CNS Summer School, Sept. 20, 2003

Looking for Three-Body Forces by the Nucleon-Deuteron Elastic Scattering at Intermediate Energy

H. Sakai

Department of Physics / CNS, The University of Tokyo RIKEN



'How about the problem of the three bodies?' whispered Kriltsov, smiling with great effort.'The solution is difficult? '

Resurrection, 1899, Chapter XX by Leo Tolstoy

Three-Body Problems in Classic Mechanics

Gravitational motion (Newton eq.)

$$\mathbf{H} = \frac{1}{2} \left(\frac{p_1^2}{m_1} + \frac{p_2^2}{m_2} + \frac{p_3^2}{m_3} \right) - \frac{m_1 m_2}{r_{12}} - \frac{m_2 m_3}{r_{23}} - \frac{m_3 m_1}{r_{31}} \right)$$

□ H. Poincaré(1892)

non-existence of analytic solution

sometime chaotic behavior \rightarrow CHAOS

Three-Body Problems in Classic Mechanics

2 special solutions are known

straight line

3rd one found **3** years ago!

Chenciner-Montgomery, Ann. Math. 152(2000)881

eight (8) shape with equal mass



Three-body problem is general and important in science related with various fields.

Are there three-body forces?



Equations of motion must be solved exactly.
To learn 3BF, 2BF must be known precisely.

$$V (P_{1}, P_{2}, P_{3}) V (P_{1}, P_{2}, P_{3}) V (P_{1}, P_{2}, P_{3})$$

Are there three-nucleon forces(3NF)?

3NF is much smaller than 2NF.3NF effects are easily masked by 2NF effects.

• Equations of motion must be solved exactly.

Faddeev eq. 2NF must be reliable.

However, numerical calc. are extremely difficult.

Three-Body Problems in Quantum Mechanics

• L.D. Faddeev, Sov. Phys. JETP 12(1961)1014 Faddeev Equation

$$H = H_{0} + V_{12}^{NN} + V_{23}^{NN} + V_{31}^{NN} + V_{123}$$

$$\Psi = \Psi_{12}^{(\pm)} + \Psi_{23}^{(\pm)} + \Psi_{31}^{(\pm)}$$

$$\begin{pmatrix} \psi_{12}^{(\pm)} \\ \psi_{23}^{(\pm)} \\ \psi_{31}^{(\pm)} \end{pmatrix} = \begin{pmatrix} 0 \\ \psi_{\alpha} \\ 0 \end{pmatrix} + G_{0} \begin{pmatrix} 0 & t_{12} & t_{12} \\ t_{23} & 0 & t_{23} \\ t_{31} & t_{31} & 0 \end{pmatrix} \begin{pmatrix} \psi_{12}^{(\pm)} \\ \psi_{23}^{(\pm)} \\ \psi_{31}^{(\pm)} \end{pmatrix}$$

can be solved exactly!

CISS02 Lecture Note by Y. Koike: http://www.cns.s.utokyo.ac.jp//summerchool/ciss02/lecturenotes





Characteristics



Independent of reaction models
 direct comparison possible between data and inputs

Two nucleon force (2NF)

Φ One π Exchange (OPE) model by Hideki Yukawa

(Proc. Phys. Math. Soc. Jpn 17(1935) 48.)

 $m_{\pi}c^2 = 140 \text{ MeV}$ (J^{π}, T)=(0⁻, 1)

$$V_{(1,2)}^{\text{OPE}} = \frac{1}{3} \frac{f_{\pi}^{2}}{\eta c} m_{\pi} c^{2} (t_{1}^{\rho} t_{2}^{\rho}) \left\{ \sigma_{1}^{\rho} \sigma_{2}^{\rho} + \left(1 + \frac{3}{x} + \frac{3}{x^{2}}\right) S_{12} \right\} \frac{\exp^{-x}}{x}$$

$$S_{12} = \frac{3(\sigma_{1}^{\rho} t_{1}^{\rho})(\sigma_{2}^{\rho} t_{1}^{\rho})}{r^{2}} - (\sigma_{1}^{\rho} \sigma_{2}^{\rho})$$

$$x = \frac{m_{\pi} c}{\eta} r$$

Realistic modern 2 nucleon forces available now.

reproduces more than 3,500 exp. NN data with $x^2 \cong 1$.

CD Bonn pot.	: strong non-locality

AV18 pot. : OPE + phenom.

□ Nijmegen I/II/93 pot. : one boson exch.

Main differences are of-shell properties

Three nucleon force (3NF)

Fujita • Miyazawa type 3NF

(Prog. Theor. Phys. 17(1957)360.)



 2π exchange type 3NF

N: proton / neutron $m_N c^2 = 940 \text{ MeV}$ $(J^{\pi}, T) = (\frac{1}{2}^+, \frac{1}{2})$ Δ : excited state of nucleon

$$\mathbf{m}_{\Delta}\mathbf{c}^{2} = \mathbf{1232}\,\mathbf{MeV}$$
$$\left(\mathbf{J}^{\pi},\mathbf{T}\right) = \left(\frac{\mathbf{3}^{+}}{\mathbf{2}},\frac{\mathbf{3}}{\mathbf{2}}\right)$$

♦<u>other type of 3NF</u>



• 3NF based on 2π exchange model



TM-3NF

S.A.Coon *et al.*, Nucl. Phys. A317(1979)242. current algebra

UR-3NF

J.Carlson *et al.*, Nucl. Phys. A401(1983)59. FM+phenomenological SR term BR-3NF

H.T. Coelho *et al.*, Phys. Rev. C28(1983)1812.
chiral Lagrangian + current algebra
Texas-3NF

U.van. Kolck *et al.*, Phys. Rev. C49(1994)2932. chiral perturbation theory

Three-Nucleon Force

$$V^{(3)} = \frac{1}{(2\pi)^{6}} \frac{g_{\pi NN}^{2}}{4m^{2}} \frac{F_{\pi NN}^{2}(q^{2})}{(q^{2} + m_{\pi}^{2})} \frac{F_{\pi NN}^{2}(q'^{2})}{(q'^{2} + m_{\pi}^{2})} \vec{\sigma}_{1} \cdot \vec{q} \vec{\sigma}_{2} \cdot \vec{q}' \left[\phi^{\alpha\beta} \tau_{\alpha} \tau_{\beta} \right]$$

$$\phi^{\alpha\beta} = \delta^{\alpha\beta} \left[a + b\vec{q}\vec{q}' + c(q^{2} + {q'}^{2}) \right] - d(\tau_{3}^{\gamma} \varepsilon^{\alpha\beta\gamma} \vec{\sigma}_{3} \cdot \vec{q} \times \vec{q}')$$

$$F_{\pi NN}^{2}(q^{2}) = \frac{\Lambda^{2} - m_{\pi}^{2}}{\Lambda^{2} + q^{2}} \qquad \pi NN \quad \text{form factor}$$
$$\underline{\Lambda : \text{ cut - off parameter}}$$

3NF model	a	b	c	d
FM	0.0	-1.15	0.0	-0.29
ТМ	1.13	-2.62	1.05	-0.60
Urbana IX	0.0	-1.20	0.0	-0.30
Brazil	1.05	-2.29	1.05	-0.77
Texas	1.87	-3.82	0.0	-1.12
Ruhr	0.51	-1.82	0.0	-0.48
TM'	-0.87	-2.62	0.0	-0.60







Faddeev type calculations

- 1 Bochum-Cracow-KIT group calculation
 - H. Witała, H. Kamada, W. Glöckle,
 - E. Epelbaum
- Input data

NN	3NF
CD Bonn	Tucson-Melbourne
AV18	Urbana IX
Nijmegen	

<u>2 Hannover group calculation</u> **P.U. Sauer, D.Deltuva Parall. Session Th-3(finisshed) Input data**

NN	3NF	
CD Bonn	Δ – isobar	

N and Δ on an equal footing! dispersive two-body effects exist.

Both calc., *nd* scattering assumed. No Coulomb force effects included.

First Evidence of 3NF Effects

• B.E. of ³H: 8.48 MeV

Faddeev calculations

Bochum-Cracow-KIT group calc.

NN pot.	NN only	NN+3NF(TM)	Λ
CD Bonn	8.00	8.483	4.86
AV18	7.65	8.479	5.22
Nijm93	7.66	8.480	5.10
Ruhr	7.64	8.459	5.31

Hannover group C.C. calc.

8.00

CDBonn

NN force only calc.
is underboud by 0.51.0 MeV.
3NF fills this gap.(but with Λ) put constraint on overall strength of 3NF.

■to study dynamical properties of 3NF

scattering

No adjustable parameter in Faddeev calc.

8.29 (CDBonn +



FIG. 1. (Color) Energies of ground or low-lying excited states of light nuclei computed with the AV18 and AV18/UIX interactions, compared to experiment. The light shading shows the Monte Carlo statistical errors. The dashed lines indicate the thresholds against breakup for each model or experiment.

To Where should we look for 3NF effects?

Nd elastic scattering is very attractive since it is simple and offers a rich set of spin observables.

•Low energy *pd* scattering



polarization degree of spin polarization

Vector polarization(rank 1 tensor)

 $P_{Z} \equiv \frac{1}{s} \sum_{m=-s}^{+s} mP(m) \qquad P(m): \text{ population of } m, \quad \sum_{m=-s}^{+s} P(m) = 1$ electron, proton(spin $\frac{1}{2}$): $P_{Z} = P(+\frac{1}{2}) - P(-\frac{1}{2})$

Tensor polarization (rank 2 tensor)

$$P_{ZZ} = \frac{1}{s(2s-1)} \left[3 \sum_{m=-s}^{+s} m^2 P(m) - s(s+1) \right]$$
$$P_{ZZ} = P(+1) - 2P(0) + P(-1)$$
$$= 1 - 3P(0) \qquad \Lambda \text{ value} \left[-2, +1 \right]$$



 $P(+1) = 0, P(0) = 1, P(-1) = 0 \implies P_Z = P(+1) - P(-1) = 0$ different from no polarization $P(+1) = P(0) = P(-1) = \frac{1}{3}$

$$\Rightarrow P_{ZZ} = -2$$





Polarization in reaction

observables in this talk

- 1. Vector polarization P^y
- 2. Vector analyzing power A_y (iT₁₁)
- 3. Tensor analyzing powerss A_{xx} , A_{yy} , A_{xz} (T_{20} , T_{21} , T_{22})

Polarization in reaction

$$\begin{array}{rcl}
\stackrel{V}{p} & : & \sigma = \sigma_0 (1 + p_y A_y) \\
\stackrel{V}{d} & : & \sigma = \sigma_0 \left[1 + \frac{3}{2} p_y A_y^d + \frac{2}{3} p_{xz} A_{xz} + \frac{1}{3} (p_{xx} A_{xx} + p_{yy} A_{yy} + p_{zz} A_{zz}) \right] \\
\quad & iT_{11} = -\frac{\sqrt{3}}{2} A_y & -1 \le A_y \le 1 \\
\quad & T_{20} = \frac{-1}{\sqrt{2}} (A_{xx} + A_{yy}) & -2 \le A_{xx}(yy) \le 1 \\
\quad & T_{21} = \frac{-1}{\sqrt{3}} A_{xz} & A_{xx} + A_{yy} + A_{zz} = 0 \\
\quad & T_{22} = \frac{1}{2\sqrt{2}} (A_{xx} - A_{yy}) & \overline{k_f} \\
\end{array}$$
Sensitive to spin-orbit interactions

 $\overline{\mathbf{k}}_i$

•Low energy *pd* scattering



To Where should we look for 3NF effects?

N*d* elastic scattering is very attractive since it is simple and offers a rich set of spin observables.

•Low energy Nd scattering

 A_y discrepancy due to 3NF?

Probably due to deficiency in ${}^{3}P_{J}$ phase shifts of NN.

•Prediction by H.Witała. PRL 81('98) 1183.

go to nd scatt. at intermediate energy.



Let's start with

135 MeV/u (E_d=270 MeV) data at RIKEN by K. Sekiguchi

d + p at 270 MeV (RIKEN)

By Kimiko Sekiguchi

d: various spin(polarization) obs. $d\sigma/d\Omega$, $P^{y'}(=-A_y^p)$, iT_{11} , T_{20} , T_{22} , T_{21}

K. Sekiguchi et al. Phys. Rev. C65 (2002) 034003.

Spin(polarization) transfer exp. by Sekiguchi $\overset{\downarrow}{d} + p \rightarrow \overset{\rho}{p} + d$ at 270 MeV (135 MeV/u) $K_{xx}^{y'}, K_{yy}^{y'}, K_{xz}^{y'}$ \longrightarrow K. Sekiguchi: to be published soon.

RIKEN Accelerator Research Facility (RARF)

Directions of polarization of *d* **beams are freely controlled!**

 Vector and tensor polarized deuteron beams are provided by the polarized ion source(PIS).
 The Spin axis is controlled by a Wien Filter prior to acceleration.
 Single-turn extraction is available both for the AVF and Ring cyclotrons.
 Beam polarizations : 60-80% of the theoretical maximum values at D-room polarimeter (A) and the Swinger polarimeter (B)



Method of spin rotation I

magnetic moment
$$\mu = s \frac{e\eta}{mc}(1+a)$$

cyclotron frequency in magnetic field B $\omega_c = \frac{eB}{m\gamma}$ angular rotation frequency in B $\omega_s = \frac{eB}{m\gamma}(\gamma a + 1)$

Larmor freq. + Thomas precession (rel. effects)

Dirac particle $(a = 0) \Rightarrow \omega_s = \omega_c$





Familiar at Tandem VdG accel...

single turn extraction needed at cyclotron

Thanks to RIKEN Accelerator Staff

• spin direction moniter needed after accel. With high efficiency.





SMART: <u>Swinger and Magnetic Analyzer with Rotator and Twister</u>

RIKEN data 270 MeV



Bochum calc. — NN









How they look like if the energy is halved ?

140 MeV(70 MeV/u) data at RIKEN



Chiral Effective Field Theory

Relation with QCD

Lagrangian L=...

Scale parameter Λ

expand in power of Q/ Λ (Q=nucl. mom.)

Chiral symmetry

Effective theory



Chiral perturbation theory





"garbage"=heavy meson, Δ

2N+3N on the same footing !





Results of 270 and 140 MeV

• $d\sigma/d\Omega$, iT_{11} : Excellent fits!

 \rightarrow clear **3NF** effects in **N***d* scatt.

 \rightarrow magnitudes of 3NF seem to be O.K.

• $A_y(=-p^{y'}), T_{20}, T_{22}, T_{21}$: Poor fits!

 \rightarrow defects in spin dependent part of 3NF.

• 3NF: TM'/ Urbana IX does better job.

 \rightarrow chiral symmetry requires: The *c* term should be zero.

 Chiral Eff. Field Theory calc. does reasonable job for *dσ/dΩ* but not much analyzing powers.

How they look like if the energy is doubled from 135 MeV/u (E_d=270 MeV) ?

250 MeV/u data

at **RCNP**

h + d at 250 MeV (RCNP)

by Yukie Maeda

- Coulomb free
- direct comparison is possible:
 - between *nd* data vs. Faddeev calc.
 - h' : secondary beam exp. \rightarrow very difficult.

⁷Li(p,n)⁷Be reaction

Forward angle : d(n,n)d NTOF+NPOLD liq. Scintillator(active target)Backward angle : d(n,d)n (n,p) facility
recoiled d dectectedAll data points are normalized by np scatt.

d(n,n)d NTOF+NPOL



Beam Swinger System



RCNP Osaka University

NPOL2

LAS Spectrometer

\ddot{h} + d at 250 MeV



p' + d at 250 MeV (RCNP)

by K. Hatanaka

• $d\sigma/d\Omega, A_y$

- complete pol. transfer meas.: $K_x^{x'}$, $K_y^{y'}$, $K_z^{z'}$, $K_z^{x'}$, $K_x^{z'}$
- direct comparison possible: between *nd* vs. *pd*

 $p + d \rightarrow p + d$ at 250 MeV

K. Hatanaka et al., Phys. Rev. C66 (2002) 044002.

h + d and h + d at 250 MeV









Data-to-data comparison between *nd vs. pd*



$$\left(\frac{d\,\sigma}{d\,\Omega}\right)_{pd} \\
\left(\frac{d\,\sigma}{d\,\Omega}\right)_{nd}$$

Coulomb effect ! ?

Calculation by Y. Koike.

Nd at 135/u MeV

Coulomb force is approx. included.

Results at 250 MeV

First hd and bd measurements at 250 MeV. • Data-to-data comparison was $m\vec{n}d$ vs. $\vec{p}d$ \rightarrow 10-20 % variation in $d\sigma/d\Omega$ \rightarrow Coulomb effect ! ? • direct comparison was made: *nd*. exp. data *vs*. Faddeev calc. • $d\sigma/d\Omega$: 50% disagreement in backward. $\bullet A_v$: large deviation in backward.

> → defects of 3NF or relativistic effects or both? OR

→ defects of nucleon exchange process? OR

defects of NN interactions?

How they look like if the energy is doubled ?

400 MeV/u data at RCNP

p + d at 400 MeV (RCNP)

by Tamii

A.Tamii: $p + d \rightarrow d + p$ at 400 MeV $d\sigma/d\Omega, A_y^p, A_y^d (= -P^{y'}), K_y^{y'}$ Limited angular range

Very preliminary calc. by Kamada

• 2NF : CD Bonn

• **3NF : TM**









Results at 400 MeV

Calc. by H. Kamada are very preliminary!

• $d\sigma/d\Omega$: agreement?

•
$$A_{y}^{p}$$
, $(= -P^{y'})$, iT_{11} : large deviation.

Experimental Summary

Precise data on p + d elastic scattering at $\mathbf{E}_p = 70 - 400$ MeV become available now, not only $d\sigma/d\Omega$, A_y^{p} , $(= -P^{y'})$, iT_{11} , T_{20} , T_{22} , T_{21} but also various Polarization Transfer Observables.

A lot of data expected at 135 MeV/u.

* Spin-correl. data from IUCF.

* Breakup data from KVI and may be from RIKEN 135 MeV/u will be the most extensive data set.

Present status of 3NF study

 3NF established firmly ? Yes and No. Magnitudes seem to be O.K. Spin dependence? Chaotic.

Defects of 3NF ?
 Yes, in TM.
 The c term violates chiral symmetry.
 Need to include, πρ and ρρ exch. 3NF.

• New development of Chiral Eff. Field Theory calc. is extremely interesting.

•Are 2NFs reliable?

Questionable in terms of off-shell properties.

No need of 3NF in BE(³H) by Y. Fujiwara!

RAPID COMMUNICATIONS

PHYSICAL REVIEW C 66, 021001(R) (2002)

Triton binding energy calculated from the SU_6 quark-model nucleon-nucleon interaction

Y. Fujiwara,¹ K. Miyagawa,² M. Kohno,³ Y. Suzuki,⁴ and H. Nemura⁵
¹Department of Physics, Kyoto University, Kyoto 606-8502, Japan
²Department of Applied Physics, Okayama Science University, Okayama 700-0005, Japan
³Physics Division, Kyushu Dental College, Kitakyushu 803-8580, Japan
⁴Department of Physics, Niigata University, Niigata 950-2181, Japan
⁵Institute of Particle and Nuclear Physics, KEK, Tsukuba 305-0801, Japan
(Received 23 May 2002; published 2 August 2002)

Properties of the three-nucleon bound state are examined in the Faddeev formalism, in which the quarkmodel nucleon-nucleon interaction is explicitly incorporated to calculate the off-shell *T* matrix. The most recent version, fss2, of the Kyoto-Niigata quark-model potential yields the ground-state energy $E({}^{3}\text{H})$ = -8.514 MeV in the 34 channel calculation, when the *np* interaction is used for the nucleon-nucleon interaction. The charge root mean square radii of the ${}^{3}\text{H}$ and ${}^{3}\text{H}$ e are 1.72 fm and 1.90 fm, respectively, including the finite size correction of the nucleons. These values are the closest to the experiments among many results obtained by detailed Faddeev calculations employing modern realistic nucleon-nucleon interaction models.

•Are 2NFs reliable?

Questionable in terms of off-shell properties. No need of 3NF in BE(³H) by Y. Fujiwara! Should be tested by scatt. Data.

Exchange term might be problematic.

Finnaly,

after a century of Tolstoy's novel, the **3NF study** has reached a new era of 'the **Renaissance**'.

This is due to recent harmonious development of both experiments and theories.

I am very happy, if I could convince you such Renaissance after my lecture. Experiments were carried out under the collaboration of researchers from University of Tokyo, RIKEN, CNS, RCNP, Saitama University, CYRIC and TIT.

This talk is supported by Kimiko Sekiguchi Yukie Maeda for preparing figures.