Nuclear Structure from First Principles: *Ab Initio* No-Core Shell Model for Light Nuclei

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- I. Introduction to the *ab initio* no-core shell model (NCSM)
  - Nucleon-nucleon (NN) interaction
  - Few-body systems
  - Methods for $A>4$ systems
  - *Ab initio* NCSM theory
    - Hamiltonian
    - Basis
    - Effective interaction
  - NCSM convergence tests
- II. Importance of three-nucleon interaction
  - NCSM application to *p*-shell nuclei
  - Failure of standard NN interactions
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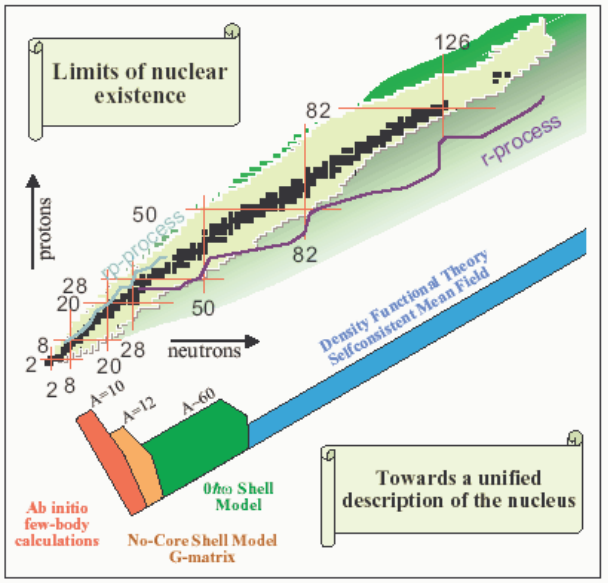
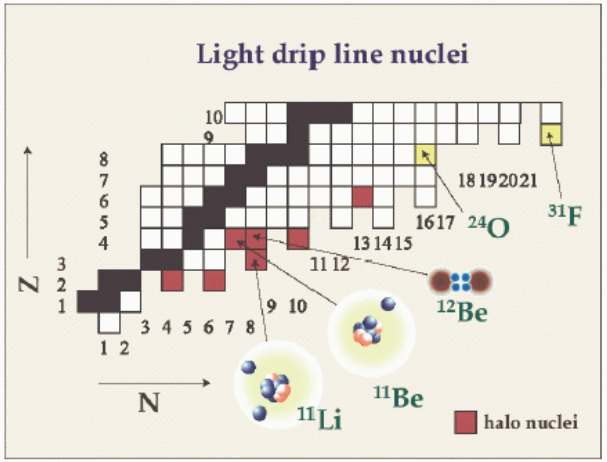
Nuclear structure from first principles

Do we really know how nuclei are put together?

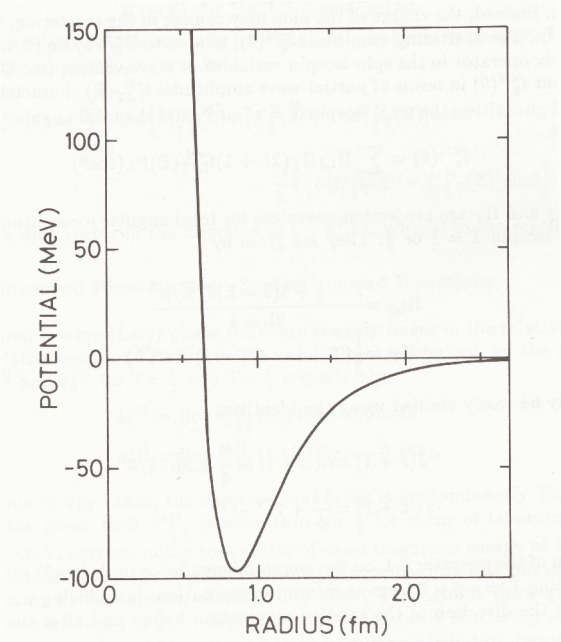
- Nuclear physics is a mature field, but there are still many unanswered questions about nuclei
- Atomic nuclei make up the vast majority of matter that we can see. How did they get there?
  - One of the key questions in science
    - Nucleosynthesis
    - Structure plays an important role
- The nuclear many-body problem is one of the hardest problems in all of physics!
  - Because nucleon-nucleon interaction is complicated
- A canonical nuclear structure problem: What are the properties of a system that consists of
  - *A* point-like nucleons
  - Nonrelativistic
  - Interact by inter-nucleon interactions that describe exactly (*or* as accurately as possible) two-nucleon system or two-and three-nucleon system





Why nuclear structure from first principles right now?

- 1993: New Nijmegen database of nucleon-nucleon data
  - Partial wave analysis of all NN scattering data below 350 MeV
    - Resulted in construction of high quality NN potentials that fit the data with $\chi^2/N \approx 1$
      ➜Nijmegen I,II, Argonne V18, Reid 93, CD-Bonn
- These potentials difficult to use in few- and many-body calculations
  - Short range repulsive core
- However: Increase of computing power makes such calculations feasible & stimulates development of new methods



- What does the two-nucleon interaction predict for *A*-nucleon system?
- Do we need higher-body forces (three-nucleon...)?

Accurate inter-nucleon interactions plus working many-body methods allow us to predict properties of light nuclei even in a situation where no experimental data exist as well as interpret known experimental data

What are the forces in nuclei?

- Start with the simplest case: Two nucleons
  - The deuteron
    - binding energy: 2.224 MeV
    - quadrupole moment: 0.282 fm^2
  - NN-scattering
    - For example: Scattering lengths and ranges for spin singlet *S*-wave pp, nn, and analog pn channels

$$a_{pp} = -17.3 \pm 0.4 \text{ fm} \quad a_{nn} = -18.9 \pm 0.4 \text{ fm} \quad a_{pn} = -23.74 \pm 0.02 \text{ fm}$$
$$r_{pp} = 2.85 \pm 0.04 \text{ fm} \quad r_{nn} = 2.75 \pm 0.11 \text{ fm} \quad r_{pn} = 2.77 \pm 0.05 \text{ fm}$$

    ➜ Unbound!

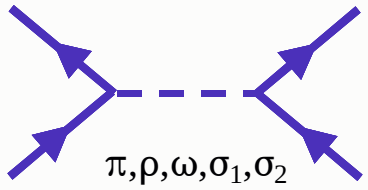
    ➜ Note that $V_{pp} \neq V_{nn} \neq V_{pn}$: Charge independence breaking (CIB), charge symmetry breaking (CSB)

- From these we infer the form of the nucleon-nucleon interaction
  - The starting point is the Yukawa hypothesis of meson exchange
    - Pion, rho, omega, two pion (sigma), etc.
- Features: short range, attractive in its intermediate range, repulsive core, tensor force, spin-orbit force
- Two-nucleon channels: spin *s*, relative orbital momentum *l*, total angular momentum *j* $|l - s| \leq j \leq l + s$, isospin *t*, parity = $(-1)^l$, antisymmetry: $(-1)^{l+s+t} = -1$
  - Coupled channels: $s=1$, $l=j-1$, $j+1$
    - ${}^3S_1$-${}^3D_1$ ... deuteron, ${}^3P_2$-${}^3F_2$, ${}^3D_3$-${}^3G_3$ ...
  - Uncoupled channels: $s=0$, $l=j$ or $s=1$, $l=j$ or $s=1$, $l=1$, $j=0$
    - ${}^1S_0$, ${}^3P_0$, ${}^1P_1$, ${}^3P_1$, ${}^1D_2$, ${}^3D_2$ ...
  - *s*, *j*, parity, *t* conserved

NN potentials

Designed to reproduce the deuteron and NN-scattering

- Argonne potentials (AV18, AV8')
  - R.B. Wiringa, V.G.J. Stoks, R. Schiavilla, PRC **51**, 38 (1995)
  - Electromagnetic + one pion exchange + intermediate- and short-range, local, in coordinates
- Bonn potential (CD-Bonn 2000)
  - R. Machleidt, PRC **63**, 024001 (2001)
  - Based on meson-exchange
  - Nonlocal, in momentum space

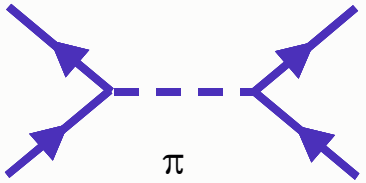


- Effective field theory
  - C. Ordóñez, L. Ray, U. van Kolck, PRC **53**, 2086 (1996);
    E. Epelbaoum, W. Glöckle, Ulf-G. Meißner, NP **A637**, 107 (1998)
  - D. R. Entem and R. Machleidt, PRC **68**, 041001(R) (2003)
    - N^3LO - fourth order of perturbation theory
  - Based on Chiral Lagrangians
  - Expansion in momentum relative to a cutoff parameter (~ 1 GeV)
  - Generally has a soft core, nonlocal, in momentum space
- Phenomenolgical nonlocal potential in coordinate space
  - INOY = Inside nonlocal, outside Yukawa
  - P. Doleschall *et al.*, PRC **67**, 064005 (2003)
  - Fits both two-nucleon and three-nucleon properties

A>2 systems and off-shell components of NN interactions

- Pion exchange is an integral part of NN interactions
  - Elastic scattering in momentum space

$$V_{local}^{\pi NN}(\mathbf{q}=\mathbf{k}'-\mathbf{k}) = -\frac{g_\pi^2}{4M^2}\frac{(\sigma_1\cdot\mathbf{q})(\sigma_2\cdot\mathbf{q})}{\mathbf{q}^2+m_\pi^2}$$



  - Or, through a Fourier transform, coordinate space ($\mu = m_\pi c/\hbar$)

$$V_\pi = \frac{g_\pi^2}{4M^2}\frac{1}{3}m_\pi\left[\sigma_1\cdot\sigma_2+\left(1+\frac{3}{\mu r}+\frac{3}{(\mu r)^2}\right)\underbrace{(3\sigma_1\cdot\hat{\mathbf{r}}\sigma_2\cdot\hat{\mathbf{r}}-\sigma_1\cdot\sigma_2)}_{\text{Tensor operator}}\right]\frac{e^{-\mu r}}{\mu r}$$

- Off-shell ($|\mathbf{k}|\neq|\mathbf{k'}|$) component present, e.g. in the Bonn or EFT potentials

$$V^{\pi NN}(\mathbf{k}',\mathbf{k}) = -\frac{g_\pi^2}{4M^2}\frac{(E'+M)(E+M)}{(\mathbf{k}'-\mathbf{k})^2+m_\pi^2}\left(\frac{\sigma_1\cdot\mathbf{k}'}{E'+M}-\frac{\sigma_1\cdot\mathbf{k}}{E+M}\right)\times\left(\frac{\sigma_2\cdot\mathbf{k}'}{E'+M}-\frac{\sigma_2\cdot\mathbf{k}}{E+M}\right)$$

  - Nonlocal
    - Depends on the energies E and E' of the initial and final state
    - Or, through a Fourier transform, on relative positions ***r*** and ***r'*** in initial and final state
  - Plays a role in many-body (A>2) applications and provides more binding

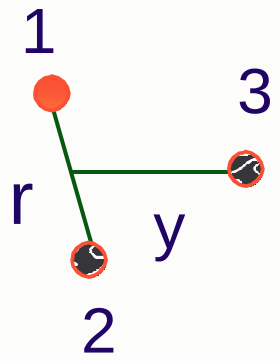
Methods for solving few-nucleon (A=3,4) systems with realistic inter-nucleon interactions

Nonrelativistic point-like nucleons interacting by realistic two- and three-nucleon forces

- A=3,4 - many exact methods
  - Faddeev equations for A=3
    - Jacobi coordinates
    - Faddeev amplitude $\Phi_3(\vec{r},\vec{y})$

$$\vec{r} = \frac{1}{\sqrt{2}}\left[\vec{r}_1 - \vec{r}_2\right]$$

$$\vec{y} = \sqrt{\frac{2}{3}}\left[\tfrac{1}{2}(\vec{r}_1 + \vec{r}_2) - \vec{r}_3\right]$$



$$\Psi = \Phi_1 + \Phi_2 + \Phi_3$$

$$\left[T + V(\vec{r})\left(1 + \tau_+ + \tau_-\right)\right]\Phi_3 = E\Phi_3$$

  - 2001: A=4 benchmark paper: 7 different approaches obtained the same ^{4}He bound state properties using the Argonne V8' nucleon-nucleon potential
    - Faddeev-Yakubovsky equations
    - CRCGV = coupled-rearrangement-channel Gaussian-basis variational method
    - SVM = stochastic variational method
    - GFMC = Green's function Monte Carlo method
    - HH variational = hyperspherical harmonic variational method
    - EIHH = effective interaction hyperspherical harmonic method
    - NCSM = no-core shell model

Methods for solving $A>4$ systems with realistic inter-nucleon interactions

Nonrelativistic point-like nucleons interacting by realistic two- and three-nucleon forces

- $A>4$ - few methods applicable when realistic two- and three-nucleon forces used
  - Green's Function Monte Carlo (GFMC)
    - S. Pieper, R. Wiringa, J. Carlson *et al.*
    - Results published up to $A=10$, ^{12}C calculations under way

$$\Psi_0 = \lim_{\tau\to\infty} e^{-(H-E_0)\tau}\Psi_T$$

  - Coupled-Cluster Method (CCM), Unitary Model Operator Approach (UMOA)
    - Applicable mostly to closed shell nuclei
      - ➜ ^{16}O results by Mihaila and Heisenberg, D. Dean and M. Hjort-Jensen (CCM)
      - ➜ K. Suzuki and R. Okamoto (UMOA)

$$\left|\Psi_0\right\rangle = e^{S^+}\left|\Phi_0\right\rangle;\ S^+ = \sum_n S_n^+$$

  - Effective Interaction Hyperspherical Harmonic Method (EIHH)
    - N. Barnea, W. Leidemann, G. Orlandini
    - Converged results for A=6,7 with semi-realistic forces, first results for ^{6}Li with AV8'
    - Now in principle capable of using realistic three-body forces
  - *Ab Initio* No-Core Shell Model (NCSM)
    - **Very simple**, when just the two-body effective interaction considered
    - **It works**
    - Much more complicated when three-body interaction included
    - D.-C. Zheng, B. R. Barrett and J. P. Vary, 1993, G-matrix
    - P. N. and B. R. Barrett, 1996
      - ➜ unitary transformation based effective interaction
      - ➜ convergence to exact solution
    - P. N. and W. E. Ormand, 2003
      - ➜ Three-body interaction included in *p*-shell nuclei calculations

Ab initio no-core shell-model

Goal: Solution of nuclear structure problem for light nuclei

- Many-body Schroedinger equation
  - *A*-nucleon wave function

$$H|\Psi\rangle = E|\Psi\rangle$$

- Hamiltonian

$$H = \sum_{i=1}^{A} \frac{\vec{p}_i^{\,2}}{2m} + \sum_{i<j}^{A} V_{NN}(\vec{r}_i - \vec{r}_j) \quad \left(+ \sum_{i<j<k}^{A} V_{ijk}^{3b} \right)$$

  - Realistic nucleon-nucleon potentials
    - Coordinate space - Argonne V18, AV8', (three-body Tucson-Melbourne)
    - Momentum space - CD-Bonn, Chiral
  - Modification by center-of-mass harmonic oscillator (HO) potential (Lipkin 1958)

$$\frac{1}{2} Am\Omega^2 \vec{R}^2 = \sum_{i=1}^{A} \frac{1}{2} m\Omega^2 \vec{r}_i^{\,2} - \sum_{i<j}^{A} \frac{m\Omega^2}{2A} (\vec{r}_i - \vec{r}_j)^2 \qquad \vec{R} = \frac{1}{A} \sum_{i=1}^{A} \vec{r}_i$$

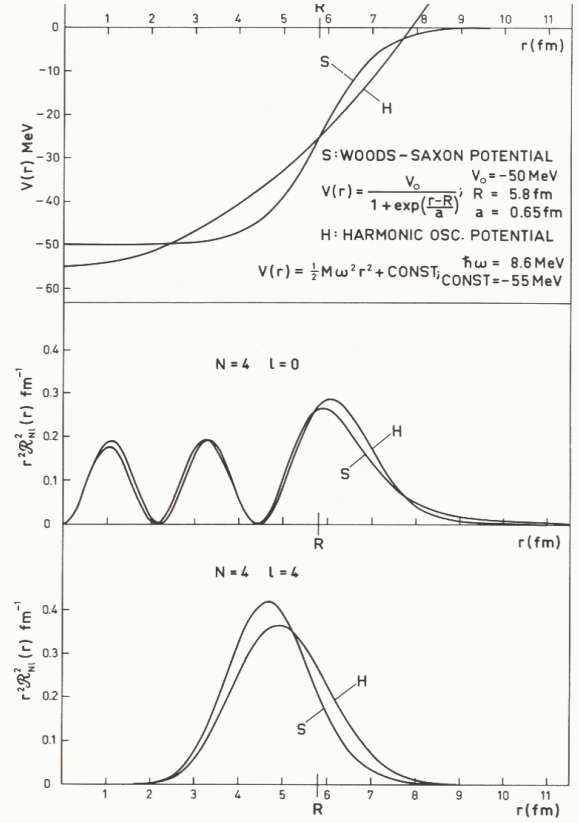
    - No influence on the internal motion (in infinite space) ☺
    - Introduces **mean field for sub-clusters** ☺
    - Convenient to work in the HO basis

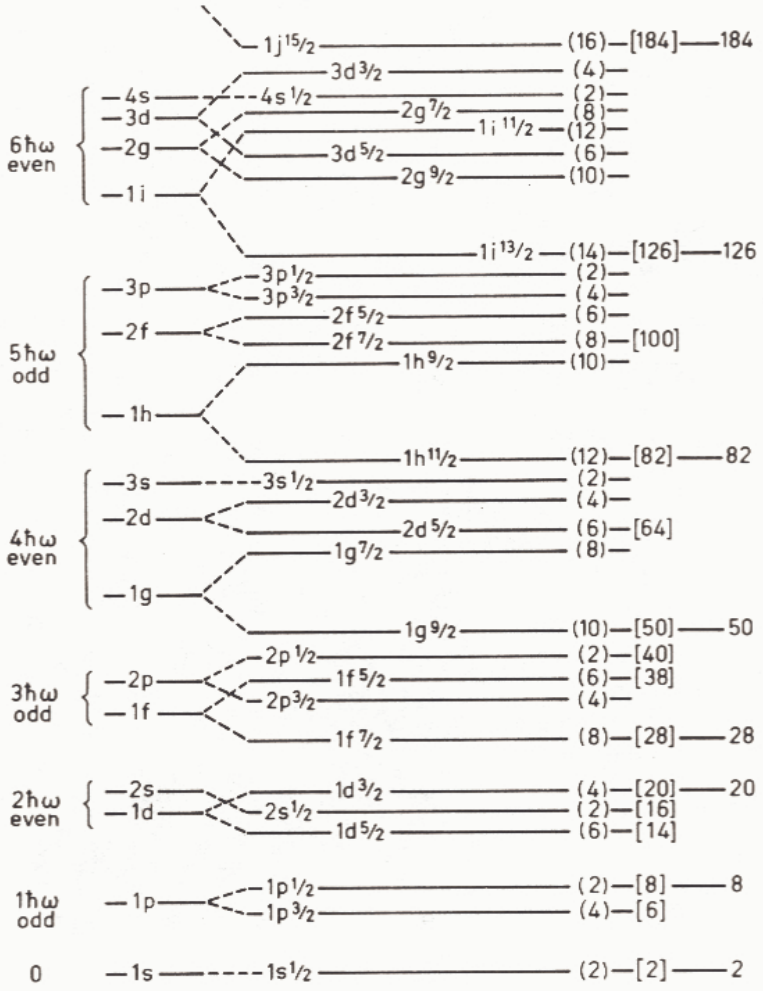
$$H^{\Omega} = \sum_{i=1}^{A} \left[\frac{\vec{p}_i^{\,2}}{2m} + \frac{1}{2} m\Omega^2 \vec{r}_i^{\,2} \right] + \sum_{i<j}^{A} \left[V_{NN}(\vec{r}_i - \vec{r}_j) - \frac{m\Omega^2}{2A} (\vec{r}_i - \vec{r}_j)^2 \right] \quad \left(+ \sum_{i<j<k}^{A} V_{ijk}^{3b} \right)$$

Origin of the shell model

- Goeppert-Mayer and Haxel, Jensen, and Suess proposed the independent-particle shell model to explain the magic numbers

Harmonic oscillator with spin-orbit is a reasonable approximation to the nuclear mean field





M.G. Mayer and J.H.D. Jensen, *Elementary Theory of Nuclear Shell Structure* , p. 58, Wiley, New York, 1955

Coordinates, basis and model space

Bound states or narrow resonances: square-integrable *A*-nucleon basis

- NN (and three-nucleon) interaction depends on relative coordinates
  - Translationally invariant system
  - We should use Jacobi (relative) coordinates
- However:
- We use (a finite) **harmonic-oscillator** basis
  - Bad asymptotics ☹
    - Large basis, renormalization
  - The only basis that allows a switch from Jacobi coordinates to Cartesian coordinates without violating translational invariance ☺
    - Use of powerful second quantization shell model techniques
    - Choice of whatever is more efficient

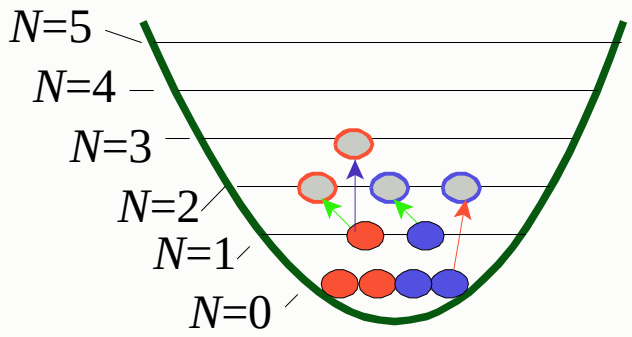
    ➜*A*>4 Cartesian coordinate Slater determinant basis
- *And* complete $N_{max}\hbar\Omega$ model space
  - Translational invariance even when Cartesian coordinate Slater determinant basis used

    ➜True only for harmonic oscillator basis

$$\varphi_{nlm}(\vec{r}) = R_{nl}(r)Y_{lm}(\hat{r})$$

$$\varphi_{nljj_z}(\vec{r},\sigma,\tau) = R_{nl}(r)\left[Y_l(\hat{r})\otimes\chi^S_{\frac{1}{2}}(\sigma)\right]^{(j)}_{j_z}\chi^T_{\frac{1}{2}m_t}(\tau)$$

$$\left[Y_l(\hat{r})\otimes\chi^S_{\frac{1}{2}}(\sigma)\right]^{(j)}_{j_z} = \sum_{ms_z}(lm\tfrac{1}{2}s_z \mid jj_z)Y_{lm}(\hat{r})\chi^S_{\frac{1}{2}s_z}$$



Translationally-invariant HO basis

Jacobi coordinates

- Larger model space ☺
  - Remove COM degrees of freedom
  - HO basis depending on Jacobi coordinates coupled to J and T
- Two-, three- and higher-body interaction ☺
  - Flexible basis rearangements, different clustering
- Limitations
  - Complicated antisymmetrization ☹
    - Profitable for few-nucleon systems A=3-4

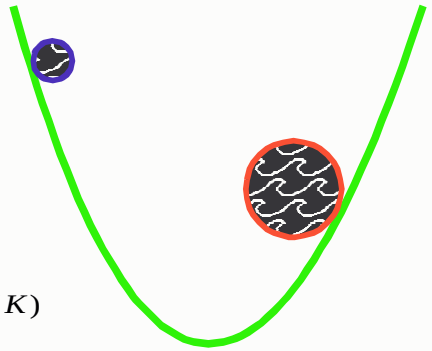
Antisymmetrization of translationally-invariant HO basis

- Transformation properties of HO states

$$\vec{r} = \sqrt{\frac{d}{1+d}}\vec{r}_1 - \sqrt{\frac{1}{1+d}}\vec{r}_2$$

$$\vec{R} = \sqrt{\frac{1}{1+d}}\vec{r}_1 + \sqrt{\frac{d}{1+d}}\vec{r}_2$$

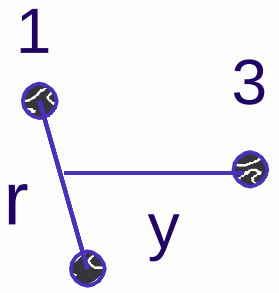
$$\left[\varphi_{n_1 l_1}(\vec{r}_1)\varphi_{n_2 l_2}(\vec{r}_2)\right]_k^{(K)} = \sum_{nlNL} \left\langle nlNLK \middle| n_1 l_1 n_2 l_2 K \right\rangle_d \left[\varphi_{nl}(\vec{r})\varphi_{NL}(\vec{R})\right]_k^{(K)}$$



- Jacobi coordinates for A=3

$$\vec{r} = \frac{1}{\sqrt{2}}\left[\vec{r}_1 - \vec{r}_2\right]$$

$$\vec{y} = \sqrt{\frac{2}{3}}\left[\tfrac{1}{2}(\vec{r}_1 + \vec{r}_2) - \vec{r}_3\right]$$

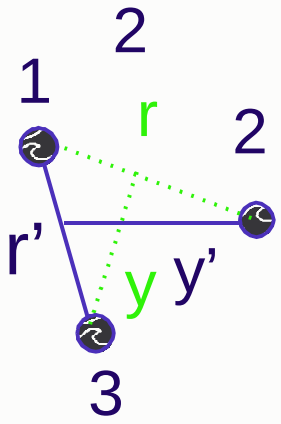


- Transposition 2↔3

$$\vec{r}' = \frac{1}{\sqrt{2}}\left[\vec{r}_1 - \vec{r}_3\right] = \frac{1}{2}\vec{r} + \frac{\sqrt{3}}{2}\vec{y}$$

$$\vec{y}' = \sqrt{\frac{2}{3}}\left[\tfrac{1}{2}(\vec{r}_1 + \vec{r}_3) - \vec{r}_2\right] = \frac{\sqrt{3}}{2}\vec{r} - \frac{1}{2}\vec{y}$$

d=3



Antisymmetrizer for A=3

- Basis $\left|nlsjt,\mathrm{NLJ},JT\right\rangle$

$$\left\langle \vec{r},\sigma_1,\sigma_2,\tau_1,\tau_2\left|nlsjt\right.\right\rangle = R_{nl}(r)\left[Y_l(\hat{r})\otimes\left[\chi^S_{\frac{1}{2}}(\sigma_1)\otimes\chi^S_{\frac{1}{2}}(\sigma_2)\right]^{(s)}\right]^{(j)}\left[\chi^T_{\frac{1}{2}}(\tau_1)\otimes\chi^T_{\frac{1}{2}}(\tau_2)\right]^{(t)}$$

$$\left\langle \vec{y},\sigma_3,\tau_3\left|\mathrm{NLJ}\right.\right\rangle = R_{\mathrm{NL}}(y)\left[Y_{\mathrm{L}}(\hat{y})\otimes\chi^S_{\frac{1}{2}}(\sigma_3)\right]^{(\mathrm{J})}\chi^T_{\frac{1}{2}}(\tau_3)$$

  - No antisymmetrization except for 1↔2: $(-1)^{l+s+t} = -1$
- $\langle X\rangle = 1/3\langle 1-\tau_{13}-\tau_{23}\rangle$
  - Eigenspace corresponding to eigenvalue 1 → *physical* eigenstates
  - Eigenspace corresponding to eigenvalue 0 → *spurious* eigenstates

$$\left\langle n_1l_1s_1t_1j_1,\mathsf{N}_1\mathsf{L}_1\mathsf{J}_1,JT\left|\tau_+ + \tau_-\right|n_2l_2s_2t_2j_2,\mathsf{N}_2\mathsf{L}_2\mathsf{J}_2,JT\right\rangle = -\delta_{N_1,N_2}\hat{t}_1\hat{t}_2\begin{Bmatrix}\frac{1}{2} & \frac{1}{2} & t_1\\ \frac{1}{2} & T & t_2\end{Bmatrix}$$

$$\times\sum_{LS}\hat{L}^2\hat{S}^2\hat{s}_1\hat{s}_2\hat{j}_1\hat{j}_2\hat{\mathsf{J}}_1\hat{\mathsf{J}}_2(-1)^L\begin{Bmatrix}l_1 & s_1 & j_1\\ \mathsf{L}_1 & \frac{1}{2} & \mathsf{J}_1\\ L & S & J\end{Bmatrix}\begin{Bmatrix}l_2 & s_2 & j_2\\ \mathsf{L}_2 & \frac{1}{2} & \mathsf{J}_2\\ L & S & J\end{Bmatrix}\begin{Bmatrix}\frac{1}{2} & \frac{1}{2} & s_1\\ \frac{1}{2} & S & s_2\end{Bmatrix}$$

$$\times\left[(-1)^{s_1+s_2+t_1+t_2-\mathsf{L}_1-l_1}\left\langle \mathsf{N}_1\mathsf{L}_1n_1l_1L\left|n_2l_2\mathsf{N}_2\mathsf{L}_2L\right.\right\rangle_{d=3} + \left\langle n_1l_1\mathsf{N}_1\mathsf{L}_1L\left|\mathsf{N}_2\mathsf{L}_2n_2l_2L\right.\right\rangle_{d=3}\right]$$

  - $\langle\tau_{13}\rangle = \langle\tau_{23}\rangle$

Slater-determinant HO basis

Single-particle coordinates

- Complete $N_{max}\hbar\Omega$ space
  - Total separation of center-of-mass and internal motion
  - Physical eigenstates contain $0\hbar\Omega$ center-of-mass wave function ☹
  - Eigenstates that contain higher than $0\hbar\Omega$ CM components spurious
    - Shifted to higher energy by Lawson projection term
- Application of powerful second quantization techniques ☺ ☺ ☺
- Limits
  - Huge dimensions
  - Three- and higher-body interactions not easy to implement ☹

Slater determinant basis and second quantization

- Second quantization is one of the most useful representations in many-body theory
- Creation and annhilation operators
  - Denote $|0\rangle$ as the state with no particles (the vacuum)
  - a^+_i creates a particle in state i;
  - a_i annhilates a particle in state i;
  - Anticommutation relations:

$$a_i^+|0\rangle = |i\rangle, \quad a_i^+|i\rangle = 0$$
$$a_i|i\rangle = |0\rangle, \quad a_i|0\rangle = 0$$

$$\{a_i^+, a_j^+\} = \{a_i, a_j\}$$
$$\{a_i, a_j^+\} = \{a_j^+, a_i\} = \delta_{ij}$$

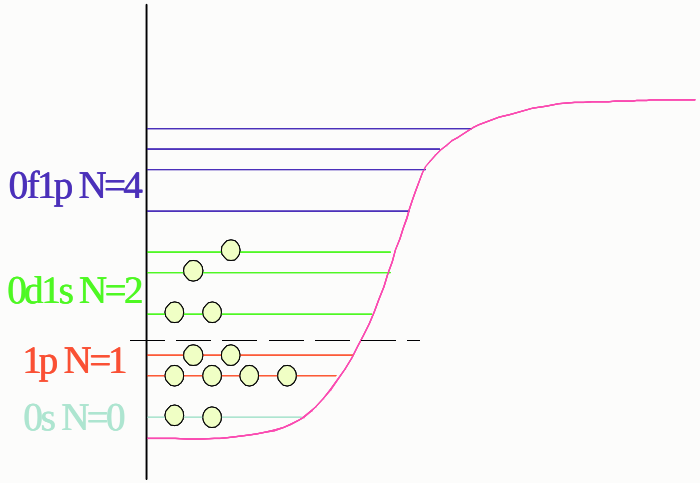
- Many-body Slater determinant

$$\phi_i(\mathbf{r}) = \langle \vec{r}, \sigma, \tau | i \equiv nljmm_t \rangle = R_{nl}(r)\left[Y_l(\hat{r}) \otimes \chi^S_{\frac{1}{2}}(\sigma)\right]^{(j)}_m \chi^T_{\frac{1}{2}m_t}(\tau)$$

$$\Phi = \frac{1}{\sqrt{A!}} \begin{vmatrix} \phi_i(\mathbf{r}_1) & \phi_i(\mathbf{r}_2) & \dots & \phi_i(\mathbf{r}_A) \\ \phi_j(\mathbf{r}_1) & \phi_j(\mathbf{r}_2) & & \phi_j(\mathbf{r}_A) \\ & \vdots & \ddots & \vdots \\ \phi_l(\mathbf{r}_1) & \phi_l(\mathbf{r}_2) & \dots & \phi_l(\mathbf{r}_A) \end{vmatrix} = \underbrace{a_l^+ \dots a_j^+ a_i^+}_{l > \dots > j > i} |0\rangle$$

Building the shell-model basis states

- Need to construct the many-body basis states to calculate matrix elements of H
- Choose states with definite parity, J_z and T_z and let the Hamiltonian do the rest
  - A very useful approach is a bit-representation known as the M-scheme



$$a^{+}_{\frac{5}{2},-\frac{1}{2}} a^{+}_{\frac{5}{2},\frac{3}{2}} a^{+}_{\frac{3}{2},-\frac{1}{2}} a^{+}_{\frac{1}{2},-\frac{1}{2}} \left|0\right\rangle =$$

	0	0	1	0	1	0	0	1	0	0	1	0
$2j_z$	-5	-3	-1	1	3	5	-3	-1	1	3	-1	1
						$0d_{5/2}$				$0d_{3/2}$		$1s_{1/2}$

$$= 2^2 + 2^4 + 2^7 + 2^{10} = 1172$$

A single integer represents a complicated Slater Determinant

Lanczos algorithm for large matrix diagonalization

- Setup Hamiltonian matrix $\langle j|H|i\rangle$ and diagonalize
- Lanczos algorithm
  - Bring matrix to tri-diagonal form

$$\hat{H}\mathbf{v}_1 = \alpha_1\mathbf{v}_1 + \beta_1\mathbf{v}_2$$
$$\hat{H}\mathbf{v}_2 = \beta_1\mathbf{v}_1 + \alpha_2\mathbf{v}_2 + \beta_2\mathbf{v}_3$$
$$\hat{H}\mathbf{v}_3 = \beta_2\mathbf{v}_2 + \alpha_3\mathbf{v}_3 + \beta_3\mathbf{v}_4$$
$$\hat{H}\mathbf{v}_4 = \beta_3\mathbf{v}_3 + \alpha_4\mathbf{v}_4 + \beta_4\mathbf{v}_5$$

After each iteration

need to re-orthogonalize

  - n^{th} iteration computes $2n^{th}$ moment
    - But you can't find eigenvalues with calculated moments
  - Eigenvalues converge to extreme (largest and smallest) values
  - ~ 100-200 iterations needed for 10 eigenvalues (even for 10^8 states)

Slater Det vs. Jacobi-coordinate HO basis

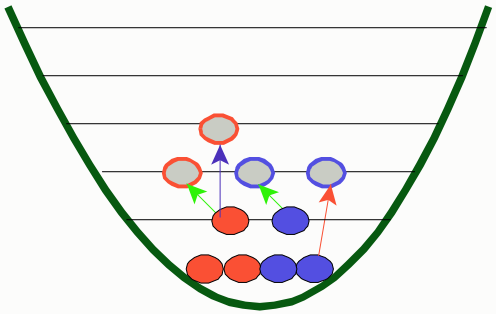
Transformations possible → choose the more efficient

	Slater Det	Jacobi coordinates
Effective interaction (V_{2eff}, V_{3eff}, V_{3b}, V_{4eff})	-	exact
A=3	~22ħΩ	60ħΩ
A=4	~22ħΩ	18ħΩ V_{2eff} , 16ħΩ V_{3eff}
A=5	~16ħΩ	8ħΩ V_{3eff}
A=6,7	~16ħΩ V_{2eff} , 6ħΩ V_{3eff}	4ħΩ V_{3eff}
A=8	10ħΩ V_{2eff} , 4ħΩ V_{3eff}	0ħΩ V_{2eff}
A=9,10	10ħΩ V_{2eff} , 4ħΩ V_{3eff}	-
A=11-16	8ħΩ V_{2eff} , 4ħΩ V_{3eff}	-
Limitations	dimension ~7×10^8 V_{3eff}, V_{4eff} complicated	antisymmetrization

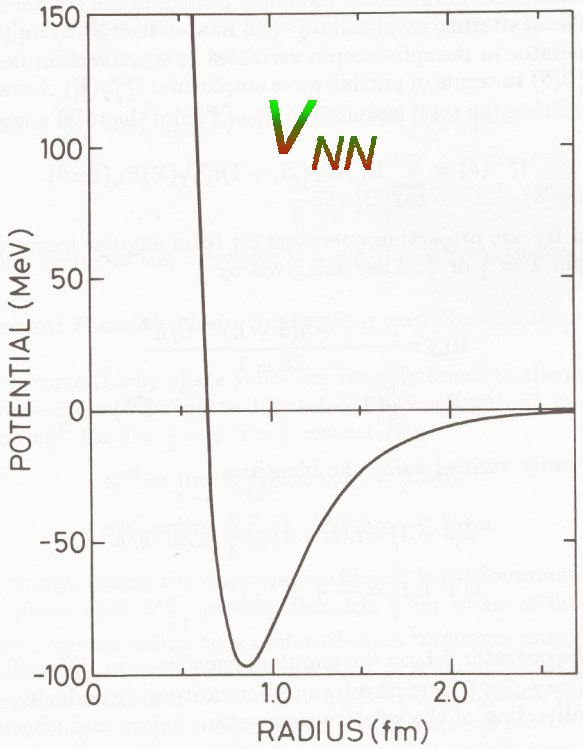
Model space, truncated basis and effective interaction

Strategy: Define Hamiltonian, basis, calculate matrix elements and diagonalize. But:

- **Finite** harmonic-oscillator *Jacobi coordinate* or *Cartesian coordinate Slater determinant* basis
  - Complete $N_{max}\hbar\Omega$ model space



Nucleon-nucleon interaction



Repulsive core in V_{NN} cannot be accomodated in a truncated HO basis

Need for the effective interaction

Non-Hermitian effective Hamiltonian from similarity transformation

Suzuki-Lee, Poves-Zuker, Krenciglowa-Kuo ...

	P	Q
P	H_{eff}	$PXHX^{-1}Q$
Q	0	$QXHX^{-1}Q$

$$H: \quad E_1, E_2, E_3, \cdots E_{dP} \cdots E_\infty$$

$$H_{eff}: \quad E_1, E_2, E_3, \cdots E_{dP}$$

Dimension of $\mathbb{P}$

$$QXHX^{-1}P = 0$$

$$H_{eff} = PXHX^{-1}P$$

- Choice of X

$$X = \exp(-\omega) \; ; \quad \omega = Q\omega P \Rightarrow \quad \omega^2 = 0 \quad \Rightarrow \quad X = 1 - \omega \; ; \quad X^{-1} = 1 + \omega$$

- Decoupling condition $QXHX^{-1}P = 0$

$$QXHX^{-1}P = Q\exp(-\omega)H\exp(\omega)P = QHP - Q\omega PHP + QHQ\omega P - Q\omega PHQ\omega P = 0$$

Non-Hermitian effective Hamiltonian: Solutions for ω

- Schroedinger equation
  - Full space $H\left|\Psi_k\right\rangle = E_k\left|\Psi_k\right\rangle;\quad k = 1,\ldots d_P,\ldots\infty$
  - Model space $H_{eff}\,P\left|\Psi_k\right\rangle = E_k P\left|\Psi_k\right\rangle;\quad k = 1,\ldots d_P$

$$H_{eff} = PHP + PHQ\omega P$$

- Formal solution for ω

$$Q\left|\Psi_k\right\rangle = Q\omega P\left|\Psi_k\right\rangle;\quad k = 1,\ldots d_P$$

$$\left\langle\alpha_Q\left|\omega\right|\alpha_P\right\rangle = \sum_{k=1}^{d_P}\left\langle\alpha_Q\left|\Psi_k\right\rangle\left\langle\tilde{\Psi}_k\right|\alpha_P\right\rangle;\quad \sum_{k=1}^{d_P}\left\langle\alpha_{P'}\left|\Psi_k\right\rangle\left\langle\tilde{\Psi}_k\right|\alpha_P\right\rangle = \delta_{PP'}$$

- Two iterative solutions ↔ $Q\omega_n PHQ\omega_{n-1}P$ *or* $Q\omega_{n-1}PHQ\omega_n P$
  - Convergence to states with largest P-space overlap - Krenciglowa-Kuo
  - Convergence to states closest to energy ε ($\pm\varepsilon H_0$) - Suzuki-Lee

Hermitian effective Hamiltonian

H hermitian $\rightarrow$ H_{cff} quasi-hermitian

- Metric operator and hermitian effective Hamiltonian

$$T = P + \omega^{+}\omega$$

$$TH_{eff} = H_{eff}^{+}T$$

$$\overline{H}_{eff} = (P+\omega^{+}\omega)^{-1/2}(P+\omega^{+}\omega)H_{eff}(P+\omega^{+}\omega)^{-1/2} = (P+\omega^{+}\omega)^{-1/2}(P+\omega^{+})H(P+\omega)(P+\omega^{+}\omega)^{-1/2}$$

$$= (P+\omega^{+}\omega)^{1/2}(PHP+PH\omega P)(P+\omega^{+}\omega)^{-1/2}$$

- Corresponding effective general operator

$$\overline{O}_{eff} = (P+\omega^{+}\omega)^{-1/2}(P+\omega^{+})O(P+\omega)(P+\omega^{+}\omega)^{-1/2}$$

  - Okubo transformation

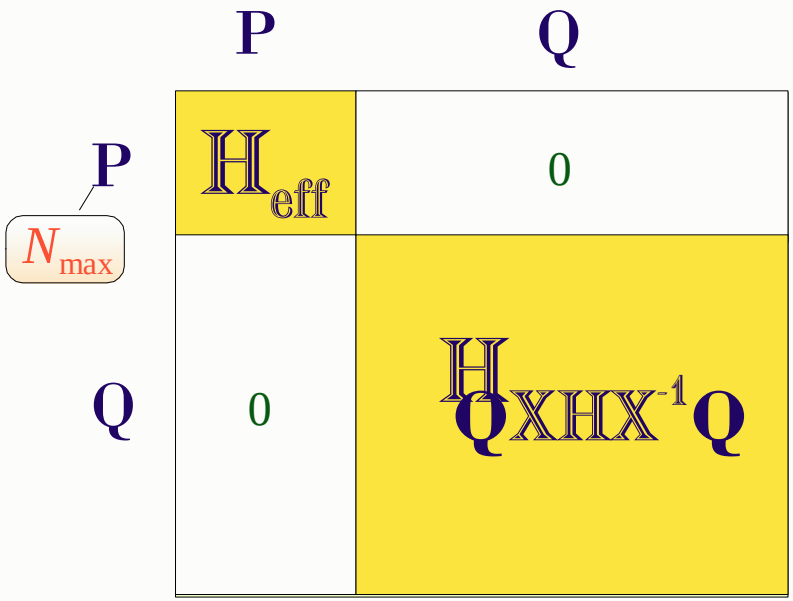
- Alternatively - Suzuki and Okamoto - unitary decoupling transformation

$$\overline{H}_{eff} = P\exp[-\arctan\mathrm{h}(\omega^{+}-\omega)]H\exp[\arctan\mathrm{h}(\omega^{+}-\omega)]P$$

$$\overline{O}_{eff} = P\exp[-\arctan\mathrm{h}(\omega^{+}-\omega)]O\exp[\arctan\mathrm{h}(\omega^{+}-\omega)]P$$

- Hermitization not unique
  - Additional **P**-space unitary transformation possible

Effective Hamiltonian in the NCSM



$$\mathbb{H}: \quad \mathbb{E}_1, \mathbb{E}_2, \mathbb{E}_3, \ldots \mathbb{E}_{d\mathbb{P}} \ldots \mathbb{E}_\infty$$

$$\mathbb{H}_{eff}: \mathbb{E}_1, \mathbb{E}_2, \mathbb{E}_3, \ldots \mathbb{E}_{d\mathbb{P}}$$

Dimension of $\mathbb{P}$

$$\mathbf{QXHX^{-1}P} = \mathbb{0}$$

$$\mathbb{H}_{eff} = \mathbf{PXHX^{-1}P}$$

Unitary $X = \exp[-\arctan h(\omega^+ - \omega)]$

- Properties of $\mathbb{H}_{eff}$ for *A*- nucleon system
  - *A*-body operator
    - Even if $\mathbb{H}$ two- (three-) body
  - For $\mathbf{P} \Rightarrow 1$ $\mathbb{H}_{eff} \Rightarrow \mathbb{H}$
- *n*-body cluster approximation, $2 \le n \le A$
  - $\mathbb{H}^{(n)}_{eff}$ *n*-body operator
  - **Two ways of convergence**:
  - For $\mathbf{P} \Rightarrow 1$ $\mathbb{H}^{(n)}_{eff} \Rightarrow \mathbb{H}$
  - For $n \Rightarrow A$ and fixed $\mathbf{P}$ $\mathbb{H}^{(n)}_{eff} \Rightarrow \mathbb{H}_{eff}$

Effective interaction calculation in the NCSM

n-body approximation

- Full space Hamiltonian

$$H^{\Omega} = \sum_{i=1}^{A} h_i + \sum_{i<j}^{A} V_{ij} + \sum_{i<j<k}^{A} V_{ijk}$$

- **Goal**: effective Hamiltonian
  P, Q, P + Q=*1*

$$PH^{\Omega}_{eff}P = P\left[\sum_{i=1}^{A} h_i + \left(\sum_{i<j}^{A} V_{ij} + \sum_{i<j<k}^{A} V_{ijk}\right)_{eff}\right]P$$

What is the best *n*-body effective interaction?

- For ***A=n* reproduces exactly the full space results** for a subset of eigenstates in the $P_n \subset P$ subspace
  - *n*=2, two-body effective interaction approximation

$$h_1 + h_2 + V_{12} \;\rightarrow\; X_2 \;\rightarrow\; P_2\left[h_1 + h_2 + V_{2eff,12}\right]P_2 \;\rightarrow\; P\left[\sum_{i=1}^{A} h_i + \sum_{i<j}^{A} V_{2eff,ij}\right]P$$

  - *n*=3, three-body effective interaction approximation

$$h_1 + h_2 + h_3 + V_{12} + V_{13} + V_{23} + V_{123} \;\rightarrow\; X_3 \;\rightarrow\; P_3\left[h_1 + h_2 + h_3 + V^{2b+3b}_{3eff,123}\right]P_3 \;\rightarrow\; P\left[\sum_{i=1}^{A} h_i + \frac{1}{A-2}\sum_{i<j<k}^{A} V^{2b}_{3eff,ijk} + \sum_{i<j<k}^{A} V^{3b}_{3eff,ijk}\right]P$$

  – Suzuki-Okamoto unitary transformation

$$\mathbf{Q}_n \mathbb{X}_n \mathbb{H}^{(n)} \mathbb{X}_n^{-1} \mathbf{P}_n = \mathbb{0}$$

Now we can subtract H_{CM}

$$H^{\Omega} = \sum_{i=1}^{A} h_i + \sum_{i<j}^{A} V_{ij} + \sum_{i<j<k}^{A} V_{ijk}$$

$$H^{\Omega} = \sum_{i=1}^{A} \left[\frac{\vec{p}_i^{\,2}}{2m} + \frac{1}{2} m\Omega^{\,2} \vec{r}_i^{\,2} \right] + \sum_{i<j}^{A} \left[V_{NN}(\vec{r}_i - \vec{r}_j) - \frac{m\Omega^{\,2}}{2A} (\vec{r}_i - \vec{r}_j)^2 \right] + \sum_{i<j<k}^{A} V_{ijk}^{3b}$$

$$h_1 + h_2 + V_{12}$$

$$H^{\Omega} = \frac{\vec{p}_1^{\,2}}{2m} + \frac{1}{2} m\Omega^2 \vec{r}_1^{\,2} + \frac{\vec{p}_2^{\,2}}{2m} + \frac{1}{2} m\Omega^2 \vec{r}_2^{\,2} + V_{NN}(\vec{r}_1 - \vec{r}_2) - \frac{m\Omega^2}{2A}(\vec{r}_1 - \vec{r}_2)^2$$

$$Q_2 X_2 [h_1 + h_2 + V_{12}] X_2^{-1} P_2 = 0$$

$$P_2 X_2 [h_1 + h_2 + V_{12}] X_2^{-1} P_2 = P_2 [h_1 + h_2 + V_{2eff,12}] P_2$$

Takes into account two-nucleon correlations from outside the model space

Dominantly short range correlations due to the repulsive core

$$P\left[\sum_{i<j}^{A}\left[\frac{(\vec{p}_i-\vec{p}_j)^2}{2Am}+\frac{m\Omega^2}{2A}(\vec{r}_i-\vec{r}_j)^2\right]+\frac{1}{A-2}\sum_{i<j<k}^{A}V_{3eff,ijk}^{2b}+\sum_{i<j<k}^{A}V_{3eff,ijk}^{3b}\right]P$$

$$+\beta P\left(H_{CM}^{HO}-\tfrac{3}{2}\hbar\Omega\right)P$$

Lawson projection term shifts higher than 0$\hbar\Omega$ CM components to higher energy

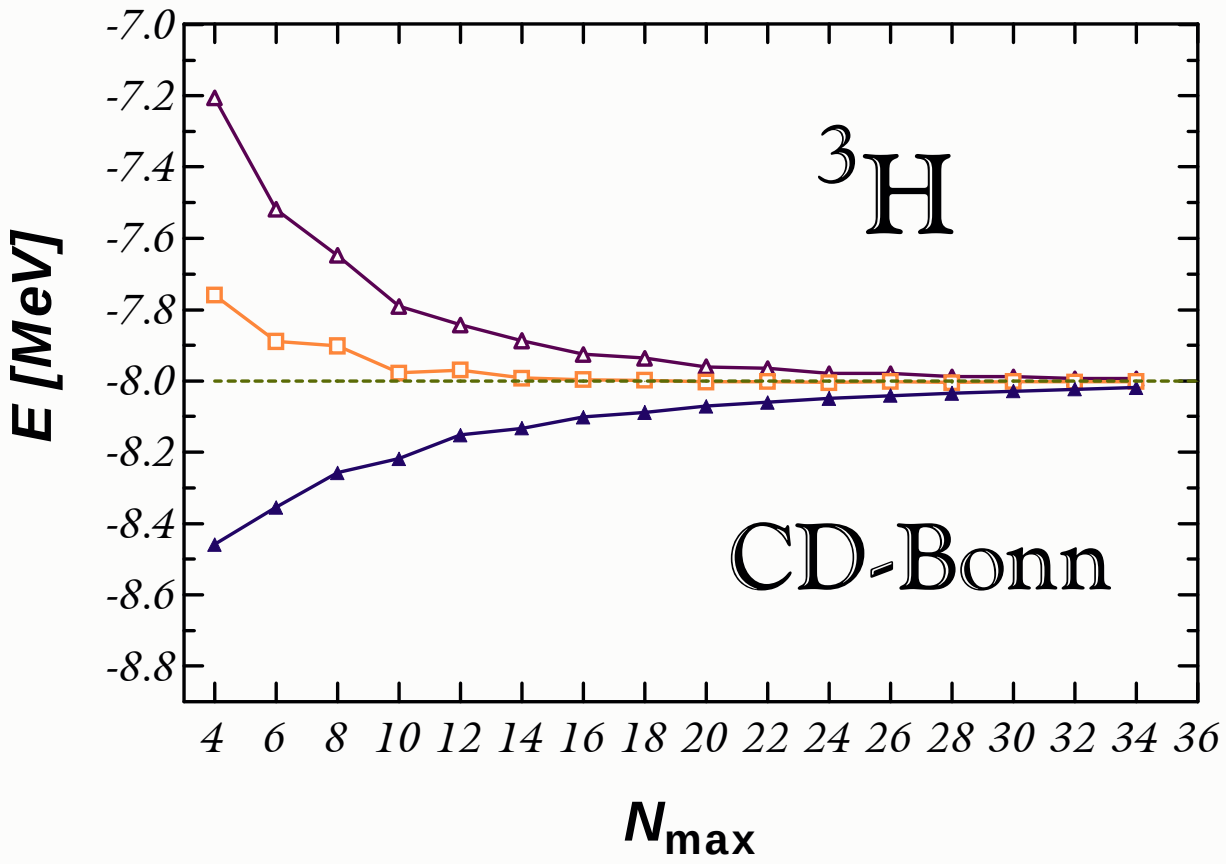
Used only in SD basis calculations

Physical energies and observables independent of β

Test of convergence

^{3}H with the CD-Bonn NN potential

Dependence on the **basis size (N_{max})** and the **HO frequency ($\hbar\Omega$)**



exact — Faddeev calculation

hΩ=32 MeV

hΩ=28 MeV

hΩ=22 MeV — Different HO frequencies

NCSM
Jacobi coordinate HO basis

n=2, two-body effective interaction approximation

not a variational calculation - neglect of three-body correlations

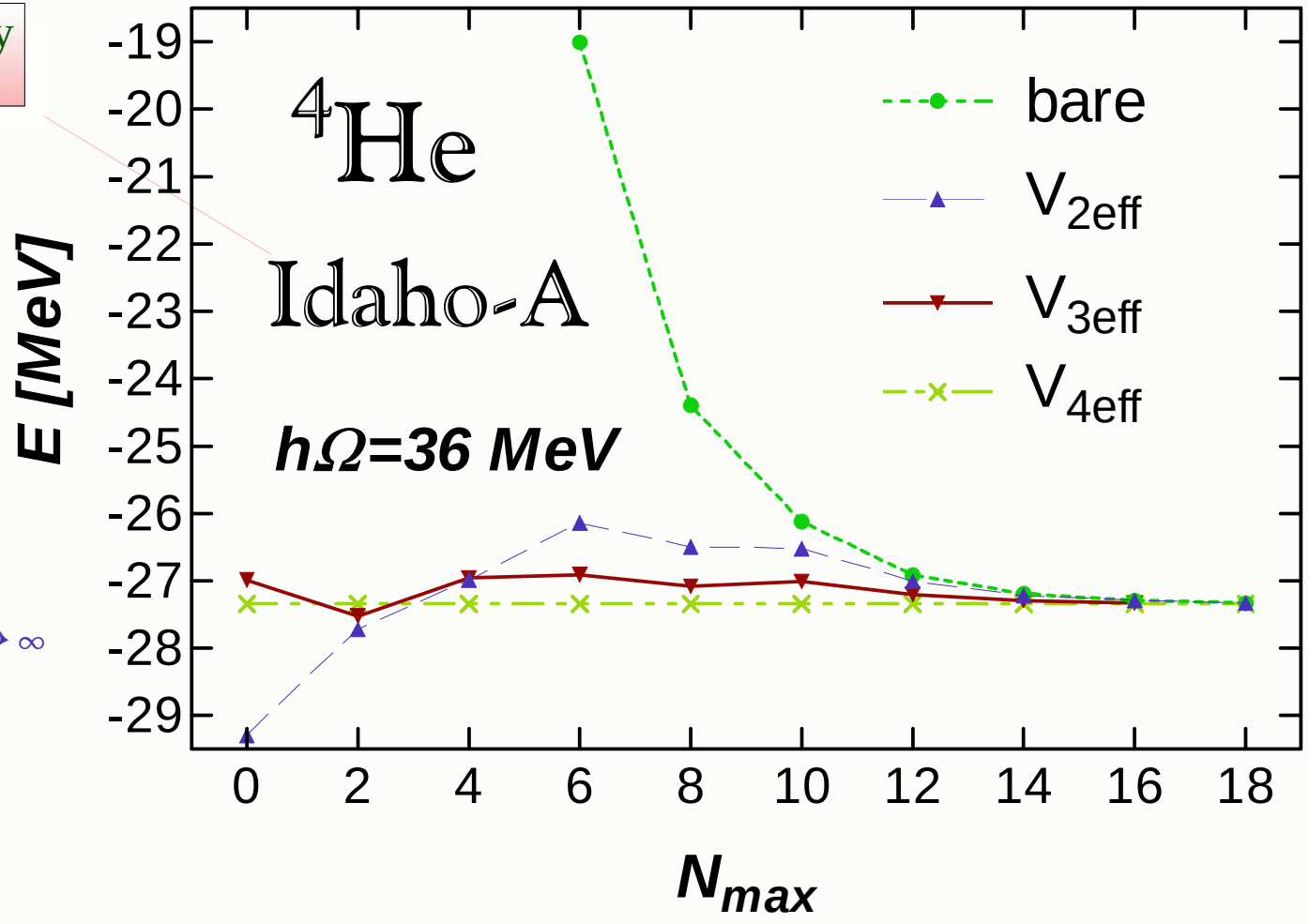
At each N_{max} and $\hbar\Omega$ point a different effective Hamiltonian

Convergence for different cluster approximations

n=2 vs. *n*=3 approximation

Effective Field Theory
NN potential

- Converged result
  -27.34 MeV
  - Independent of
    N_{max}
    when $n \rightarrow A$
    Ω
    when $n \rightarrow A$ or $N_{max} \rightarrow \infty$



Convergence to the same result for either bare or any effective interaction.
For fixed N_{max} higher cluster approximation improves convergence.

Many-body effective interaction code

manyeff

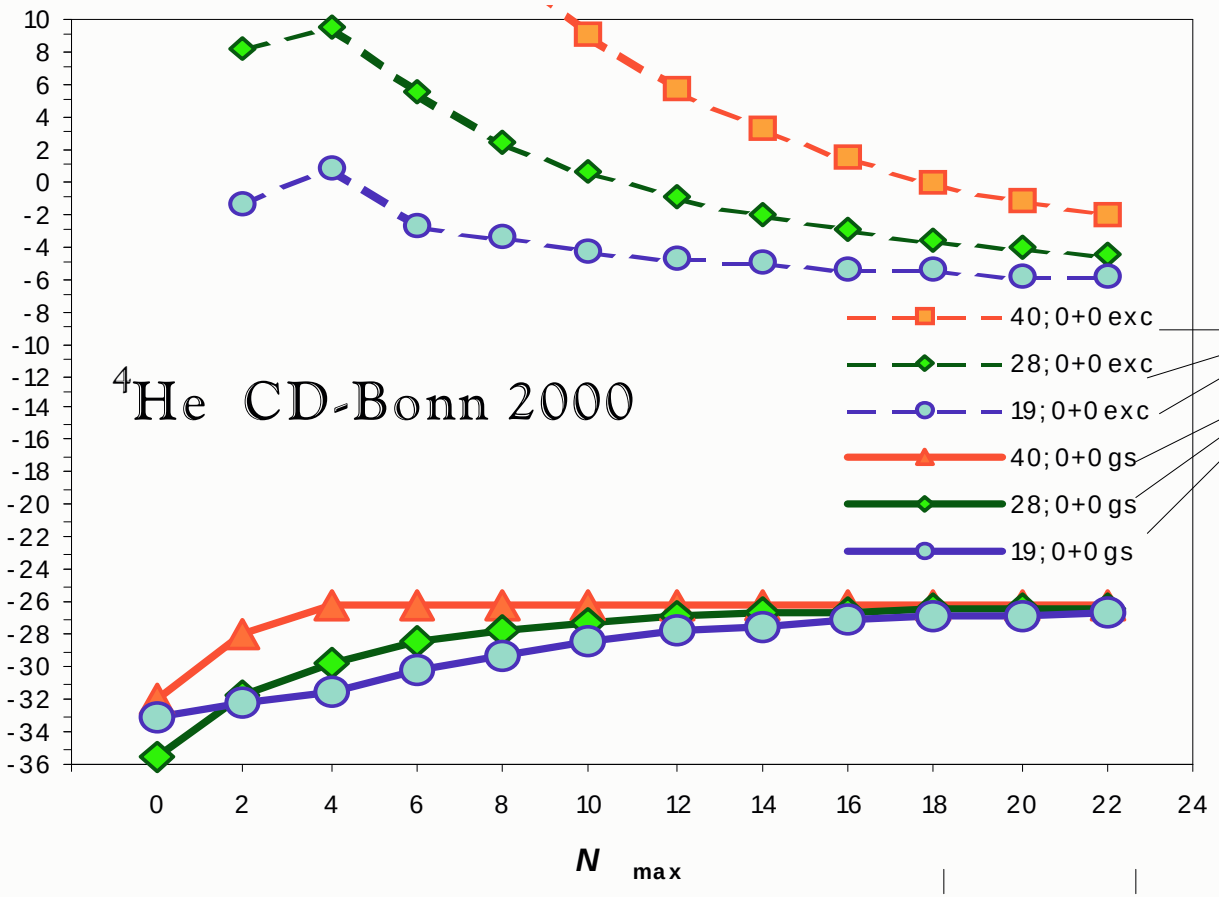
- Jacobi-coordinate HO basis
  - Antisymmetrization for any A (in practice up to A=6)
    - Starts with A=3→A=4→...→A
  - Basis transformations for different sets of Jacobi coordinates
- Calculates effective interaction by the Lee-Suzuki method
  - Two-nucleon solutions → $V_{2\mathrm{eff}}$
  - Three-nucleon solutions → $V_{3\mathrm{eff}}$
  - $V_{4\mathrm{eff}}$ generalization possible
- Solves A-nucleon system using the effective interactions
  - A=3 up to 60$\hbar\Omega$, A=4 up 18$\hbar\Omega$
    - Sufficient for convergence
  - A=5 up to 8$\hbar\Omega$
- Fortran90
  - OpenMP parallelization
  - *NN*-potential codes as subroutines
  - Eispack routines used for diagonalization of matrices
    - dimensions a few thousand

Now includes Tucson-Melbourne *NNN*-interaction

Yet another test of convergence

^{4}He with the CD-Bonn 2000 NN interaction

Dependence of the 0^+ 0 ground state and the 0^+ 0 excited state energies on the basis size (N_{max}) and the HO frequency ($\hbar\Omega$)



SD basis d=9 601 891
Jacobi basis d=4 750

SD basis
d=49 864 544
code Antoine

Ground-state energy agrees with the Faddeev-Yakubovsky result -26.2 MeV

Different HO frequencies

NCSM
Jacobi coordinate *or* Slater determinant HO basis

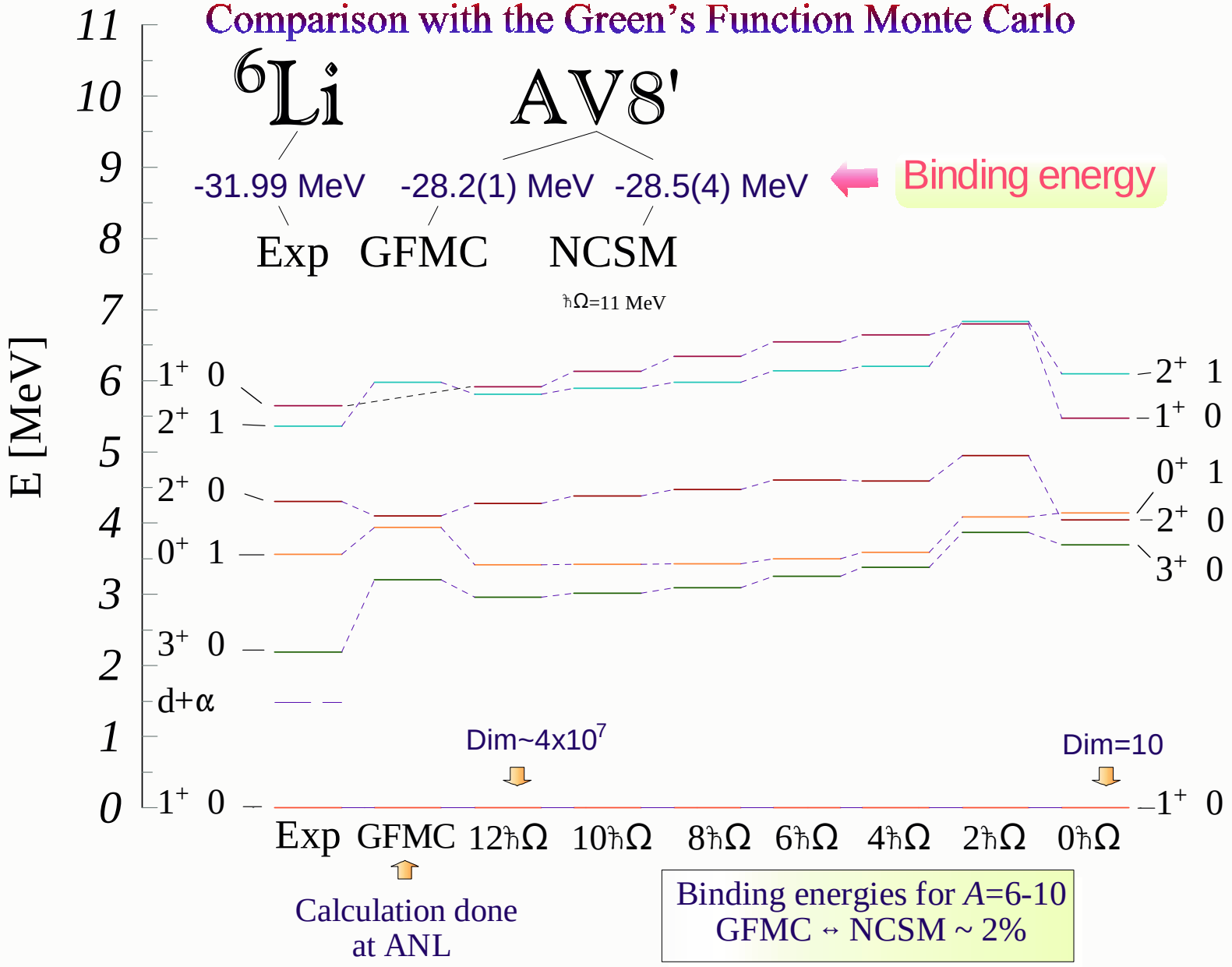
n=2, two-body effective interaction approximation

not a variational calculation - neglect of three- and four-body correlations

Shell model codes

- NCSM two-body effective interaction code
  - Two-nucleon solutions in all NN channels
  - LS two-body effective interaction (in relative coordinates)
  - Transformation to single-particle basis ⇨ input to shell model codes
- M-scheme shell model codes with Lanczos diagonalization
  - Many-fermion dynamics (MFD) shell model code
    - James P. Vary (ISU), D.-C. Zheng, P. N.
    - In 1st Lanczos iteration calculates non-zero matrix elements and stores them on disk
    - In subsequent Lanczos iterations matrix elements read from the disk
    - Three-body interaction capability
  - Antoine (no-core version)
    - Etienne Caurier (Strasbourg)
    - Non-zero matrix elements calculated on the fly - no disk storage
    - Sofisticated basis ordering and matrix elements enumeration
    - Exploits the fact that the shell model problem consists of proton and neutron subsystems
    - Reaches huges dimensions: 10^9
  - Redstick
    - W. E. Ormand
    - Similar algorithm as Antoine
    - Three-body interaction capability
- Wave function processing
  - Specialized transition density code
    - Reads wave functions obtained by the MFD or Antoine

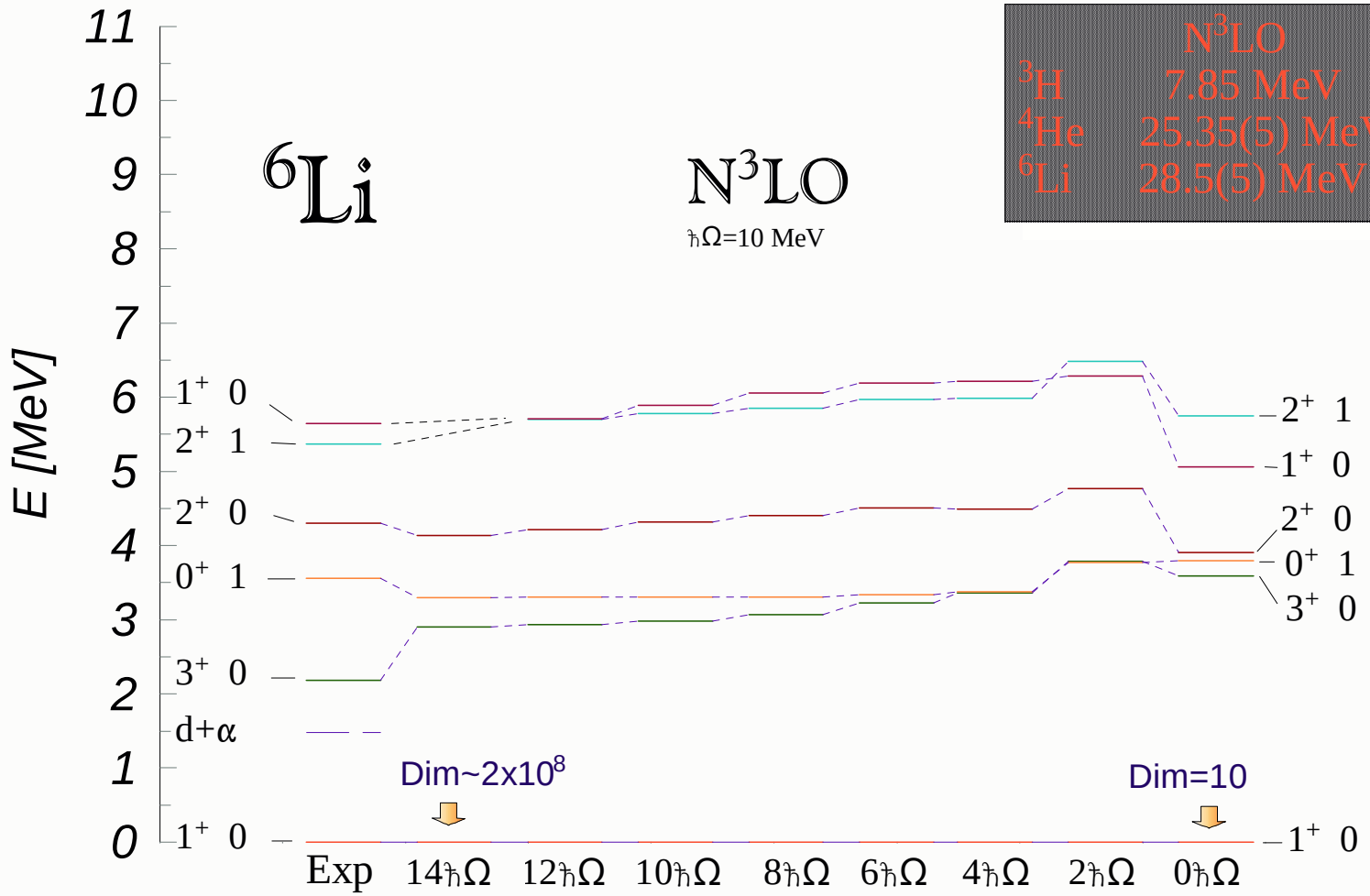
^{6}Li with a realistic NN potential



NCSM calculations with the EFT NN interactions

^{6}Li with accurate NN potential at fourth order of chiral-perturbation theory (N^{3}LO)

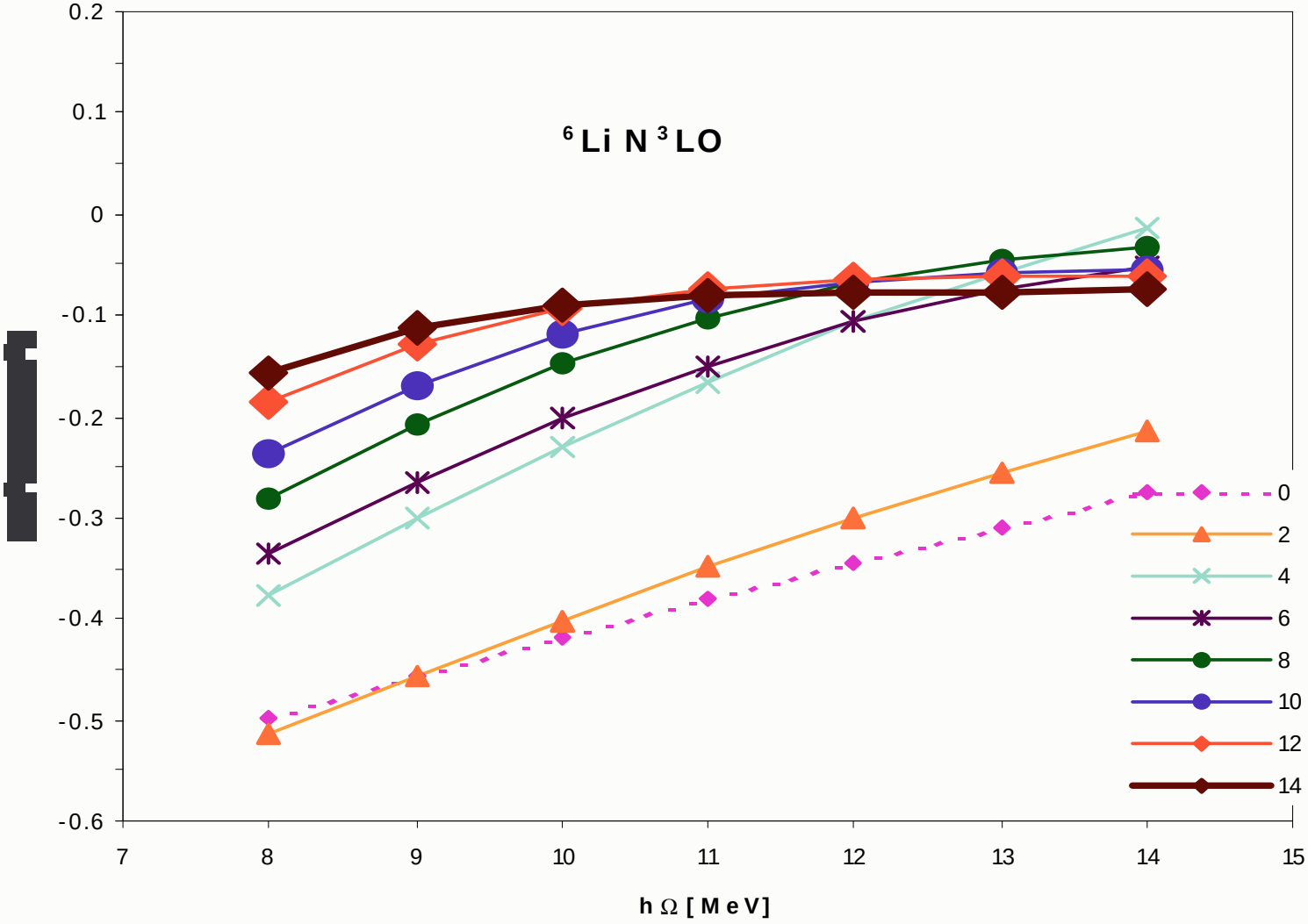
	N^{3}LO	Exp
^{3}H	7.85 MeV	8.48 MeV
^{4}He	25.35(5) MeV	28.30 MeV
^{6}Li	28.5(5) MeV	31.99 MeV



Converged excitation energies
Correct level ordering, level spacing not right

^{6}Li quadrupole moment

EFT N^{3}LO NN potential

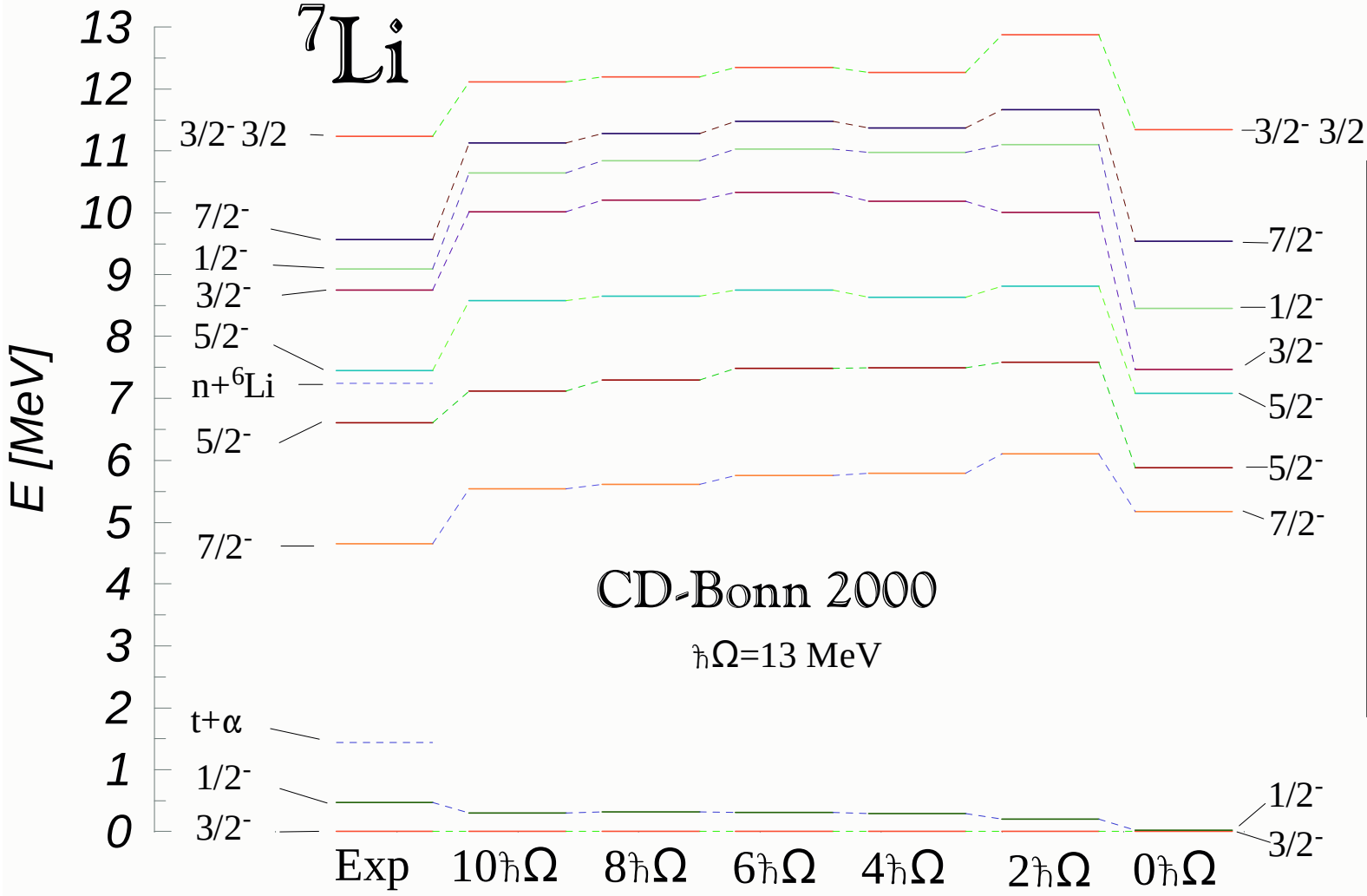


Exp.
-0.08 *e* fm^2

NCSM:
good
convergence
with N_{max}

p-shell nuclei with realistic NN forces

Correct level ordering for light *p*-shell nuclei

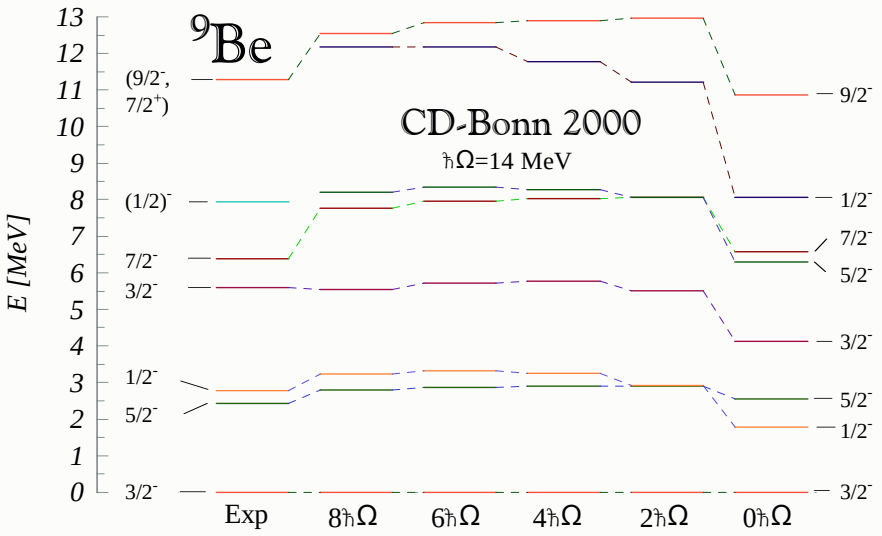


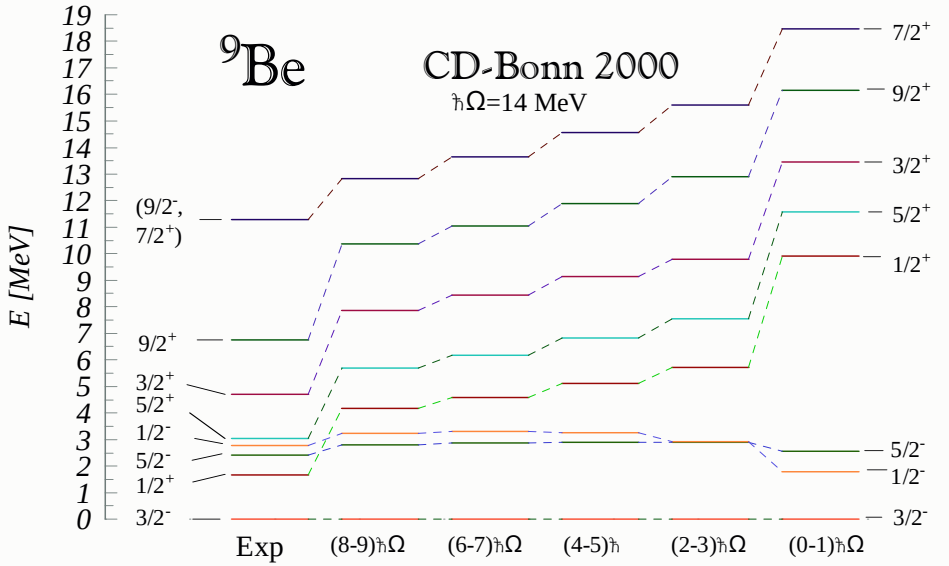
Old evaluation NPA**490**,1(1988) No $1/2^-_2$ and $3/2^-_2$, $7/2^-_2$ reversed

New evaluation NPA**708**,3(2002) introduces $1/2^-_2$ and orders the states as in calculation

Convergence of excitation energies
Realistic NN interactions provide reasonable description of nuclear structure

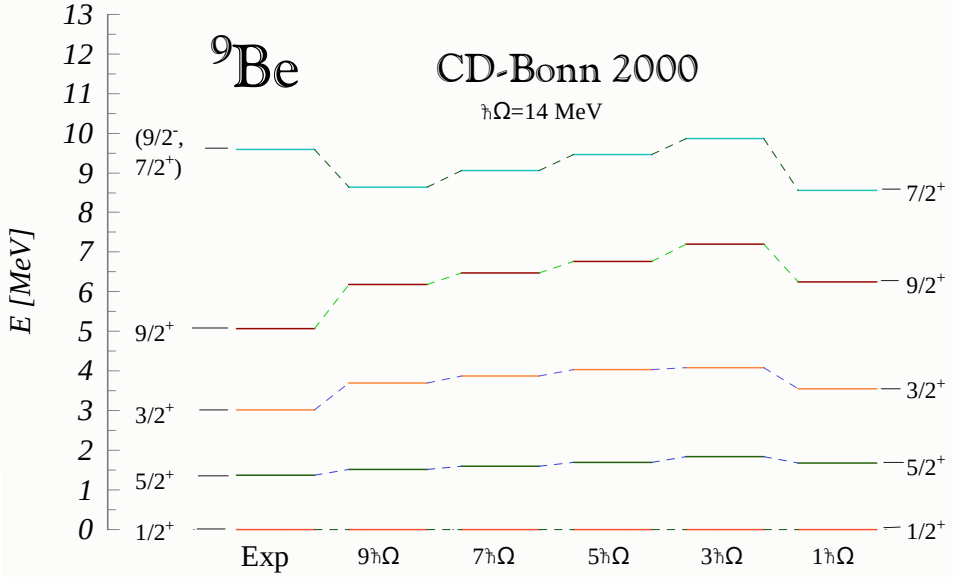
^{9}Be - negative- & positive-parity states





- Levels of both parity at low excitation enegy
- Experimental spin and parity assignments uncertain
- Existence of second low-lying $1/2^-$, $3/2^-$, $5/2^-$ states
- Cluster calculations (Koike) suggest the 11.28 MeV state to be $9/2^-$ rather than experimentally assigned $7/2^+$

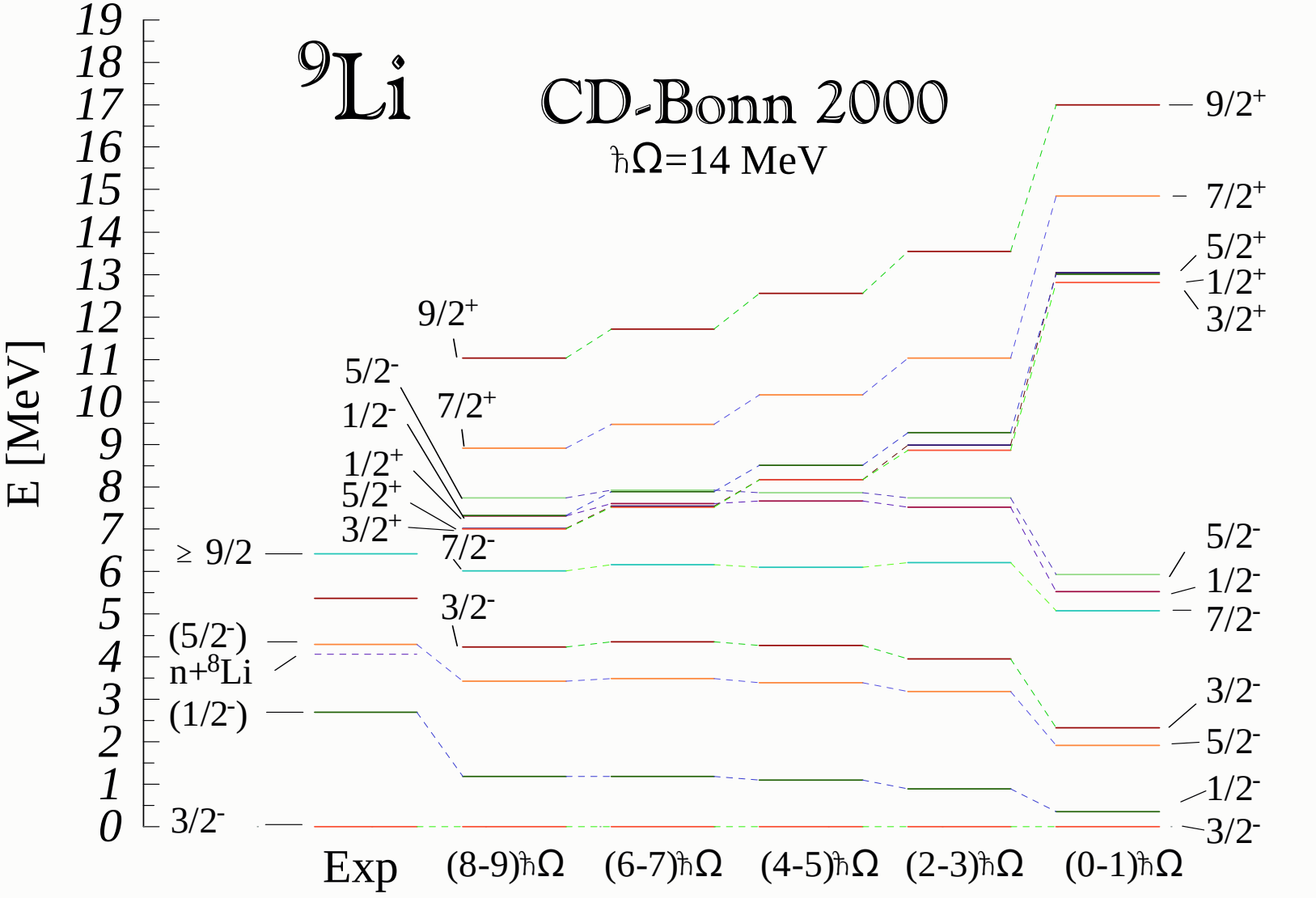
NCSM supports low-lying $3/2^-_2$, $5/2^-_2$ states
$9/2^-$ for the 11.28 MeV state



Nuclear structure of exotic nuclei

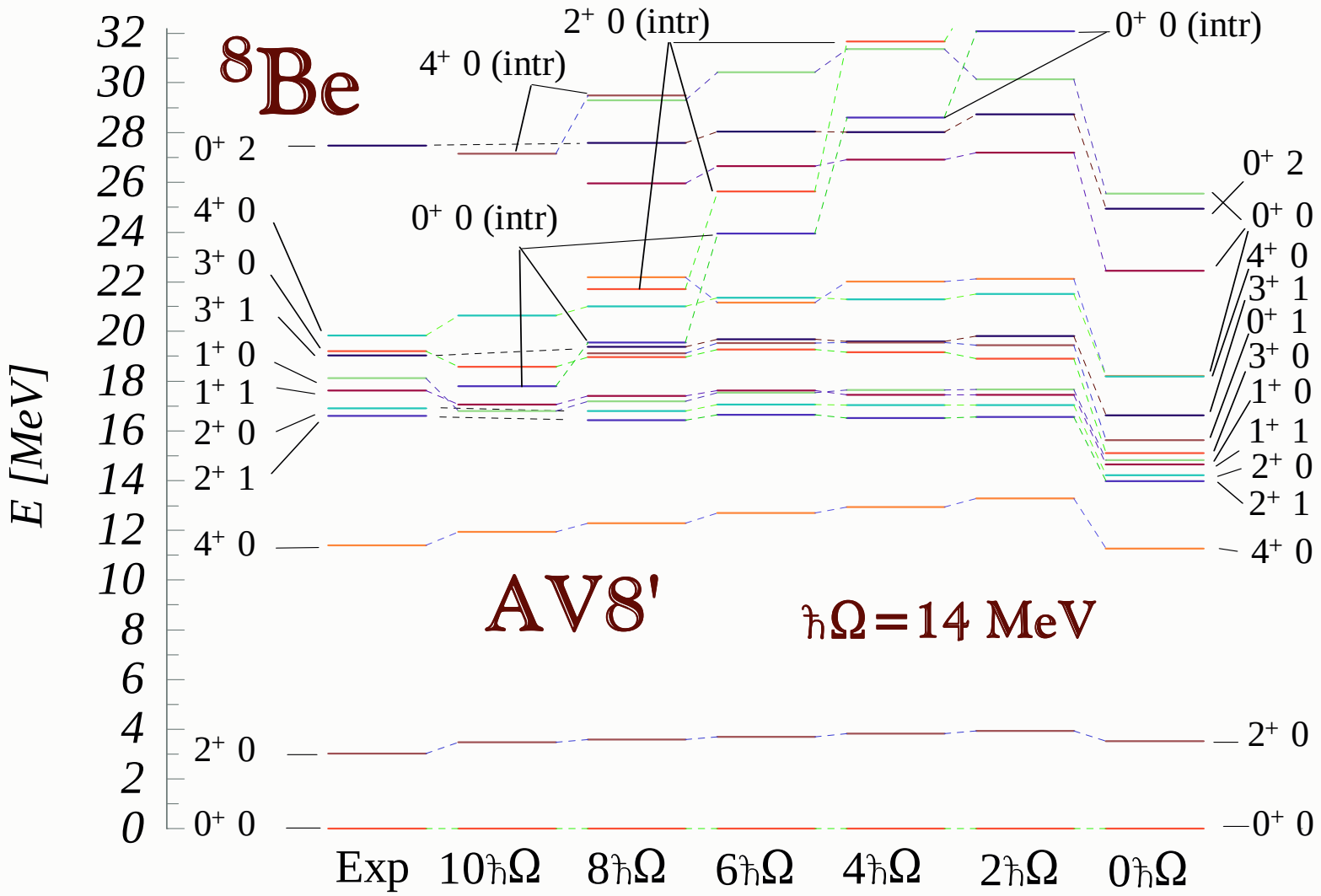
^{9}Li - very few states known experimentally

Excited states states relevant for astrophysically important ^{8}Li(n,γ)^{9}Li



^{8}Be

Intruder states relevant to the R-matrix analysis



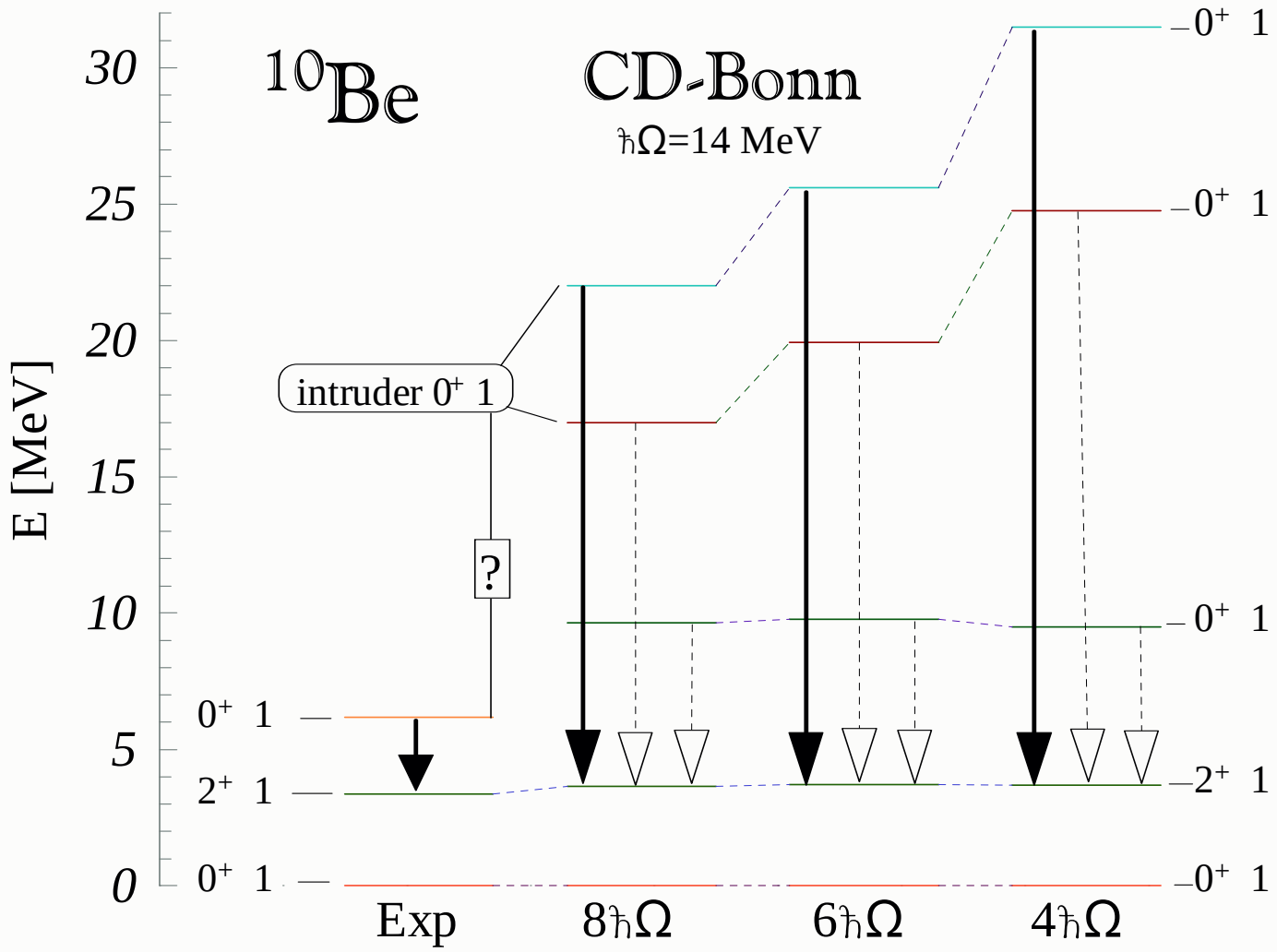
Intruder T=0 states in ^{8}Be

- Required in R-matrix fits of α+α scattering (Barker)
- Disputed in shell model calculations (Zamick)
- NCSM - up to 10ħΩ basis space (10^8 dimensions)
  - Fast decrease of intruder state excitation energy
  - 0^+ 0 state below 18 MeV and below 0^+ 1 state
    - Likely a two-α cluster *l=0* state
    - **Configuration**

	0ħΩ	2ħΩ	4ħΩ	6ħΩ	8ħΩ	10ħΩ
0^+ 0(intruder)	9%	41%	21%	16%	8%	5%
	0ħΩ	2ħΩ	4ħΩ	6ħΩ	8ħΩ	10ħΩ
0^+ 0 (gr. st.)	55%	19%	12%	7%	4%	3%

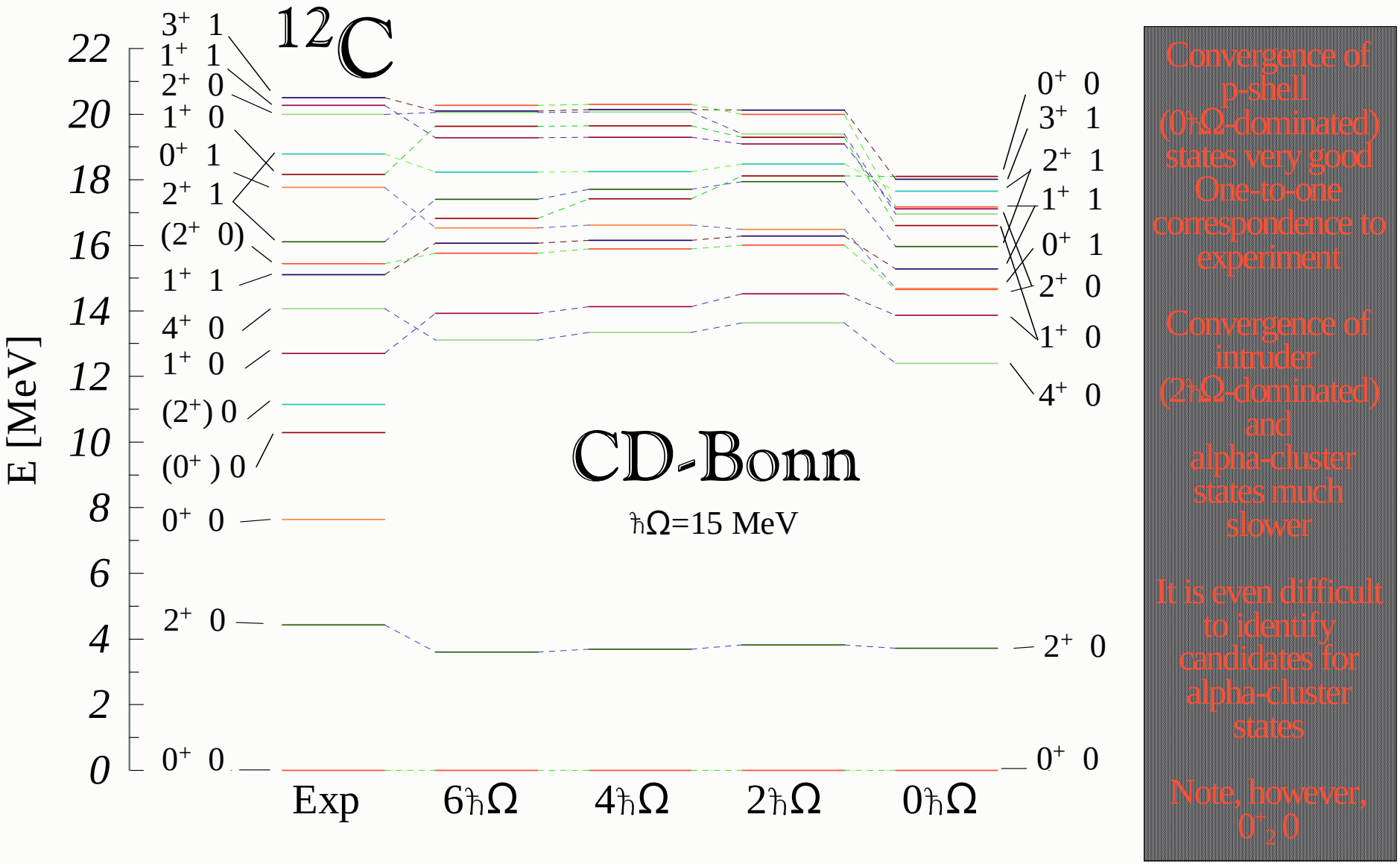
  - Relation to R-matrix analysis
    - Wave function overlap of ^{8}Be and ^{4}He
      - ➜ Possible even using the Slater determinant basis

Intruder states in ^{10}Be



Convergence of p-shell (0ħΩ-dominated) states good
Convergence of intruder (2ħΩ-dominated) states much slower

NCSM description of ^{12}C



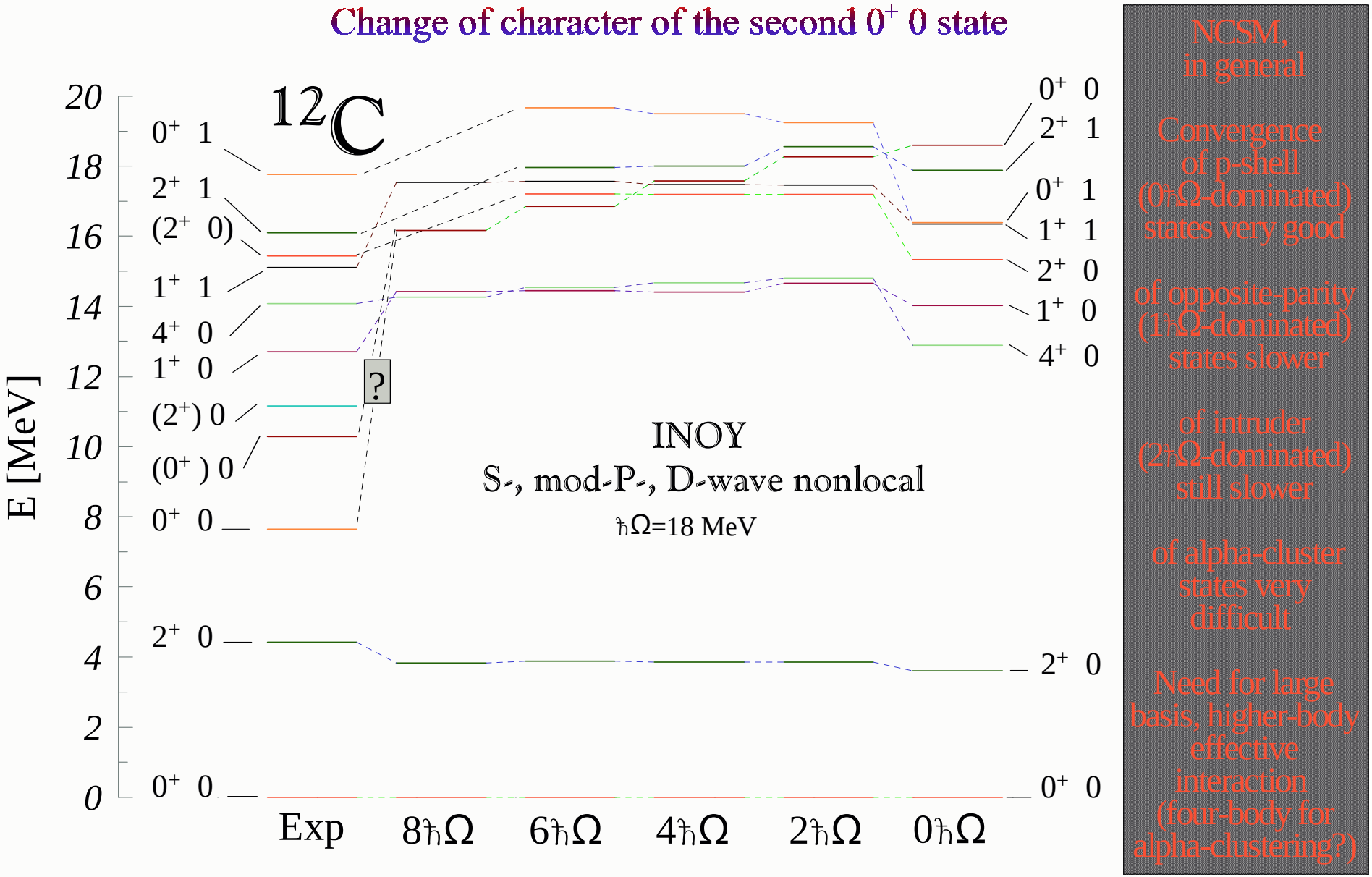
Convergence of p-shell (0ħΩ-dominated) states very good
One-to-one correspondence to experiment

Convergence of intruder (2ħΩ-dominated) and alpha-cluster states much slower

It is even difficult to identify candidates for alpha-cluster states

Note, however, 0^-_2 0

NCSM description of ^{12}C



NCSM, in general

Convergence of p-shell (0ħΩ-dominated) states very good

of opposite-parity (1ħΩ-dominated) states slower

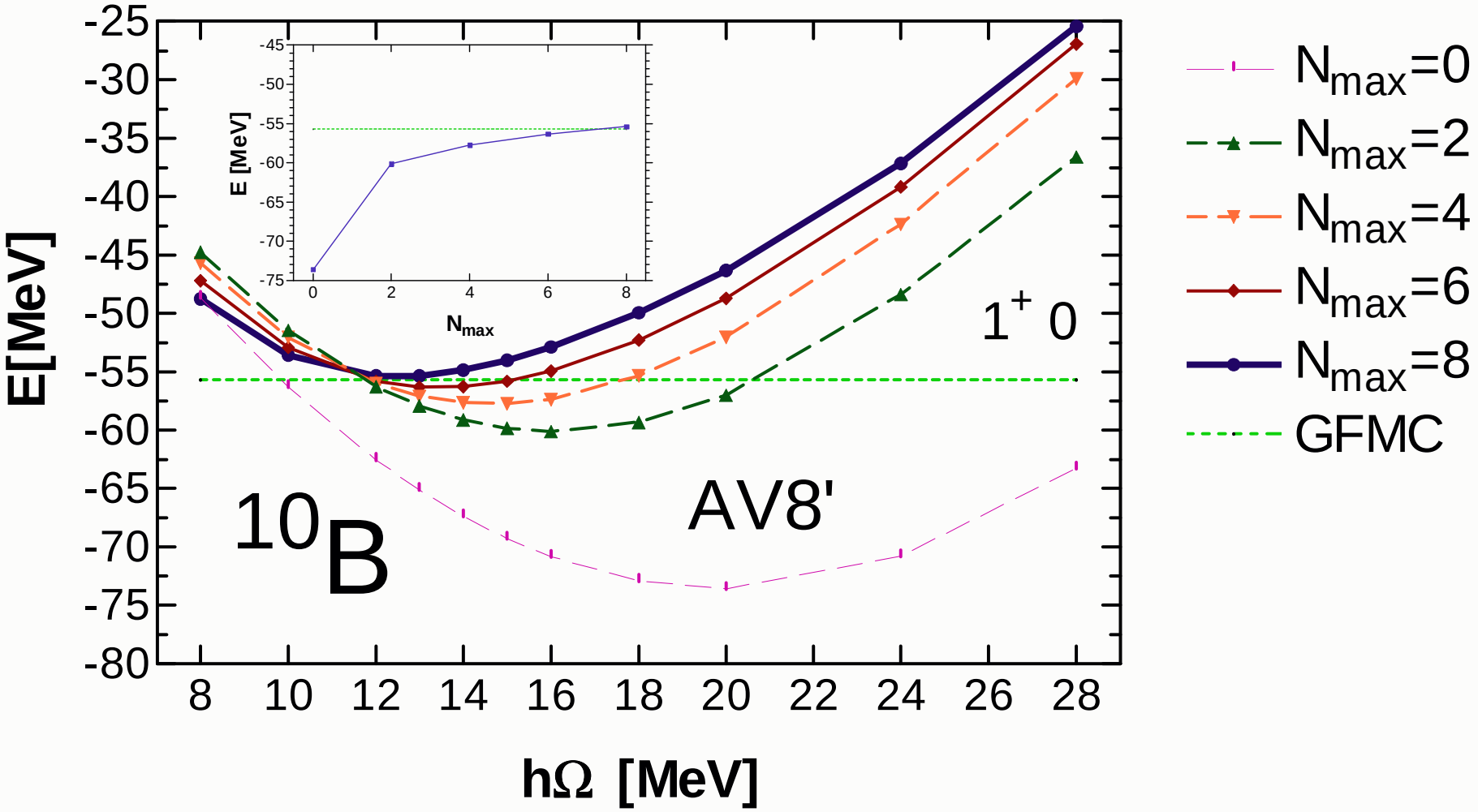
of intruder (2ħΩ-dominated) still slower

of alpha-cluster states very difficult

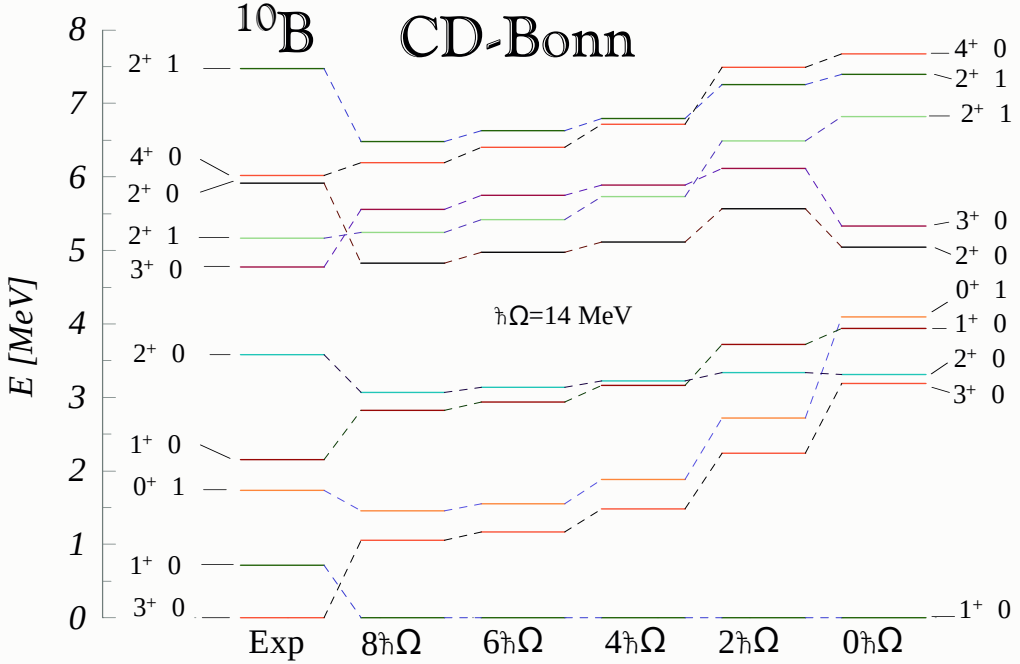
Need for large basis, higher-body effective interaction (four-body for alpha-clustering?)

^{10}B binding energy

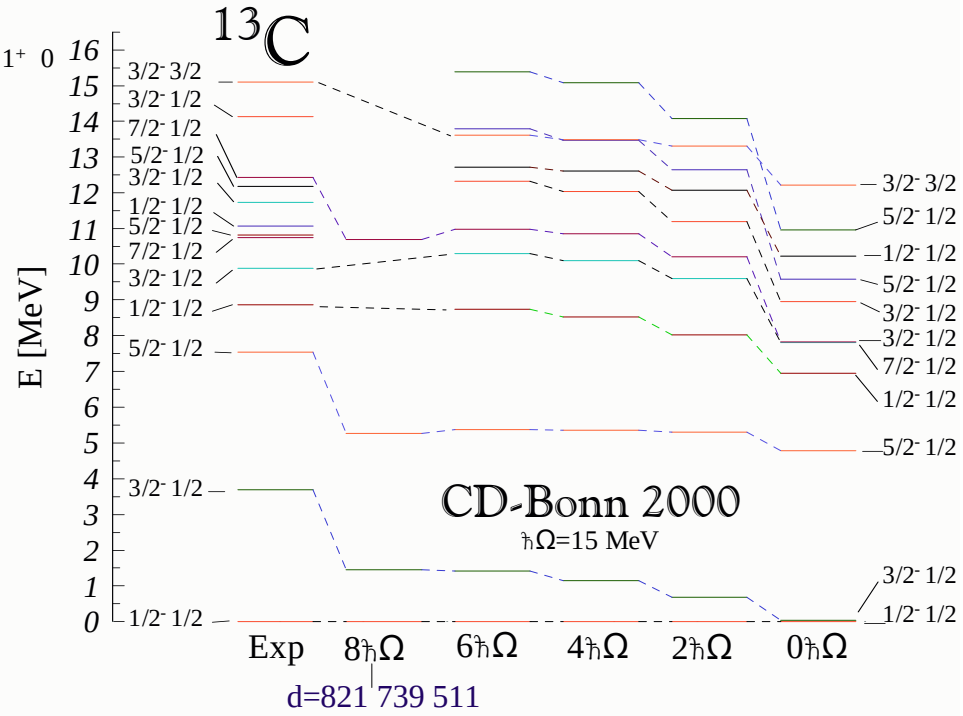
Comparison with the Green's Function Monte Carlo for AV8'



Problems in complex *p*-shell nuclei with realistic NN interactions



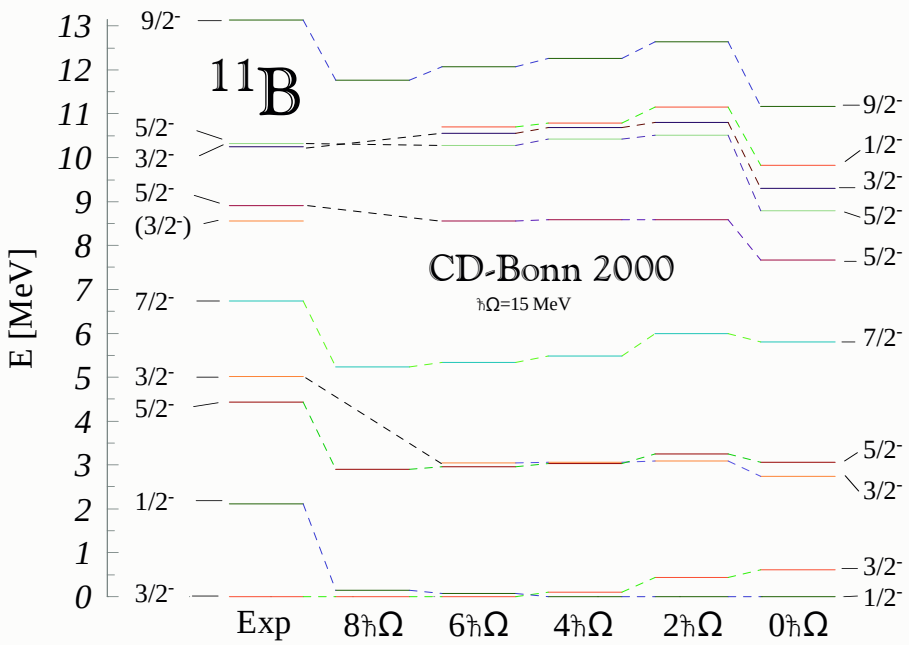
^{10}B
Experimental ground-state spin 3^+ 0
NCSM - calculations up to $10\hbar\Omega$ in a range of HO frequencies:
Ground state spin 1^+ 0
The same observed in the GFMC calculations

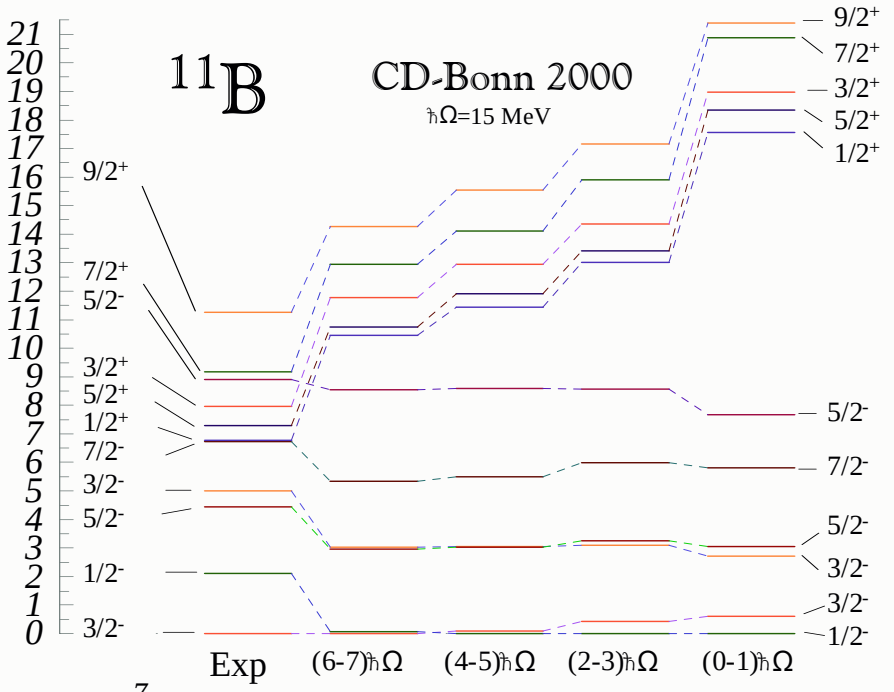


^{13}C
NCSM - no parameters to adjust
5 lowest levels in correct order
Problems remain:
Splittings underestimated for the lowest levels

Need for the three-body interaction

^{11}B negative & positive-parity states





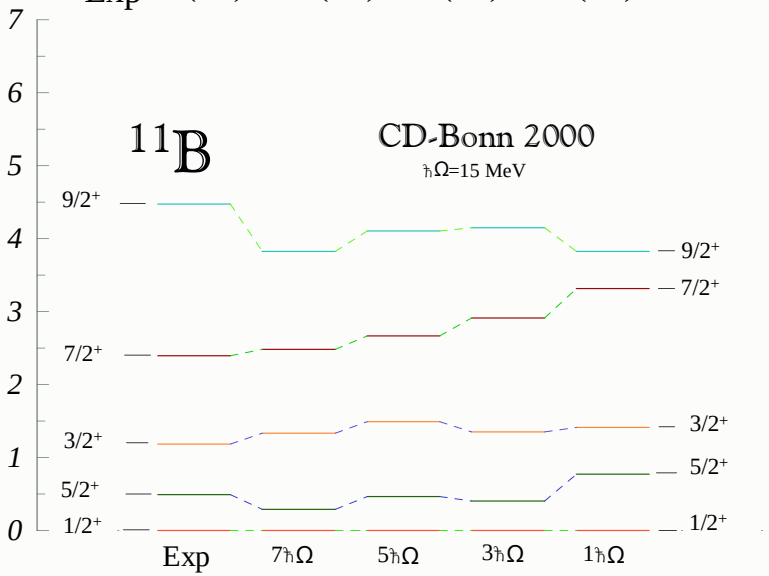
Binding energy

CD-Bonn 2000
68 MeV

Exp
76.2 MeV

NCSM

- ☞ correct level ordering for both parities
- ☞ visible effects of weak spin-orbit force in the NN potential
- ☞ positive-parity state excitation energy slower convergence



Why we need a three-nucleon interaction

- Realistic two-nucleon potentials highly accurate
  - Coordinate space potentials
    - Local: AV18, AV8', Nijmegen II
    - Nonlocal: Nijmegen I
  - Momentum space nonlocal potentials
    - Boson-exchange potentials: CD-Bonn
    - Effective field theory potentials: N^{3}LO

☜ High Quality

- Exact few-body calculations with these potentials show that ^{3}H, ^{3}He, ^{4}He underbound by 5-10%
- Problems in A=3 scattering: p+d, n+d A_y puzzle
- Nuclear structure calculations with realistic two-nucleon potentials in the *p*-shell
  - GFMC, NCSM, CCM
  - Underbinding
  - Level spacing not quite right
  - Ordering of lowest states not correct for some nuclei

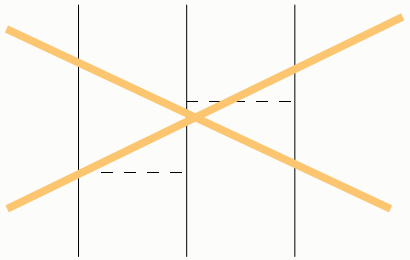
Regardless of which realistic two-nucleon potential used problems remain:

Need to include a three-nucleon interaction

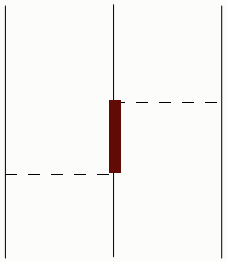
What is a three-body interaction?

Two-pion exchange among three-nucleons

- Not a sequence of two two-body interactions



- Virtual Δ excitation
  - Fujita-Miazawa term



- Detailed form, spin, isospin dependence and parameters not well known
- Several three-body interactions used in few-body calculations:
  - Urbana IX: Argonne group ☹
  - Illinois II: Argonne group, three-pion loops ☺
  - Tucson-Melbourne': S. Coon *et al.*, two-pion exchange only ☺

^{10}B problem

- Effective field theory (S. Weinberg)
  - Nucleon and pion degrees of freedom (low energies)
  - Chiral symmetry of QCD
  - Systematic expansion: Predicts three-nucleon interaction,TM' is a leading term

General form of the three-nucleon interaction

NCSM: Matrix element in Jacobi-coordinate HO three-nucleon basis

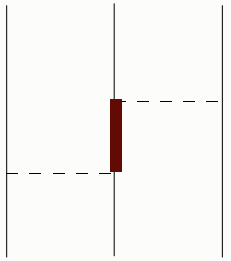
- Three-nucleon interaction depends on two sets of Jacobi coordinates (or momenta)

$$W = W_1 + W_2 + W_3$$

$$W_1 = \sum c_i^{k_1 k_2 K} \left[f_{3i}^{(k_1)}(\vec{x}_3)\, f_{2i}^{(k_2)}(\vec{x}_2) \right]^{(K)} . \; S_i^{(K)}(\sigma_1,\sigma_2,\sigma_3)\; T_i^{(0)}(\tau_1,\tau_2,\tau_3)$$

$$\vec{x}_3 = \vec{r}_1 - \vec{r}_2 \quad ; \quad \vec{y}_3 = \tfrac{1}{2}(\vec{r}_1 + \vec{r}_2) - \vec{r}_3$$

$$\vec{x}_2 = \vec{r}_3 - \vec{r}_1 \quad ; \quad \vec{y}_2 = \tfrac{1}{2}(\vec{r}_3 + \vec{r}_1) - \vec{r}_2$$



- Basis depends on a single set of Jacobi coordinates
  - $\vec{x}_3 \, ; \vec{y}_3$
- Two tricks
  - Using antisymmetry, apply permutation operators on initial state
    - $\vec{x}_3 \, ; \vec{y}_3 \;\rightarrow\; \vec{x}_2 \, ; \vec{y}_2$
  - Insert complete set of coordinate HO states depending on $\vec{x}_3 \, ; \vec{y}_3$ between $f_{3i}^{(k_1)}(\vec{x}_3) f_{2i}^{(k_2)}(\vec{x}_2)$ and for the bra HO functions use Moshinsky transformations to $\vec{x}_2 \, ; \vec{y}_2$

Realistic three-nucleon interaction in the NCSM

The lowest possible approximation $n=3$ ↔ three-body effective interaction

- Calculations performed in four steps
  - 1) Three-nucleon solutions for all relevant $n=3$ $J^{\pi}T$ channels with and without V^{3b}

$$H^{\Omega} = \sum_{i=1}^{3}\left[\frac{\vec{p}_i^{\,2}}{2m} + \frac{1}{2}m\Omega^{\,2}\vec{r}_i^{\,2}\right] + \sum_{i<j}^{3}\left[V_{NN}(\vec{r}_i - \vec{r}_j) - \frac{m\Omega^{\,2}}{2A}(\vec{r}_i - \vec{r}_j)^2\right] + V_{123}^{3b}$$

  - 2) Three-body effective interaction by unitary transformation method
    - X_3
  - 3) Effective interaction in Jacobi coordinate HO basis, *p*-shell nuclei calculations more efficient in Cartezian coordinate Slater determinant basis
    → transformation must be performed
  - 4) *A*-nucleon calculation performed by a shell model code with a three-body capability
    - MFD, REDSTICK

$$h_1 + h_2 + h_3 + V_{12} + V_{13} + V_{23} + V_{123} \;\rightarrow\; X_3 \;\rightarrow\; P_3\left[h_1 + h_2 + h_3 + V_{3eff,123}^{2b+3b}\right]P_3 \;\rightarrow\; P\left[\sum_{i=1}^{A} h_i + \frac{1}{A-2}\sum_{i<j<k}^{A} V_{3eff,ijk}^{2b} + \sum_{i<j<k}^{A} V_{3eff,ijk}^{3b}\right]P$$

n=3, $V_{3\mathrm{eff}}$: I. Exact Solutions

- Finding exact solutions

$$H^{(n=3)} = h_1 + h_2 + h_3 + V_{12} + V_{13} + V_{23} + V_{123}$$

$$H^{(n=3)}|k\rangle = E_k|k\rangle$$

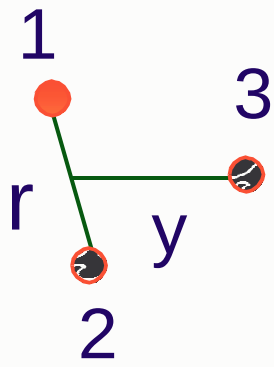
  - For A>3 three nucleons bound in a HO well
  - All $J^\pi T$ n=3 channels:
    - J=1/2 ... $N_{\max}$+3/2; T=1/2,3/2; π=+,-
  - Number of needed exact eigenstates ↔ dimension of the model space (defined by $N_{\max}$) in each $J^\pi T$ n=3 channel
  - Faster convergence achieved by

$$V_{ij} \rightarrow V_{2eff,ij}$$

    with $N_{3\max} \approx 30\text{-}40 >> N_{\max}$
  - Separate contribution of the three-body interaction
    - Calculations with and without V_{123}
  - Antisymmetrized Jacobi coordinate HO basis

$$\vec{r} = \frac{1}{\sqrt{2}}\left[\vec{r}_1 - \vec{r}_2\right]$$

$$\vec{y} = \sqrt{\frac{2}{3}}\left[\tfrac{1}{2}(\vec{r}_1 + \vec{r}_2) - \vec{r}_3\right]$$



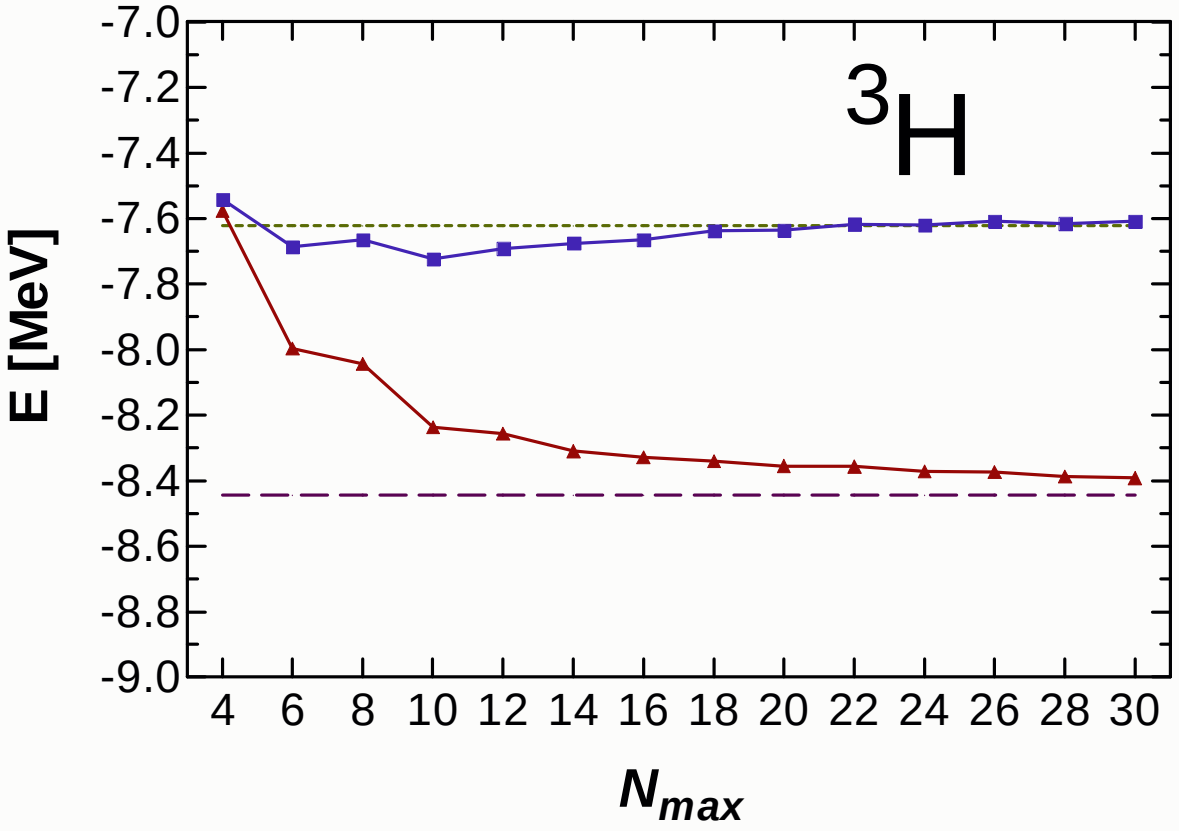
$$\mathcal{A}\left(\varphi_{nlsjt}(\vec{r})\varphi_{\mathcal{NLJ}}(\vec{y})\right)^{(J^\pi T)}$$

$$2n + l + 2\mathcal{N} + \mathcal{L} \le N_{3\max}$$

Convergence for ^{3}H with a real three-body interaction

Tucson-Melbourne force

Needed to reproduce experimental binding energy



AV18
AV18 +TM'(81)
AV18 exact
AV18+TM'(81) exact

NCSM

Faddeev calculation

NCSM
Jacobi coordinate HO basis

AV18 ↔ V_{2eff}
TM' ↔ bare

Paves the way for the inclusion of real three-body interactions in the NCSM calculations for A>3

n=3, $V_{3\mathrm{eff}}$: II. Lee-Suzuki method

- Model space P_3: $2n + l + 2\mathcal{N} + \mathscr{L} \le N_{\max}$

$$Q_3 e^{-\omega} H^{(n=3)} e^{\omega} P_3 = 0 \; ; \quad \omega = Q_3 \omega P_3$$

$$H^{(n=3)} \left| k \right\rangle = E_k \left| k \right\rangle$$

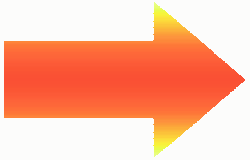
$$\left\langle \alpha_Q \left| \omega \right| \alpha_P \right\rangle = \sum_{k \in K} \left\langle \alpha_Q \left| k \right\rangle \left\langle \tilde{k} \right| \alpha_P \right\rangle , \quad k \in K \qquad \sum_{\alpha_P} \left\langle \tilde{k} \right| \alpha_P \left\rangle \right\langle \alpha_P \left| k' \right\rangle = \delta_{k,k'}$$

$$\left\langle \gamma_P \left| H_{eff}^{(n=3)} \right| \alpha_P \right\rangle = \sum_{k \in K} \left\langle \gamma_P \left| (P_3 + \omega^+ \omega)^{-1/2} \right| \alpha_P^{'} \right\rangle \left\langle \alpha_P^{'} \left| \tilde{k} \right\rangle E_k \left\langle \tilde{k} \right| \alpha_P^{''} \right\rangle \left\langle \alpha_P^{''} \left| (P_3 + \omega^+ \omega)^{-1/2} \right| \alpha_P \right\rangle$$

- Properties of $V_{3\mathrm{eff}}$
  - Depends on A, $N_{\max}$ and $\hbar\Omega$
  - For $N_{\max} \to \infty$ interaction $V_{3\mathrm{eff}} \to V_{\mathrm{NN}} + V_{3\mathrm{b}}$

n=3, $V_{3\mathrm{eff}}$: III. Transform to M-scheme

- $V_{3\mathrm{eff}}$ calculated in Jacobi coordinate HO basis
- For p-shell nuclei the Slater determinant single-particle HO basis more efficient



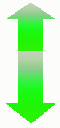
Transformation needed

$$\vec{r} = \frac{1}{\sqrt{2}}\left[\vec{r}_1 - \vec{r}_2\right]$$

$$\vec{y} = \sqrt{\frac{2}{3}}\left[\tfrac{1}{2}(\vec{r}_1 + \vec{r}_2) - \vec{r}_3\right]$$

$$\vec{R} = \frac{1}{\sqrt{3}}\left[\vec{r}_1 + \vec{r}_2 + \vec{r}_3\right]$$

$$\left[\mathcal{A}\left(\varphi_{nlsjt}(\vec{r})\varphi_{\mathcal{NLJ}}(\vec{y})\right)^{(JT)}_{M_J M_T}\varphi_{N_C L_C M_{L_C}}(\vec{R})\right]_{MM_T\pi}$$

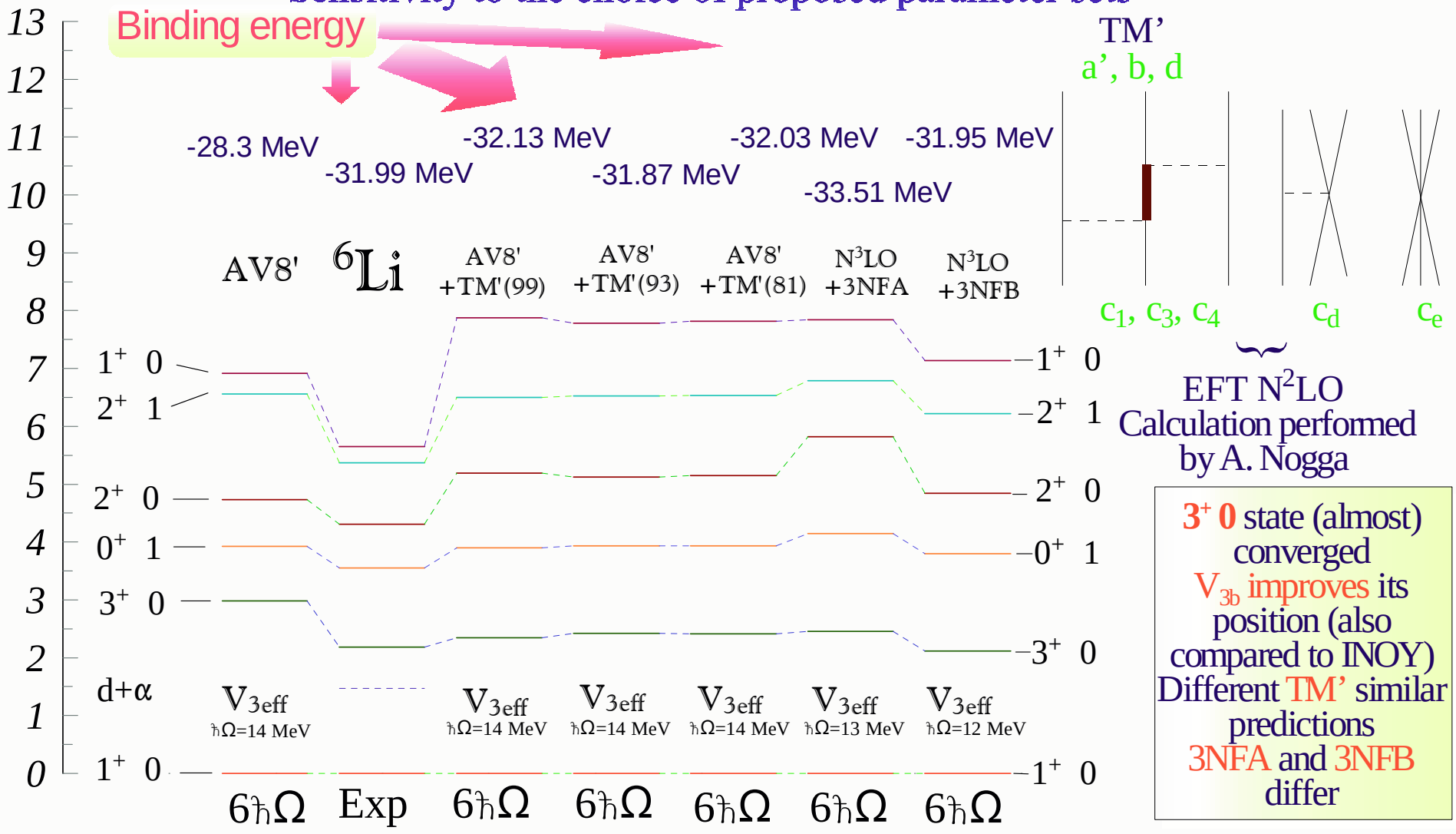


$$Det\left[\varphi_{n_a l_a j_a m_a m_{ta}}(\vec{r}_1)\varphi_{n_b l_b j_b m_b m_{tb}}(\vec{r}_2)\varphi_{n_c l_c j_c m_c m_{tc}}(\vec{r}_3)\right]_{MM_T\pi}$$

- Complicated sum depending on
  - Products of two Brody-Moshinsky brackets (mass ratios 1, ½)
  - CFP from Jacobi coordinate antisymmetrization
  - Clebsh-Gordan coefficients

^{6}Li with the Tucson-Melbourne and the EFT N^{2}LO three-body force

Sensitivity to the choice of proposed parameter sets



3$^+$ 0 state (almost) converged

V_{3b} improves its position (also compared to INOY)

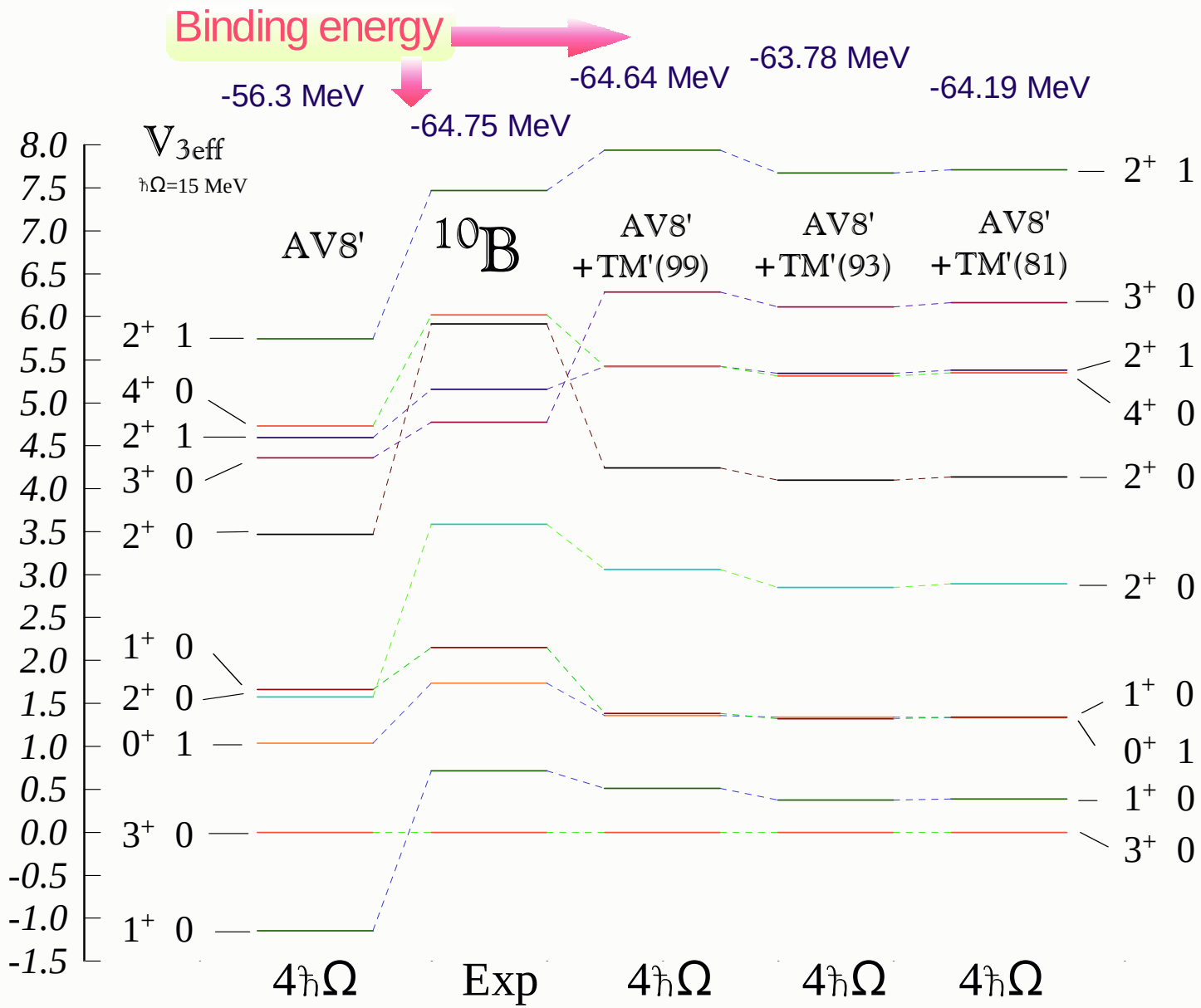
Different TM' similar predictions

3NFA and 3NFB differ

Dim=197 822; # non-zero matrix elements: V_{3eff} → 981 142 479
V_{2eff} → 60 236 339

For A>6, 6ħΩ, # input three-body matrix elements: 741 823 056

^{10}B with the Tucson-Melbourne force



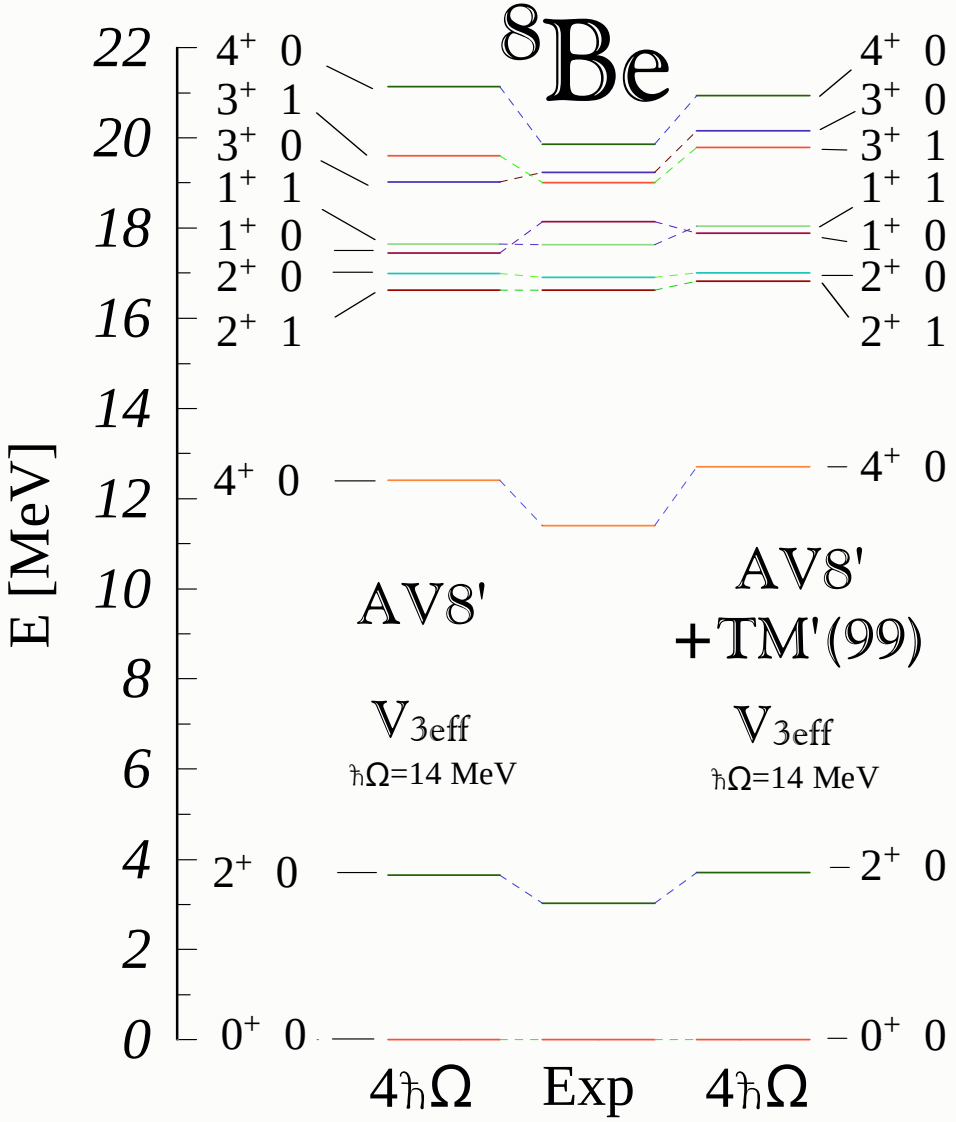
TM' ☺
resolves the ^{10}B ground-state problem similarly as **Illinois** 3NF ☺
but
Urbana IX ☹
does not

TM'(81,93,99)
Differences small but more significant than for ^{6}Li

Issues of convergence

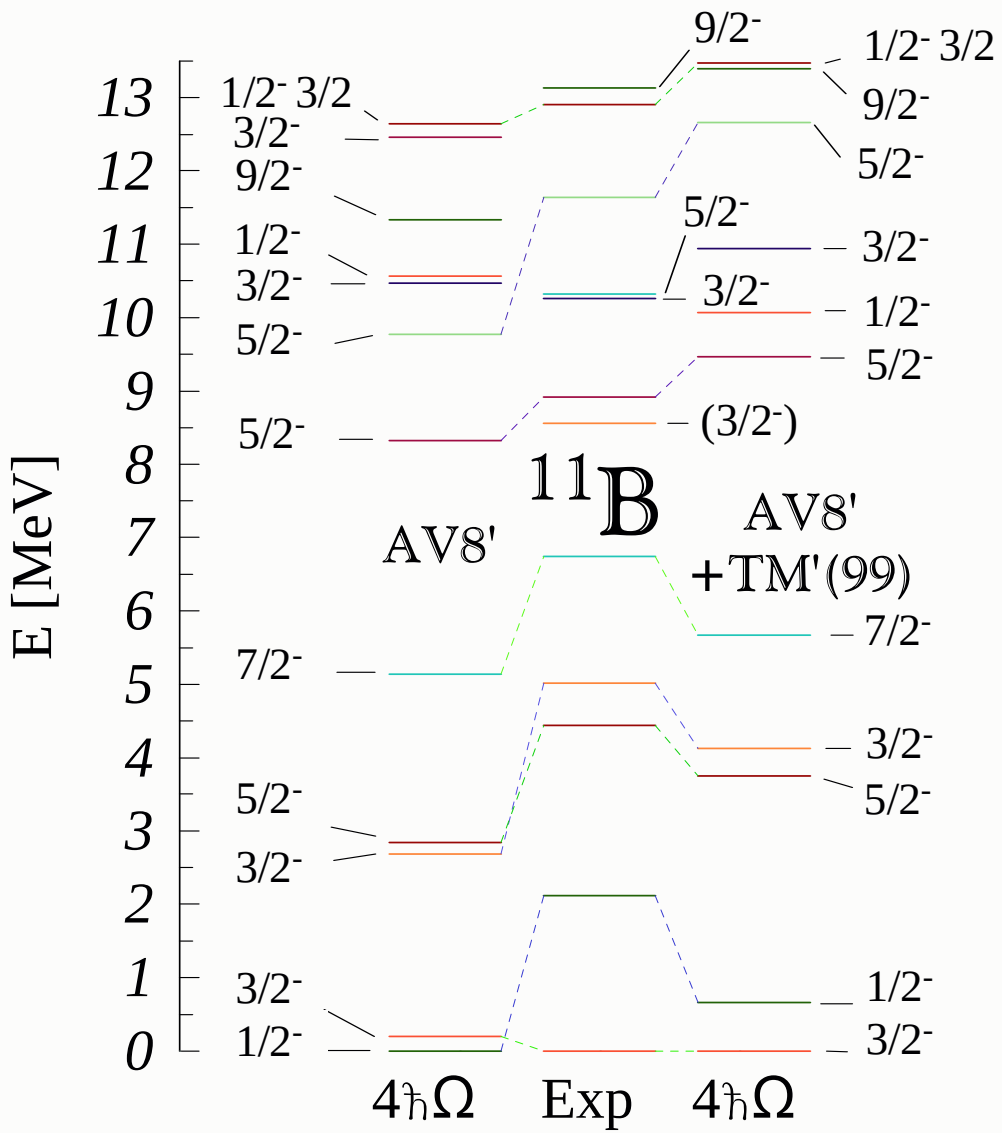
Cutoff adjustments

^{8}Be the Tucson-Melbourne force



V_{3b} : Good description of ^{8}Be unchanged

^{11}B with the Tucson-Melbourne force



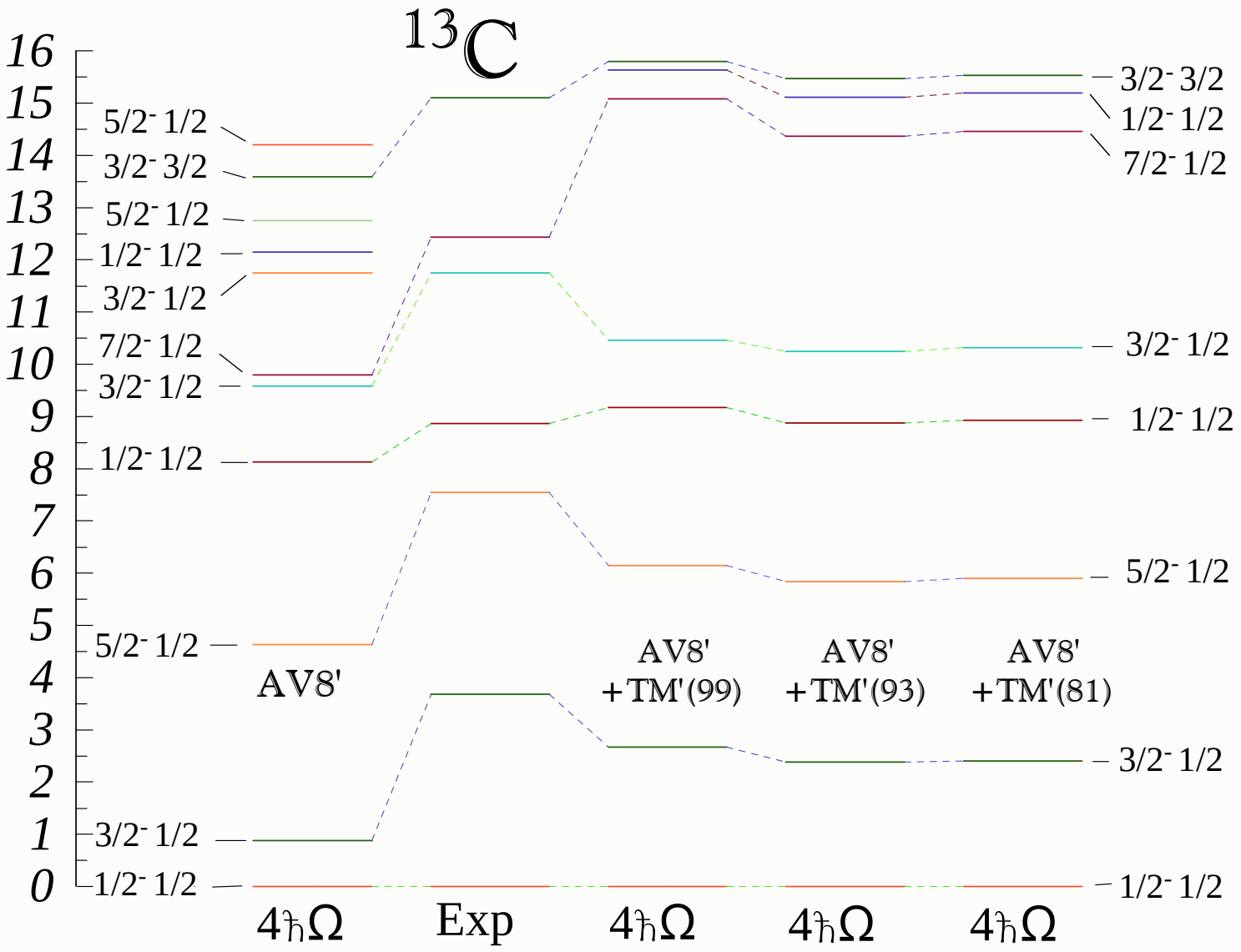
Gamow-Teller transitions
^{11}B → ^{11}C
B(GT; $3/2^-_1 \rightarrow J^-_f$)

J^-_f	AV8'	AV8'+TM'(99)	Exp
$3/2^-_1$	0.765	0.315	0.345
$1/2^-_1$	0.909	0.591	0.440
$5/2^-_1$	0.353	0.517	0.526
$3/2^-_2$	0.531	0.741	0.525
$5/2^-_2$	0.197	0.625	0.461

New (^{3}He,t) experiment at RCNP Osaka, Y. Fujita *et al.*, PRC **70**, 011306(R) (2004).

V_{3b} : Bad description of ^{11}B greatly improved

^{13}C with the Tucson-Melbourne force



TM'(81,93,99)
Differences more significant for heavier nuclei

Issues of convergence

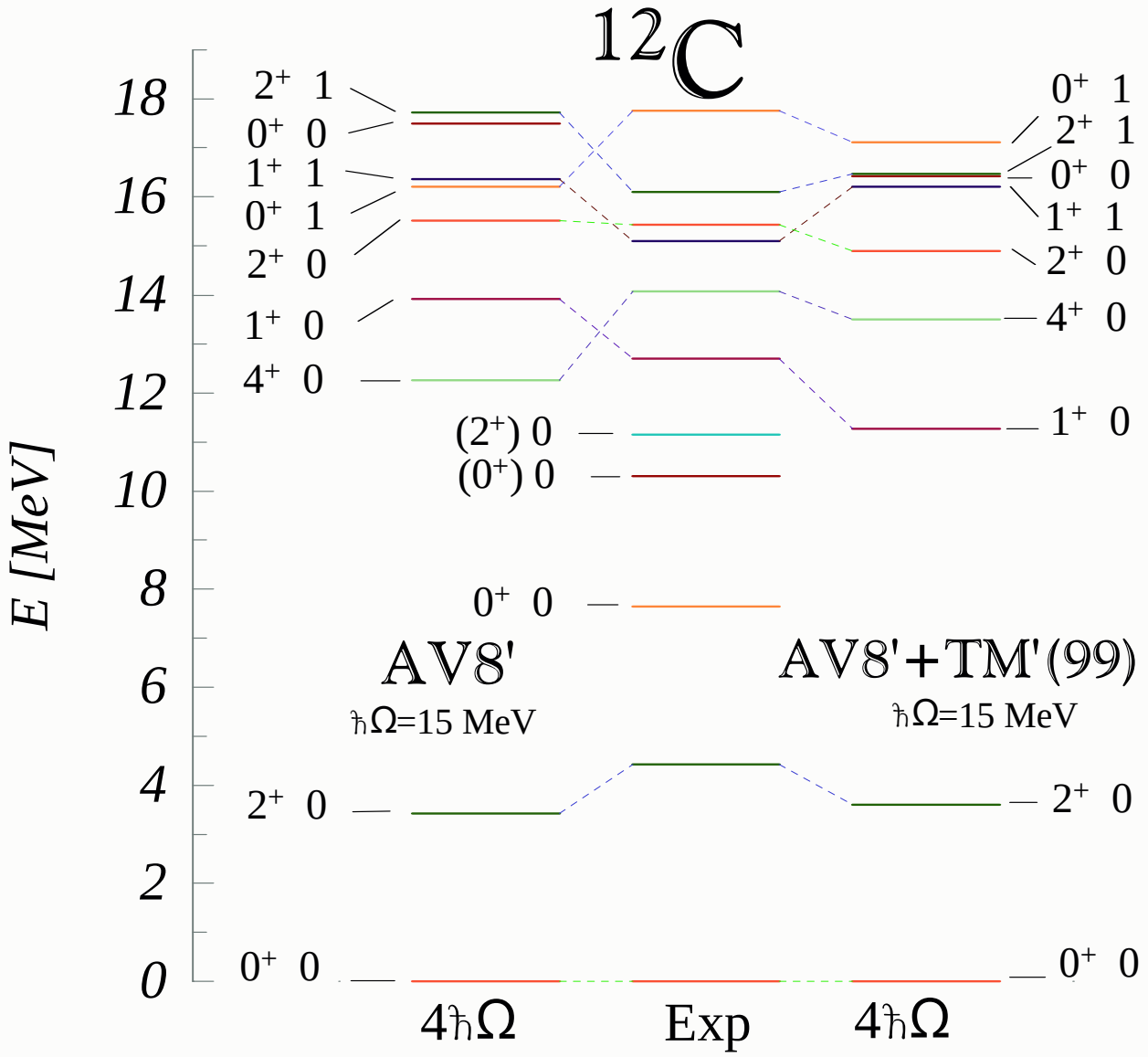
Cutoff adjustments

Need for the 6ħΩ calculation
☞ very challenging

☼ Nuclear structure can be used to learn about the form of and can constrain the parameters of the three-nucleon forces

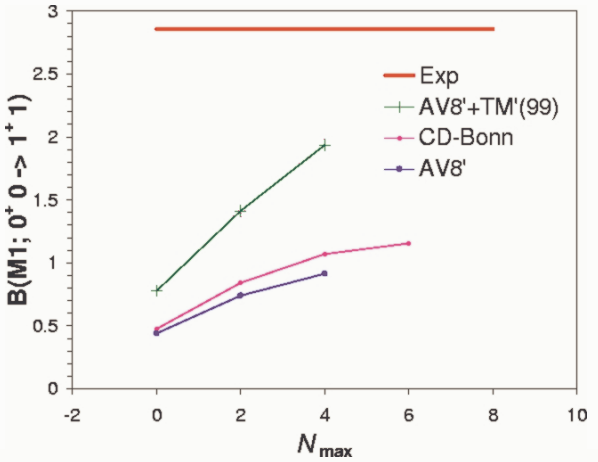
^{12}C with the Tucson-Melbourne force

- Binding energy increase by 6 MeV
- Improved level ordering
  - $1^+\ 0 \leftrightarrow 4^+\ 0$
  - T=1 states
- B(GT; $0^+\ 0 \rightarrow 1^+\ 1$) & B(M1; $0^+\ 0 \rightarrow 1^+\ 1$) stronger
  - Closer to experiment



Neutrino scattering on ^{12}C

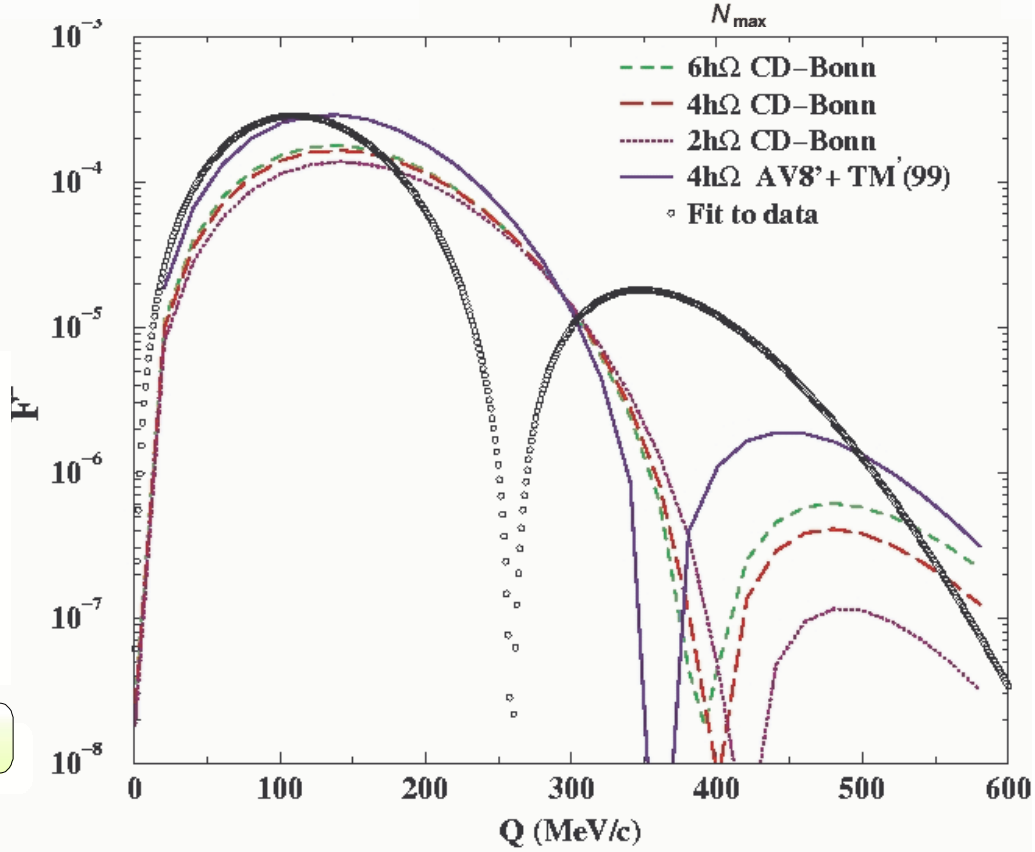
Exclusive $0^+\,0 \rightarrow 1^+\,1$ cross section & transistions



- Extremely sensitive to the spin-orbit interaction strength
  - B(GT) (B(M1)) $<\sigma\tau>$
    - No spin-orbit → 0^+ 0 and 1^+ 1 in different SU(4) irreps
      - ➜ no transition
    - ^{12}C ground state 8 nucleons in $p3/2$
      - ➜ Transition overestimated by a factor of six
- NCSM - no fit, no free parameters
  - V^{2b} up to $6\hbar\Omega$ - saturation
    - Underestimates by a factor of 2-3
  - $V^{2b}+V^{3b}$ up to $4\hbar\Omega$
    - Significant improvement
  - Different processes dominated by different Q
    - Correlation with M1 transverse form factor

	AV8'	AV8'+TM'(99)	Exp
B(GT)	0.26	0.67	0.88
	CD-Bonn	AV8'+TM'(99)	Exp
(ν_e,e^-)	3.69	6.8	8.9±0.3±0.9
(ν_μ,μ^-)	0.312	0.537	0.56±0.08±0.1
μ-capture	2.38	4.43	6.0±0.4

V^{3b} increases spin-orbit interaction strength



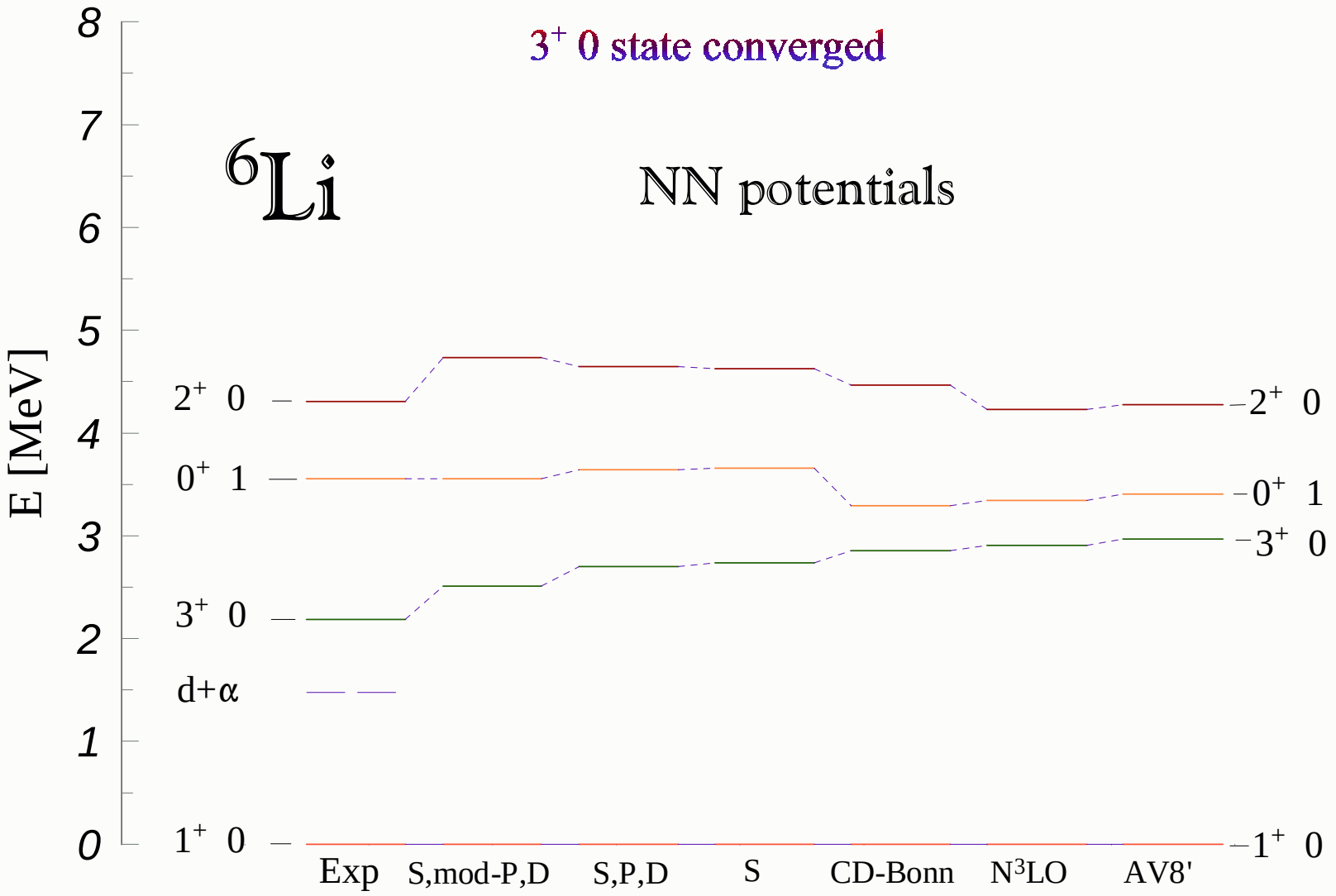
Do we really need a three-body interaction?

P. Doleschall, I. Borbely, Z. Papp and W. Plessas:
Nonlocal NN interaction that fits two-nucleon data and ^{3}H, ^{3}He binding energies
Phys. Rev. C **67**, 064005 (2003)

- Two-nucleon interaction in coordinate space
  - Local at long ranges: Yukawa tail
  - Nonlocal at shorter distances (up to 3 fm)
- Published version: Nonlocal $^1S_0$, $^3S_1$-$^3D_1$ channels
  - Remaining channels taken from AV18
    - Denoted as INOY *S*
- Two more versions:
- i) Nonlocal in all channels containing *S*, *P* and *D* waves
  - Remaining channels taken from AV18
    - Denoted as INOY *SPD*
- ii) Nonlocal in all channels containing *S*, *P* and *D* waves
  - Triplet *P* waves modified to reproduce three-nucleon analysing powers
  - Remaining channels taken from AV18
    - Denoted as INOY *Smod**PD*

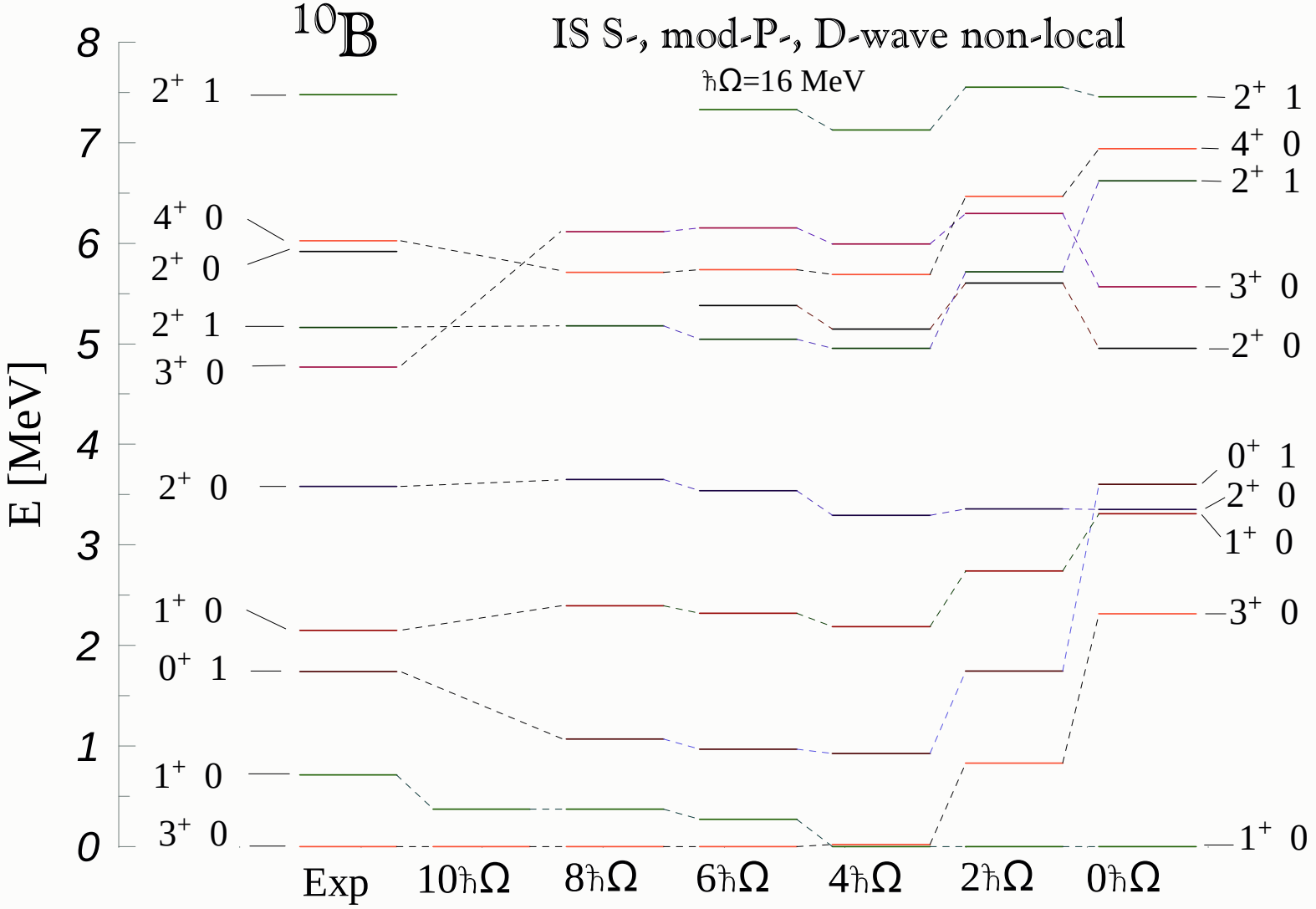
Straightforward to use in the NCSM and apply to *p*-shell nuclei

^{6}Li spectrum sensitive to the NN potential



INOY nonlocal NN potential with modified triplet P-waves
⇨ best agreement with experiment
⇨ very similar effect as adding the three-nucleon interaction to standard NN potentials

^{10}B with the INOY nonlocal NN potential



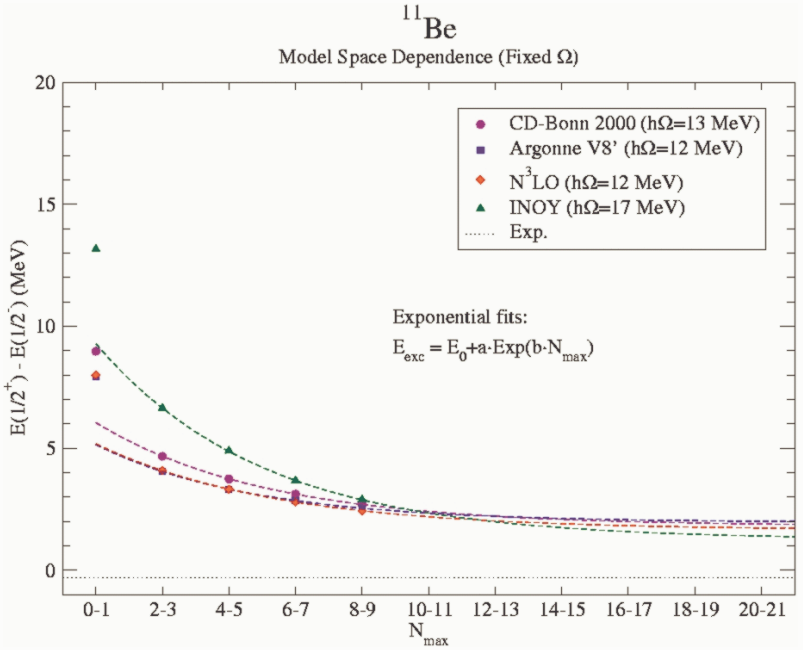
At least part of the three-nucleon interaction can be traded for nonlocality in two-nucleon interaction

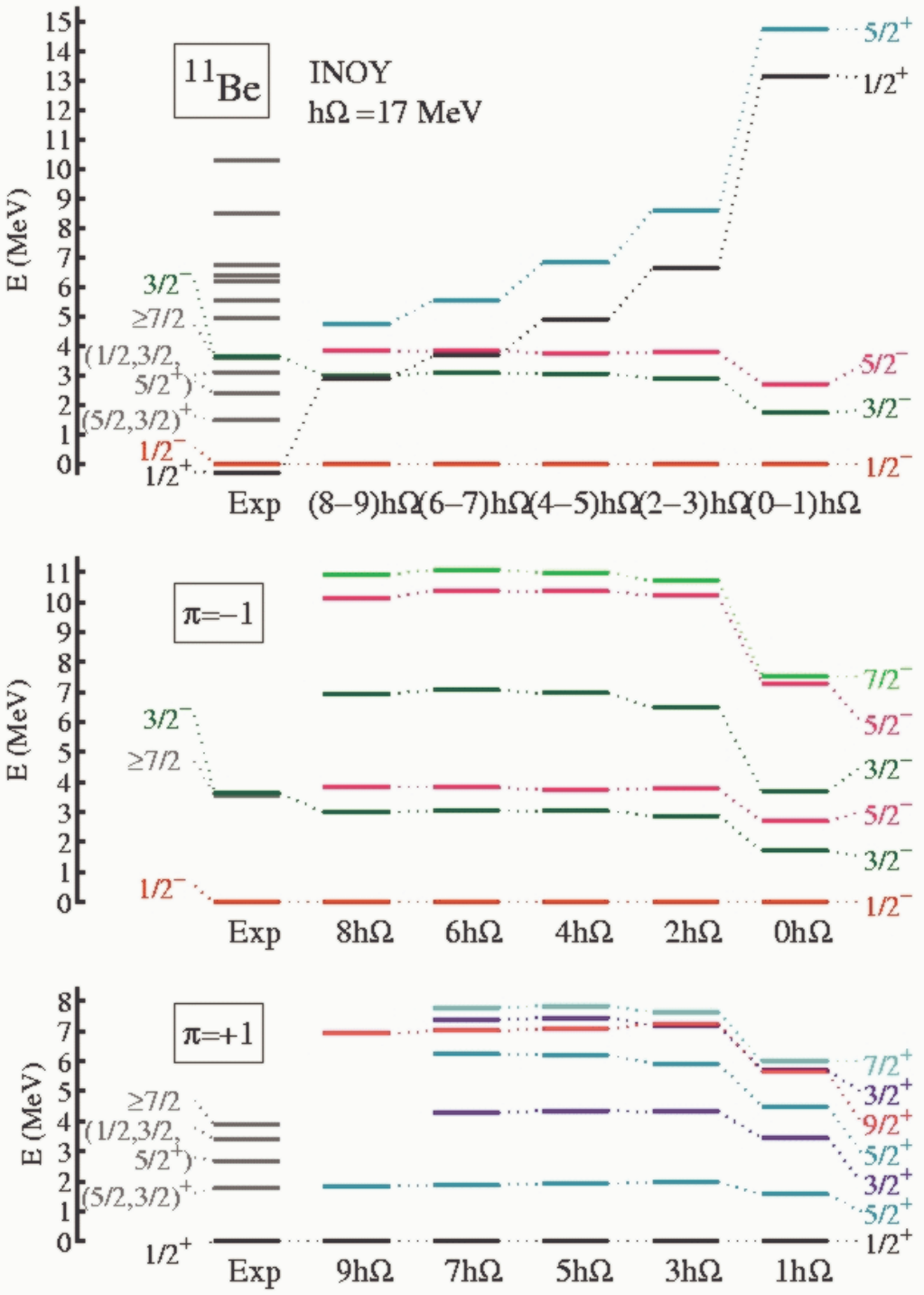
INOY nonlocal NN potential with modified triplet P-waves
⇨ better agreement with experiment than with any standard NN potentials

Parity inversion in ^{11}Be

Does the three-body force play a role?

- NCSM calculations performed up to 9ħΩ basis space
  - Code Antoine
  - Dim reaching 7×10^8 (706 087 842 in the 9ħΩ)
- Four different realistic NN potentials
  - CD-Bonn 2000
  - Argonne V8'
  - Chiral N^{3}LO
  - Nonlocal INOY with modified *P*-waves
    - Includes a part of the three-body effects
- INOY convergence rate differs from the other NN potentials
  - Three-body force might play a role

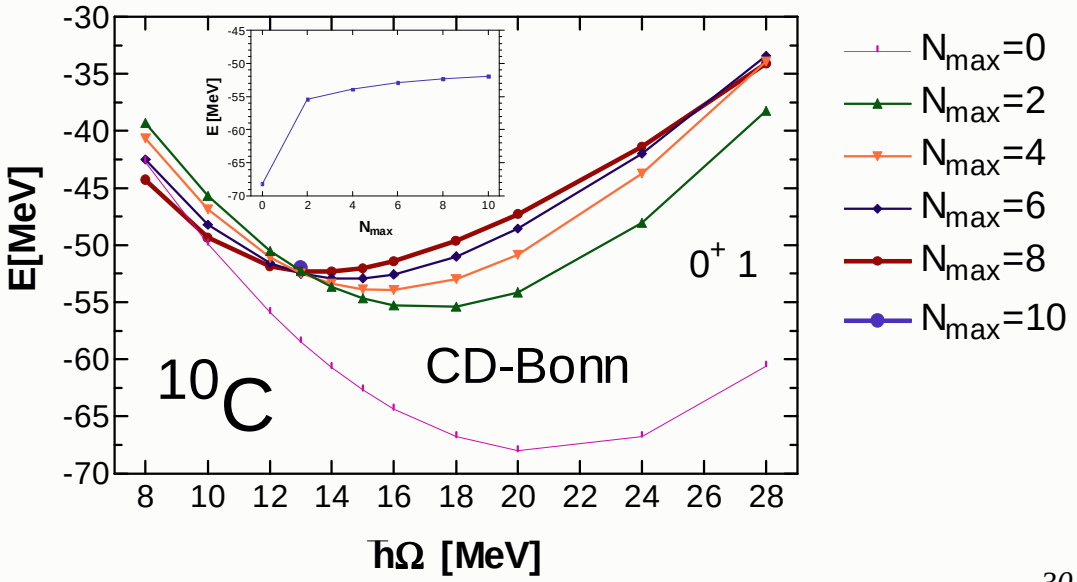


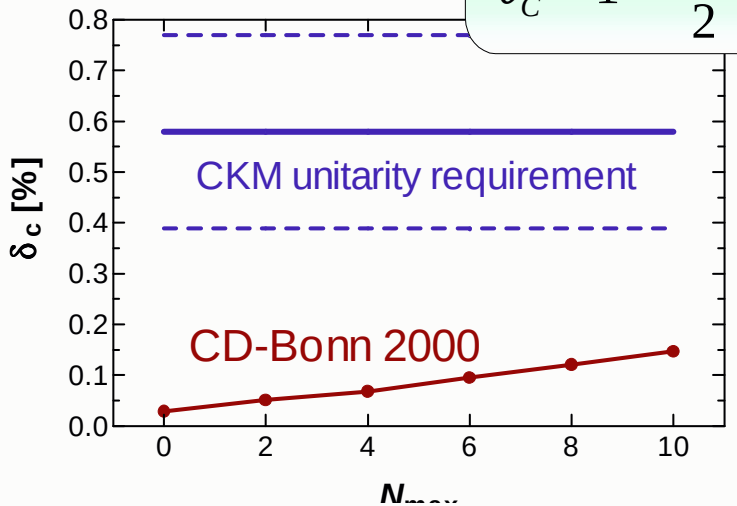


Isospin mixing correction of $^{10}C \rightarrow {}^{10}B$ Fermi matrix element

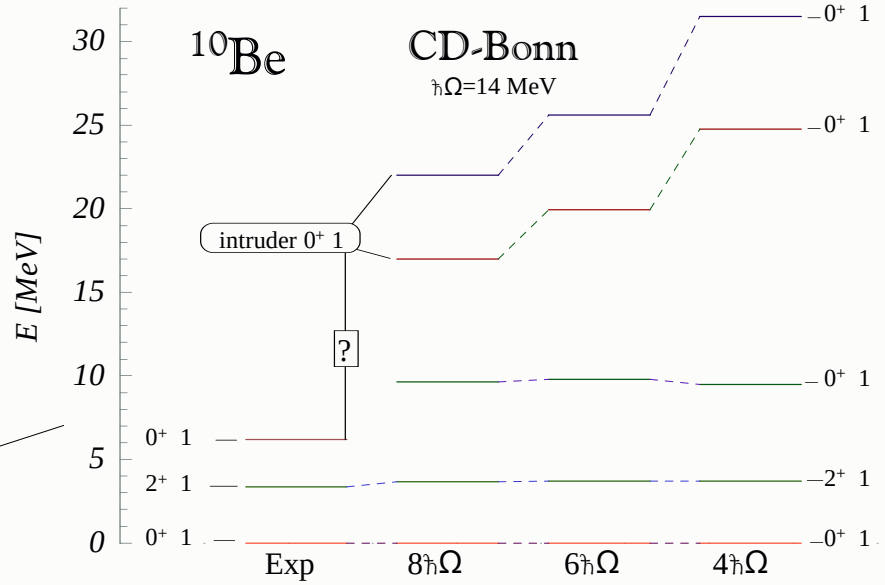
Relevant to CKM matrix unitarity ($v_{ud}^2 + v_{us}^2 + v_{ub}^2 = 1$?)

$$\delta_C = 1 - \frac{M_F^2}{2}$$





- ^{10}C and ^{10}B 0^+ 1 calculations up to 10ħΩ
  - Code Antoine (E. Caurier) - dimension 8×10^8
- $$M_F = \left\langle {}^{10}B;0^+1 \middle| T_- \middle| {}^{10}C;0^+1 \right\rangle$$
  - $=\sqrt{2}$ for an isospin invariant system
- Needed to extract v_{ud} m.e. from experimental Fermi β- decay ft values



Extrapolated value: $\delta_C \approx 0.19$ %
Insufficient for CKM unitarity

Low-energy reactions: Cluster overlaps

Consider *a*-nucleon projectile, *(A-a)*-nucleon target and *A*-nucleon composite system

- Jacobi coordinates

Center-of-mass of *A* nucleons

$$\vec{\xi}_0 = \frac{1}{\sqrt{A}}\left[\vec{r}_1 + \vec{r}_2 + \ldots + \vec{r}_A\right] \quad \ldots$$

$$\vec{\xi}_1 = \frac{1}{\sqrt{2}}\left[\vec{r}_1 - \vec{r}_2\right]$$

$$\vec{\xi}_2 = \sqrt{\frac{2}{3}}\left[\tfrac{1}{2}(\vec{r}_1 + \vec{r}_2) - \vec{r}_3\right] \quad \ldots$$

$$\vec{\xi}_{A-a} = \sqrt{\frac{(A-a)a}{A}}\left[\tfrac{1}{A-a}(\vec{r}_1 + \vec{r}_2 + \ldots + \vec{r}_{A-a}) - \tfrac{1}{a}(\vec{r}_{A-a+1} + \ldots + \vec{r}_A)\right]$$

Relative motion of projectile and target

- Channels described by ΓJ , $\Gamma \equiv A\text{-}a\,\alpha I_1\, a\,\beta I_2\, sl$
  - $\boldsymbol{s} = \boldsymbol{I}_1 + \boldsymbol{I}_2$ ... channel spin
  - l ... orbital momentum
- Channel states

$$\left|\Phi_{\Gamma}^{JM}\right\rangle = \sum (I_1 M_1 I_2 M_2 | s m_s)(s m_s l m_l | JM) \left| A-a\,\alpha I_1 M_1 \right\rangle \left| a\,\beta I_2 M_2 \right\rangle Y_{l m_l}(\hat{\xi}_{A-a})$$

^{6}Li(^{4}He) d(t)

- Overlap with the composite state

$$u^{J}_{A\lambda;\,A-a\Gamma}(\xi_{A-a}) = \left\langle A\lambda JM \left| \mathcal{A} \right| \Phi_{\Gamma}^{JM} \right\rangle$$

^{8}Be(^{7}Li) Antisymmetrizer

Calculation of cluster overlaps using the NCSM wave functions

Consider target, projectile and composite system as eigenstates of NCSM effective Hamiltonians with a consistent definition of basis (N_{max}, $\hbar\Omega$)

- Cluster overlap defined in Jacobi-coordinate basis
- Projectile states for a=2,3,4 are described in Jacobi-coordinate HO basis
- NCSM calculations for A>4 more efficient in single-particle Slater Determinant HO basis
  - Composite system+target in SD HO basis
- Coordinate relation for a=1

Target center-of mass

$$\vec{R}_{A-1} = \sqrt{\frac{1}{A}}\vec{\xi}_{A-1} + \sqrt{\frac{A-1}{A}}\vec{\xi}_0$$

$$\vec{r}_A = -\sqrt{\frac{A-1}{A}}\vec{\xi}_{A-1} + \sqrt{\frac{1}{A}}\vec{\xi}_0$$

Composite center-of mass

- In the SD basis both the composite and the target states in the $0\hbar\Omega$ center-of-mass state

$$\varphi_{000}(\vec{R}_{A-1})\varphi_{nlm_l}(\vec{r}_A) = \langle nl00l|00nll\rangle_{\frac{1}{A-1}}\,\varphi_{nlm_l}(\vec{\xi}_{A-1})\varphi_{000}(\vec{\xi}_0)$$

a=1

- Cluster overlap for a=1

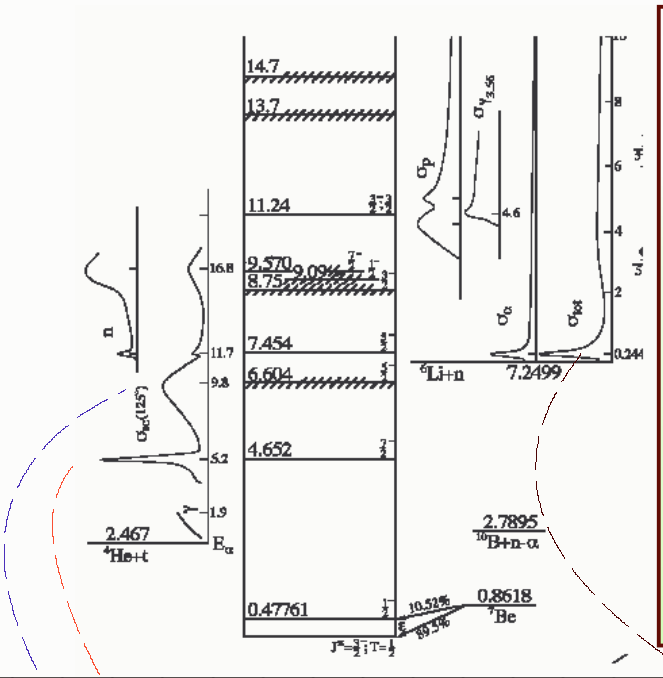
$$\left\langle A\lambda J\middle|\mathcal{A}\Phi^{J}_{\Gamma}\right\rangle = \sum_n R_{nl}(\xi_{A-1})\frac{1}{\langle nl00l|00nll\rangle_{\frac{1}{A-1}}}\frac{1}{\hat{J}}\sum_j \begin{Bmatrix} I_1 & \frac{1}{2} & s \\ l & J & j \end{Bmatrix}\hat{s}\hat{j}(-1)^{I_1+J-j}$$

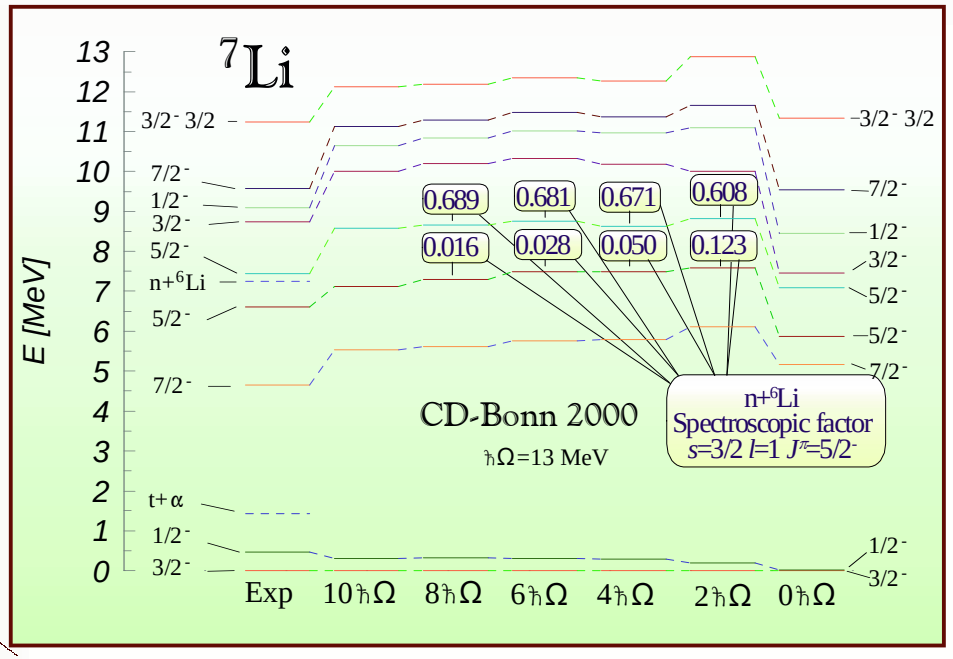
$$\times\ {}_{SD}\left\langle A\lambda J\middle\| a^{+}_{nlj}\middle\| A-1\alpha I_1\right\rangle_{SD}$$

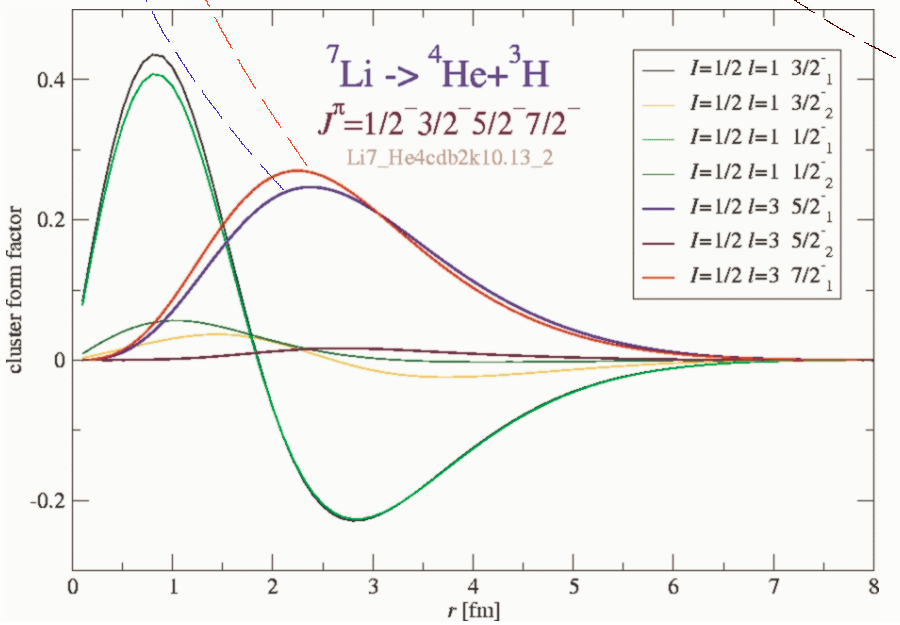
$$(-1)^l\left(\frac{A-1}{A}\right)^{\frac{2n+l}{2}}$$

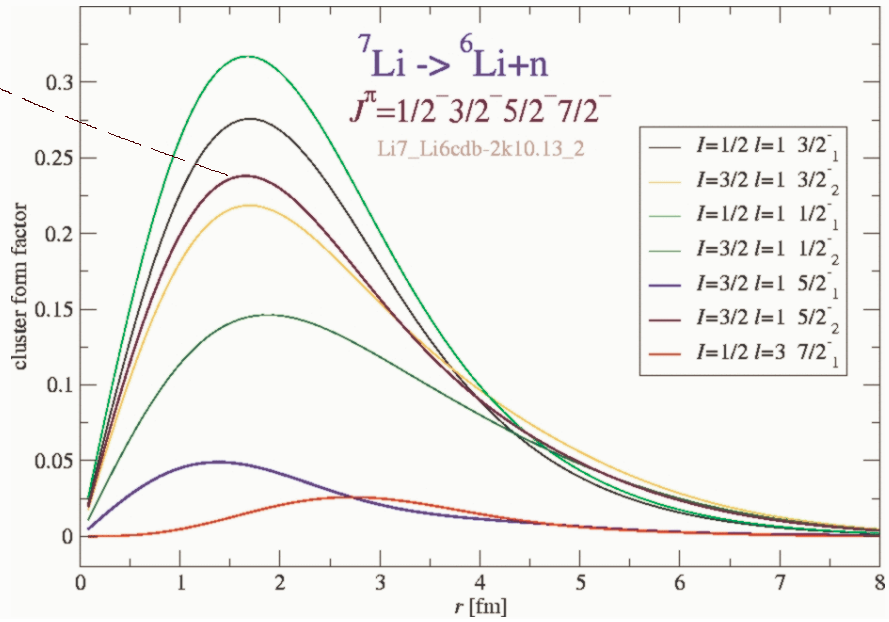
Generalization for a=2,3,4 straightforward

Clustering in light nuclei & resonant reactions

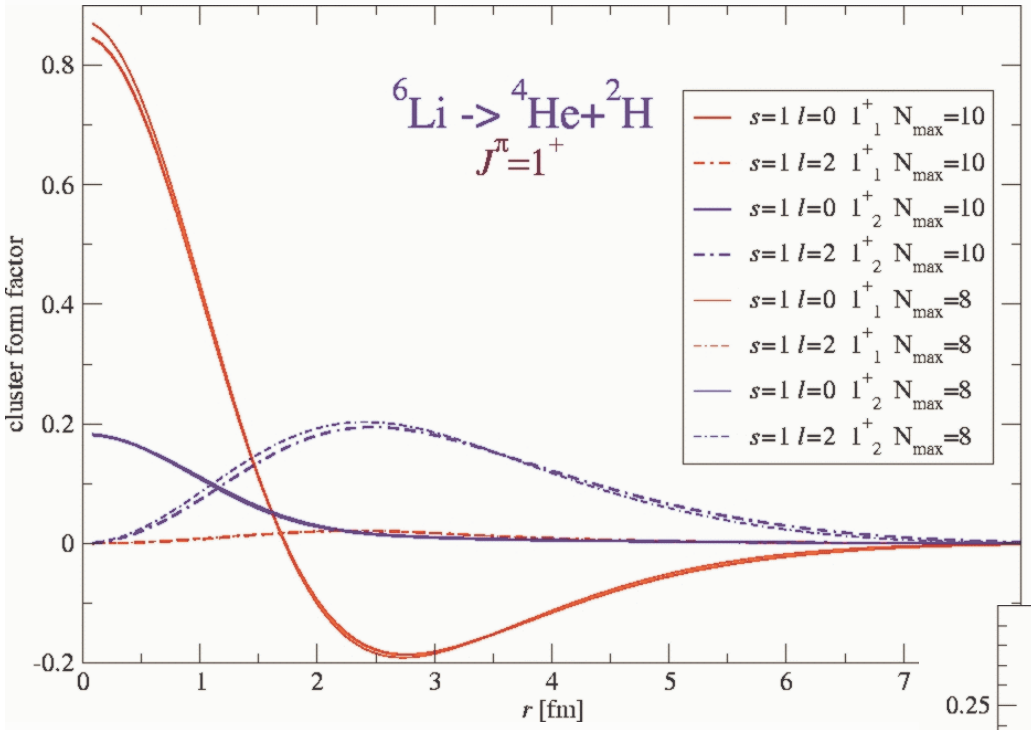








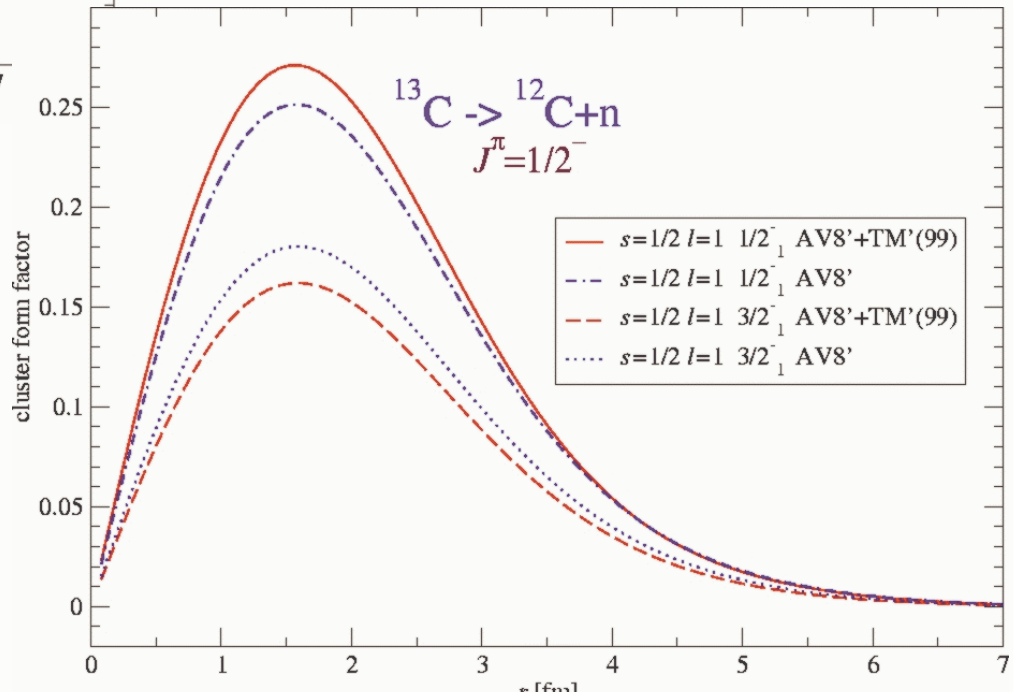
Cluster form factors for ^{6}Li→^{4}He+d and ^{13}C→^{12}C+n



^{6}Li ground state dominated by S-wave (s=1, l=0) ^{4}He+d channel

The first excited 1^{+} 0 state dominated by D-wave (s=1, l=2) channel

Effects of the three-nucleon interaction in the ^{13}C→^{12}C+n overlaps:
Stronger spin-orbit splitting increases the J=1/2^{-}
and
reduces the J=3/2^{-}
cluster form factors
and spectroscopic factors



JLM microscopic folding optical potential

J.-P. Jeukenne, A. Lejeune and C. Mahaux, Phys. Rev. C **16**, 80 (1977)

- Complex optical potential in infinite nuclear matter from BHF approximation and Reid NN potential
- Applied to finite nuclei using local density approximation
  - Non-uniform medium effects of finite range of the effective interaction included in a semiphenomenological way suggested by Hartree approximation
- Monopole diagonal optical potential:

$$U(r,E) = \frac{1}{(t\sqrt{\pi})^3}\int \rho(r')\,\mathcal{U}^{JLM}\left(\rho(r_i),E\right)\exp\left(-|\vec{r}-\vec{r}'|/t^2\right)d\vec{r}'$$

$$\mathcal{U}^{JLM}\left(\rho,E\right) = V^{JLM}\left(\rho,E\right)/\rho \;;\quad r_i = \begin{cases} r \\ r' \\ (r+r')/2 \end{cases}$$

- Transition optical potential:

$$U_{tr}(\vec{r},E) = \frac{1}{(t\sqrt{\pi})^3}\int \rho_{tr}(\vec{r}')\left(1+\rho\frac{d}{d\rho}\right)\mathcal{U}^{JLM}\left(\rho,E\right)\Big|_{r_i}\exp\left(-|\vec{r}-\vec{r}'|/t^2\right)d\vec{r}'$$

Spin-orbit potential

- Original JLM only the central optical potential
- Complex spin-orbit potential added by E. Bauge *et al.*, Phys. Rev. C **58**, 1118.

$$V^{so}_{n(p)} = \lambda_{so}\left(\frac{\hbar}{m_\pi c}\right)^2 \vec{l}\cdot\vec{\sigma}\,\frac{1}{2}\frac{1}{r}\frac{d}{dr}\left(\frac{2}{3}\rho_{p(n)} + \frac{1}{3}\rho_{n(p)}\right)$$

- Other forms available in direct reaction codes (Fresco)
  - Wood-Saxon

$$V^{so}_{n(p)} = \lambda_{so}\left(\frac{\hbar}{m_\pi c}\right)^2 \vec{l}\cdot\vec{\sigma}\,\frac{1}{r}\frac{d}{dr}\left(\frac{1}{1+\exp[(r-R)/a]}\right)$$

  - Squared Wood-Saxon

$$V^{so}_{n(p)} = \lambda_{so}\left(\frac{\hbar}{m_\pi c}\right)^2 \vec{l}\cdot\vec{\sigma}\,\frac{1}{r}\frac{d}{dr}\left(\frac{1}{1+\exp[(r-R)/a]}\right)^2$$

  - Gaussian

$$V^{so}_{n(p)} = \lambda_{so}\left(\frac{\hbar}{m_\pi c}\right)^2 \vec{l}\cdot\vec{\sigma}\,\frac{1}{4}\frac{1}{r}\frac{d}{dr}\left(\exp[-(r-R)^2/a^2]\right)$$

  - Yukawa

$$V^{so}_{n(p)} = \lambda_{so}\left(\frac{\hbar}{m_\pi c}\right)^2 \vec{l}\cdot\vec{\sigma}\,\frac{1}{4}\frac{1}{r}\frac{d}{dr}\left(\frac{1}{r}\exp[-(r-R)/a]\right)$$

  - Exponential

$$V^{so}_{n(p)} = \lambda_{so}\left(\frac{\hbar}{m_\pi c}\right)^2 \vec{l}\cdot\vec{\sigma}\,\frac{1}{4}\frac{1}{r}\frac{d}{dr}\left(\exp[-(r-R)/a]\right)$$

Density calculation in the NCSM

NCSM wave functions for the target nucleus

- Density operator

$$\hat{\rho}(\vec{r}) = \sum_{i=1}^{A} \delta(\vec{r} - \vec{r}_i) = \sum_{i=1}^{A} \frac{\delta(r - r_i)}{r r_i} \sum_{Kk} Y_{Kk}(\hat{r}_i)\, Y^*_{Kk}(\hat{r})$$

- Standard calculation in the Slater determinant basis

$$\left\langle I_f M_f \left| \hat{\rho}(\vec{r}) \right| I_i M_i \right\rangle = \frac{1}{\hat{I}_f} \sum_{Kk} (I_i M_i Kk \,|\, I_f M_f)\, Y^*_{Kk}(\hat{r}) \sum_{\substack{n_1 l_1 j_1 \\ n_2 l_2 j_2}} R_{n_1 l_1}(r) R_{n_2 l_2}(r) \frac{-1}{\hat{K}} \left\langle l_1 j_1 \| Y_K \| l_2 j_2 \right\rangle \left\langle I_f \left\| (a^+_{n_1 l_1 j_1} \tilde{a}_{n_2 l_2 j_2})^{(K)} \right\| I_i \right\rangle$$

  - Contains spurious center of mass components ☹

- Translationally invariant calculation

$$\left\langle I_f M_f \left| \hat{\rho}(\vec{r}) \right| I_i M_i \right\rangle = \left(\frac{A}{A-1}\right)^{3/2} \frac{1}{\hat{I}_f} \sum_{Kk} (I_i M_i Kk \,|\, I_f M_f)\, Y^*_{Kk}(\hat{x}) \sum_{\substack{nl, n_1 l_1 j_1 \\ n'l', n_2 l_2 j_2}} R_{nl}(x) R_{n'l'}(x)$$

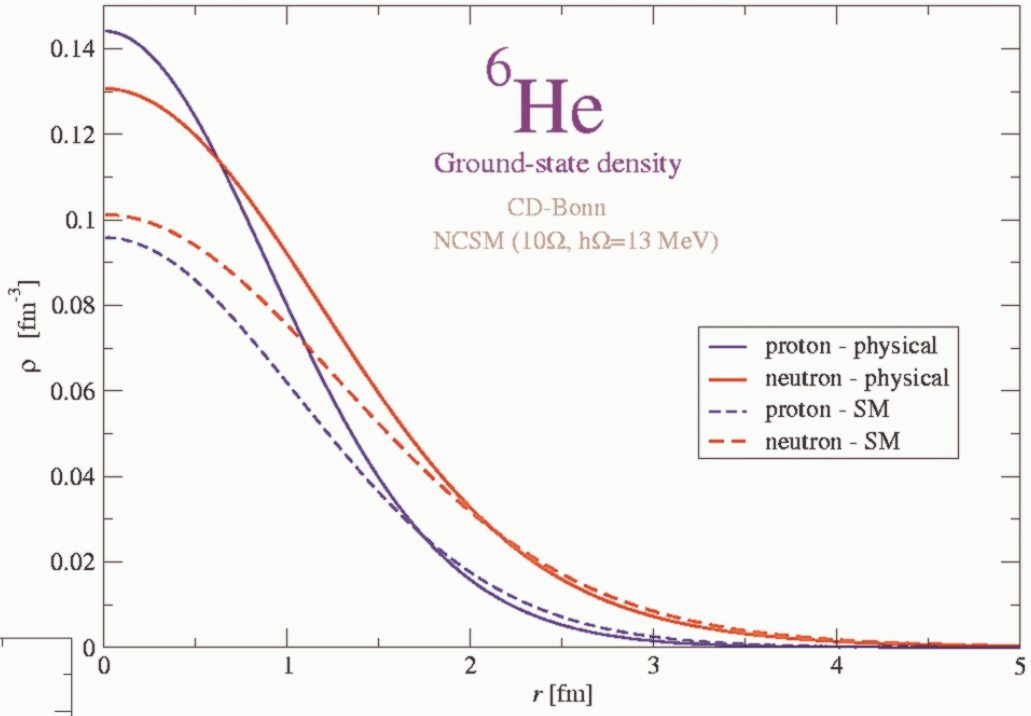
$$\times (-1)^K \frac{\hat{l}\hat{l}'(l0l'0|K0)}{\hat{l}_1 \hat{l}_2 (l_1 0 l_2 0|K0)} (M^K)^{-1}_{nln'l', n_1 l_1 n_2 l_2} \frac{-1}{\hat{K}} \left\langle l_1 j_1 \| Y_K \| l_2 j_2 \right\rangle \left\langle I_f \left\| (a^+_{n_1 l_1 j_1} \tilde{a}_{n_2 l_2 j_2})^{(K)} \right\| I_i \right\rangle$$

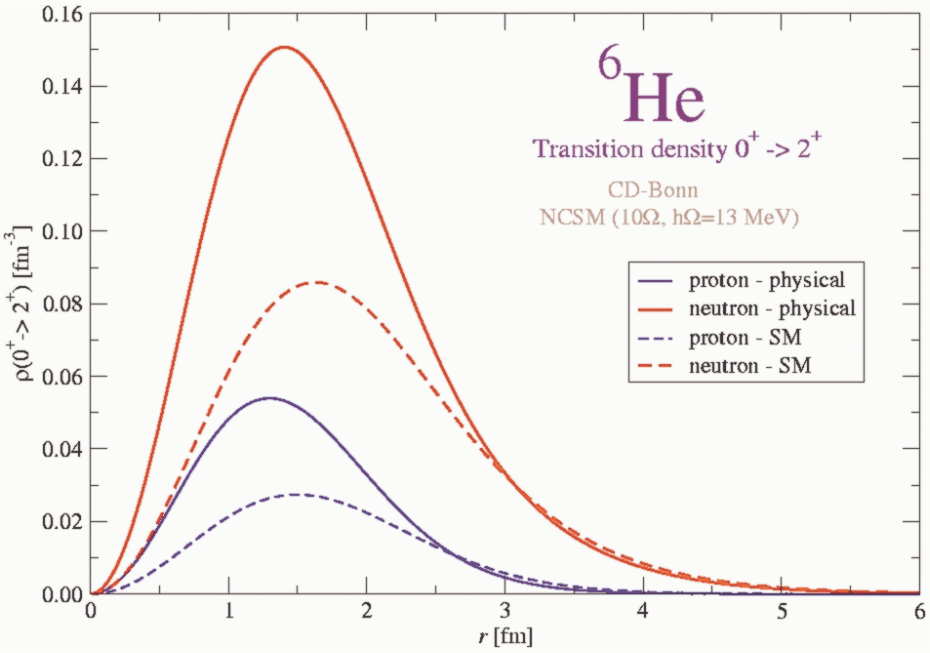
$$\vec{x} = \sqrt{\frac{A}{A-1}} (\vec{r} - \vec{R})\,; \qquad M^K_{n_1 l_1 n_2 l_2, nln'l'} = \sum_{N_1 L_1} (-1)^{l+l'+K+L_1} \begin{Bmatrix} l_1 & L_1 & l \\ l' & K & l_2 \end{Bmatrix} \hat{l}\hat{l}' \left\langle nl00l \left| N_1 L_1 n_1 l_1 l \right\rangle_{\frac{1}{A-1}} \left\langle n'l'00l' \right| N_1 L_1 n_2 l_2 l' \right\rangle_{\frac{1}{A-1}}$$

  - Physical density ☺

Importance of translational invariance

Ground state monopole density
Only physical density gives correct point-nucleon radius
Dramatic impact on spin-orbit optical potential proportional to the derivative of the density



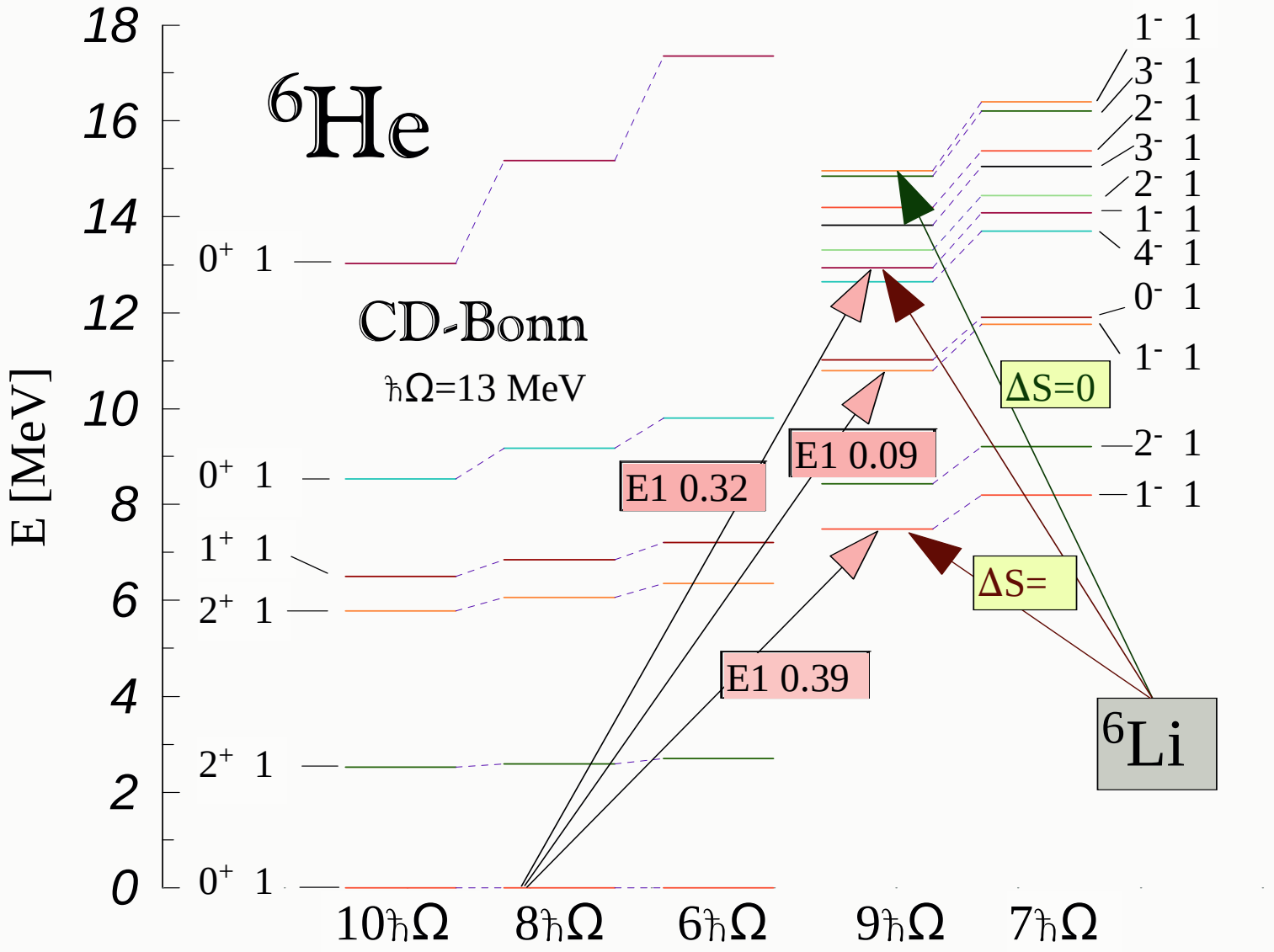


Transition density to the first 2^+ state

Both SM and physical densities integrate to the same *B(E2)*

^{6}He spectrum obtained in the NCSM

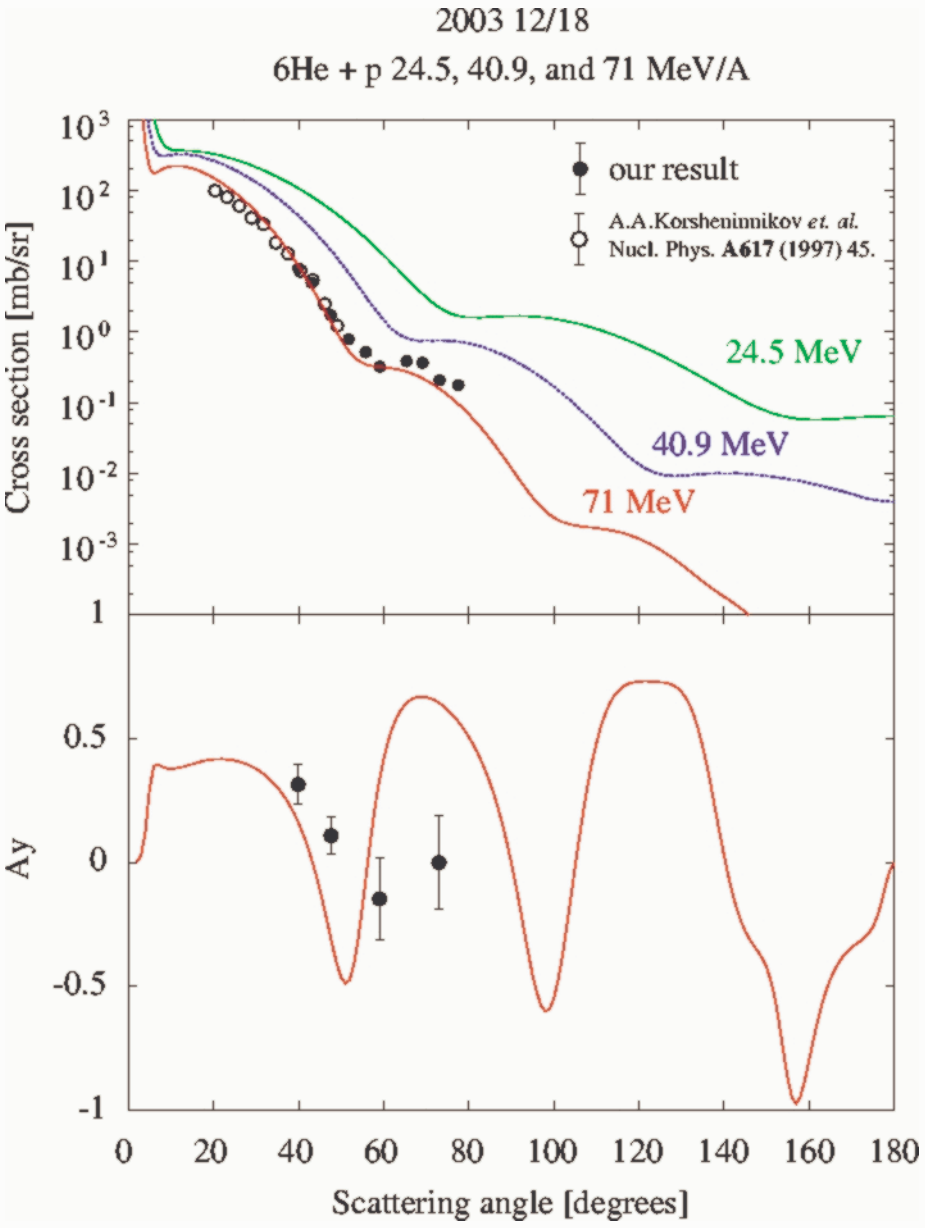
Relevance to the soft-dipole mode in halo nuclei



^{6}He on polarized proton target

CNS experiment at 71 MeV/A

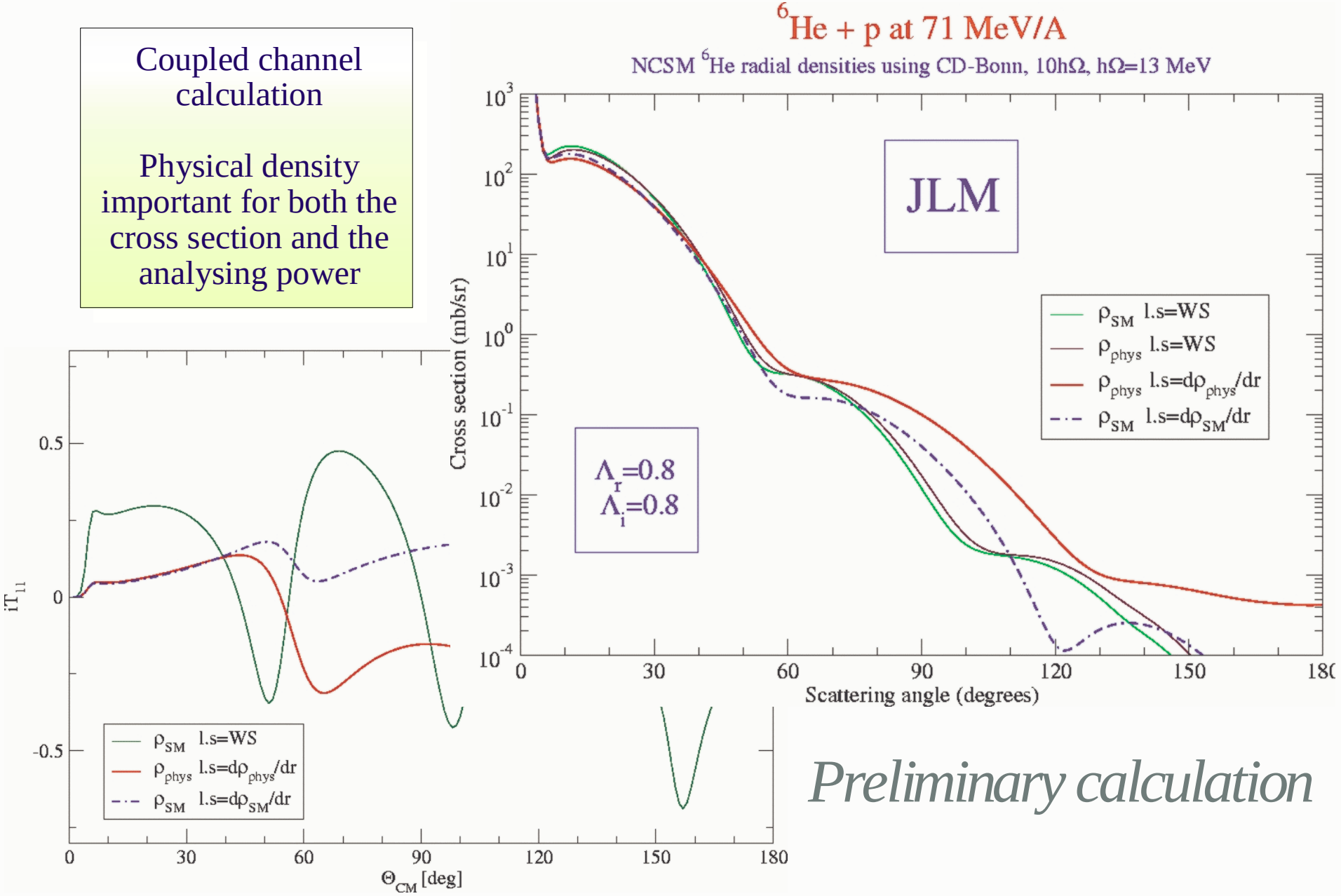
- NCSM ^{6}He ground state density
- JLM folding optical potential
  - Parameters as obtained by earlier studies of light nuclei:
    λ_V=0.8, λ_W=0.8, t=1 fm,
    ρ evaluated at target position,
    original JLM parametrization
- Spin-orbit term with Wood-Saxon shape
  - Parameters:
    λ_{SO}=5 MeV, R=1.3 $A^{1/3}$ fm, a=0.8 fm
- Direct reaction code FRESCO (I. Thompson)
- Reasonable agreement with experiment also for other energies (24.5 MeV - Dubna, 40.9 MeV - Ganil)



^{6}He on polarized proton target revisited

Coupled channel calculation

Physical density important for both the cross section and the analysing power



Preliminary calculation

Open issues

- Description of opposite parity states, intruder states and alpha-cluster states
  - Still larger basis
  - Four-body effective interaction to describe alpha-clustering?
- Further development of effective interactions (two- and higher-body)
  - Freedom of unitary transformations
    - Reduction of the HO frequency dependence
    - Speed up of convergence
- Increase of basis in calculations with realistic three-body interaction
  - Technical issue

Collaborators

- B. R. Barrett (University of Arizona), J. P. Vary (Iowa State University), D.-C. Zheng (University of Arizona: 1993-1995)
  - Started the program in 1992-1993
- E. Caurier (Strasbourg)
  - Shell model code Antoine
- A. Nogga (INT Seattle)
  - Three-body interaction, code cross-checking
- W. E. Ormand (LLNL)
  - Shell model code Redstick
- A. C. Hayes (LANL), S.A. Coon (DOE), G. P. Kamuntavicius (Vilnius)
- C. Forssen (LLNL), I. Stetcu (University of Arizona), V. Gueorguiev (LLNL)
  - New postdocs
- D. C. J. Marsden (University of Arizona: 1995-2001)
  - Ph.D. student

Conclusions and outlook

- *Ab initio* no-core shell model
  - Method for solving the nuclear structure problem for light nuclei
  - Apart from the GFMC the only working method for $A>4$ at present
  - Advantages
    - applicable for any NN potential
      - ➜Effective field theory
    - Easily extendable to heavier nuclei
    - Calculation of complete spectra at the same time
  - Success - importance of three-nucleon forces for nuclear structure

Work in progress

- Calculations with realistic three-body forces in the *p*-shell
  - Better determination of the structure of the three-body force itself
- Coupling of the NCSM to nuclear reactions theories
  - Direct reactions
    - Density from NCSM plus folding approaches
  - Low-energy resonant and nonresonant reactions
    - RGM-like approach
      - ➜Exotic nuclei: RIA
      - ➜Thermonuclear reaction rates: Astrophysics

Future plans

- Extensions to heavier nuclei
  - Effective ineraction for valence nucleons
    - RIKEN, RIA