Heavy-ion Fusion Reactions Around the Coulomb Barrier

Kouichi Hagino (Tohoku University)

1. *Potential Model*

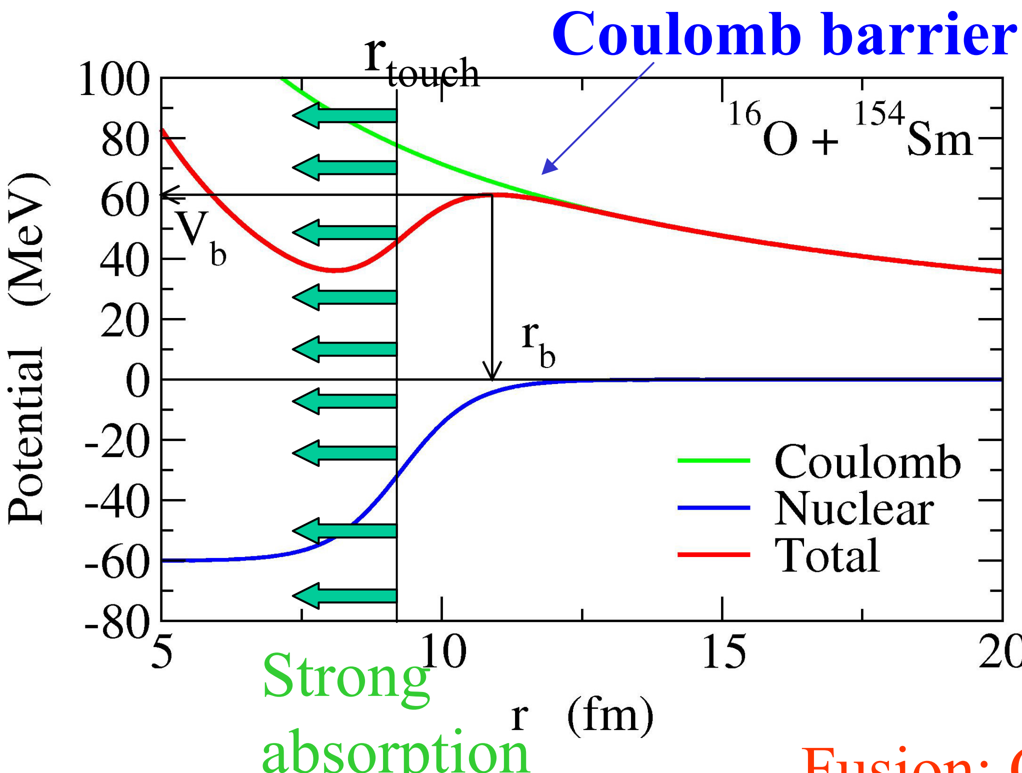
   Large enhancement of sub-barrier cross sections

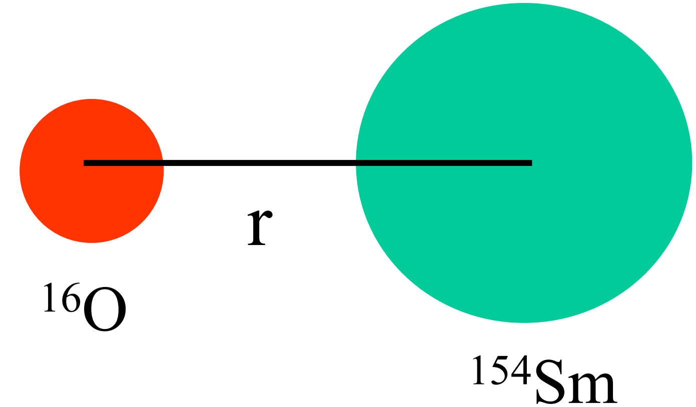
2. *Coupled-channels Approach to H.I. Fusion*

   Vibrational and Rotational couplings

3. *Fusion Barrier Distributions*
4. *Alternative: Quasi-Elastic Barrier Distributions*
5. *Fusion of neutron-rich nuclei*

Potential model for fusion





$$V(r) = V_N(r) + V_C(r)$$

Fusion: Compound nucleus formation

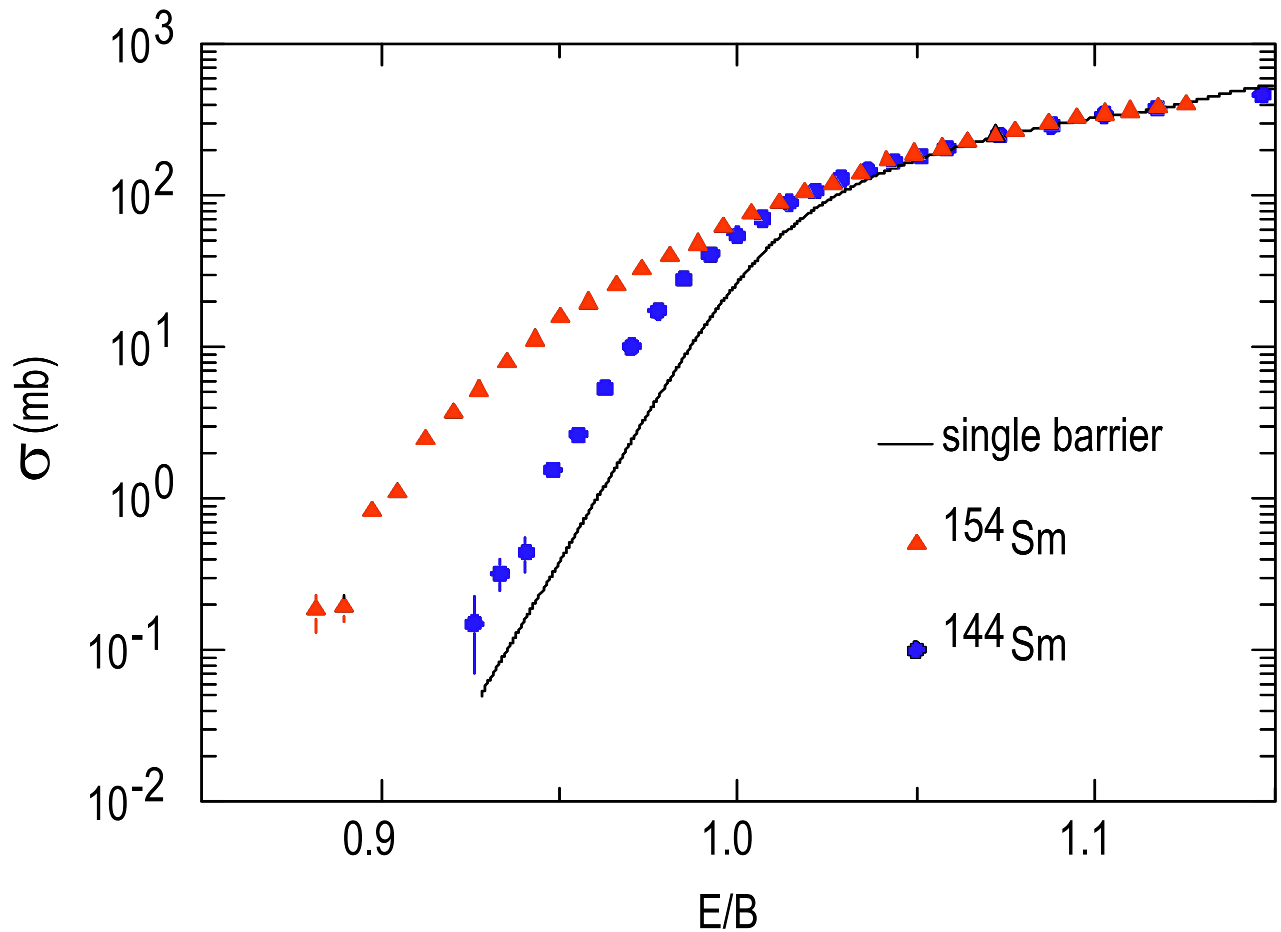
$$^{16}O + {}^{154}Sm \longrightarrow {}^{170}Yb^*$$

$$\sigma_{\text{fus}}(E) = \frac{\pi}{k^2} \sum_l (2l+1) P_l(E)$$

Barrier Penetration Model

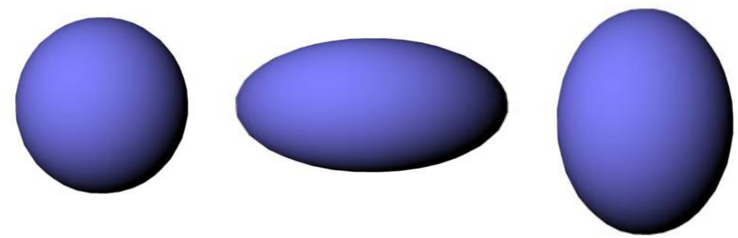
Experimental data vs. potential model

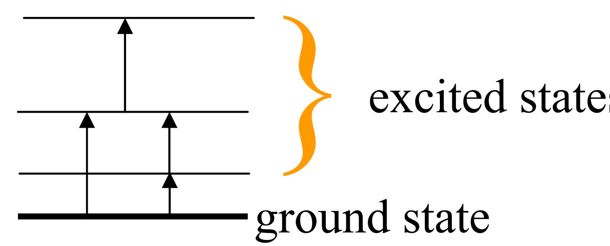
$^{16}O + ^{144,154}Sm$



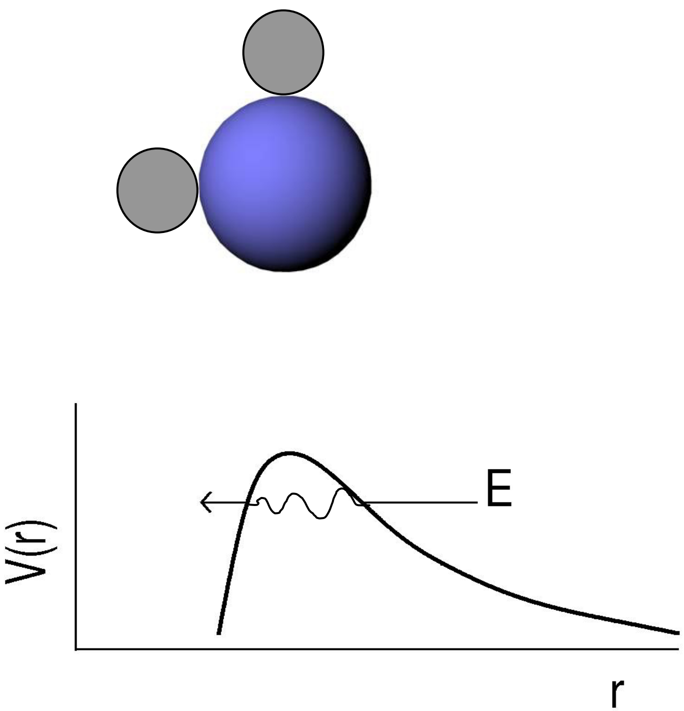
Missing Physics!

Missing Physics – Nuclear Structure Information

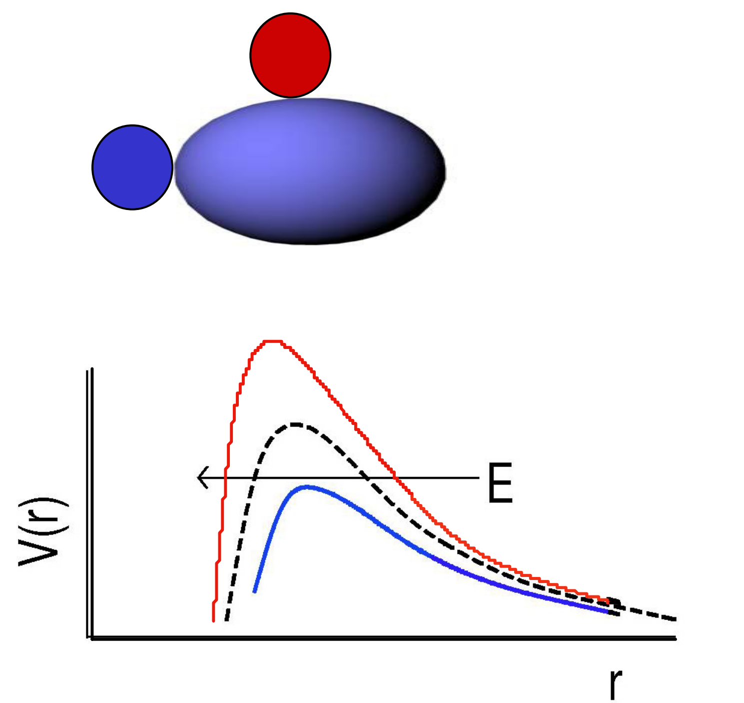
Potential model	Reality
	
Nuclei: spherical structure-less objects	Different shapes, internal structure



Nuclear Structure Effect – a classical demonstration

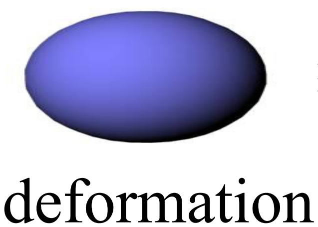


Single barrier

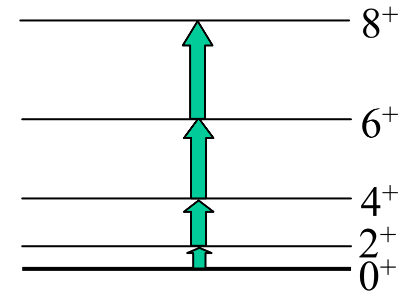


Distribution of barriers heights depending on the orientation

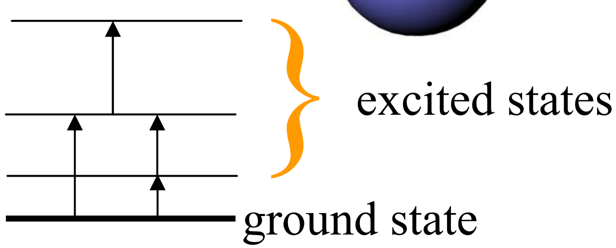
Nuclear Structure Effect – a quantal treatment

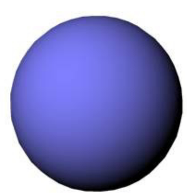


≡ Rotational excitation



In general



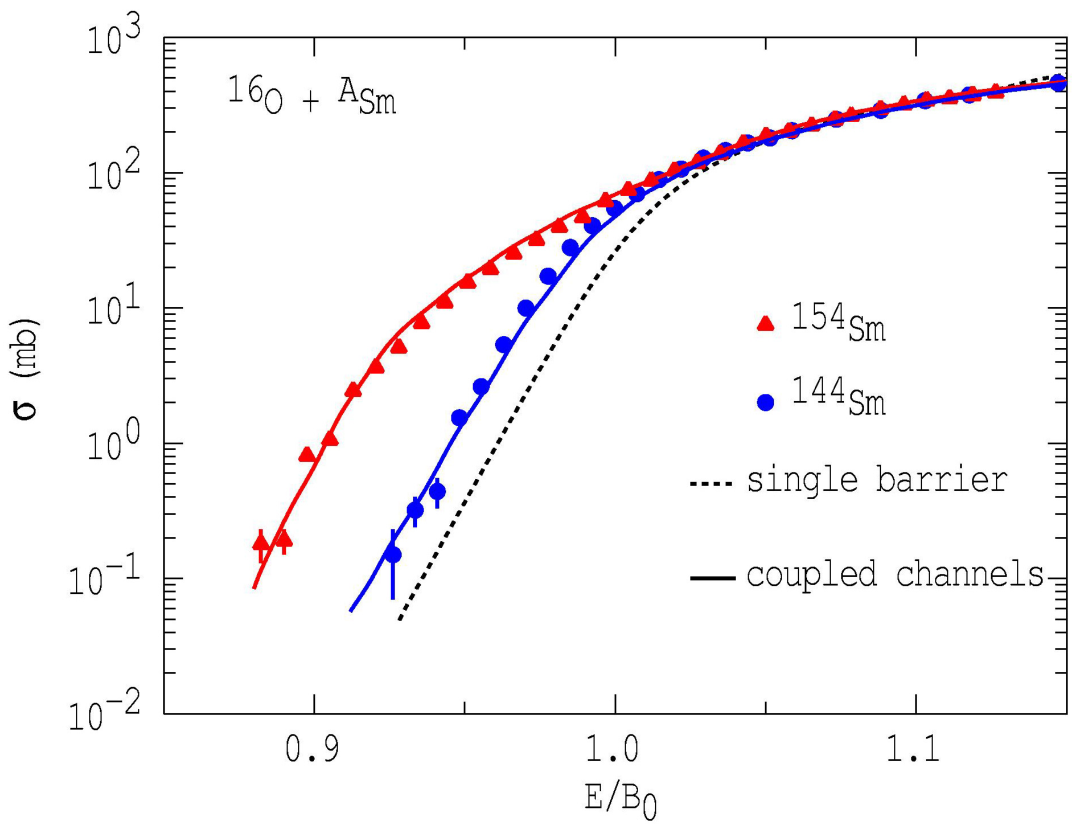


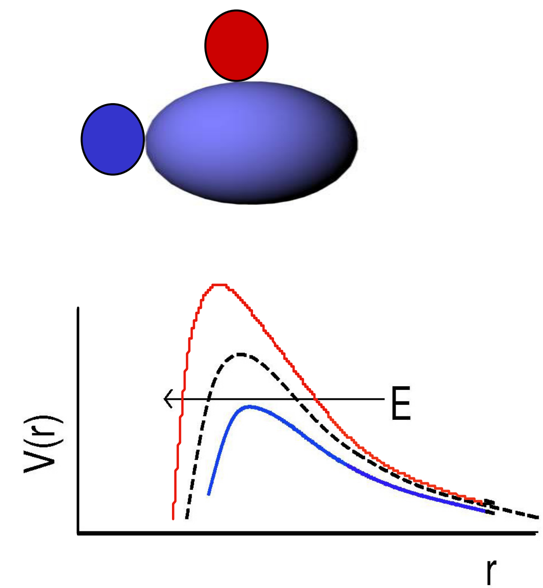
≡ Coupling between the relative motion and intrinsic degrees of freedom

Rotational, vibrational states **(Low-lying collective excitations)**

Coupled channels model

Effect of couplings on fusion cross sections



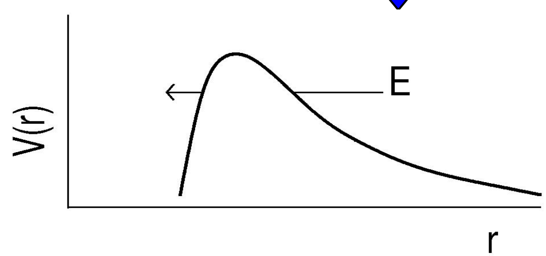


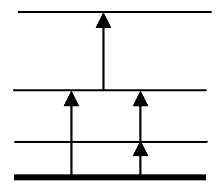
Couplings ⟶ enhancement of σ_{fus} by factors of 10-100

Fusion: an interesting tool to probe nuclear structure

Coupled-channels Method

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) + H_0(\xi) + V_{\rm coup}(\boldsymbol{r},\xi)$$



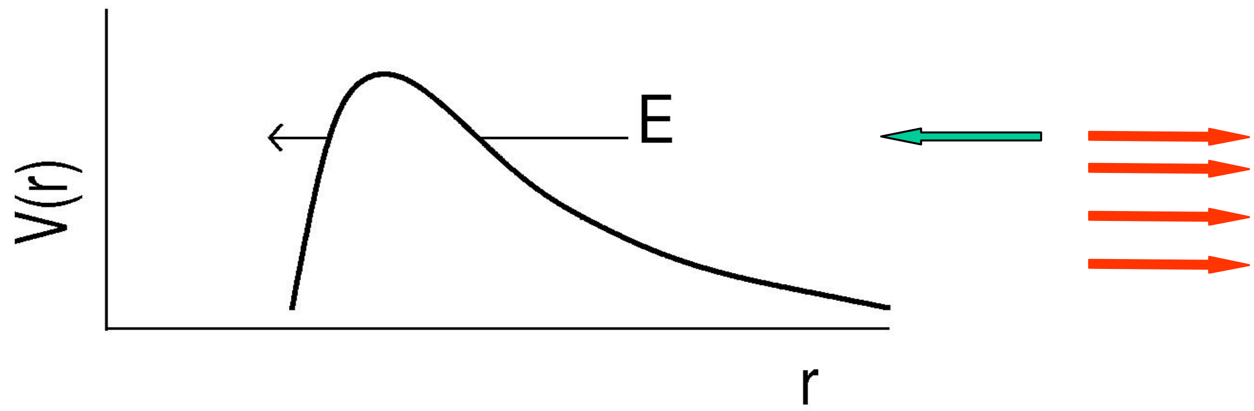


$$H_0(\xi)\phi_{nIm_I}(\xi) = \epsilon_{nI}\,\phi_{nIm_I}(\xi)$$

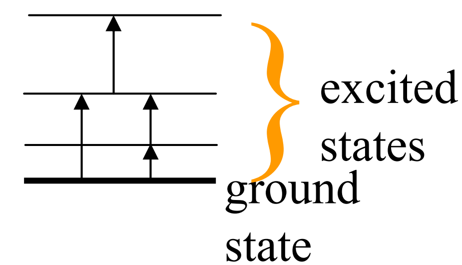
$$\Psi(\boldsymbol{r},\xi) = \sum_{n,l,I}\frac{u_{nlI}(r)}{r}[Y_l(\hat{\boldsymbol{r}})\phi_{nI}(\xi)]^{(JM)}$$

$$\langle[Y_l\phi_{nI}]^{(JM)}|H - E|\Psi\rangle = 0$$

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{l(l+1)\hbar^2}{2\mu r^2} + V_0(r) - E + \epsilon_{nI}\right]u_{nlI}(r) + \sum_{n'l'I'}\langle[Y_l\phi_{nI}]^{(JM)}|V_{\rm coup}(r)|[Y_{l'}\phi_{n'I'}]^{(JM)}\rangle u_{n'l'I'}(r) = 0$$



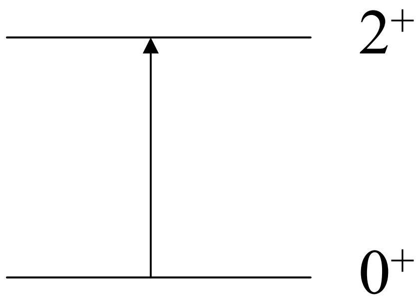
$$\Psi(\boldsymbol{r},\xi) = \sum_{n,l,I} \frac{u_{nlI}(r)}{r}[Y_l(\hat{\boldsymbol{r}})\phi_{nI}(\xi)]^{(JM)}$$



$$u_{nlI}(r) \to H_l^{(-)}(k_{nI}r)\delta_{n,n_i}\delta_{l,l_i}\delta_{I,I_i} - \sqrt{\frac{k_0}{k_{nI}}}\, S_{nlI}\, H_l^{(+)}(k_{nI}r)$$

$$P_l(E) = 1 - \sum_{nI} |S_{nlI}|^2 \qquad \sigma_{\mathrm{fus}}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E)$$

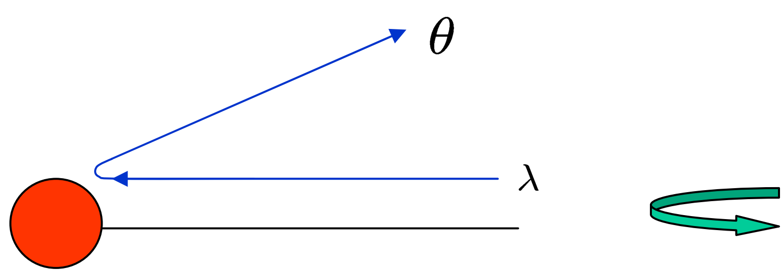
Iso-centrifugal approximation



l=J-2, J, J+2

l=J

J=l+I



Truncation	Dimension
2^+	$4 \longrightarrow 2$
4^+	$9 \longrightarrow 3$
6^+	$16 \longrightarrow 4$
8^+	$25 \longrightarrow 5$

Iso-centrifugal approximation:

λ : independent of excitations

$$\frac{l(l+1)\hbar^2}{2\mu r^2} \to \frac{J(J+1)\hbar^2}{2\mu r^2}$$

$$V_{\rm coup}(\boldsymbol{r},\xi) = f(r)Y_\lambda(\hat{\boldsymbol{r}})\cdot T_\lambda(\xi)$$

$$\to \sqrt{\frac{2\lambda+1}{4\pi}}f(r)T_{\lambda 0}(\xi)$$

"Spin-less system"

Coupling Potentials

K.H., N. Rowley, and A.T. Kruppa,
Comp. Phys. Comm. 123('99)143

Deformed Woods-Saxon model (collective model)

$$V_{\rm coup}(r,\hat{O}) = V^{(N)}_{\rm coup}(r,\hat{O}) + V^{(C)}_{\rm coup}(r,\hat{O})$$

Nuclear coupling:

$$V^{(N)}_{\rm coup}(r,\hat{O}) = -\frac{V_0}{1+\exp[(r-R-R\hat{O})/a]}$$

Coulomb coupling:

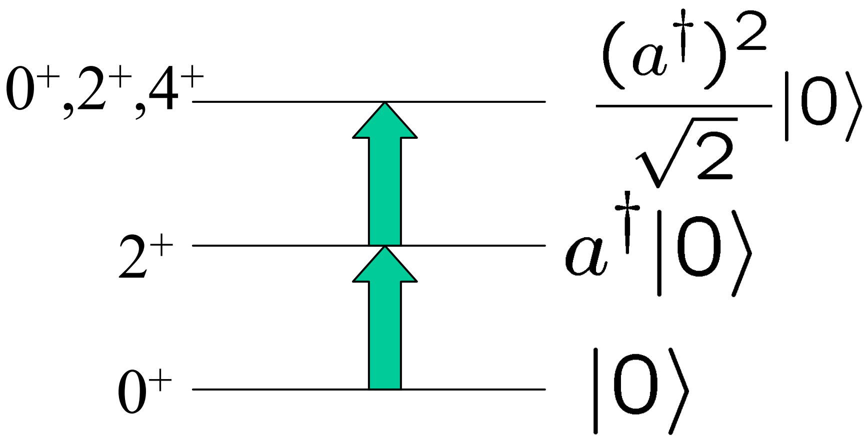
$$V^{(C)}_{\rm coup}(r,\hat{O}) = \frac{3}{2\lambda+1} Z_P Z_T e^2 \frac{R^\lambda}{r^{\lambda+1}}\hat{O}$$

Rotational coupling: $\hat{O} = \beta Y_{20}(\theta)$

Vibrational coupling: $\hat{O} = \frac{\beta}{\sqrt{4\pi}}(a + a^\dagger)$

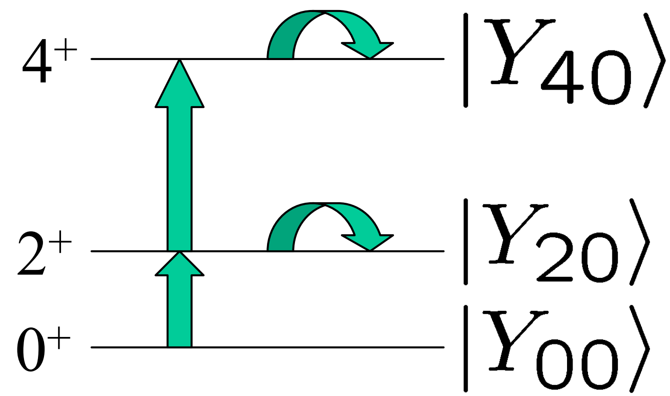
Vibrational coupling

$$\hat{O} = \frac{\beta}{\sqrt{4\pi}}(a + a^\dagger)$$



Rotational coupling

$$\hat{O} = \beta Y_{20}(\theta)$$



$$\begin{pmatrix} 0 & F & 0 \\ F & \epsilon & \sqrt{2}F \\ 0 & \sqrt{2}F & 2\epsilon \end{pmatrix}
\qquad
\begin{pmatrix} 0 & F & 0 \\ F & \epsilon + \frac{2\sqrt{5}}{7}F & \frac{6}{7}F \\ 0 & \frac{6}{7}F & \frac{10\epsilon}{3} + \frac{20\sqrt{5}}{77}F \end{pmatrix}$$

$$F = \frac{\beta}{\sqrt{4\pi}}$$

Two limiting cases: (i) Adiabatic limit

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) + H_0(\xi) + V_{\rm coup}(\boldsymbol{r},\xi)$$

When ϵ is large,

$$H_0(\xi) + V_{\rm coup}(\boldsymbol{r},\xi) \to \epsilon_0(r)$$

where

$$[H_0(\xi) + V_{\rm coup}(\boldsymbol{r},\xi)]\varphi_0(\xi;\boldsymbol{r}) = \epsilon_0(r)\,\varphi_0(\xi;\boldsymbol{r})$$

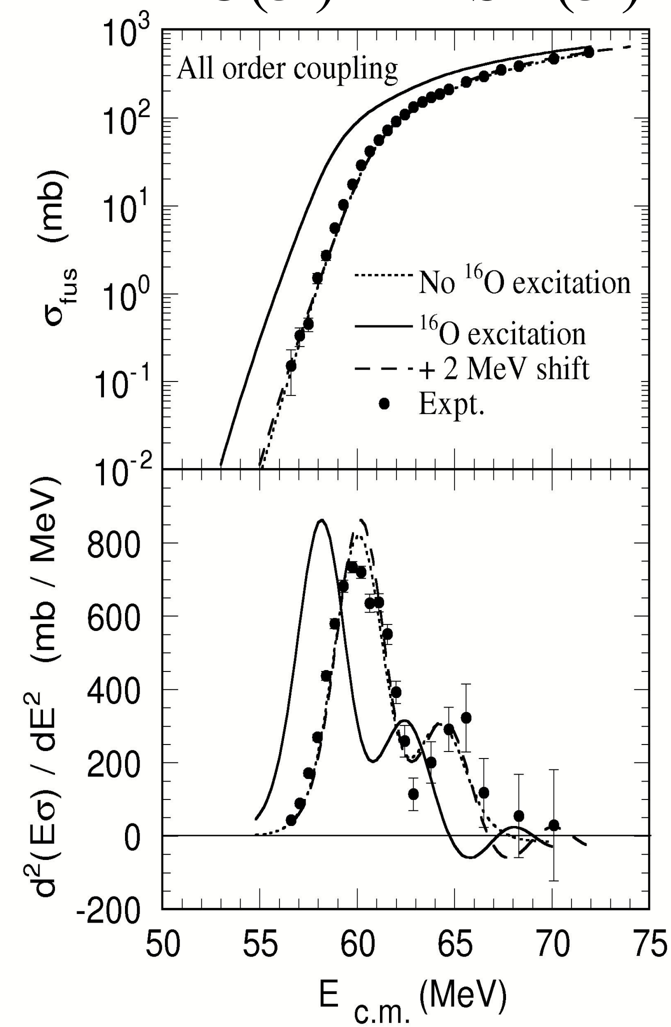
Fast intrinsic motion

→ Adiabatic potential renormalization

$$V_{\rm ad}(r) = V_0(r) + \epsilon_0(r)$$

Giant Resonances, ^{16}O(3$^-$) [6.31 MeV]

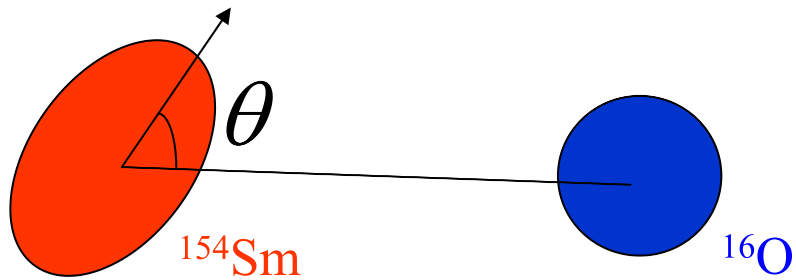
^{16}O(3$^-$) + ^{144}Sm(3$^-$)



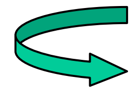
K.H., N. Takigawa, M. Dasgupta, D.J. Hinde, J.R. Leigh, PRL79('99)2014

Two limiting cases: (ii) Sudden limit

$$\epsilon \to 0$$



$$\epsilon_I = I(I+1)\hbar^2/2\mathcal{J}$$



$$\mathcal{J} \to \infty$$

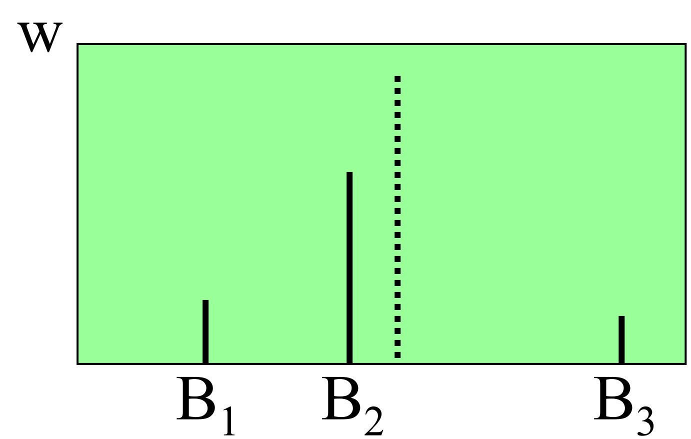
$$\sigma_{\rm fus}(E) = \int_0^1 d(\cos\theta)\sigma_{\rm fus}(E;\theta)$$

Coupled-channels:

$$\begin{pmatrix} 0 & f(r) & 0 \\ f(r) & \frac{2\sqrt{5}}{7}f(r) & \frac{6}{7}f(r) \\ 0 & \frac{6}{7}f(r) & \frac{20\sqrt{5}}{77}f(r) \end{pmatrix} \xrightarrow{\text{diagonalize}} \begin{pmatrix} \lambda_1(r) & 0 & 0 \\ 0 & \lambda_2(r) & 0 \\ 0 & 0 & \lambda_3(r) \end{pmatrix}$$

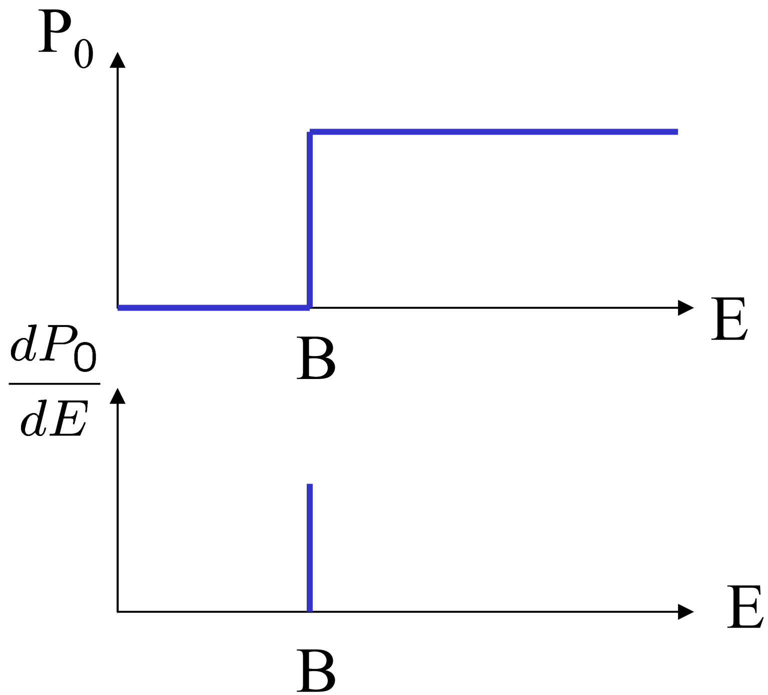
$$P(E) = \sum_i w_i P(E; V_0(r) + \lambda_i(r))$$

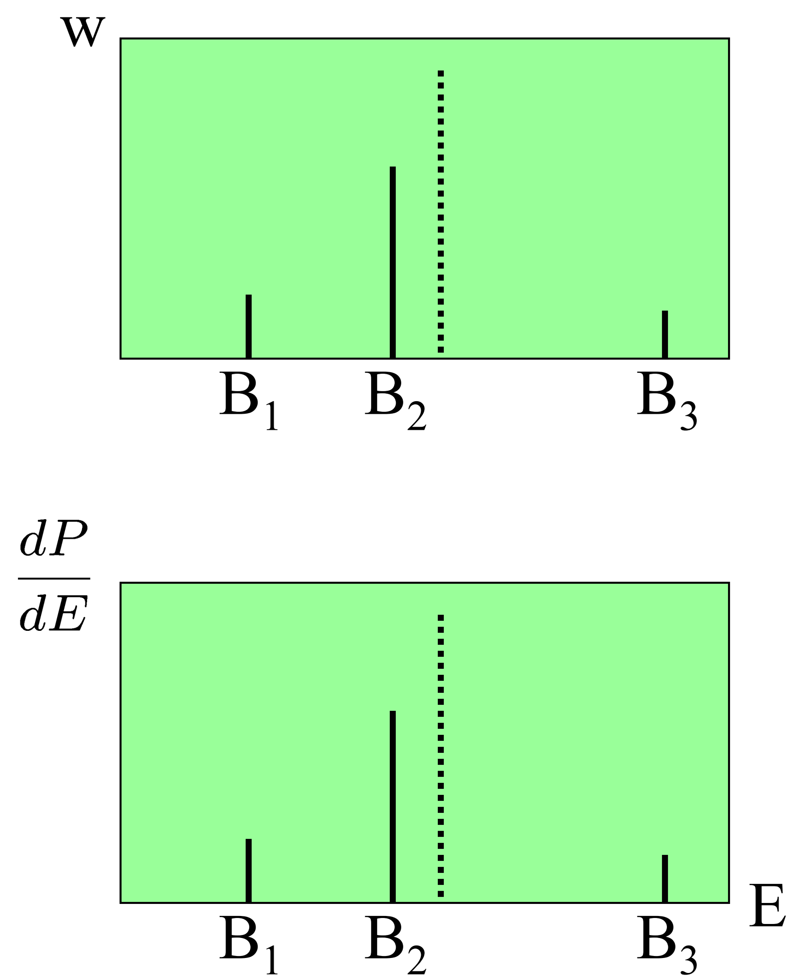
Slow intrinsic motion
→ Barrier Distribution



Fusion Barrier Distribution

$$P(E) = \sum_i w_i \, P(E; V_0(r) + \lambda_i(r))$$



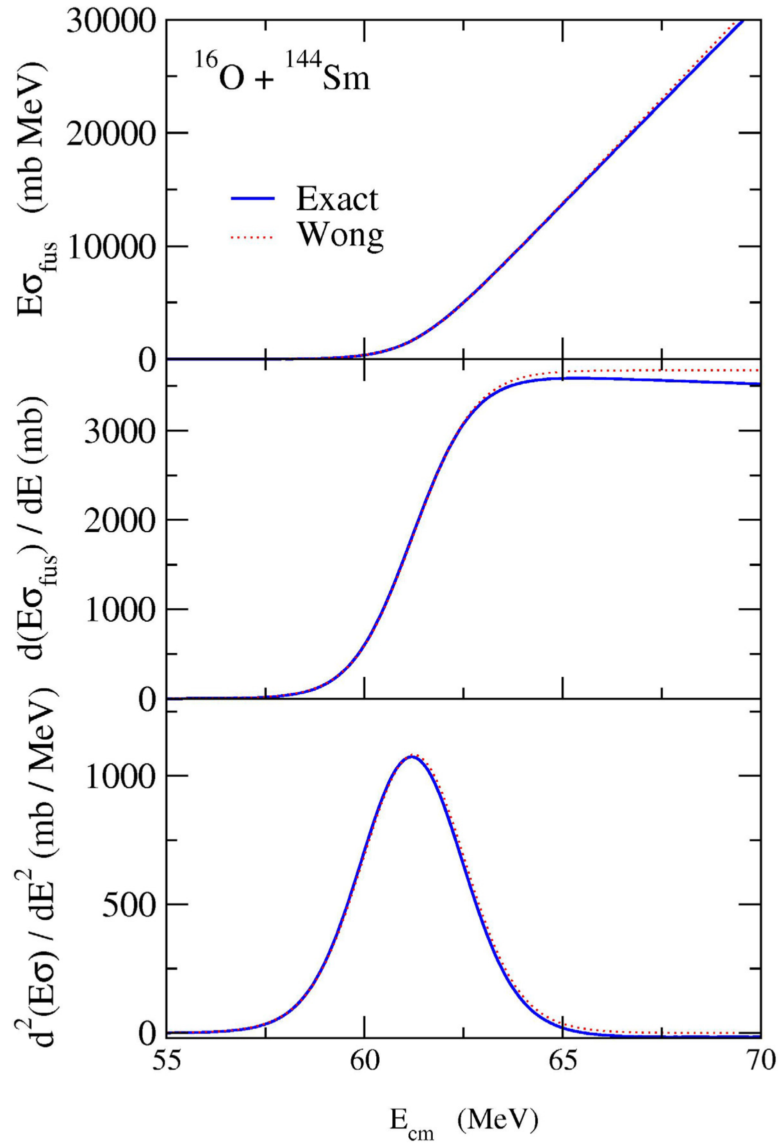


Representation of fusion barrier distribution from experimental data:

$$D_{\rm fus}(E) = \frac{d^2(E\sigma)}{dE^2}$$

N. Rowley, G.R. Satchler, P.H. Stelson, PLB254('91)25

Fusion test function



Classical fusion cross section:

$$\sigma^{cl}_{\text{fus}}(E) = \pi R_b^2 \left(1 - \frac{V_b}{E}\right) \theta(E - V_b)$$

$$\frac{d}{dE}[E\sigma^{cl}_{\text{fus}}(E)] = \pi R_b^2\, \theta(E - V_b)$$

$$= \pi R_b^2\, P_{cl}(E)$$

$$\frac{d^2}{dE^2}[E\sigma^{cl}_{\text{fus}}(E)] = \pi R_b^2\, \delta(E - V_b)$$

Tunneling effect

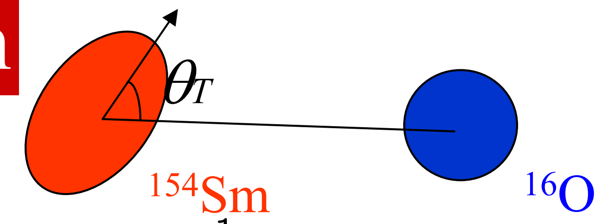
⟶ smears the delta function

Fusion test function:

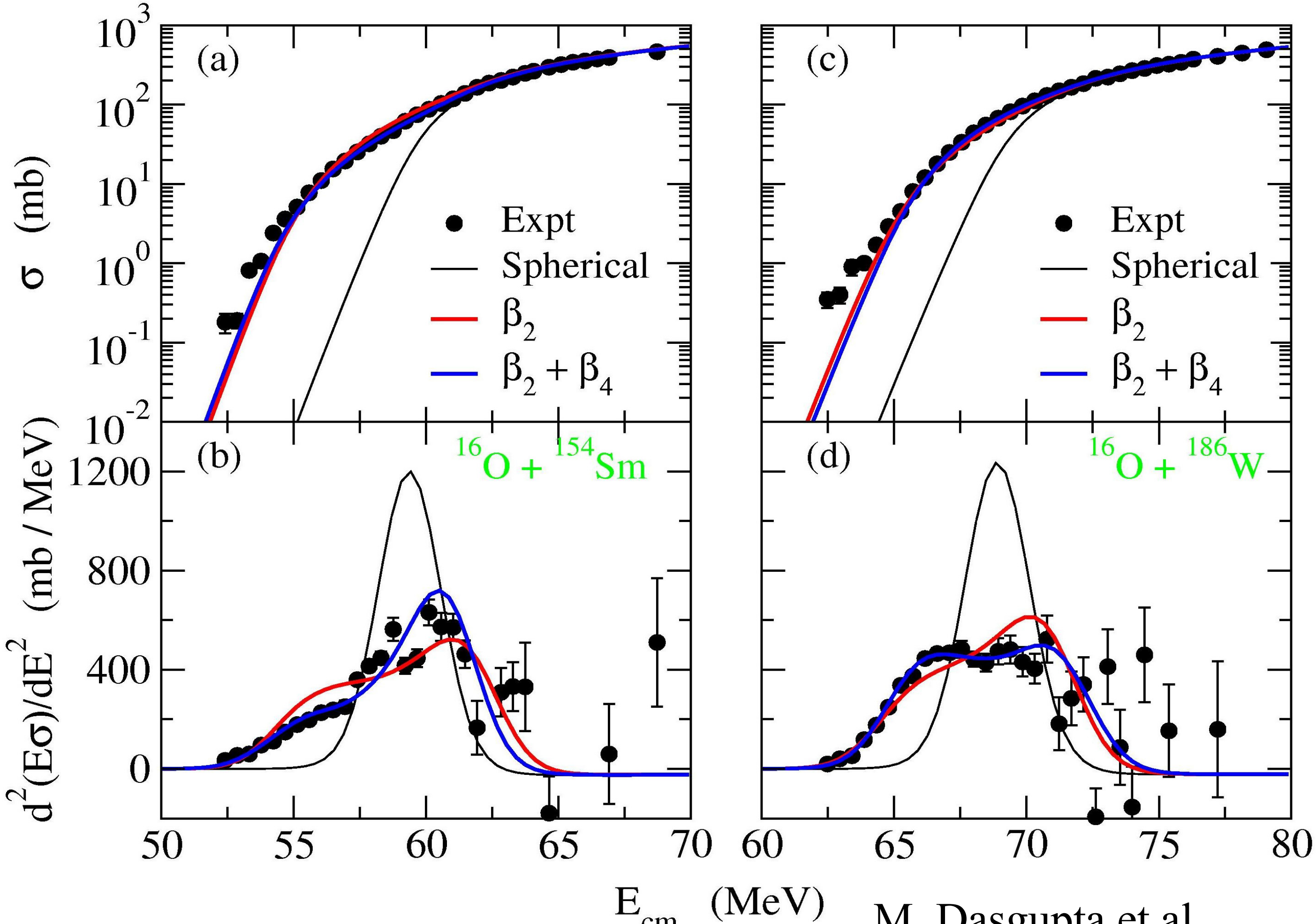
- Symmetric around E=V_b
- Centered on E=V_b
- Its integral over E is πR_b^2
- Has a relatively narrow width $(\sim 0.56\hbar\Omega)$

Experimental Barrier Distribution

Requires high precision data



$$\sigma_{\rm fus}(E) = \int_0^1 d(\cos\theta_T)\sigma_{\rm fus}(E;\theta_T)$$



M. Dasgupta et al.,
Annu. Rev. Nucl. Part. Sci. 48('98)401

$D_{fus}(E)$ for finite ε

$$U(r)\begin{pmatrix} 0 & f(r) & 0 \\ f(r) & \epsilon & \sqrt{2}f(r) \\ 0 & \sqrt{2}f(r) & 2\epsilon \end{pmatrix} U^{-1}(r) = \begin{pmatrix} \lambda_1(r) & 0 & 0 \\ 0 & \lambda_2(r) & 0 \\ 0 & 0 & \lambda_3(r) \end{pmatrix}$$

r dependent

$$P(E) = \sum_i w_i(E)\, P(E; V_0(r) + \lambda_i(r))$$

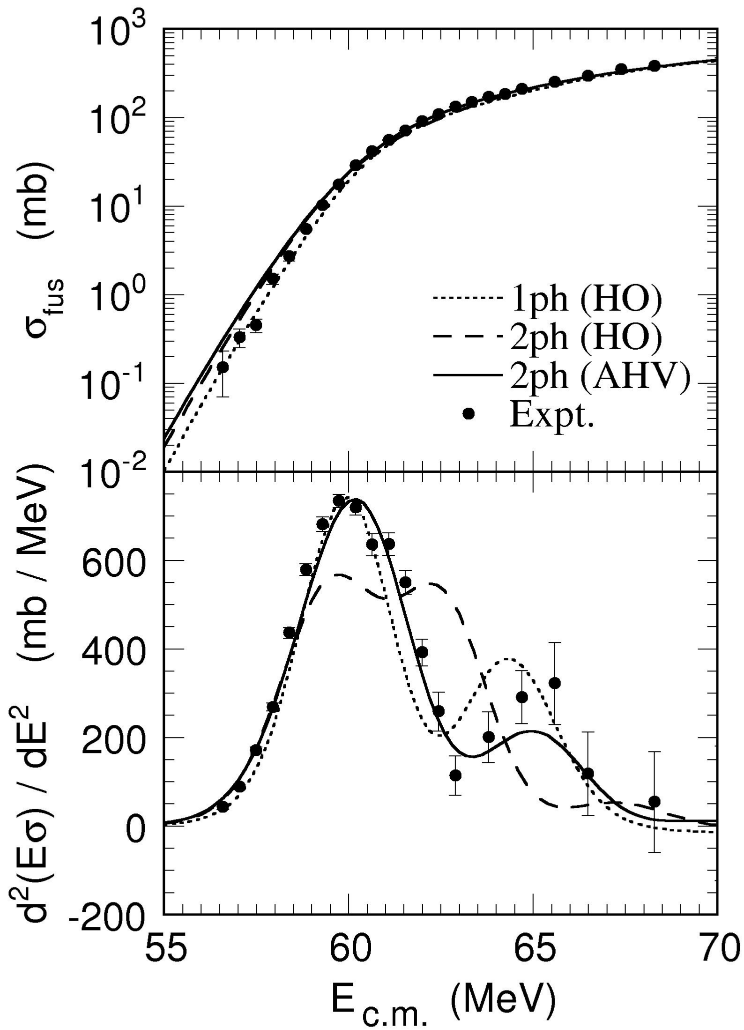
E dependent

K.H., N. Takigawa, A.B. Balantekin, PRC56('97)2104

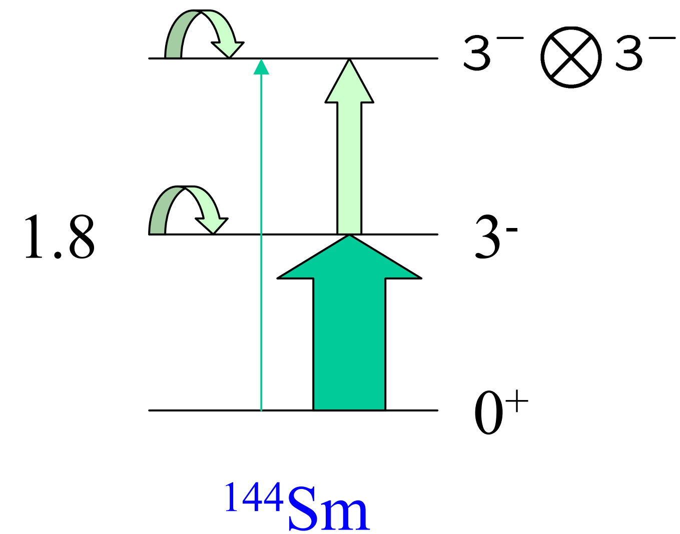
$$w_i(E) \sim \text{constant}$$

The concept of b.d.: approximately holds even for finite ε

$^{16}O + ^{144}Sm$ reaction



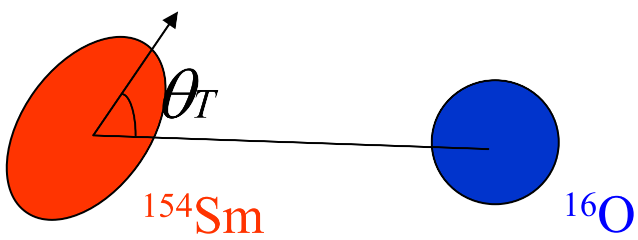
K.H., N. Takigawa, and S. Kuyucak, PRL79('97)2943



Quadrupole moment:

$Q(3^-) = -0.70 \pm 0.02b$

Alternative Representation: Quasi-elastic Barrier Distribution



$$\sigma_{\mathrm{fus}}(E) = \int_0^1 d(\cos\theta_T)\sigma_{\mathrm{fus}}(E;\theta_T)$$

$$D_{\mathrm{fus}}(E) = \frac{d^2(E\sigma_{\mathrm{fus}})}{dE^2}$$

Quasi-elastic scattering:

A sum of all the reaction processes other than fusion (elastic + inelastic + transfer + ……)

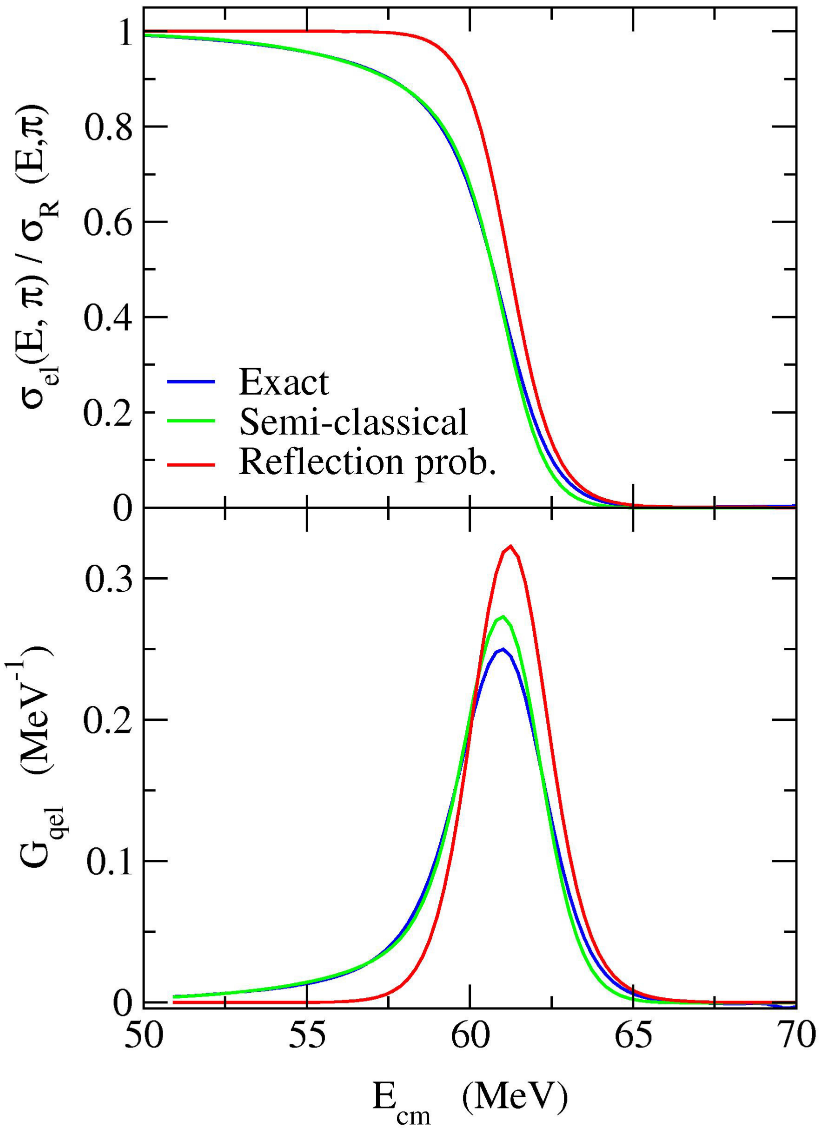
$$\sigma_{\mathrm{qel}}(E,\theta) = \sum_I \sigma(E,\theta) = \int_0^1 d(\cos\theta_T)\sigma_{\mathrm{el}}(E,\theta;\theta_T)$$

Quasi-elastic barrier distribution:

$$D_{\mathrm{qel}}(E) = -\frac{d}{dE}\left(\frac{\sigma_{\mathrm{qel}}(E,\pi)}{\sigma_R(E,\pi)}\right)$$

H. Timmers et al., NPA584('95)190

Quasi-elastic test function



Classical elastic cross section
(in the limit of a strong Coulomb):

$$\sigma_{el}^{cl}(E,\pi) = \sigma_R(E,\pi)\,\theta(V_b - E)$$

$$\frac{\sigma^{cl}_{\rm el}(E,\pi)}{\sigma_R(E,\pi)} = \theta(V_b - E) = R(E)$$

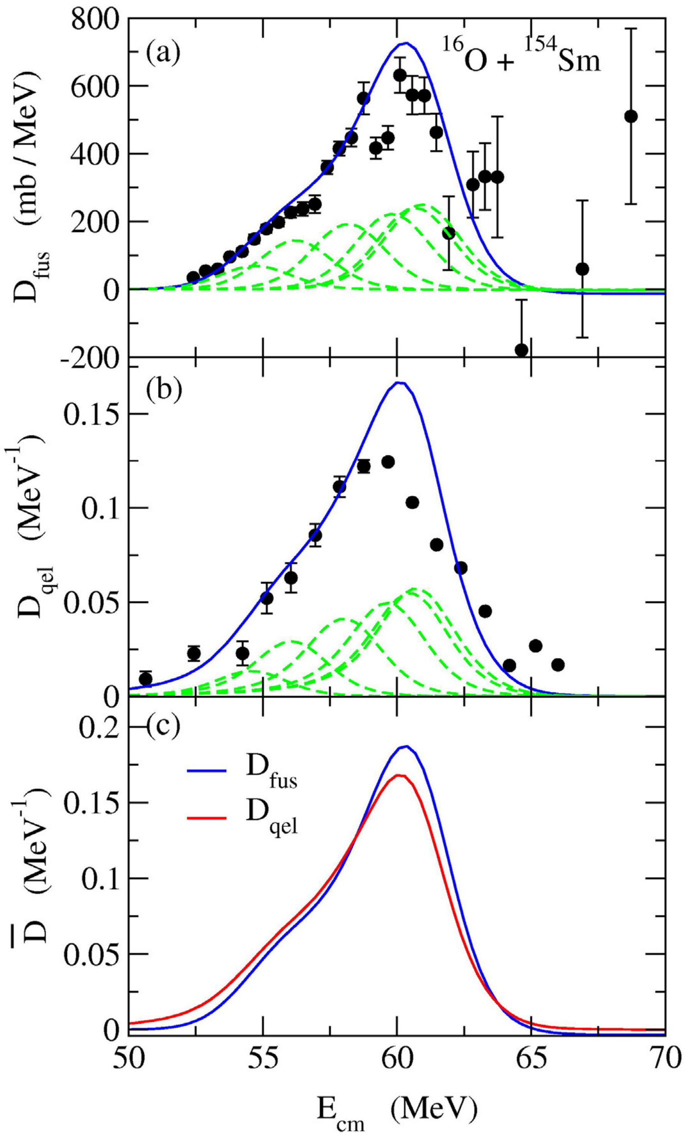
$$-\frac{d}{dE}\left(\frac{\sigma^{cl}_{\rm el}(E,\pi)}{\sigma_R(E,\pi)}\right) = \delta(E - V_b)$$

Nuclear effect

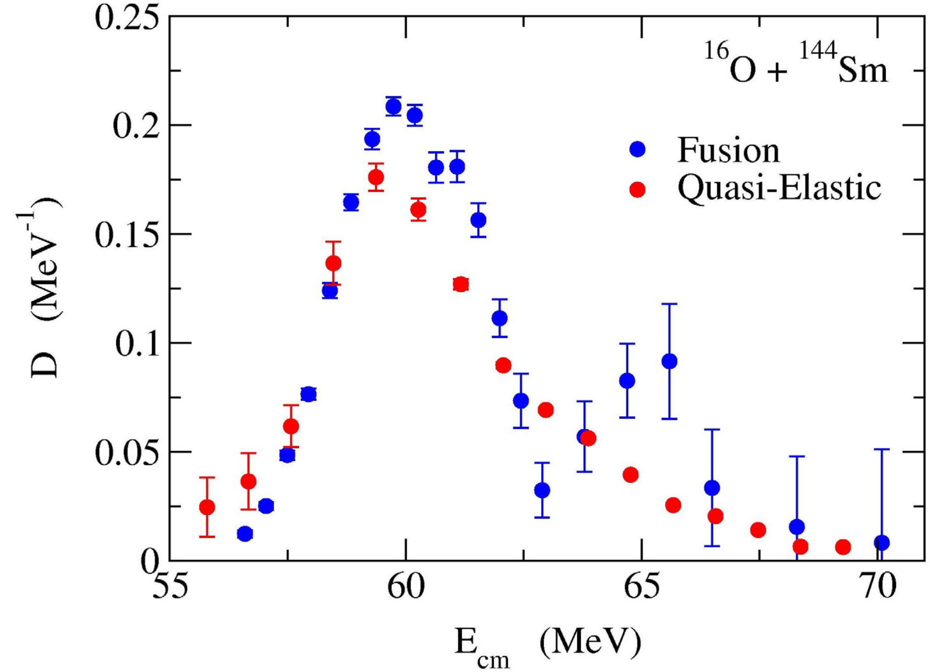
- ➢The peak position slightly deviates from V_b
- ➢Low energy tail
- • Integral over E: unity
- • Relatively narrow width

➡ Close analog to fusion b.d.

Comparison between D_{fus} and D_{qel}



K.H. and N. Rowley, PRC69('04)054610



H. Timmers et al., NPA584('95)190

Experimental advantages for D_{qel}

$$D_{\rm qel}(E) = -\frac{d}{dE}\left(\frac{\sigma_{\rm qel}(E,\pi)}{\sigma_R(E,\pi)}\right) \qquad D_{\rm fus}(E) = \frac{d^2(E\sigma_{\rm fus})}{dE^2}$$

- less accuracy is required in the data (1^{st} vs. 2^{nd} derivative)
- much easier to be measured

Qel : a sum of everything

→ a very simple charged-particle detector

Fusion : requires a specialized recoil separator to separate ER from the incident beam

ER + fission for heavy systems

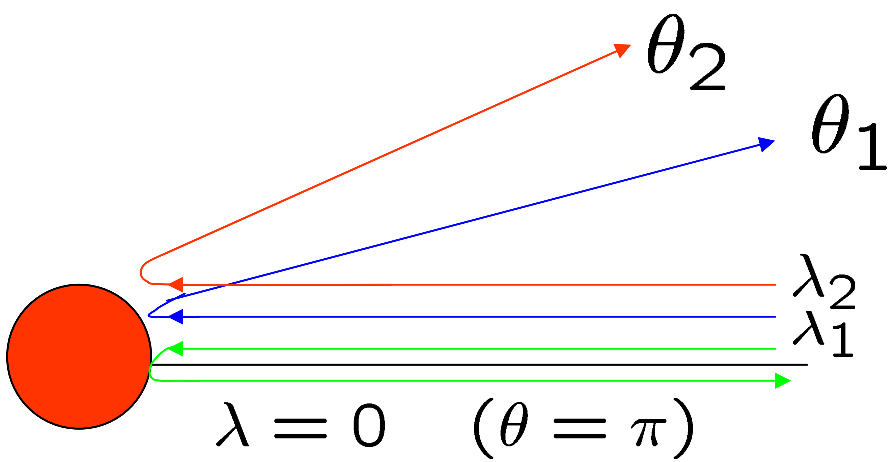
- several effective energies can be measured at a single-beam energy ↔ relation between a scattering angle and an impact parameter

⇒ measurements with a cyclotron accelerator: possible

⇒ Suitable for low intensity exotic beams

Scaling property of D_{qel}

Expt.: impossible to perform
at $\theta = \pi$
→ Relation among different θ?



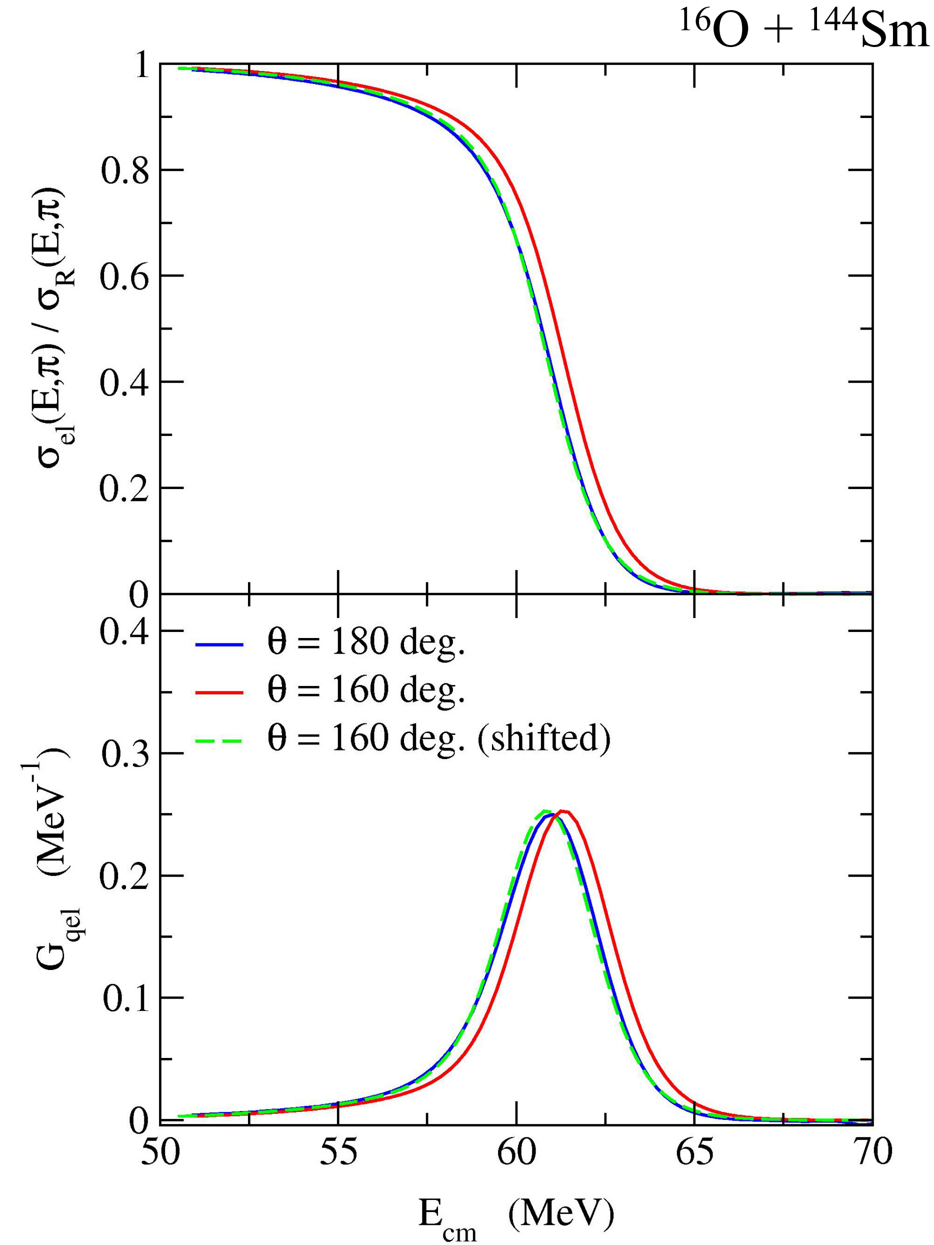
Effective energy:

$$E_{\rm eff} \sim E - \frac{\lambda_c^2 \hbar^2}{2\mu r_c^2}$$

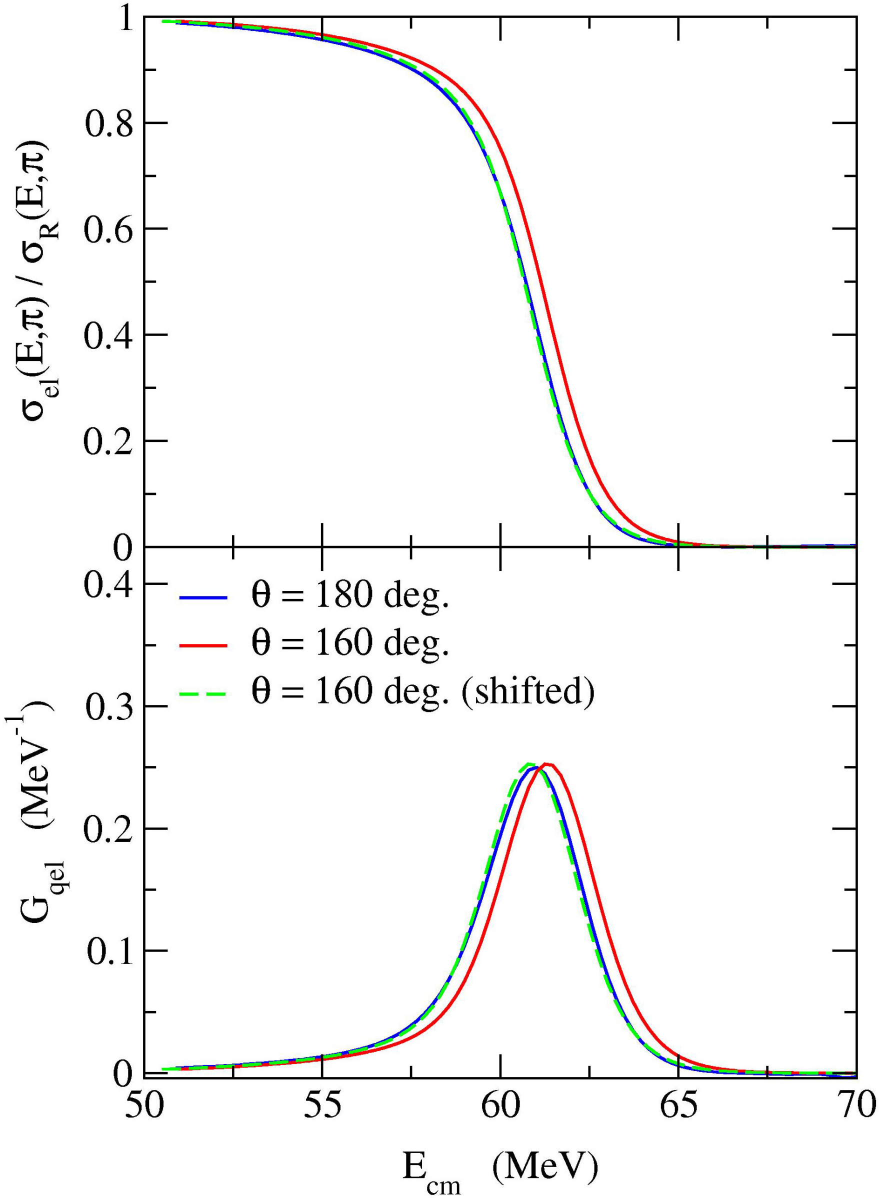
$$= 2E \frac{\sin(\theta/2)}{1+\sin(\theta/2)}$$

$$D_{\rm qel}(E,\theta) \sim D_{\rm qel}(E_{\rm eff},\pi)$$

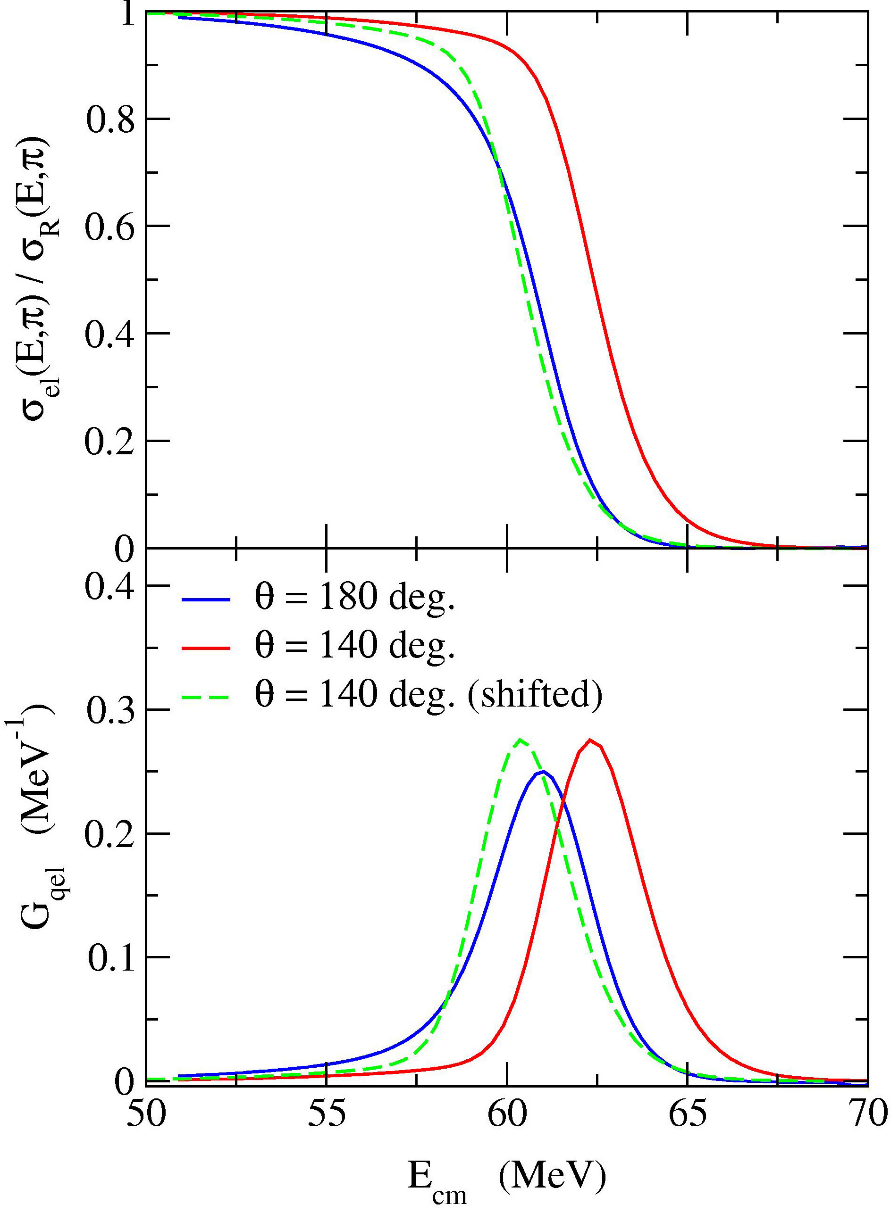
$$\lambda_c = \eta \cot(\theta/2)$$



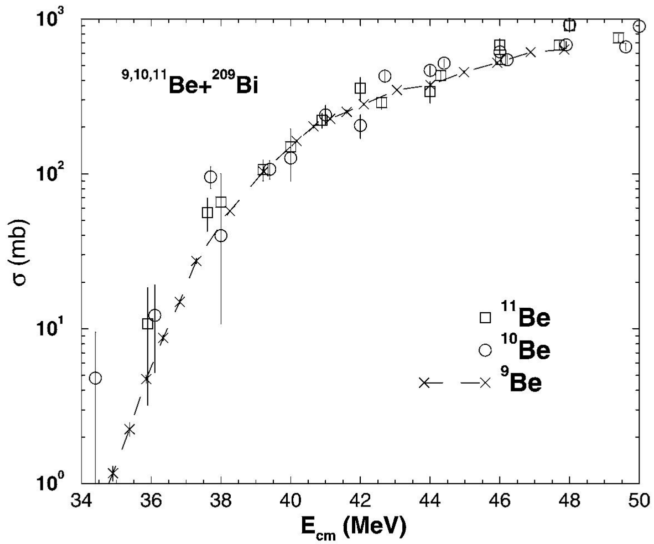
Scaling property of D_{qel}



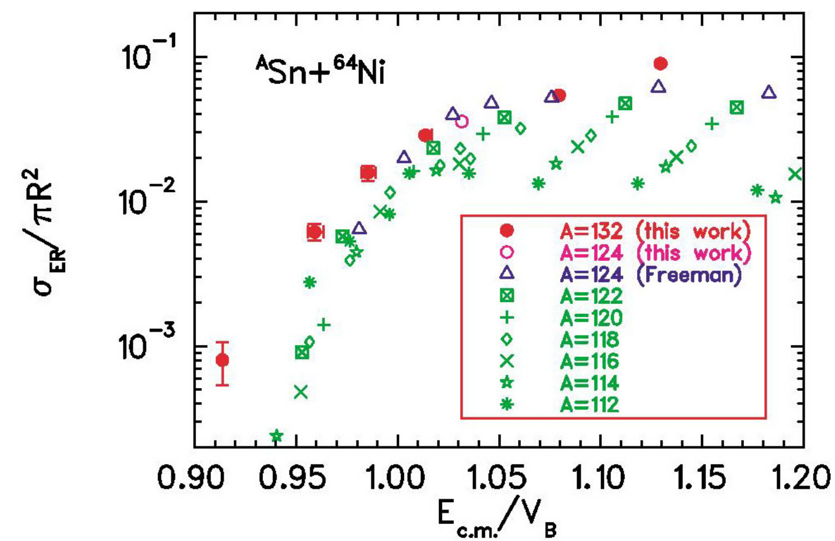
$^{16}O + ^{144}Sm$



Fusion of neutron-rich nuclei



C. Signorini et al., NPA735('04)329



J.F. Liang et al., PRL 91('03) 152701

Theoretical works:

- M.S. Hussein, M.P. Pato, L.F. Canto, and R. Donangelo, PRC46('92)377
- N. Takigawa, M. Kuratani, and H. Sagawa, PRC 47('93) 2470
- C.H. Dasso and A. Vitturi, PRC50('94)R12
- K. H., A. Vitturi, C.H. Dasso, and S.M. Lenzi, PRC61('00)037602
- A. Diaz-Torres and I.J. Thompson, PRC65('02)024606
- K. Yabana, M. Ueda, and T. Nakatsukasa, NPA722('03)261c

Physics of fusion induced by exotic beams

1) Synthesis of heavy drip-line nuclei

→ requires to use radioactive neutron-rich beams

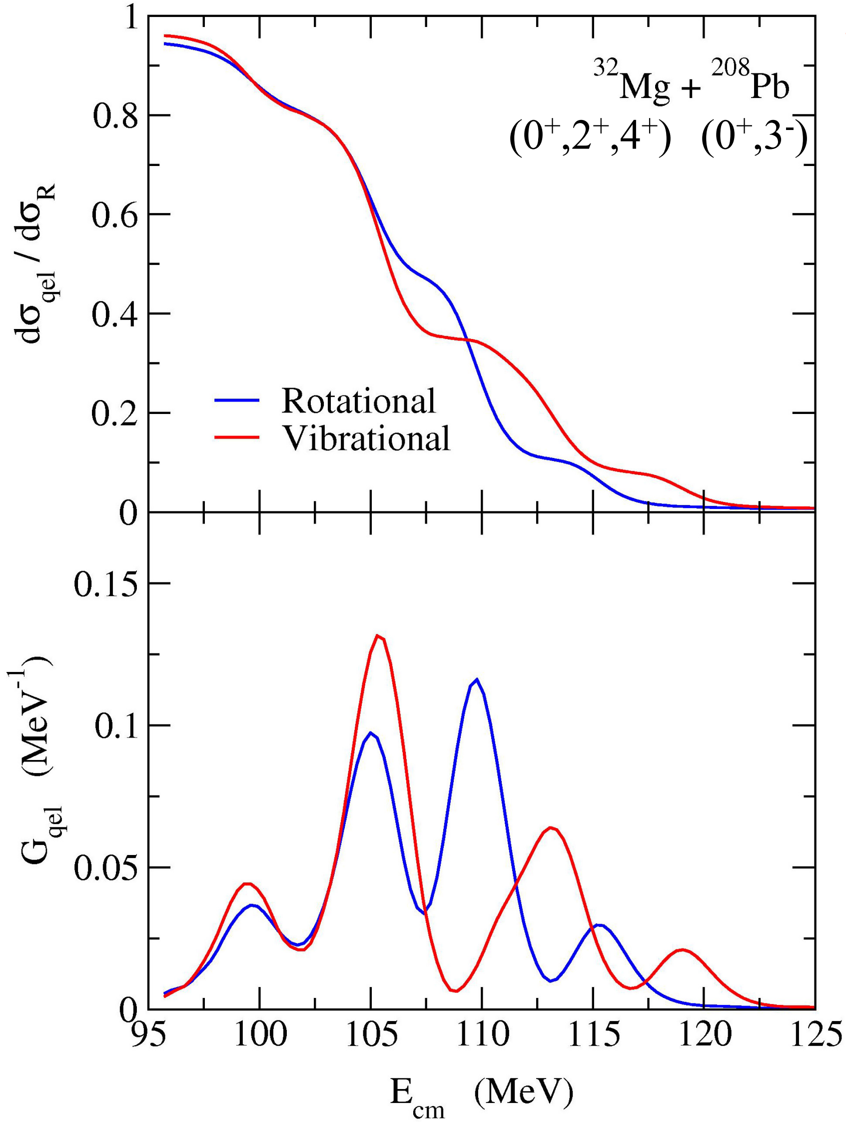
→ need to know the reaction mechanisms

2) Reaction dynamics itself: interesting

- Interplay between the break-up and the halo/skin structure
- Effect of irreversible process on quantum tunneling
- **Possibility of probing unique collective excitations in neutron-rich nuclei**

Quasi-elastic reaction: counter part of fusion

D_{qel} measurements with radioactive beams



K.H. and N. Rowley, PRC69('04)054610

Low intensity radioactive beams:

High precision fusion measurements
→ still difficult

Quasi-elastic measurements
→ may be possible

(possible examples)

^{32}Mg:

Expt. at RIKEN and GANIL:
large B(E2) and small E_{2+}
↔ deformation?
MF calculations → spherical?

^{16}C:

Opposite (static) deformation
between proton and neutron?
Neutron vibrational mode?

Reference cross sections

Does break-up hinder/enhance fusion cross sections?

How to choose reference cross sections?

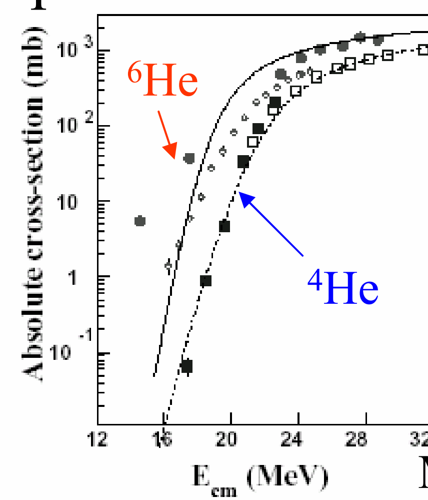
→ Fusion enhancement/hindrance compared to ***what***?

i) Comparison to tightly-bound systems

$^{11}Be + ^{209}Bi \longleftrightarrow ^{10}Be + ^{209}Bi$

$^{6}He + ^{238}U \longleftrightarrow ^{4}He + ^{238}U$

Separation between static and dynamical effects?



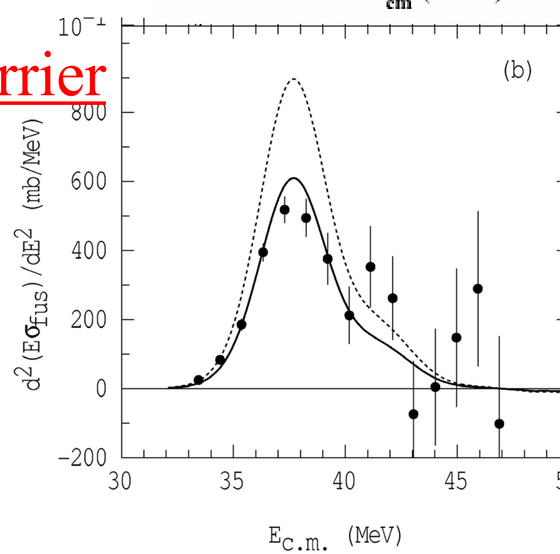
M. Trotta et al. PRL84('00)2342

ii) Measurement of average fusion barrier

← Fusion barrier distribution

$^{9}Be + ^{208}Pb$

$^{6,7}Li + ^{209}Bi$



M. Dasgupta et al. PRL82('99)1395

Neutron-rich nuclei ⟶ $D_{qel}(E)$

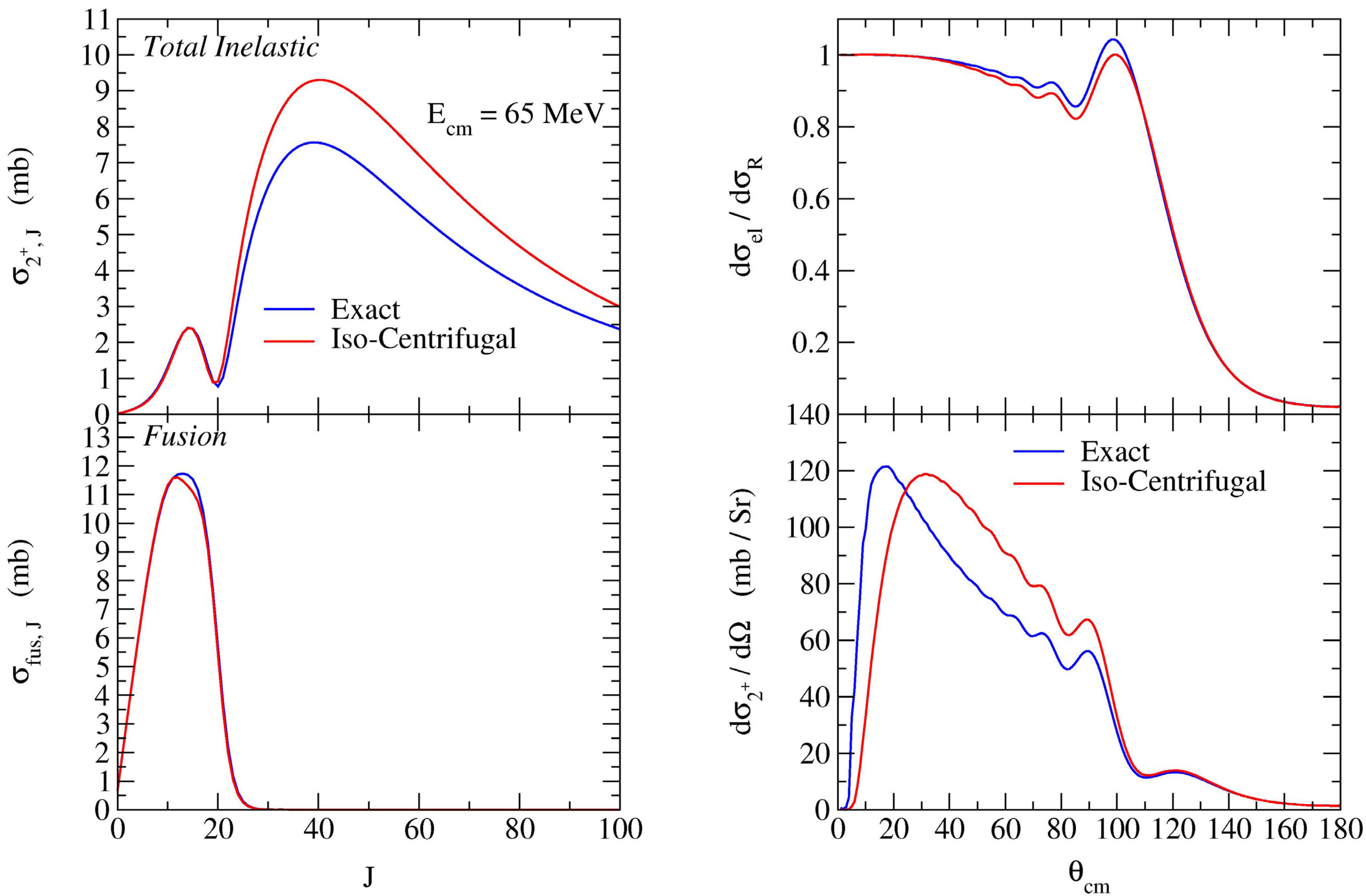
Summary

$$D_{\mathsf{fus}}(E) = \frac{d^2(E\sigma_{\mathsf{fus}})}{dE^2} \qquad D_{\mathsf{qel}}(E) = -\frac{d}{dE}\left(\frac{\sigma_{\mathsf{qel}}(E,\pi)}{\sigma_R(E,\pi)}\right)$$

- Barrier distributions: sensitive to nuclear structure
- Quasi-elastic b.d. $D_{qel}(E)$ ⟷ counterpart of fusion b.d. $D_{fus}(E)$
- $D_{qel}(E)$: allows measurements with a cyclotron accelerator
- $D_{qel}(E)$: well suited to future expt. with low-intensity exotic beams

$D_{qel}(E)$: may be opening up a novel way to probe the structure of neutron-rich nuclei

$^{16}O + {}^{144}Sm(2^+)$



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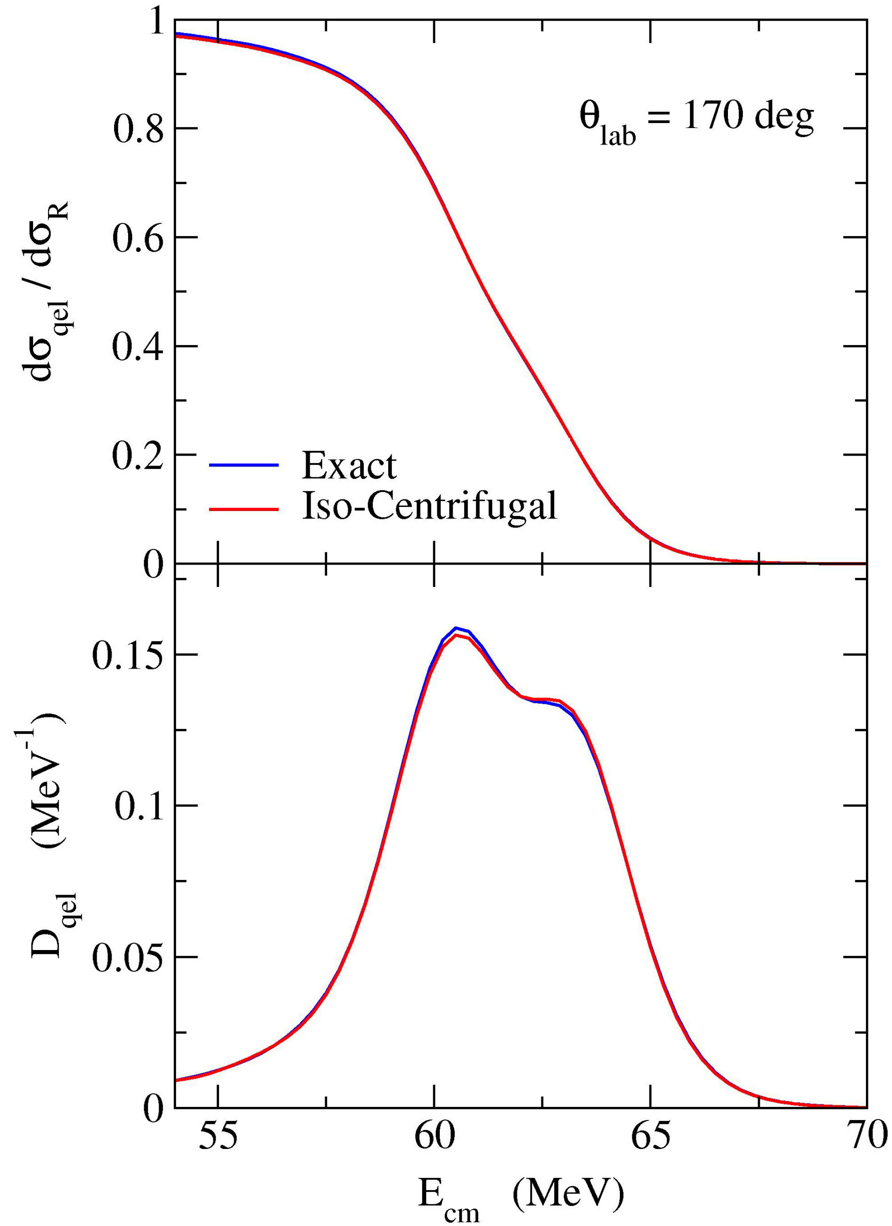
$^{16}O + ^{144}Sm(2^+)$



θlab = 170 deg
dσqel / dσR
Exact
Iso-Centrifugal
Dqel (MeV-1)
Ecm (MeV)
0
0.2
0.4
0.6
0.8
1
0.05
0.1
0.15
55
60
65
70