Sep. 17. 2003
@ RIKEN

QCD and Chiral Symmetry

Lecture I : What is QCD ?

Lecture II : Chiral effective field theory

(+ some application if time allows)

1. What is QCD?

- quarks & gluons

q^{α} ← color $\underline{3}$ (r, b, g); q ← flavor $\begin{pmatrix}u\\d\end{pmatrix}\begin{pmatrix}c\\s\end{pmatrix}\begin{pmatrix}t\\b\end{pmatrix}$

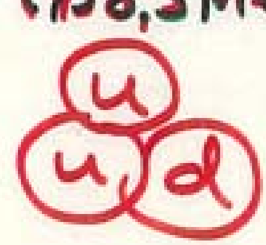
A_{μ}^{a} ← color $\underline{8}$; μ ← Lorentz

$SU_c(3)$ → color

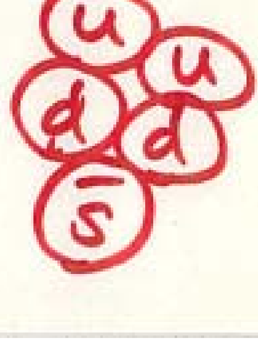
	color	charge	mass
q	$\underline{3}$	$\begin{pmatrix}2/3\\-1/3\end{pmatrix}e$	$m_{u,d} \sim 10\,\text{MeV}$ $m_s \sim 100\,\text{MeV}$ $m_c \sim 1.3\,\text{GeV}$ $m_b \sim 5\,\text{GeV}$ $m_t \sim 175\,\text{GeV}$
A_μ	$\underline{8}$	0	0

π^+ (139.6 MeV)

P (938.3 MeV)

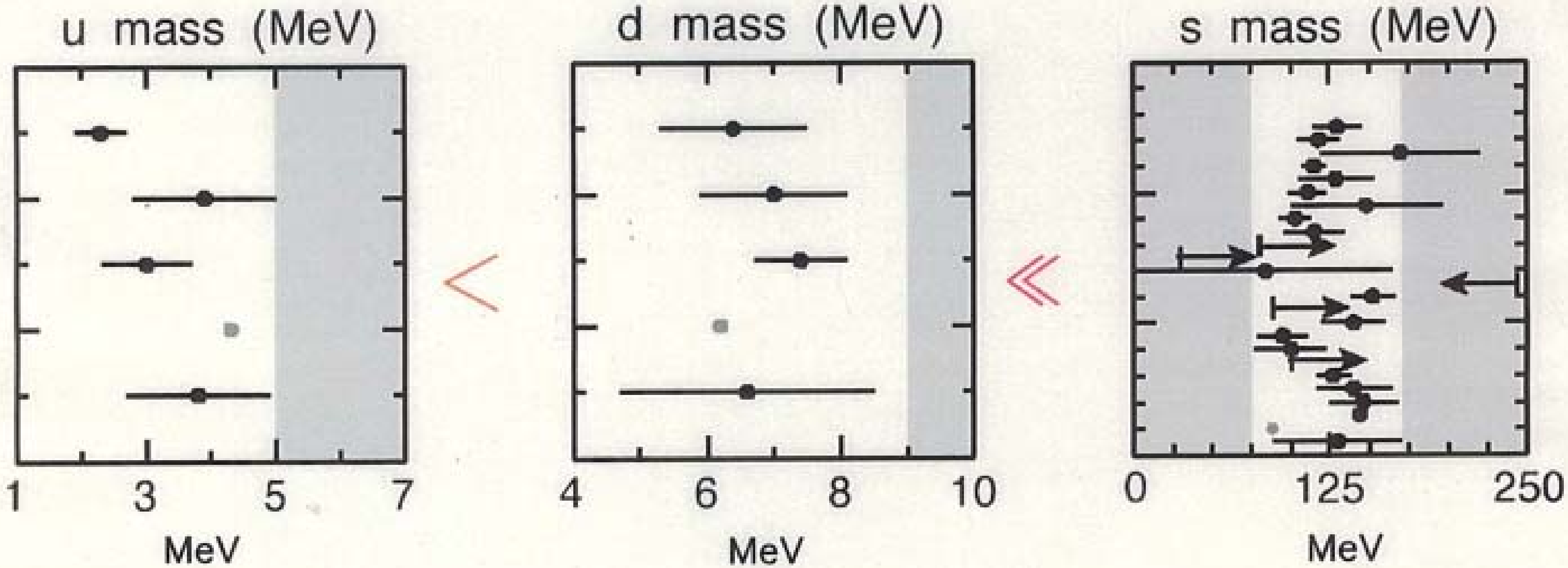


Θ^+ (1530 MeV)



← spring8 Nakano et al PRL ('03) July

Quark Masses & SU(3) breaking



- n (ddu), p(uud) ~ 940 MeV : not from $m_{u,d}$
- n (ddu) − p (uud) ~ 1.5 MeV : due to $m_d > m_u$
- our world is mostly made of n, p, π : because $m_s >> m_{u,d}$

QCD Lagrangian (Nambu '67)

$$\mathcal{L} = \bar{q}(i\gamma_\mu D^\mu - m)q - \frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu}$$

– Dirac – – Yang-Mills –

$$\begin{cases} D_\mu = \partial_\mu + ig\,t^a A^a_\mu \equiv \partial_\mu + igA_\mu \\ F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \end{cases}$$

– local gauge symmetry –

$$\begin{cases} q \to Vq \quad (\bar{q} \to \bar{q}V^\dagger) \\ D_\mu \to VD_\mu V^\dagger \quad (gA_\mu \to V(gA_\mu - i\partial_\mu)V^\dagger) \\ F_{\mu\nu} \to VF_{\mu\nu}V^\dagger \end{cases}$$

covariant!

* $F_{\mu\nu} \equiv t^a F^a_{\mu\nu}$
  $A_\mu \equiv t^a A^a_\mu$

SU(3) generators ($a = 1 \sim 8$)

$$t^a = \frac{\lambda^a}{2}$$

$$[t^a, t^b] = i f_{abc} t^c, \quad tr(t^a t^b) = \frac{1}{2}\delta^{ab}$$

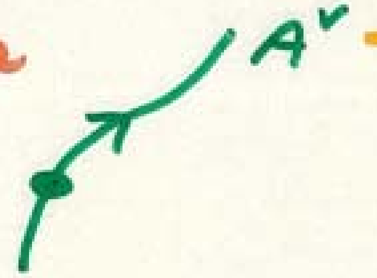
- quantizing QCD

- canonical quantization (Kugo-Ojima)
- path integral quantization (Faddeev-Popov)
- .....

$$Z = \langle 0_+ | 0_- \rangle = \int [dA\, d\bar{q}\, dq]\, e^{i\int d^4x\, \mathcal{L}(A,q)}$$

$$= \left(\int [dV]\right) \int [dA\, d\bar{q}\, dq\, d\bar{c}\, dc]\, e^{i\int d^4x\, \mathcal{L}_{gf}(A,q,c)}$$

gauge volume



$$\left(\mathcal{L}_{gf} = \bar{q}(i\gamma\cdot D - m)q - \frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} - \bar{c}\,\partial_\mu D^\mu c - \frac{1}{2\xi}(\partial_\mu A^\mu_a)^2 \right)$$

Basic parameters

$$g,\ m,\ \xi$$

$$Z[g_0, m_0, \xi_0] = Z[g(\kappa), m(\kappa), \xi(\kappa); \kappa]$$

Renormalizating group invariance

perturbation theory

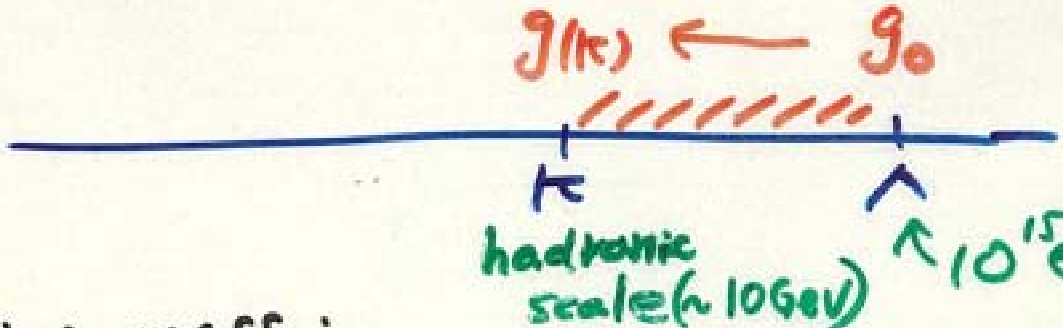
$$\mathcal{L}_{gf} = \mathcal{L}^0 + \mathcal{L}_{int}$$

free part

$O(g)$ $O(g)$ $O(g^2)$ $O(g)$

- effective (running) coupling:

$$g(\kappa) = g_0 + g_0^3\left(\text{const} + \log\left(\frac{\Lambda}{\kappa}\right)\right) + \cdots$$



- running mass:

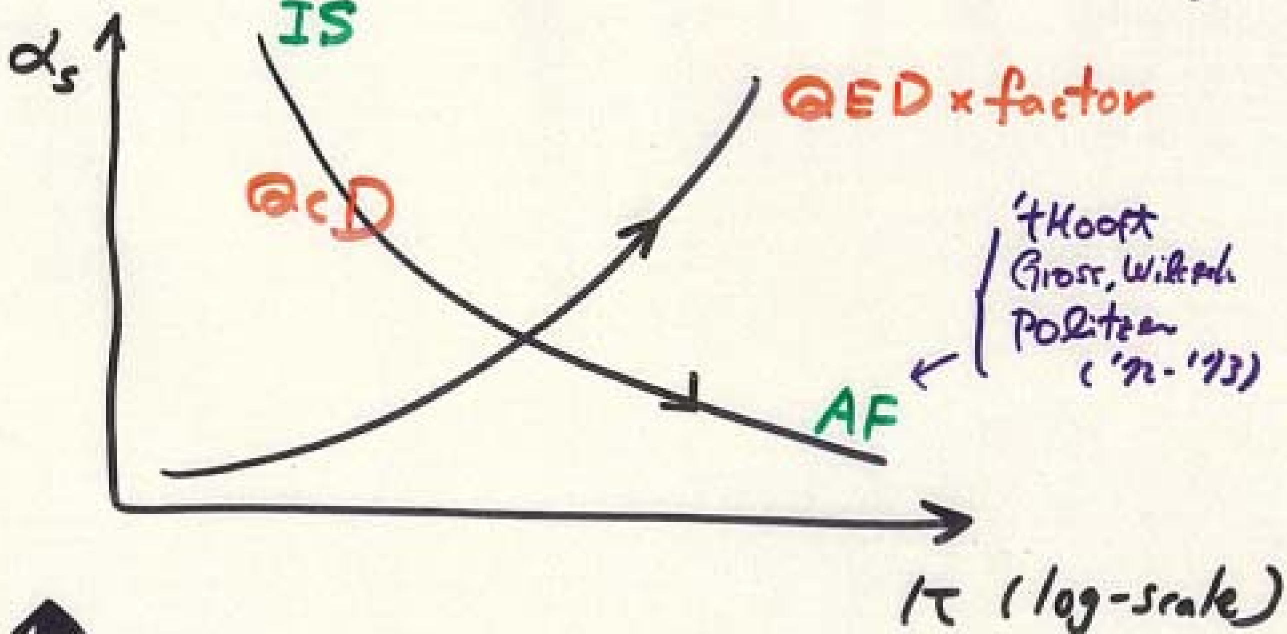
$$m(\kappa) = m_0\left(1 + \underset{g_0^2}{\hat{}}\log\left(\frac{\Lambda}{\kappa}\right) + \cdots\right)$$

exp. $\begin{cases} g(\kappa = 10\text{GeV}) = \cdots \\ m(\kappa = 10\text{GeV}) = \cdots \end{cases}$

$\Rightarrow$ others are predictions

Asymptotic freedom

$$\left(\alpha_s(k) \equiv \frac{g^2(k)}{4\pi}\right)$$

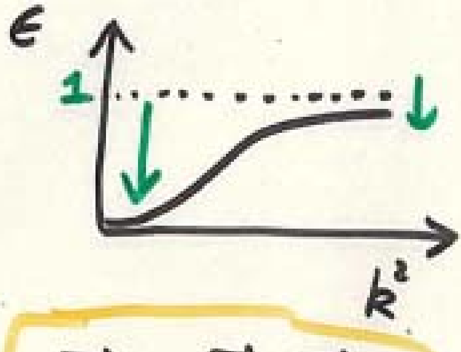


⇕

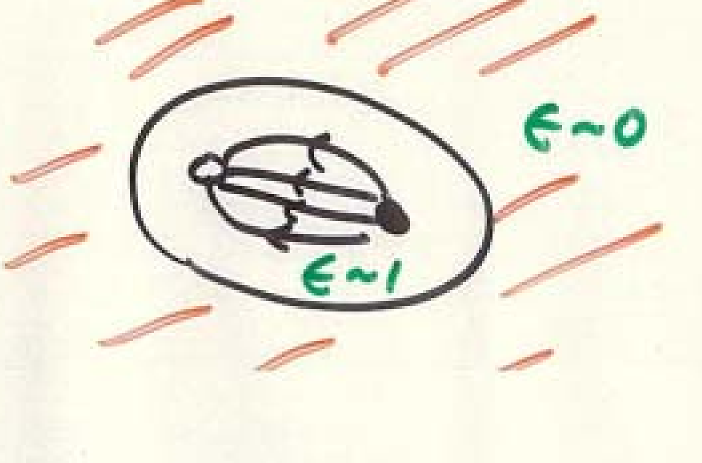
anti-screening of color charge
(screening) (electric charge)

⇕

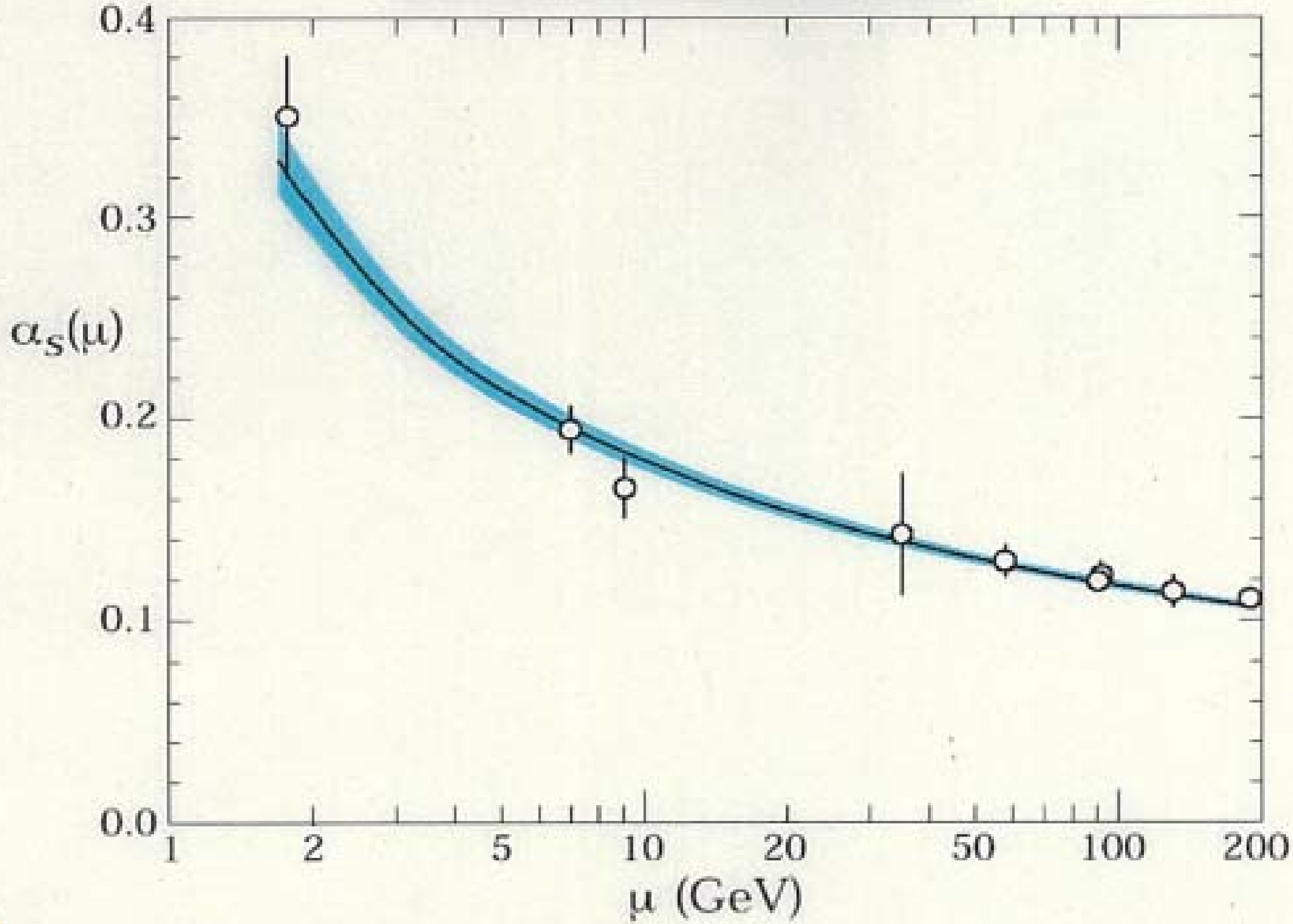
dielectric constant $\epsilon_{QCD} < 1$



$$\vec{D} = \vec{E} + \vec{P} = \epsilon \vec{E}$$



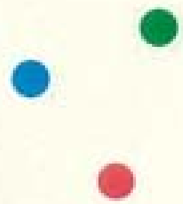
QCD running coupling

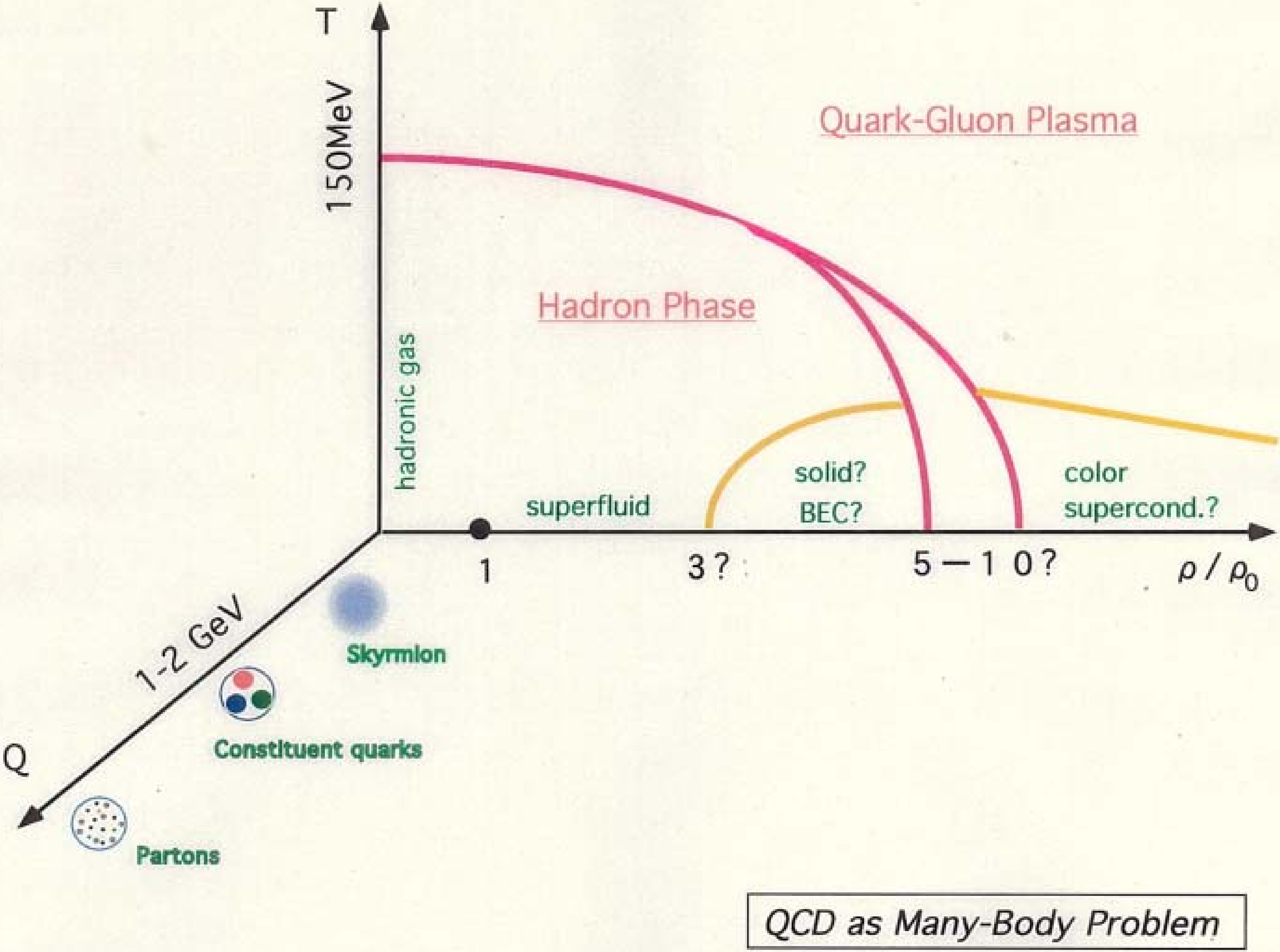


10^{-13}cm

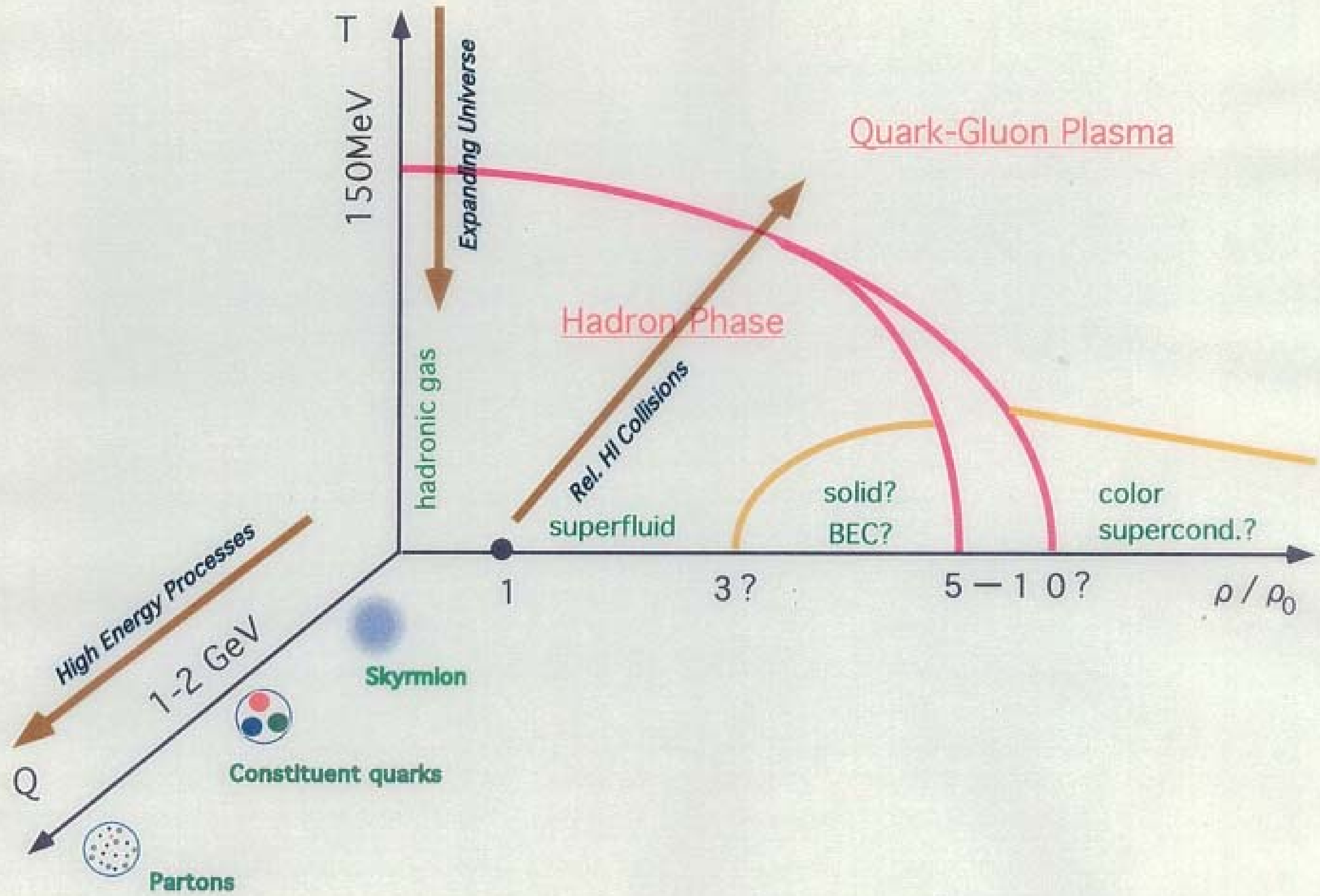
⇩ confinement of quarks & gluons at Λ_{QCD}~200 MeV

⇩ asymptotic free quarks & gluons





T
150MeV
Quark-Gluon Plasma
Hadron Phase
hadronic gas
superfluid
solid?
BEC?
color
supercond.?
1
3 ?
5 — 1 0 ?
ρ / ρ0
1-2 GeV
Q
Skyrmion
Constituent quarks
Partons
QCD as Many-Body Problem



QCD as Many-Body Problem

Basic symmetry in QCD

1. local color gauge symmetry $SU_c(3)$
   - unbroken at $T=\mu=0$ — Vafa-Witten [redacted] theorem
   - can be broken at $\mu\neq0$ — Color superconductivity

2. Dilatational / conformal symmetry
   - always broken for $g\neq0$

3. Chiral symmetry (global)
   - dynamically broken at $T=\mu=0$

   $SU_L(N_f)\times SU_R(N_f)\rightarrow SU_V(N_f)$

⇩

what is chiral symmetry ?

Chiral Symmetry in QCD (1)

$$q = q_L + q_R \quad \Bigg| \quad \begin{array}{l} q_L = \frac{1}{2}(1-\gamma_5)\, q \\ q_R = \frac{1}{2}(1+\gamma_5)\, q \end{array}$$

$\Downarrow$

- Interaction:

$$\bar{q}\gamma_\mu A^\mu q = \bar{q}_L \gamma_\mu A^\mu q_L + \bar{q}_R \gamma_\mu A^\mu q_R$$

$\Leftarrow$ chirality NON-flip

- mass term:

$$\bar{q}q = \bar{q}_L q_R + \bar{q}_R q_L$$

$\Leftarrow$ chirality flip

$$\mathcal{L}_{QCD} = \mathcal{L}_{m=0}(q_L, A) + \mathcal{L}_{m=0}(q_R, A) + m\,\bar{q}_L q_R + m\,\bar{q}_R q_L$$

IF $m \ll \Lambda_{QCD}$,

THEN <u>L</u> and <u>R</u> do <u>not</u> talk with each other directly!

Chiral Symmetry in QCD (2)

$m_{u,d} \sim 10\,\mathrm{MeV} \ll \Lambda_{QCD} \sim 200\,\mathrm{MeV}$

$m_s \sim 100\,\mathrm{MeV} \lesssim \Lambda_{QCD} \sim 200\,\mathrm{MeV}$

— $m_{u,d,s} = 0$ world —

$\mathcal{L}_{QCD}$ is invariant under

$$q_L \to e^{-i\lambda^i\theta_L^i} q_L, \quad q_R \to e^{-i\lambda^i\theta_R^i} q_R$$

independent rotation!

global

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix}_{L,R} \xrightarrow{\text{global}} V_{L,R} \begin{pmatrix} u \\ d \\ s \end{pmatrix}_{L,R}$$

$$\begin{pmatrix} V_L \in SU_L(3) \\ V_R \in SU_R(3) \end{pmatrix}$$

Very good symmetry for u, d
O(20%) breaking for s

$SU_L(3) \times SU_R(3)$

Conserved current

$$SU_L(3) \longrightarrow \partial^\mu(\bar{q}_L \gamma_\mu \lambda^i q_L) = 0$$

$$SU_R(3) \longrightarrow \partial^\mu(\bar{q}_R \gamma_\mu \lambda^i q_R) = 0$$

$V = L+R$
$A = L-R$ $\longrightarrow$

$$\partial^\mu(\bar{q} \gamma_\mu \lambda^i q) = 0$$

$$\partial^\mu(\bar{q} \gamma_\mu \gamma_5 \lambda^i q) = 0$$

$(i = 1 \sim 8)$

Vector current

$$V_\mu^i = \bar{q} \gamma_\mu \frac{\lambda^i}{2} q$$

axial current

$$A_\mu^i = \bar{q} \gamma_\mu \gamma_5 \frac{\lambda^i}{2} q$$

---

current algebra

$$[Q_V^i, V_\mu^j] = i f_{ijk} V_\mu^k$$

$$[Q_A^i, A_\mu^j] = i f_{ijk} V_\mu^k$$

$$[Q_V^i, A_\mu^j] = i f_{ijk} A_\mu^k$$

$$[Q_A^i, V_\mu^j] = -i f_{ijk} A_\mu^k$$

QCD vacuum

Non trivial condensations of quarks and gluons

- gluon condensate:

$$\left\langle \frac{\alpha_s}{\pi} F^a_{\mu\nu} F_a^{\mu\nu} \right\rangle_{vac} \simeq (300\,\text{MeV})^4$$

$\leftrightarrow$ confinement?

- quark condensate:

$$\langle \bar{u}u \rangle_{vac} \simeq \langle \bar{d}d \rangle_{vac} \simeq \langle \bar{s}s \rangle_{vac} \simeq -(250\,\text{MeV})^3$$

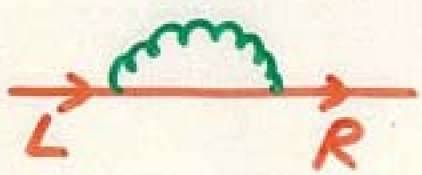
$\leftrightarrow$ dynamical breaking of chiral symmetry

$$SU_L(3) \times SU_R(3) \longrightarrow SU_V(3)$$

Dynamical Breaking of Chiral Symmetry

Consider $m=0$:

- perturbation theory



forbidden $m=0$

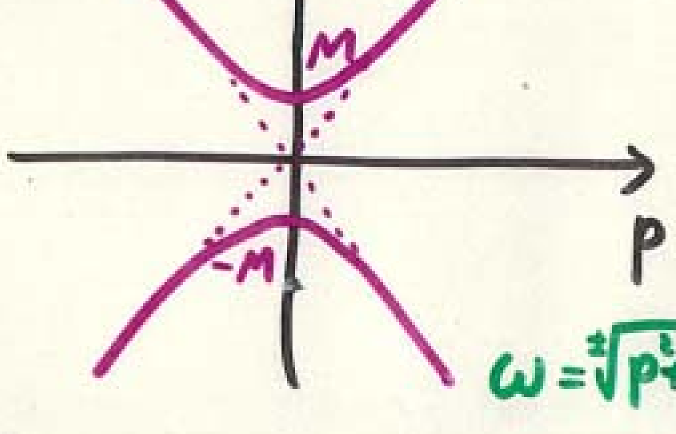
- non-pert. gluons

allowed

$M \sim 300\,\mathrm{MeV}$

„generation of an "effective" mass
or "constituent" mass

$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$



$\omega = \pm\sqrt{p^2+M^2}$

proton

$300 \times 3 = 900$ MeV!

Pion as Nambu-Goldstone (NG) boson

① Nambu-Goldstone theorem

$$\left.\begin{array}{l} m=0 \\ \langle\bar{q}q\rangle\neq 0 \end{array}\right] \longrightarrow m_\pi = 0$$

② Gell-Mann-Oakes-Renner (GOR) relation

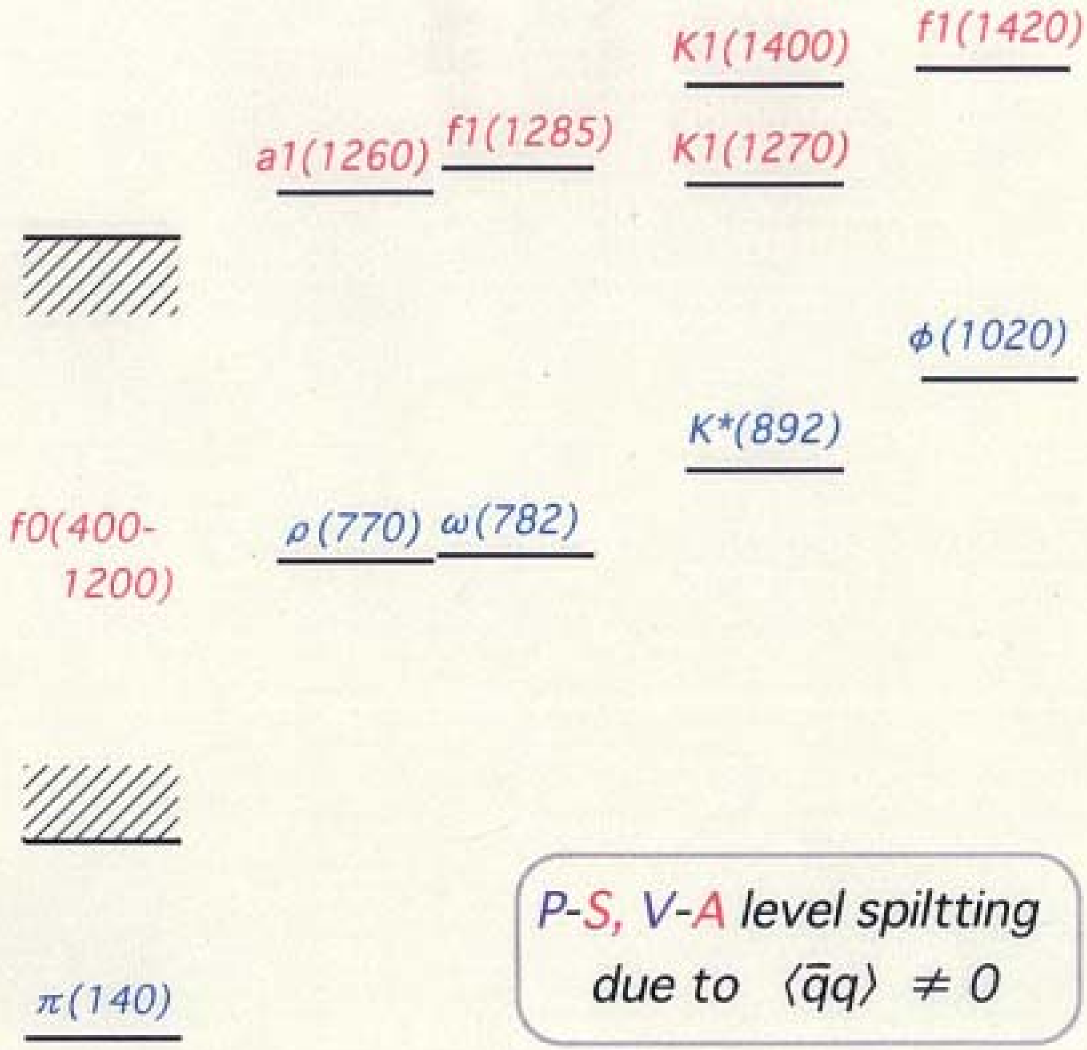
$$\left.\begin{array}{l} m\neq 0 \\ \langle\bar{q}q\rangle\neq 0 \end{array}\right] \to f_\pi^2 m_{\pi^\pm}^2 = -\frac{(m_u+m_d)}{2}\langle\bar{u}u+\bar{d}d\rangle_{vac}$$

i.e. $m_\pi^2 \sim O(m\Lambda_{QCD})$ small!

$(\leftrightarrow m_\rho^2 \sim O(\Lambda_{QCD}^2))$

an example of the low energy theorems obtained by $\partial_\mu A_\mu^a \simeq O(m)$

$q\bar{q}$ excitations



P-S, V-A level spiltting due to $\langle\bar{q}q\rangle \neq 0$

Chiral mass formulas

$$m_\pi^2 \approx \hat{m}\langle\bar{q}q\rangle_0 / f_\pi^2 \qquad m_\rho^2 \approx \left[\frac{448}{27}\pi^3\alpha_s\langle\bar{q}q\rangle_0^2\right]^{1/3}$$

$$m_{a1}^2 \approx \left[\frac{2816}{27}\pi^3\alpha_s\langle\bar{q}q\rangle_0^2\right]^{1/3}$$

⇧ small quark-mass

⇧ QCD sum rules in large N_C

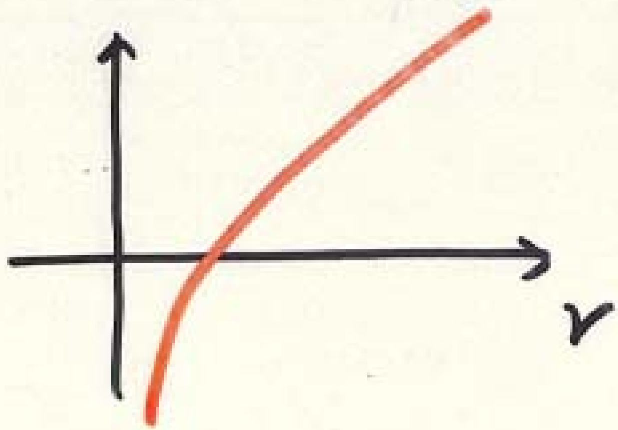
How to solve QCD at low energies ?

$Q \ll 1\,\mathrm{GeV}$: confinement
chiral symmetry breaking

① potential model (for heavy quarks)
charm, bottom

$$H = \sum_q m_q + \sum_q \frac{p^2}{2m_q} + V_q(r)$$

$$V_{q\bar{q}}(r) = -\frac{4}{3}\frac{\alpha_s}{r} + kr + \cdots$$



k: string tension

$\simeq (0.42\,\mathrm{GeV})^2 = 0.9\,\mathrm{GeV}\cdot\mathrm{fm}^{-1}$

② constituent quark model (for light quarks)

u, d, s

$$H = \sum_q M_q + \sum_q \frac{p^2}{2M_q} + V_q(r)$$

$M_q \sim 350\,\mathrm{MeV}$ (u, d), $\sim 500\,\mathrm{MeV}$ (s)

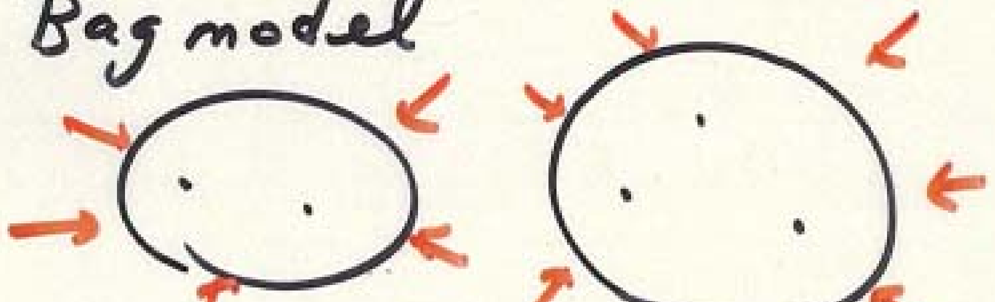
constituent masses

$\updownarrow$

$m_{u,d} \sim 10\,\mathrm{MeV}$, $m_s \sim 150\,\mathrm{MeV}$

current masses

③ Bag model



relativistic quarks inside a cavity

④ Nambu-Jona-Lasinio model

dynamical model to generate M_q

($\leftrightarrow$ BCS theory)

(5) Chiral perturbation theory

Lecture II (↔ IBM)

(6) QCD sum rules

operator product expansion + dispersion relation (↔ EW sum rules)

(7) Lattice QCD (↔ SMMC)

1st principle approach to solve QCD

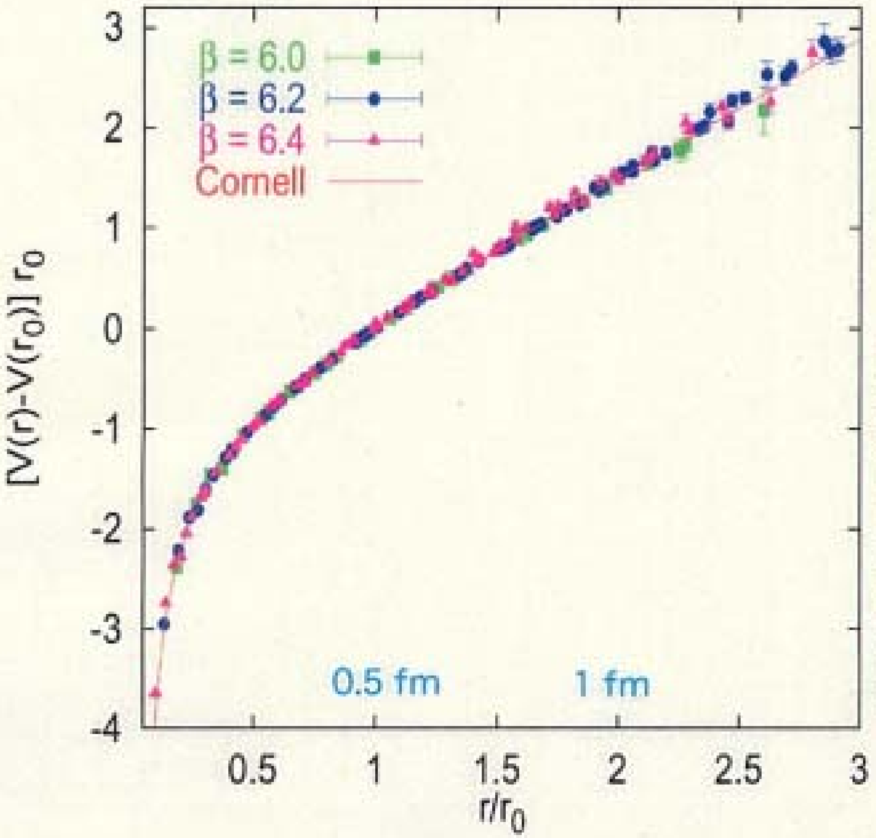
present status ← examples

- hadron spectra: 5–10% accuracy
- QCD phase structure (T, μ)
- hadron matrix elements

....

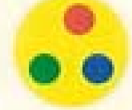
not for nucleus yet

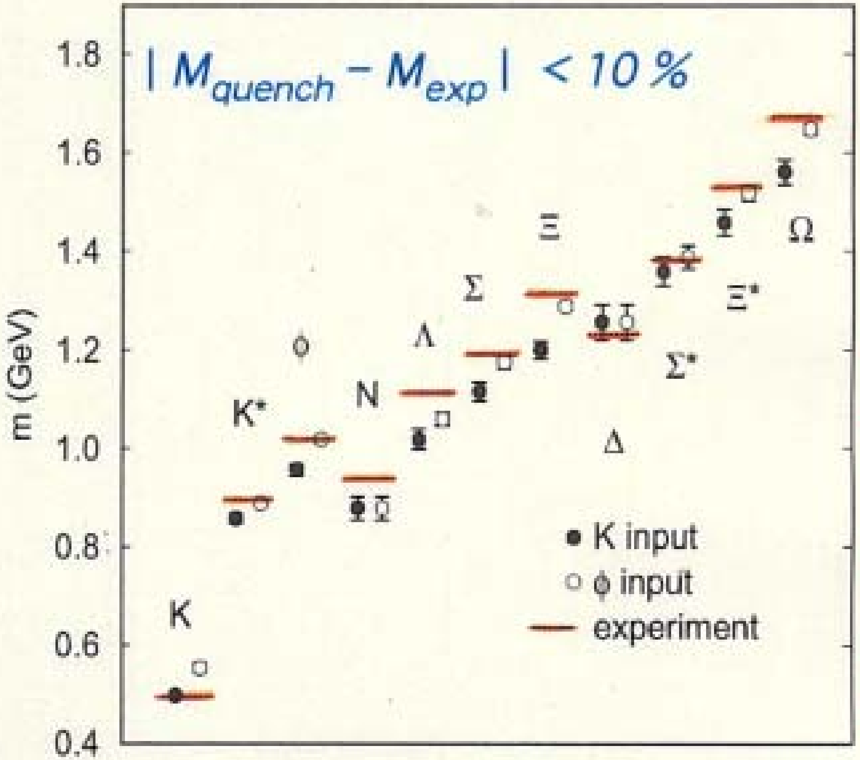
quenched heavy-quark potential



G.S. Bali, Phys. Rep. ('01)

quenched light-hadron spectrum





CP-PACS Coll., PRL 84 ('00)

Open problems

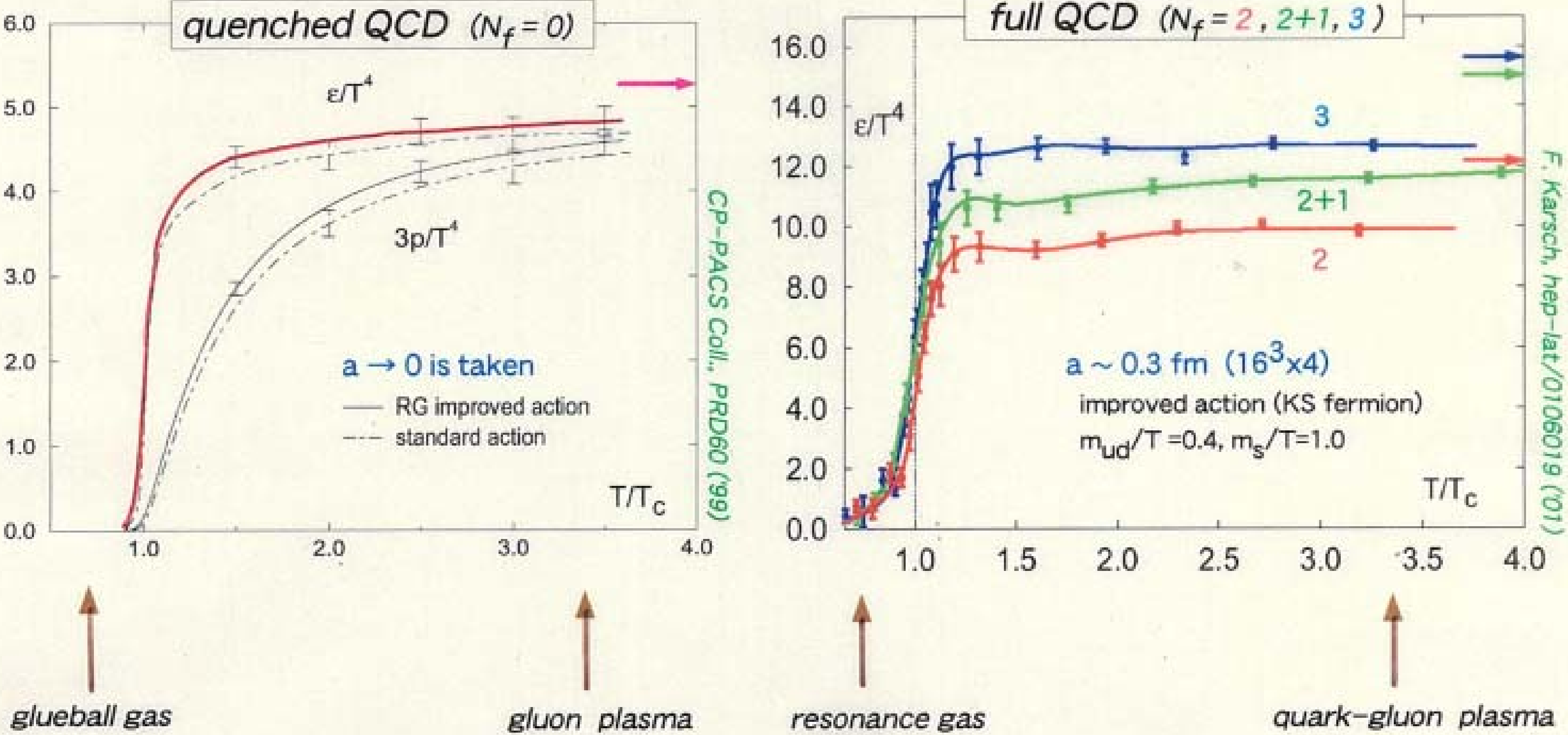
String breaking in full QCD ?

SESAM &T$_\chi$L, UKQCD, MILC Coll.

$|M_{full} - M_{exp}| < |M_{quench} - M_{exp}|$?

CP-PACS Coll., hep-lat/0105015 ('01).

Energy density on the Lattice

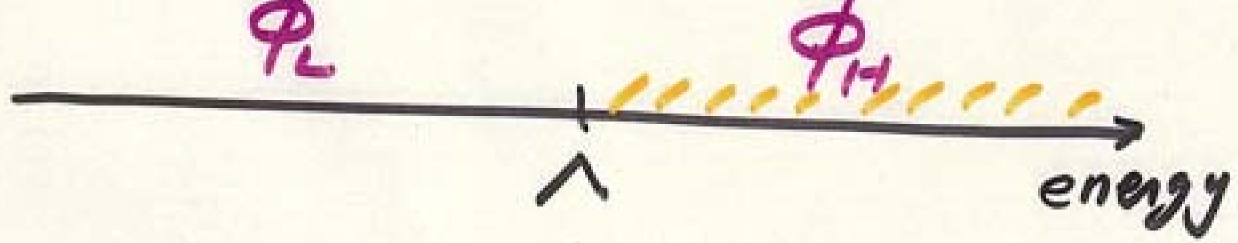


Are they phase transitions ?

What is the order parameter ?   Tc ?

2. What is effective field theory?
(EFT)

(Wilson)

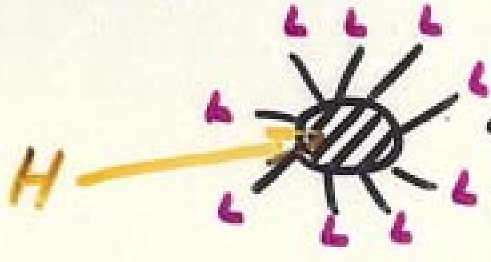


$$Z = \int [d\phi]\, e^{iS(\phi)} = \int [d\phi_L d\phi_H]\, e^{iS(\phi_L, \phi_H)}$$

$$= \int_{k<\Lambda} [d\phi_L]\, e^{i S_{eff}(\phi_L)}$$

infinite series

$$S_{eff}(\phi_L, g_4(\Lambda), g_6(\Lambda), \cdots\cdots)$$



$$\sim g_n(\Lambda) \frac{1}{\Lambda^n} \phi_L^{n+4}$$

Z is Λ-indep.

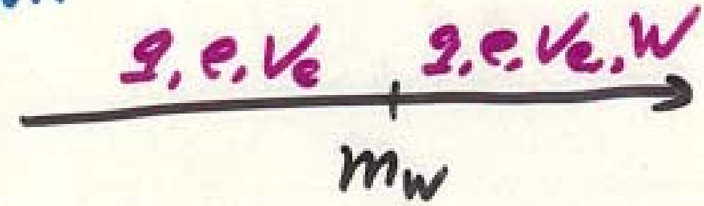
loops of $\phi_L \leftrightarrow g_n(\Lambda)$

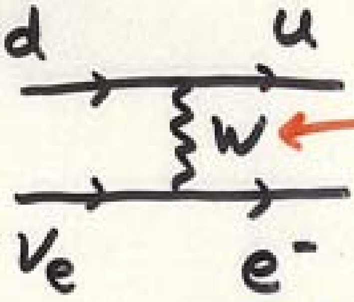
cancel

$$\frac{dZ}{d\Lambda} = 0$$ Renormalization group equation

– Some examples –

1) Fermi interaction





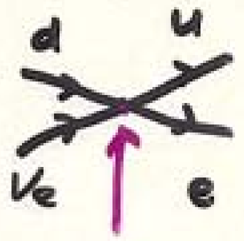
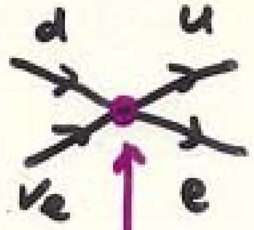
$\frac{1}{k^2+m_W^2}$

$$\simeq \frac{e_W^2}{m_W^2}\left[\bar{u}\gamma_\mu(1-\gamma_5)d\right]\left[\bar{e}\gamma_\mu(1-\gamma_5)\nu_e\right]$$

$$+ \frac{e_W^2}{m_W^2}\left[\ \text{''}\ \right]\frac{k^2}{m_W^2}\left[\ \text{''}\ \right]$$

$$+ \cdots .$$

$\frac{e_W^2}{m_W^2} = \frac{8G_F}{\sqrt{2}}$

$\sim G_F$ [ +  + ...]

1

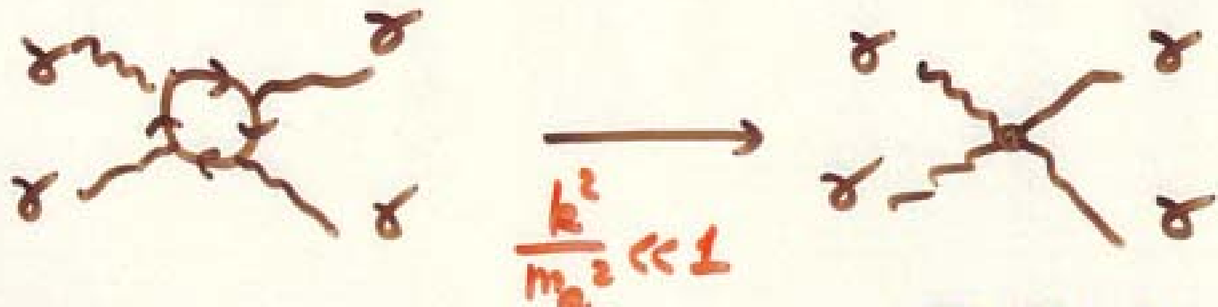
$\frac{\partial^2}{m_W^2} \sim \left(\frac{1\,\text{MeV}}{100\,\text{GeV}}\right)^2$

$\sim 10^{-10}$

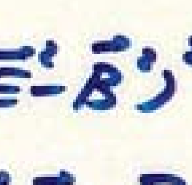
small !!

> 4-fermi int.
> is an EFT
> for $k \ll m_W$

② Heisenberg-Euler action

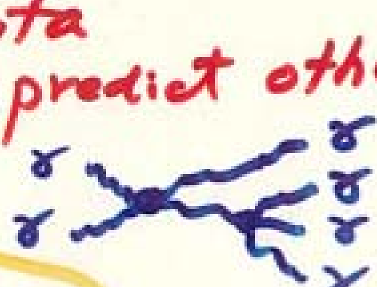


$$\mathcal{L}_{eff} = \frac{1}{2}(\vec{E}^2-\vec{B}^2) + \frac{e^4}{360\pi^2 m_e^4}\begin{pmatrix}(\vec{E}^2-\vec{B}^2)^2 \\ +7(\vec{E}\cdot\vec{B})^2\end{pmatrix} + \cdots$$

What if we do not know how to evaluate  ?

$$\mathcal{L}_{eff} = \frac{1}{2}(\vec{E}^2-\vec{B}^2) + \frac{c_1}{\Lambda^2}(\vec{E}^2-\vec{B}^2)^2 + \frac{c_2}{\Lambda^2}(\vec{E}\cdot\vec{B})^2 + \frac{c_3}{\Lambda^2}(\vec{E}\cdot\vec{B})\frac{\Box}{\Lambda^2}(\vec{E}\cdot\vec{B}) + \cdots$$

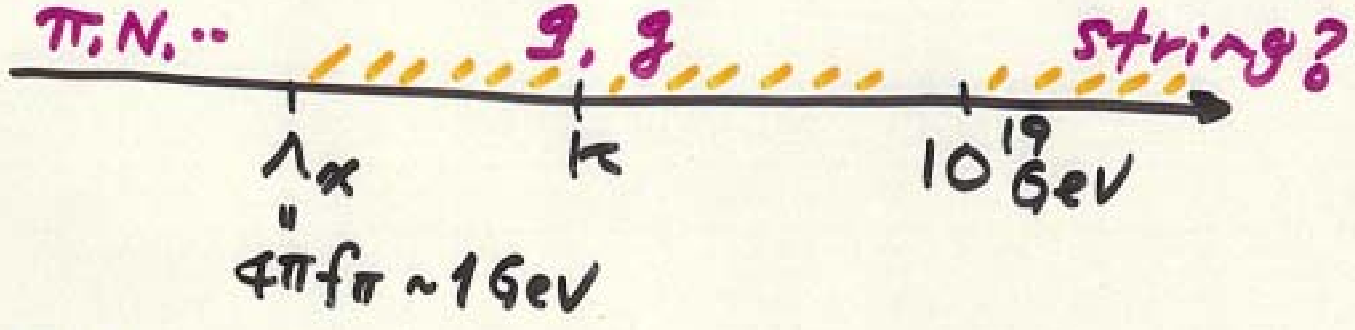
1. Write down **all** **possible** **terms** with correct Lorentz and CPT structure as an expansion of $\Box/\Lambda^2$
2. Observables are **Λ-indep.**
3. **Fit** a few exp. data → predict others



⇩

QCD? (Weinberg '79)

Effective field theory ($\neq$ model) in QCD



Model:

1. Choose appropriate fields (e.g. π, N)
2. Write down an action by guess

$$\mathcal{L}(\pi, N;\, g_1, g_2) \longrightarrow \text{fit data}$$

3. if failed → add more parameters or fields

(variational)

EFT:

1. Choose fields (e.g. π, N)
2. Write down an action with chiral symmetry and as an expansion of $\Box/\Lambda^2$

$$\mathcal{L}(\pi, N;\, g_1(\Lambda), g_2(\Lambda), \cdots g_n(\Lambda), \cdots) \longrightarrow \text{fit data}$$

$$\updownarrow$$

$$(\partial^2/\Lambda^2)^n$$

3. Systematic Improvent by increasing n

(Systematic pert.)

= chiral perturbation theory =

Chiral perturbation theory

(π, πN, NN ....)

- Systematic pert. scheme based on P/Λ-expansion
- Infinite # of couplings $g_n(\Lambda)$ ordered by $(P/\Lambda)^n$
- Physics is Λ-indep.

  loops $\leftrightarrow$ $g(\Lambda)$

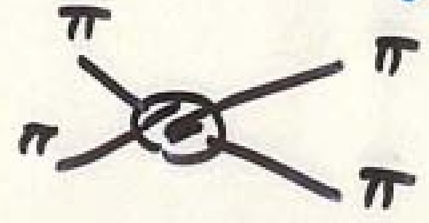
Λ-dep. Λ-dep. Λ-indep.

- Severe constraint from chiral symmetry

  ↓
  - relates various amplitudes
  - reduces the number of couplings

Importance of Chiral Symmetry –

consider π-π scattering



"wrong" model:

$$\mathcal{L}(\pi) = \frac{1}{2}(\partial\pi)^2 - \frac{\lambda}{4}\vec{\pi}^4$$

$T(s) \to \lambda$

wrong!

"chiral" model:

$$\mathcal{L}(\sigma,\vec{\pi}) = \frac{1}{2}(\partial\sigma)^2 + \frac{1}{2}(\partial\pi)^2 - \frac{\lambda}{4}(\sigma^2+\vec{\pi}^2-f_\pi^2)^2$$

$\Rightarrow T(s) = \lambda - \lambda + O(s/f_\pi^2)$

o.k.

chiral EFT:

$$\mathcal{L}(U) = \frac{f_\pi^2}{4}\underset{\mathrm{Tr}}{\wedge}(\partial U^\dagger \partial U) + \ldots$$

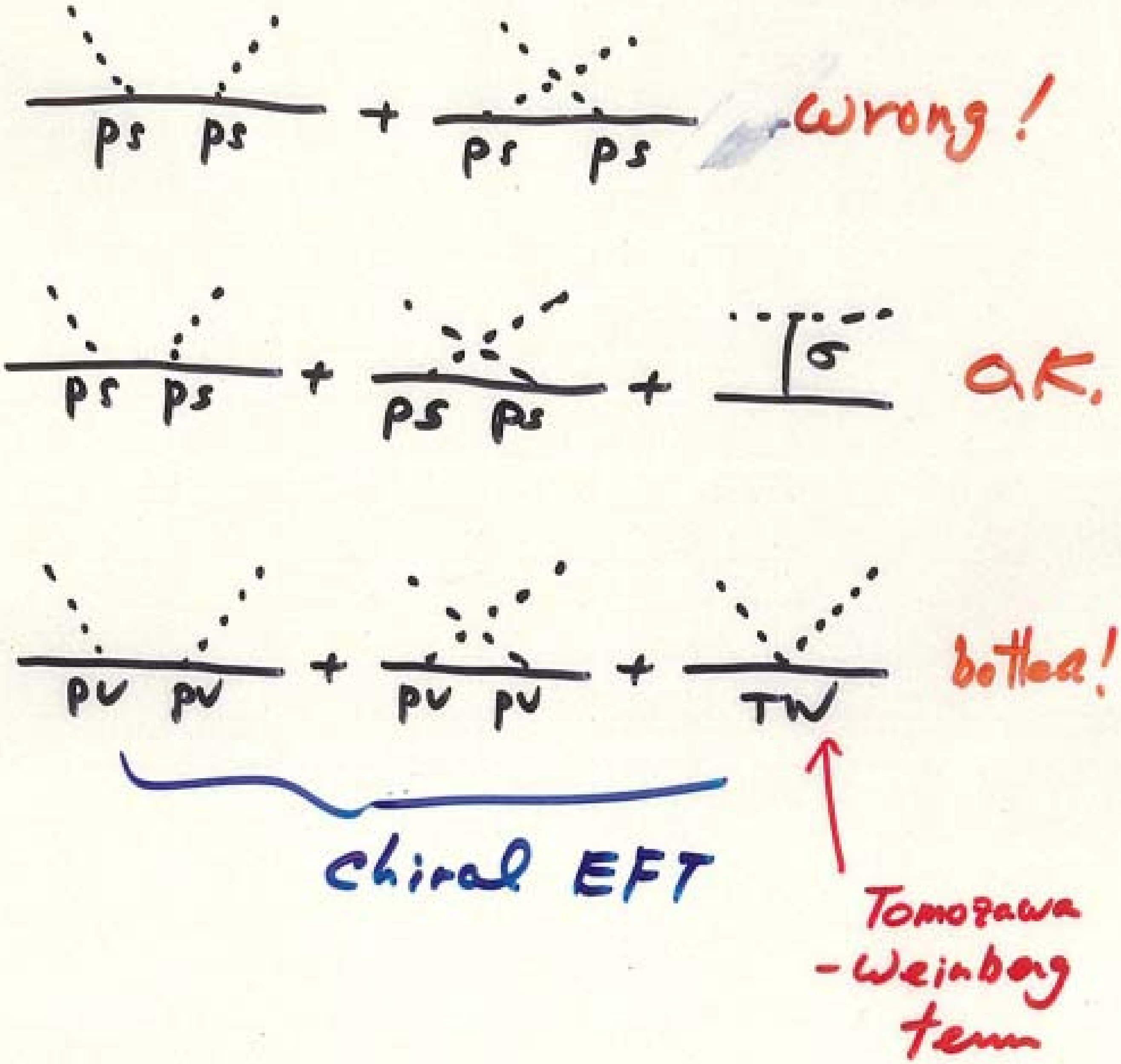
$$U = \exp(i\vec{\tau}\cdot\vec{\phi}/f_\pi) = 1 - i\vec{\tau}\cdot\vec{\phi}/f_\pi + \ldots$$

$\Rightarrow T(s) = \frac{s}{f_\pi^2} + \ldots$

better!

EFT (with π only) is an efficient way of realizing correct behavior without thinking about heavy resonances.

another example πN scattering



Chiral pert. theory for pions

(Gasser & Leutwyler '84)

$SU_L(3) \times SU_R(3)$

exp. by p^2/Λ_χ^2 , m_π^2/Λ_χ^2

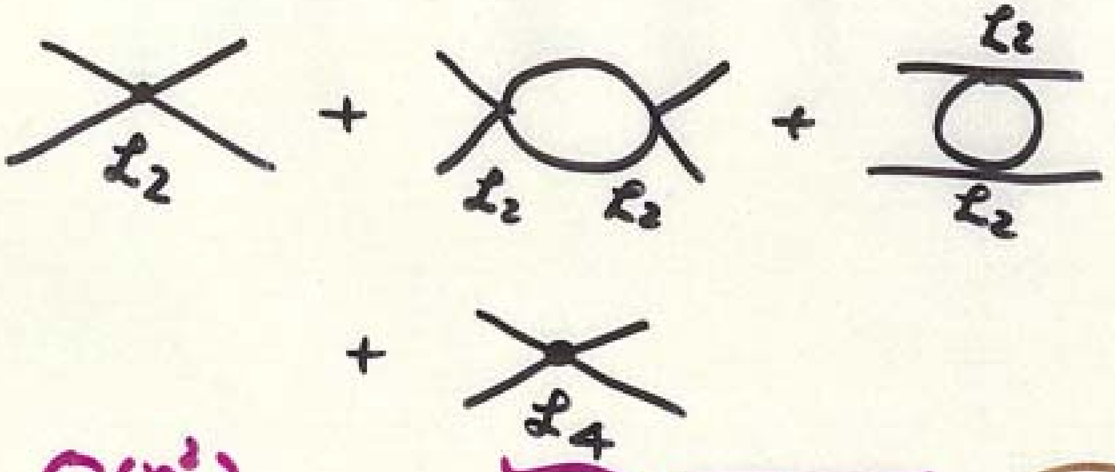
$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \cdots$$

$$\mathcal{L}_2 = \frac{1}{4} \underline{f_\pi}^2 \left[Tr(\partial_\mu U \partial_\mu U^\dagger) + \underline{\chi}^\dagger U + \underline{\chi} U^\dagger \right]$$

$$\begin{aligned}
\mathcal{L}_4 = & \ \underline{L_1} Tr(\partial_\mu U^\dagger \partial_\mu U)^2 \\
& + \underline{L_2} Tr(\partial_\mu U^\dagger \partial_\nu U) Tr(\partial_\mu U^\dagger \partial_\nu U) \\
& + \underline{L_3} Tr(\partial_\mu U^\dagger \partial_\mu U \partial_\nu U^\dagger \partial_\nu U) \\
& + \underline{L_4} Tr(\partial_\mu U^\dagger \partial_\mu U) Tr(\underline{\chi}^\dagger U + \underline{\chi} U^\dagger) \\
& + \underline{L_5} Tr[(\partial_\mu U^\dagger \partial_\mu U)(\underline{\chi}^\dagger U + \underline{\chi} U^\dagger)] \\
& + \underline{L_6} [Tr(\underline{\chi}^\dagger U + \underline{\chi} U^\dagger)]^2 \\
& + \underline{L_7} [Tr(\underline{\chi}^\dagger U - \underline{\chi} U^\dagger)]^2 \\
& + \underline{L_8} Tr(\underline{\chi}^\dagger U \underline{\chi}^\dagger U + \underline{\chi} U^\dagger \underline{\chi} U^\dagger)
\end{aligned}$$

$$U = \exp(i \lambda^a \phi^a / f_\pi)$$

Couplings $L_i^{(\Lambda)} = L_i^{finite\,(\Lambda)} + \log\Lambda$



$O(p^2)$ $O(p^4)$

$L_i^{finite} \times 10^3$	
1	0.4 ± 0.3
2	1.35 ± 0.3
3	-3.5 ± 1.1
4	-0.3 ± 0.5
5	1.4 ± 0.5
6	-0.2 ± 0.3
7	-0.4 ± 0.2
8	0.9 ± 0.3

$$\mathrm{Re}\, t^I_\ell = q^{2\ell}\left(a^I_\ell + q^2 b^I_\ell + \dots\right)$$

⇧

	$O(p^2)$	$O(p^4)$	Exp.
a_0^0	0.16	0.22 ± 0.005	0.26 ± 0.05
b_0^0	0.18	0.25 ± 0.02	0.25 ± 0.03
a_1^1	0.03	0.038 ± 0.003	0.038 ± 0.002
b_1^1	0	$(5 \pm 3) \times 10^{-3}$	–

expansion parameter

$$\frac{m_\pi^2}{(4\pi f_\pi)^2} \sim 0.015 \qquad \text{good!}$$

$$\frac{m_K^2}{(4\pi f_\pi)^2} \sim 0.18 \qquad \text{not too bad.}$$

$$\frac{m_\rho^2}{(4\pi f_\pi)^2} \sim 0.43 \qquad \text{not good}$$

> naive chiral pert. works
>
> $2M_\pi < \sqrt{s} < m_\rho$

↓

resonance regim

a. include ρ explicitly $\mathcal{L}(\pi, \rho)$

b. unitarization to generate ρ dynamically

General way to construct chiral Lagrangian

$$\mathcal{L}(\pi, N) = \mathcal{L}_{\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN}$$

Expansion parameter:

$$Q \sim \frac{p}{4\pi f_\pi}, \frac{m_\pi}{4\pi f_\pi}, \frac{p}{m_N} \leftrightarrow NR$$

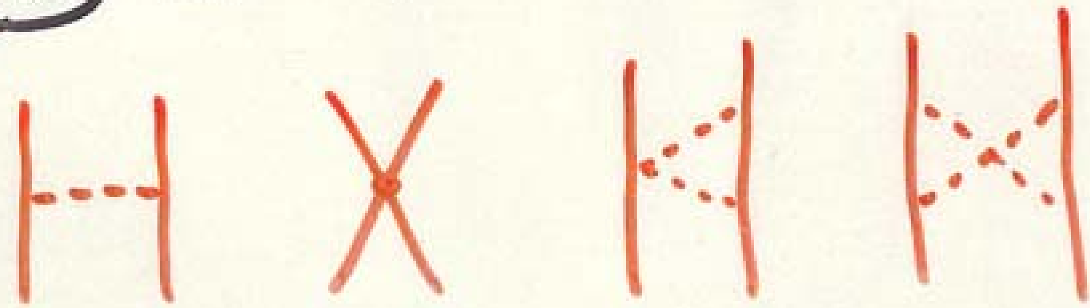
Weinberg's counting rule:

$$Q^\nu : \quad \nu = 2 - \frac{E_N}{2} + 2L + \sum_i V_i$$

$$V_i = d_i + \frac{1}{2} n_i - 2$$

- E_N : external nucleon lines
- L : number of loops
- d_i : number of ∂ or m_π at each vertex i
- n_i : number of nucleons attached to vertex i

(NN) $E_N = 4,\ n_i = 2, 4$



$2-2+0$	$2-2+0$	$2-2+1$	$2-2+1$
$+(1+1-2)$	$+(2-2)$	$+(1+1-2)$	$+4(1+1-2)$
$+(1+1-2)$		$+(1+1-2)$	$=1$
	$=0$	$+(1+1-2)$	
$=0$		$=1$	
Q^0	Q^0	Q^1	Q^1

LO (first two) — NLO (last two)

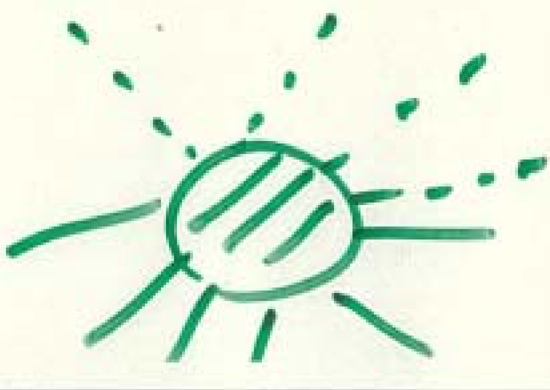
$$\mathcal{L} = \sum_\nu \mathcal{L}^{(\nu)}$$

→ calculate G up to ν'

renormalize (= Λ-indep.) G

↕

compare with exp.



Explicit form of $\mathcal{L}$ up to $\nu = 2$

(with 2 pion field)

$$\mathcal{L} = \frac{f_\pi^2}{4} \mathrm{Tr}\left(\partial U \partial U^+ + \chi U^+ + U \chi^+ \right)$$

Q^2

$$+ \bar{N} \Big[i\partial_0 - \frac{\nabla^2}{2m_N}$$

$$+ \frac{g_A}{2f_\pi} (\vec{\sigma}\,\tilde{\tau}\;\vec{\nabla}\tilde{\pi})$$

Q^0

$$+ \frac{1}{2f_\pi^2} \vec{\tau}\cdot(\vec{\pi} \times \partial_0 \vec{\pi}) \Big] N$$

Q^1

$$+ \sum_\Gamma C_\Gamma (\bar{N}\Gamma N)^2$$

Q^0

⇩

Do loops!

Summary

1. QCD : fundamental theory of strong interaction

   "2" parameters : α_s , m_q

2. QCD : pert. at high energy (AF)

   non pert. at low energy (IS)

$$\alpha_s \sim \frac{\#}{\ln(Q^2/\Lambda^2_{QCD})}$$

3. Confinement & chiral symmetry breaking

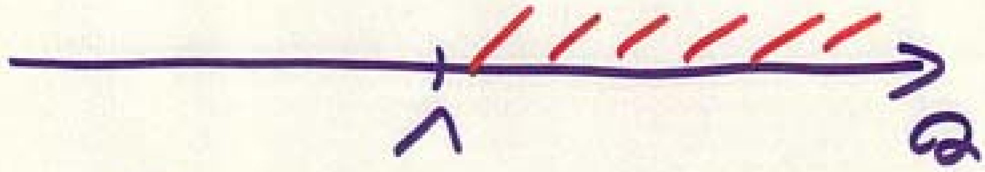
   Confinement ↓ origin of hadrons

   chiral symmetry breaking ↓ origin of hadron masses

4. Chiral symmetry :

   severe constraints on hadron interactions

5. Effective field theory



$$\frac{dZ}{d\Lambda} = 0 \quad \text{RG-inv.}$$

whole basis of
- modern field theory
- modern renormalization
- modern chiral theory

6. Chiral pert. theory

π, K, η → well developed

$\left.\begin{array}{l}\pi N \\ NN\end{array}\right\}$ → progresses going on

⇒ may give a systematic $\left\{\begin{array}{l}\text{approach} \\ \text{basis}\end{array}\right\}$ to Nuclear Physics

(do not forget lattice QCD though!)

Further reading

- QCD general:

  F.J. Ynduráin, Theory of quark and gluon interactions (Springer, 1993)

- Modern renormalization:

  G.P. Lepage, What is renormalization? TASI '89 Lecture (Boulder, Co, June, 1989)

- general effective field theory:

  H. Georgi, Effective field theory. Ann. Rev. Nucl. Part. Sci. **43** (1993) 209.

- Chiral pert. theory (foundation):

  S. Weinberg, Phenomenological Lagrangians Physica **96A** (1979) 327

- Nuclear applications:

  V. Bernard, N. Kaiser & U. Meissner, Chiral dynamics in nucleons & Nuclei Int. J. Mod. Phys. **E4** (1995) 193.