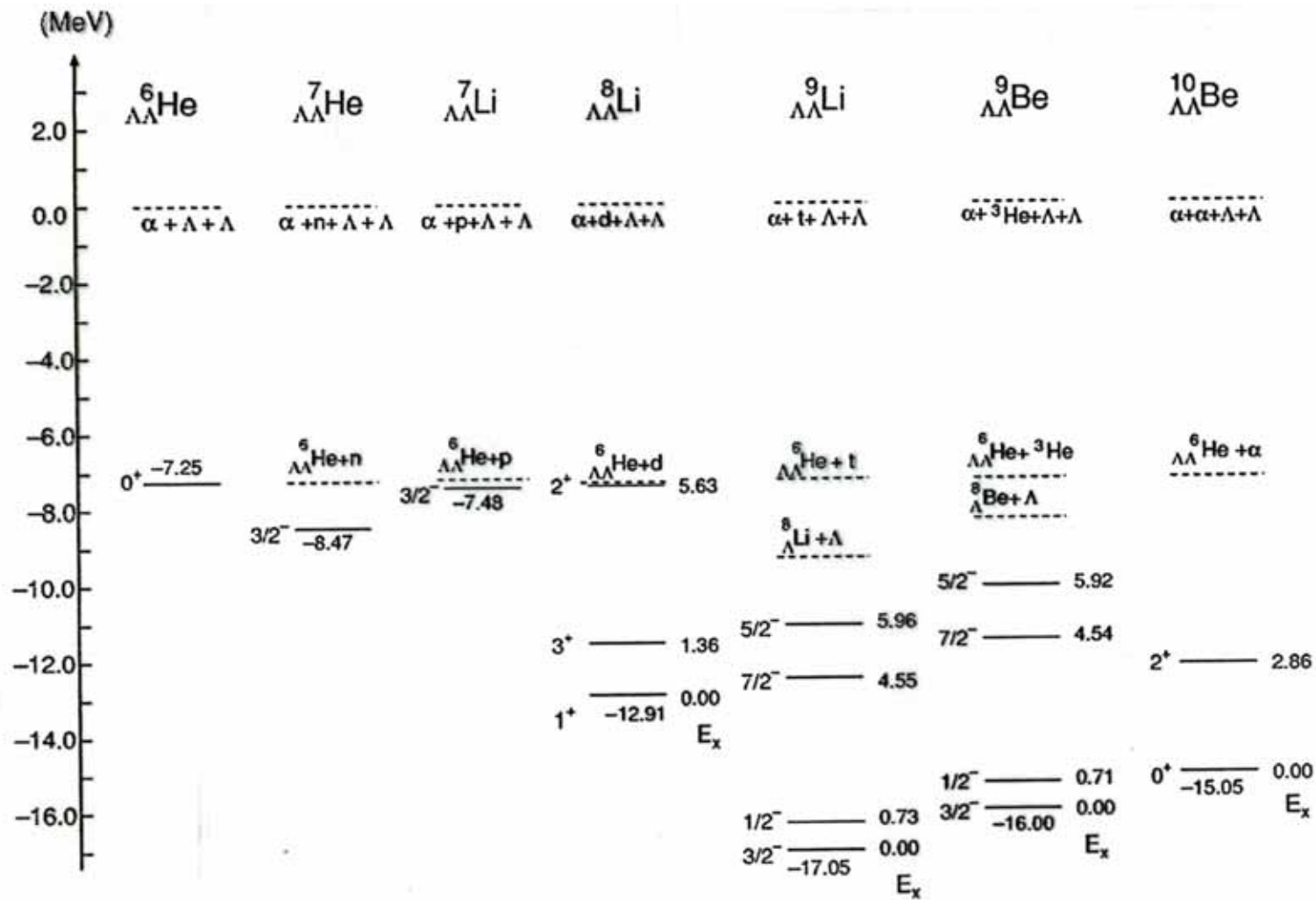


# $^{12}\text{C}(\pi^-, \text{K}^+)$ spectrum from the $\text{CH}_2$ target

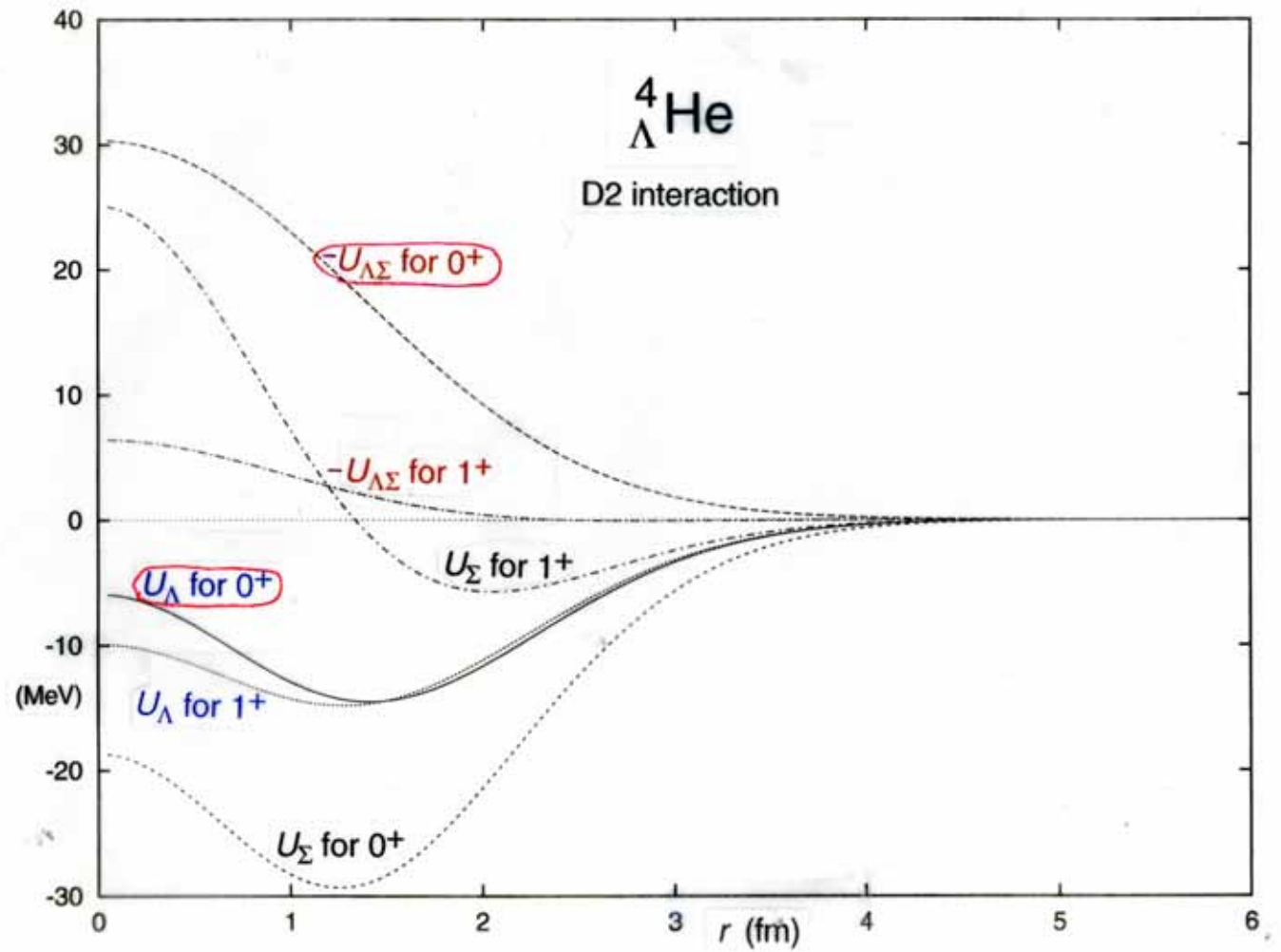
by Saha, Fukuda, Noumi



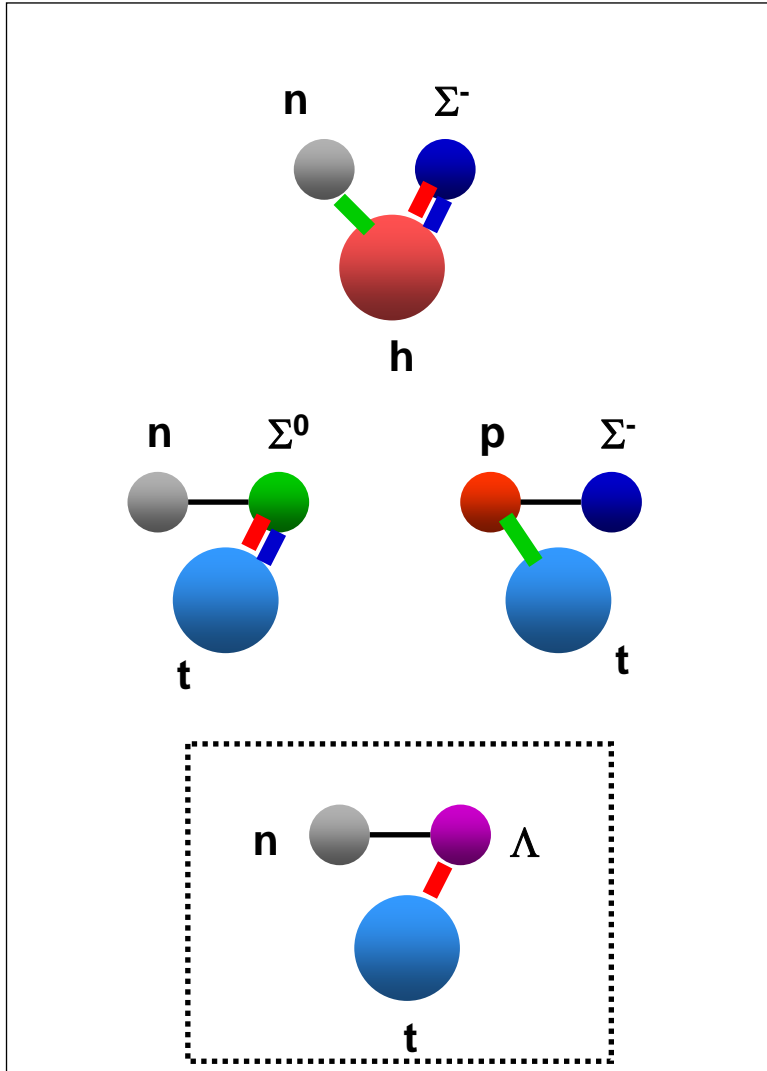
$^{10}\text{B}(\pi^-, \text{K}^+)$ ; W. Imoto, P.K. Saha et al.; this evening



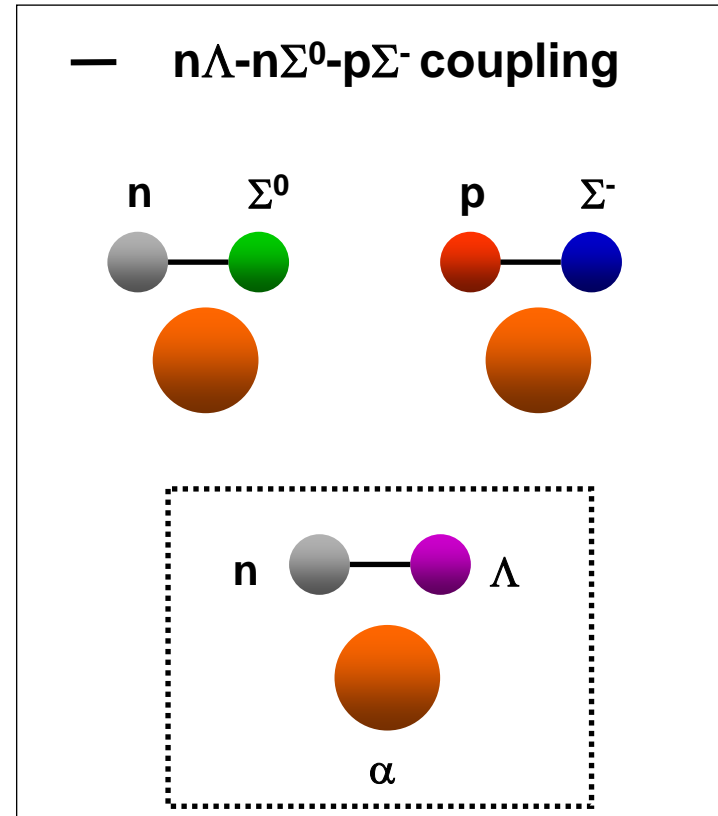
E. Hiyama, M. Kamimura, T. Motoba, T. Yamada & Y. Yamamoto, Phys. Rev. C **66** (2002) 024007.



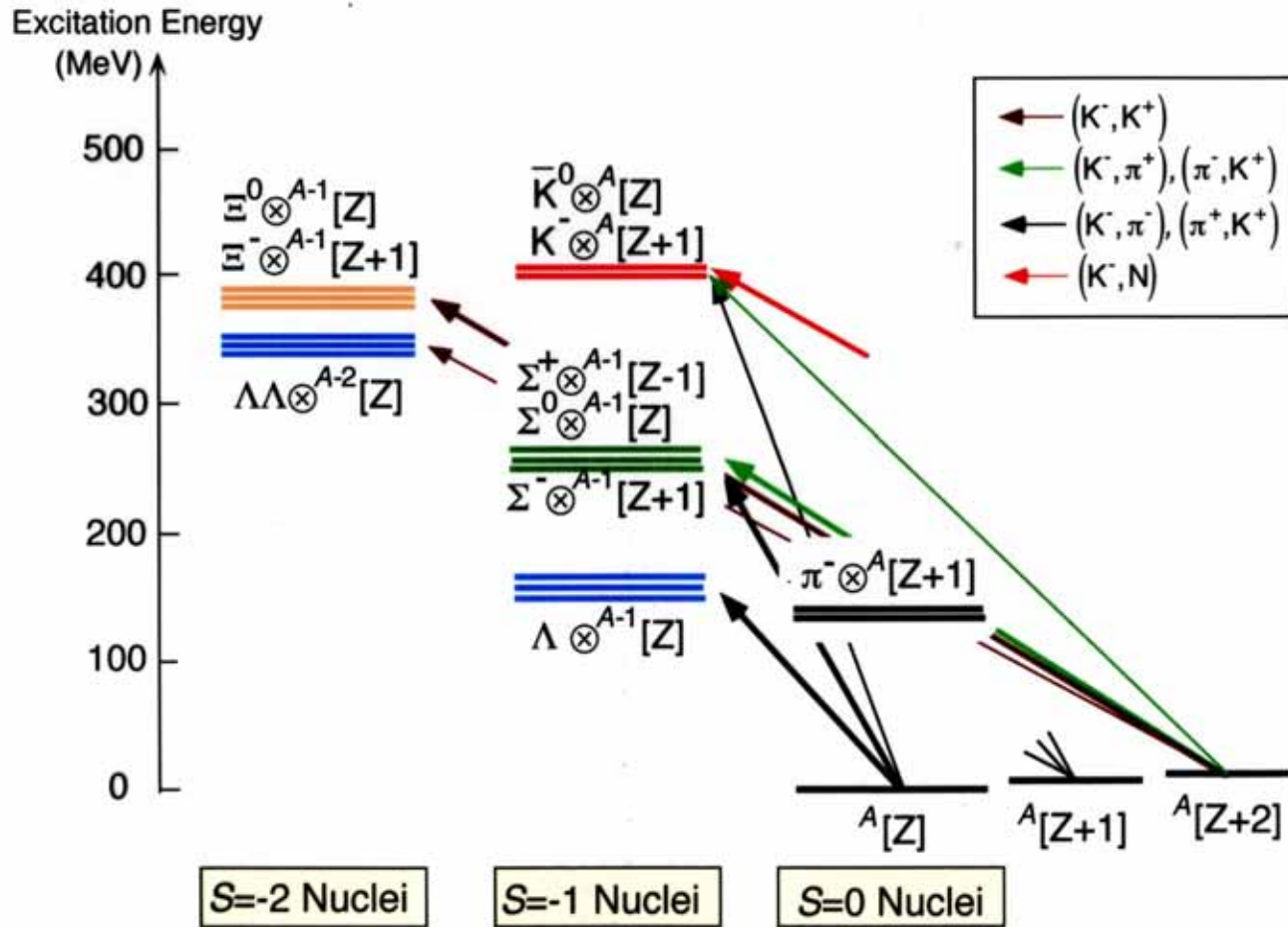
# Coupling Scheme



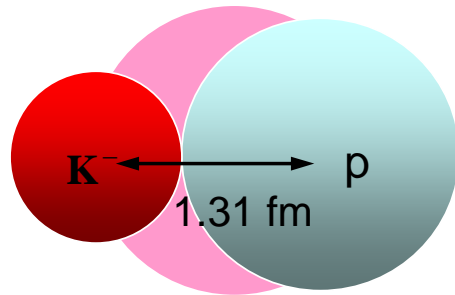
- Coherent  $\Lambda$ - $\Sigma$  coupling
- $\Sigma$ -nuclear Lane term
- $\alpha$  formation



# Yamazaki's diagram



$\Lambda(1405)$



$p p K^-$

