CNS Summer School, 3rd Course,
17-20, August, 2004

Evolution of Shell Structure and Spin-Isospin Interaction

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Single-particle (or shell) structure is the basis of many nuclear properties such as (sub)magic number, deformation, and even the existence of the nucleus.

The single-particle levels in exotic nuclei can be different from those of stable nuclei due to the following aspects:

Loose binding *Woods-Saxon potential*

Neutron skin *Mean-Field models*

***NN* interaction, particularly, spin-isospin interactions (also in nuclei not very close to drip lines)**

Major subject :

Such shell evolution due to

tensor and 2-body *LS* interactions

Something simple enough that everyone can remember

1. Shell evolution due to Tensor interaction
2. Tensor effect in shell-model effective interaction
3. Tensor-implemented mean field model
4. Shell evolution due to 2-body *LS* interaction
5. Example : (partial) origin of carbon mystery
6. Summary
7. Exercise

Effective single particle energy

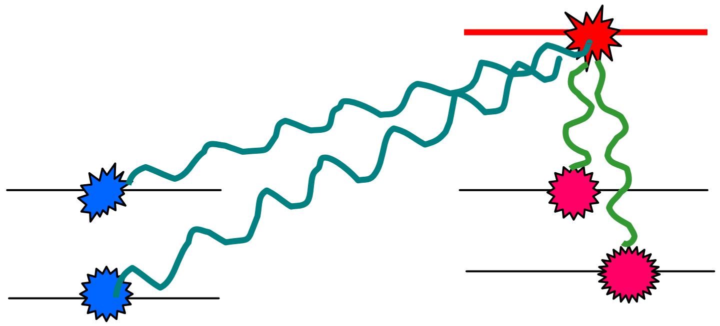
- Monopole part of the *NN* interaction

$$V_{ab}^{T} = \frac{\sum_{J}(2J+1)V_{abab}^{JT}}{\sum_{J}(2J+1)}$$

Angular averaged interaction

➡ spherical single particle energies

- Effective single-particle energy (ESPE)



Shift of single-particle energies due to interaction with other valence nucleons

Major origin of shell evolution
due to spin-isospin *NN* interaction :

tensor interaction

Tensor Interaction

$$V_T = (\tau_1 \tau_2)\ (\ [\sigma_1 \sigma_2]^{(2)}\ Y^{(2)}(\Omega)\ )\ Z(r)$$

contributes only to **S=1** states

relative motion

π meson : dominant source

ρ meson (~ π+π) : minor cancellation for smaller *r*

Important for binding *e.g.* B.S. Pudliner et al., Phys. Rev. **C**56, 1720 (1997)

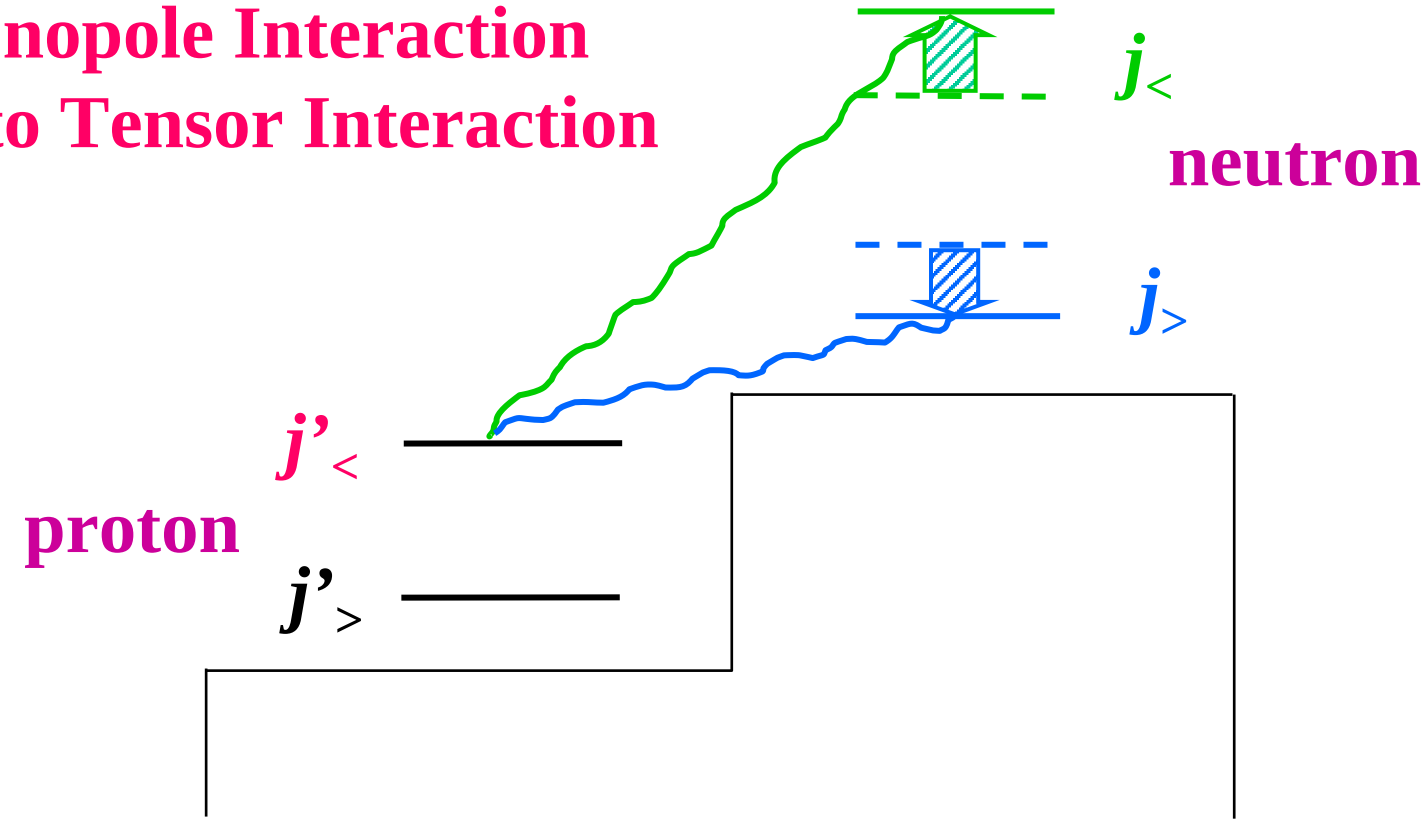
Has never shown up directly (or in the first order) in nuclear spectroscopy (e.g. levels, etc)

second-order effect on spin-orbit splitting

T. Terasawa, Prog. Theor. Phys. **23**, 87 (1960);

A. Arima and T. Terasawa, Prog. Theor. Phys. **23**, 87 (1960)

Monopole Interaction due to Tensor Interaction



Identity

$$(2j_> + 1)\, v_{m,T}^{(j'\, j_>)} + (2j_< + 1)\, v_{m,T}^{(j'\, j_<)} = 0$$

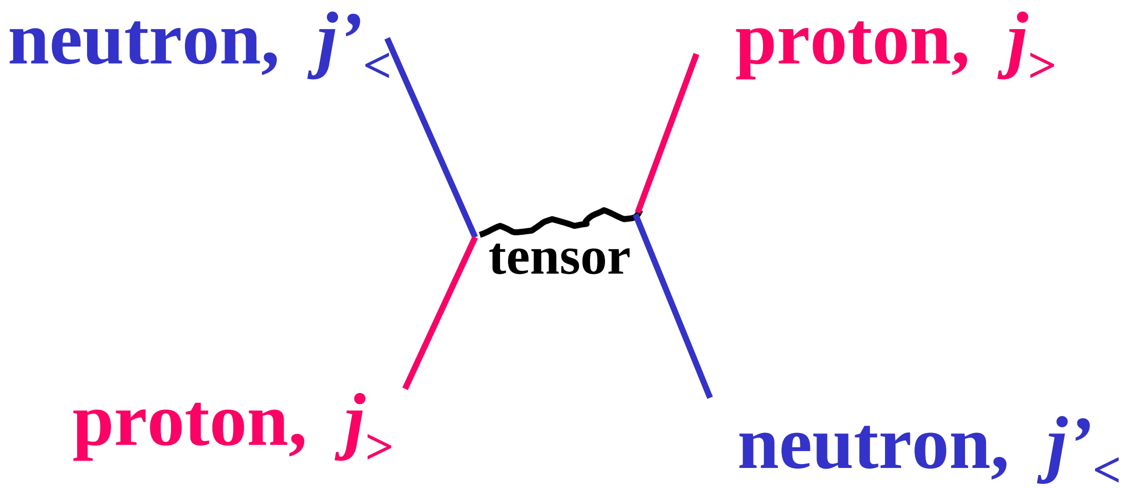
$v_{m,T}$: monopole strength for isospin T

Major features

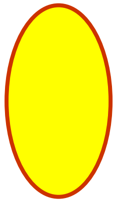
Opposite signs ⟹ spin-orbit splitting varied

T=0 : T=1 = 3 : 1 (same sign)

Only exchange terms

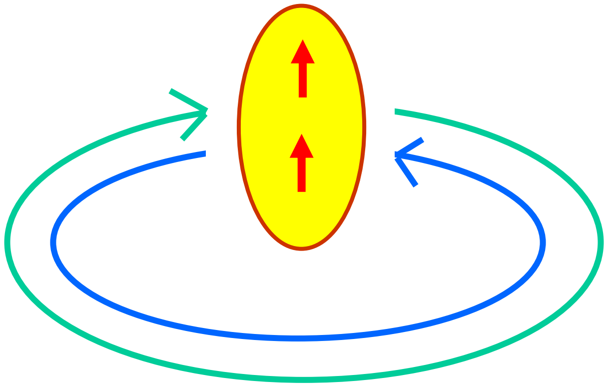


Intuitive Picture

 wave function of relative motion

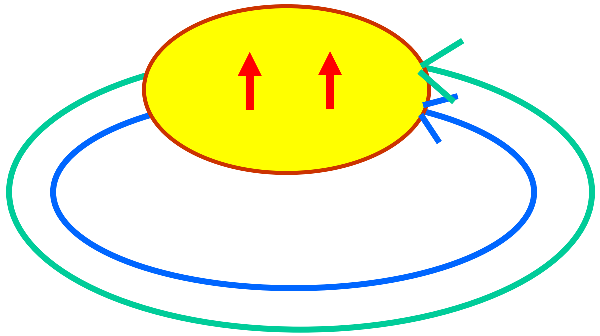
↑↑ spin of nucleon

large relative momentum



$j_>$ $j'_<$

small relative momentum



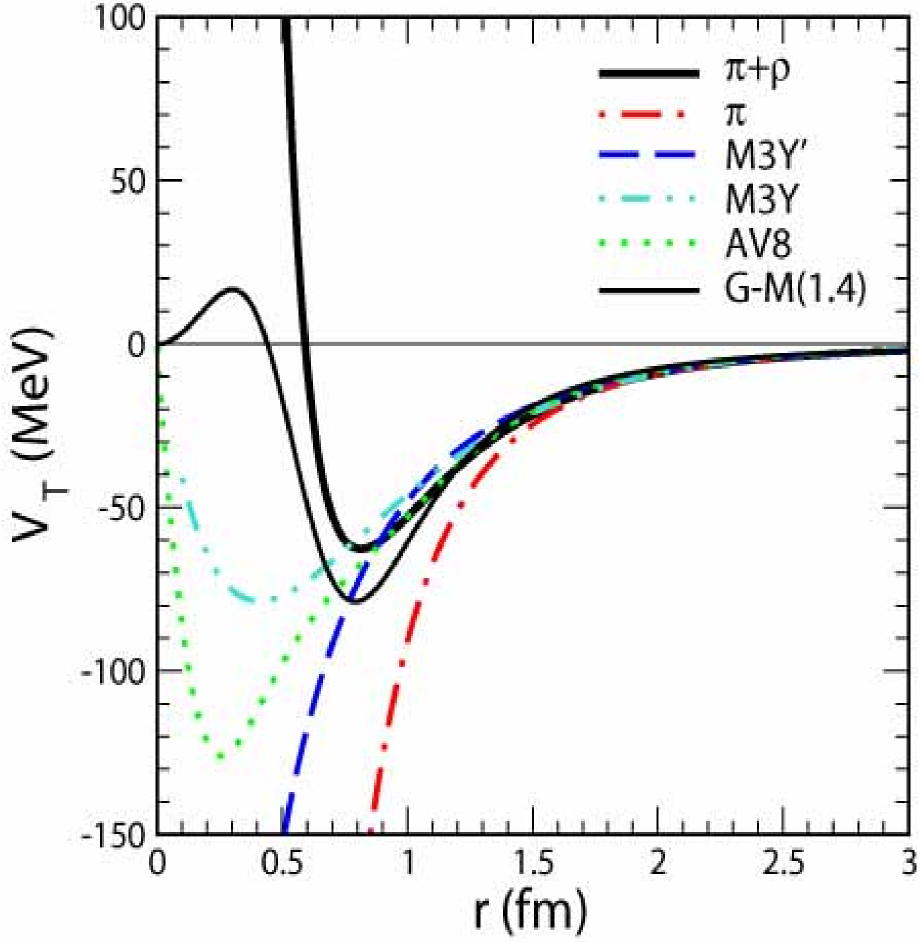
$j_>$ $j'_>$

deuteron ⇨ attractive

repulsive

Effect sizable for (i) same l or (ii) large l and l'

Tensor potential



tensor

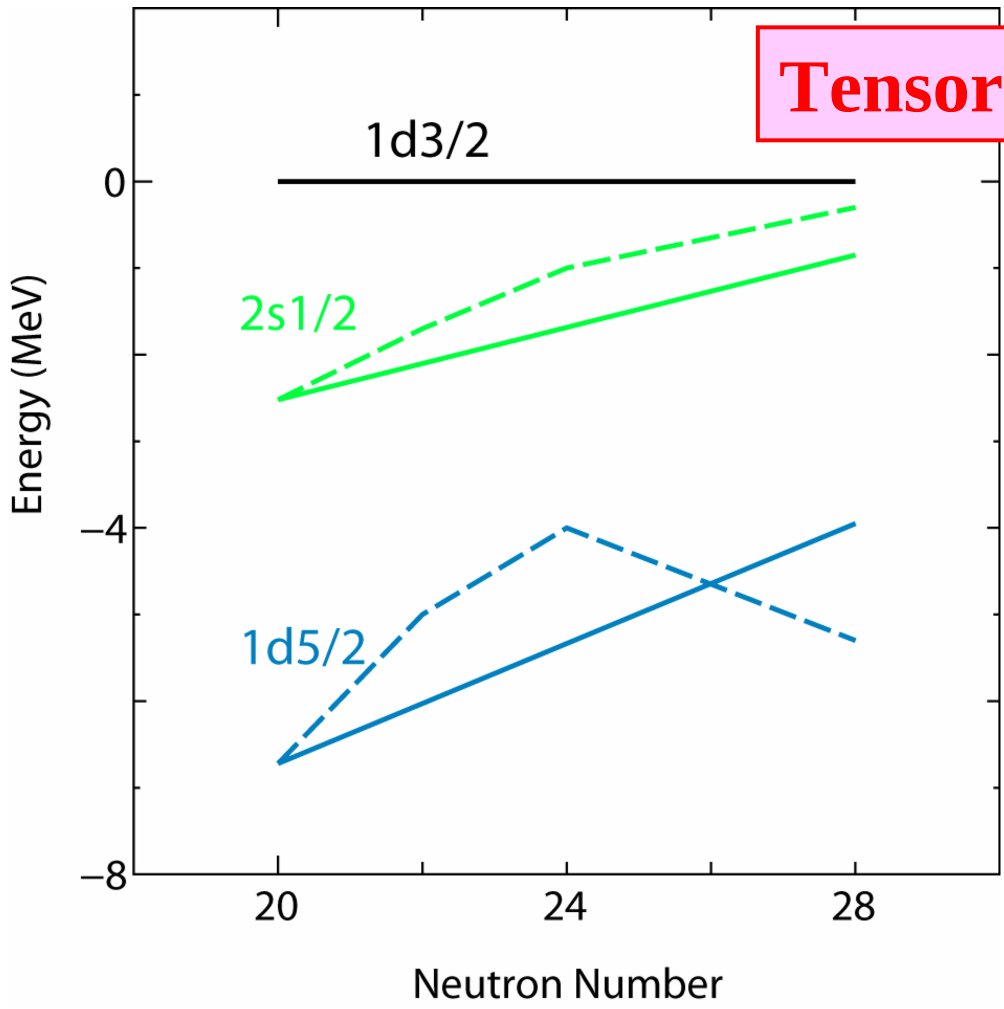
⇩

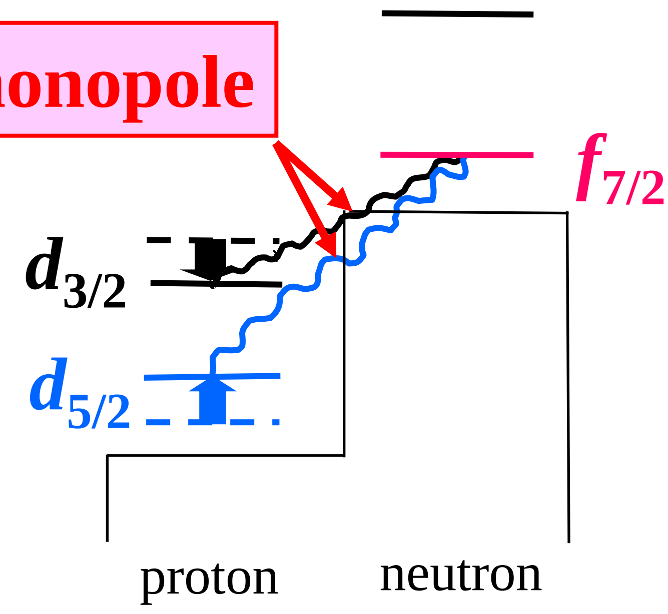
no s-wave to s-wave coupling

⇩

differences in short distance : irrelevant

Proton effective single-particle levels (relative to $d_{3/2}$)





—— $\pi + \rho$ **meson tensor**

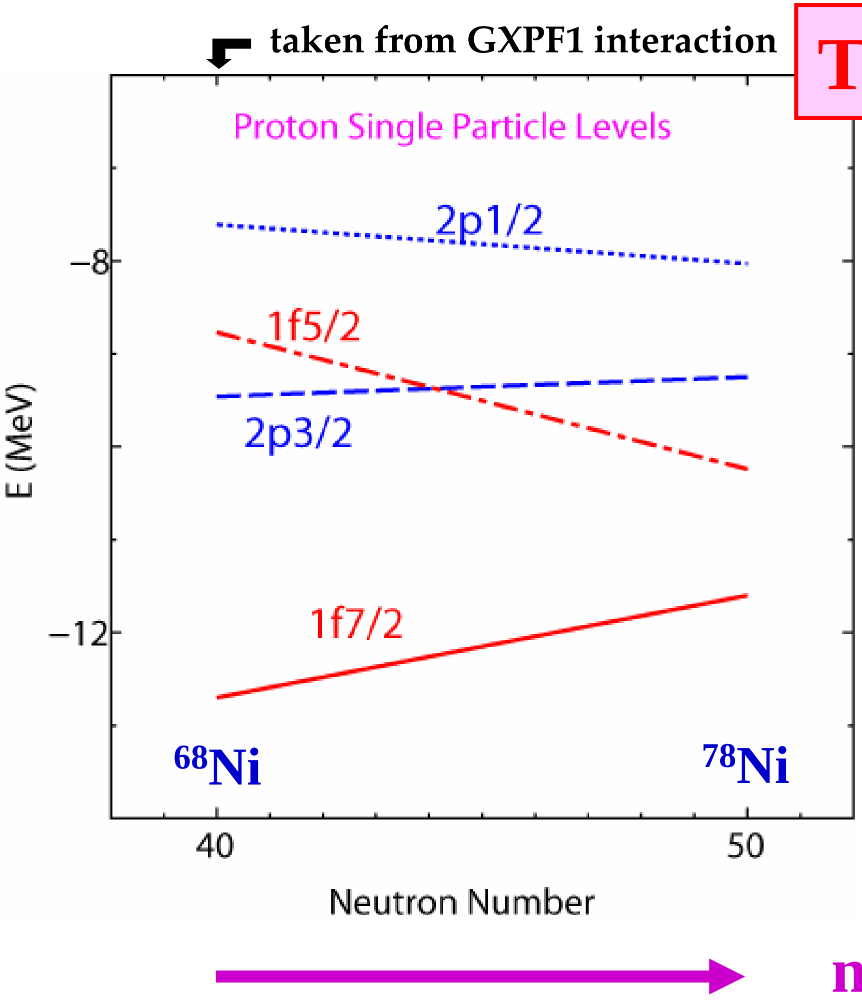
- - - **exp.**

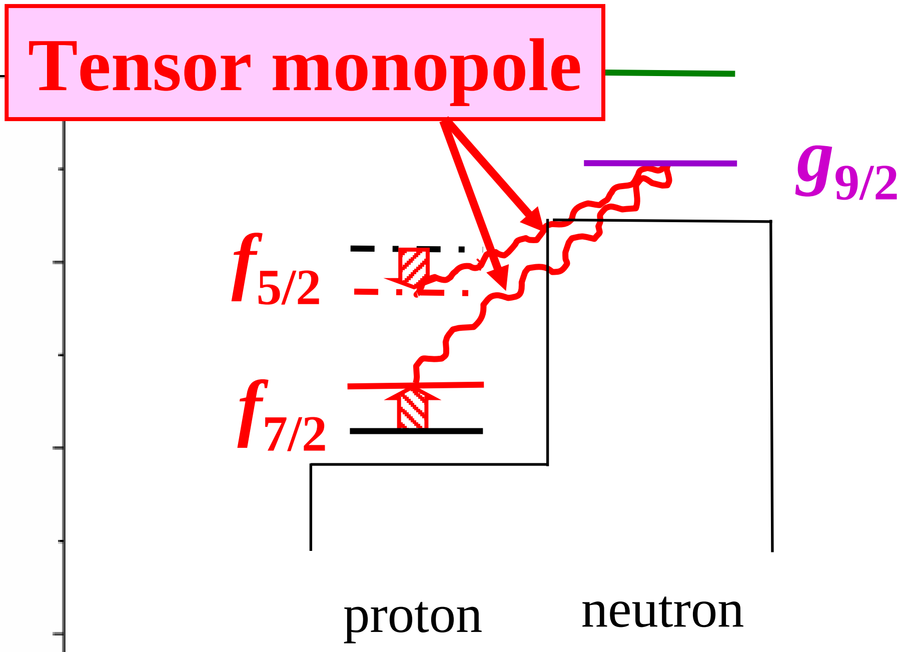
Cottle and Kemper, Phys. Rev. C58, 3761 (98)

→ **neutrons in $f_{7/2}$**

Systematic variation of proton effective single-particle energies due to the tensor interaction (π + ρ meson)

calculation only



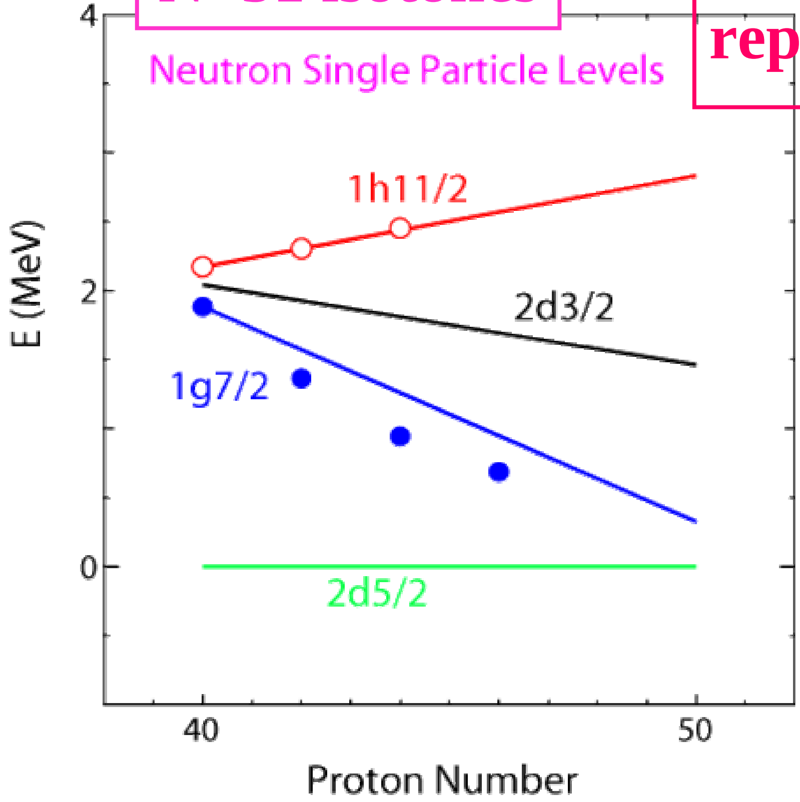


➡ Low-lying 2^+ levels in Ni, M. Sawicka et al., Phys. Rev. C68, 044304 (03)

neutrons in $g_{9/2}$

Systematic variation of neutron effective single-particle energies due to the tensor interaction ($\pi + \rho$ meson)

N=51 isotones



repulsion between $g_{9/2}$ and $h_{11/2}$

widening between $h_{11/2}$ and $g_{7/2}$, not spin-orbit partner

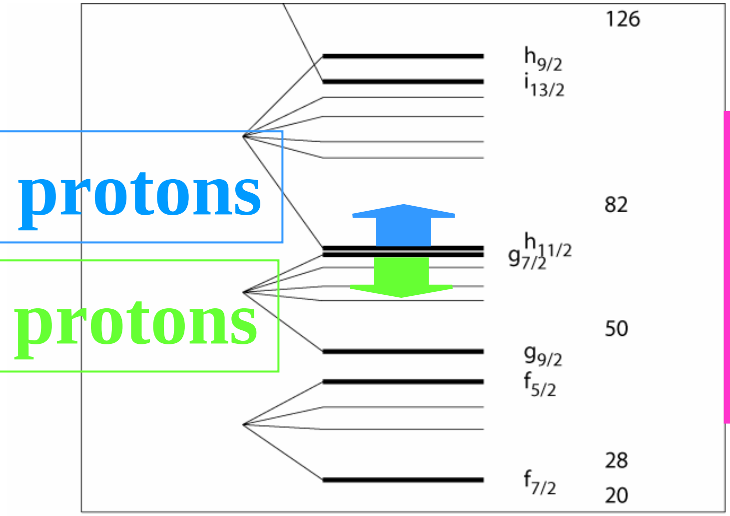
Lines : $\pi + \rho$ meson

Points : corresponding exp. level

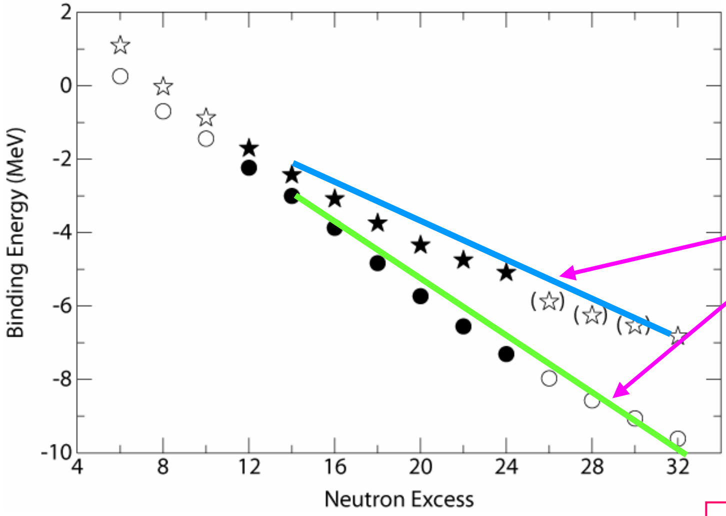
protons in $g_{9/2}$

$1h_{11/2}$ protons

$1g_{7/2}$ protons



monopole effect due to tensor interaction



$\pi + \rho$ mesons

\+ common effect

$1h_{11/2}$ neutrons

Exp. data from J.P. Schiffer et al., Phys. Rev. Lett. 92, 162501 (2004)
Also, C. Baktash, Paestum talk.

How the tensor interaction is included in effective shell model interaction ?

1. pf shell GXPF1

(2. sd shell SDPF-M (USD+corr))

3. p shell SFO (CK+corr)

pf shell

GXPF1 interaction

M. Honma et al., PRC65 (2002) 061301(R)

G-matrix + polarization correction + **empirical refinement**

- Modify realistic G interaction

  M. Hjorth-Jensen, et al., Phys. Repts. 261 (1995) 125

  - **Bonn-C potential** ← ***Tensor is here***
  - 3rd order Q-box + folded diagram
- Vary 70 well-determined LC's of 195 TBME and 4 SPE
- Fit to **699 experimental energy data of 87 nuclei**
- Mass dependence $V(A) = V(A=42) \times \left(\frac{A}{42}\right)^{-0.3}$
- Data selection to avoid intruder: $47 \leq A, \quad Z \leq 32$
- Energy evaluation by FDA* ⟶ 168keV rms error

G-matrix vs. GXPF1

two-body matrix element

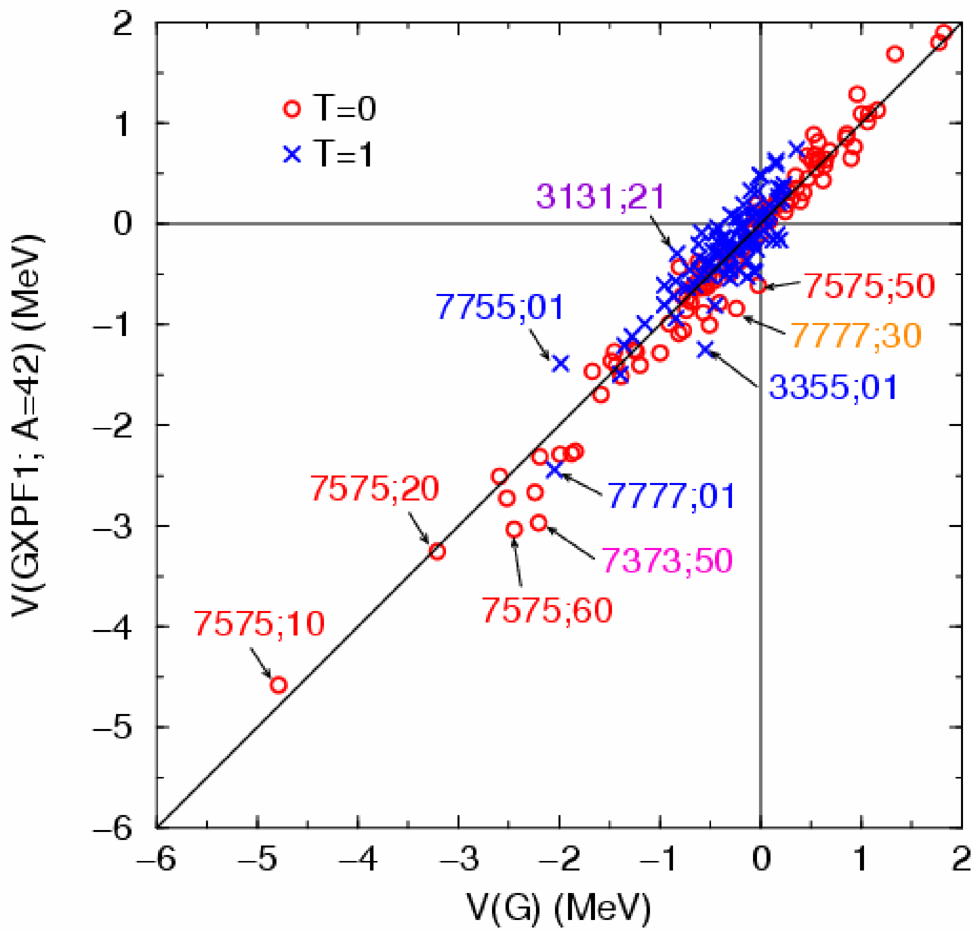
<*ab*; *JT* | *V* | *cd* ; *JT* >

7= $f_{7/2}$, 3= $p_{3/2}$, 5= $f_{5/2}$, 1= $p_{1/2}$

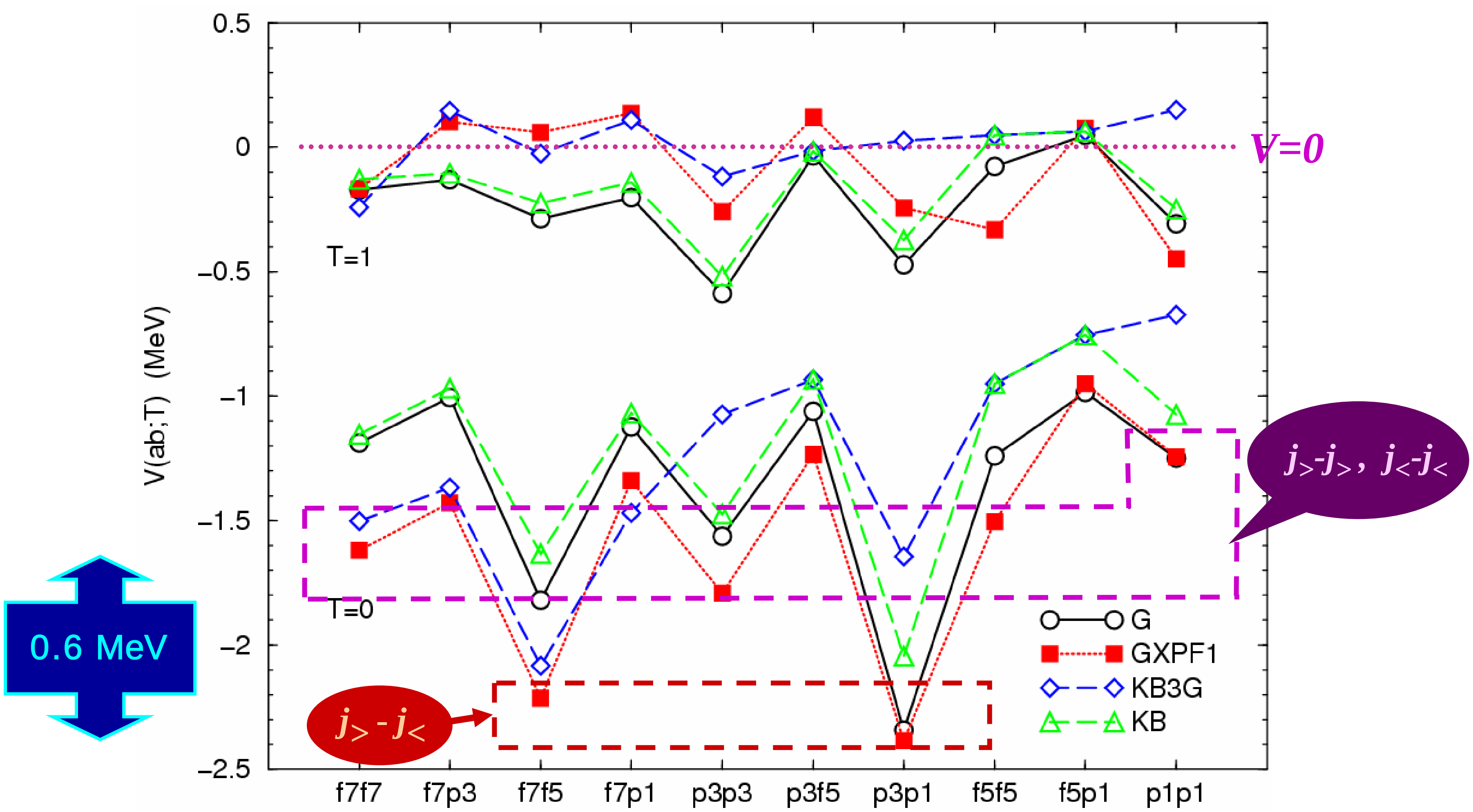
- T=0 … attractive
- T=1 … repulsive
- Relatively large modifications in

  V(*abab* ; *J*0) with large *J*

  V(*aabb* ; *J*1) pairing



Monopole part of various interactions



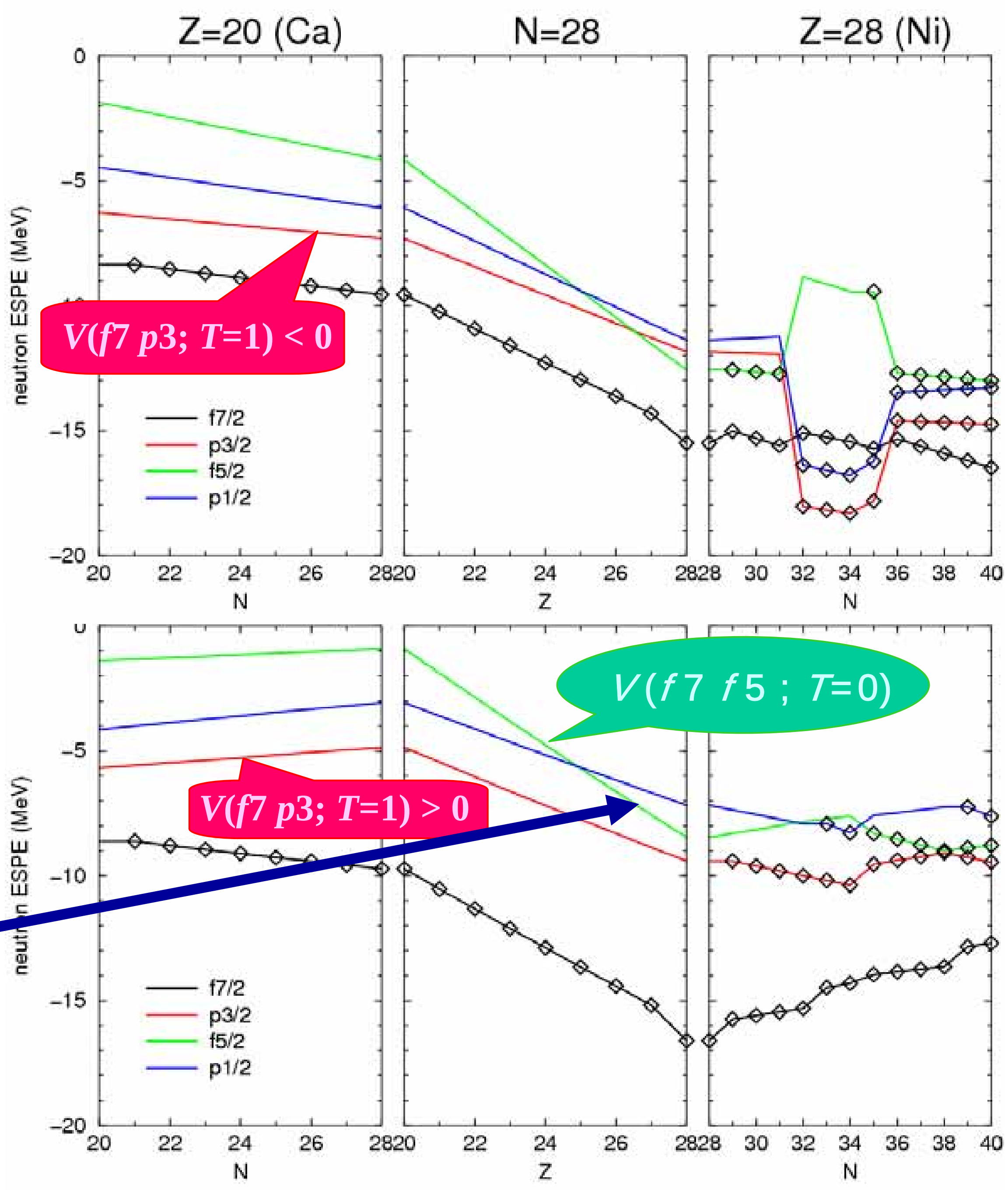
E S P E

Effective Single-particle Energy

of neutrons

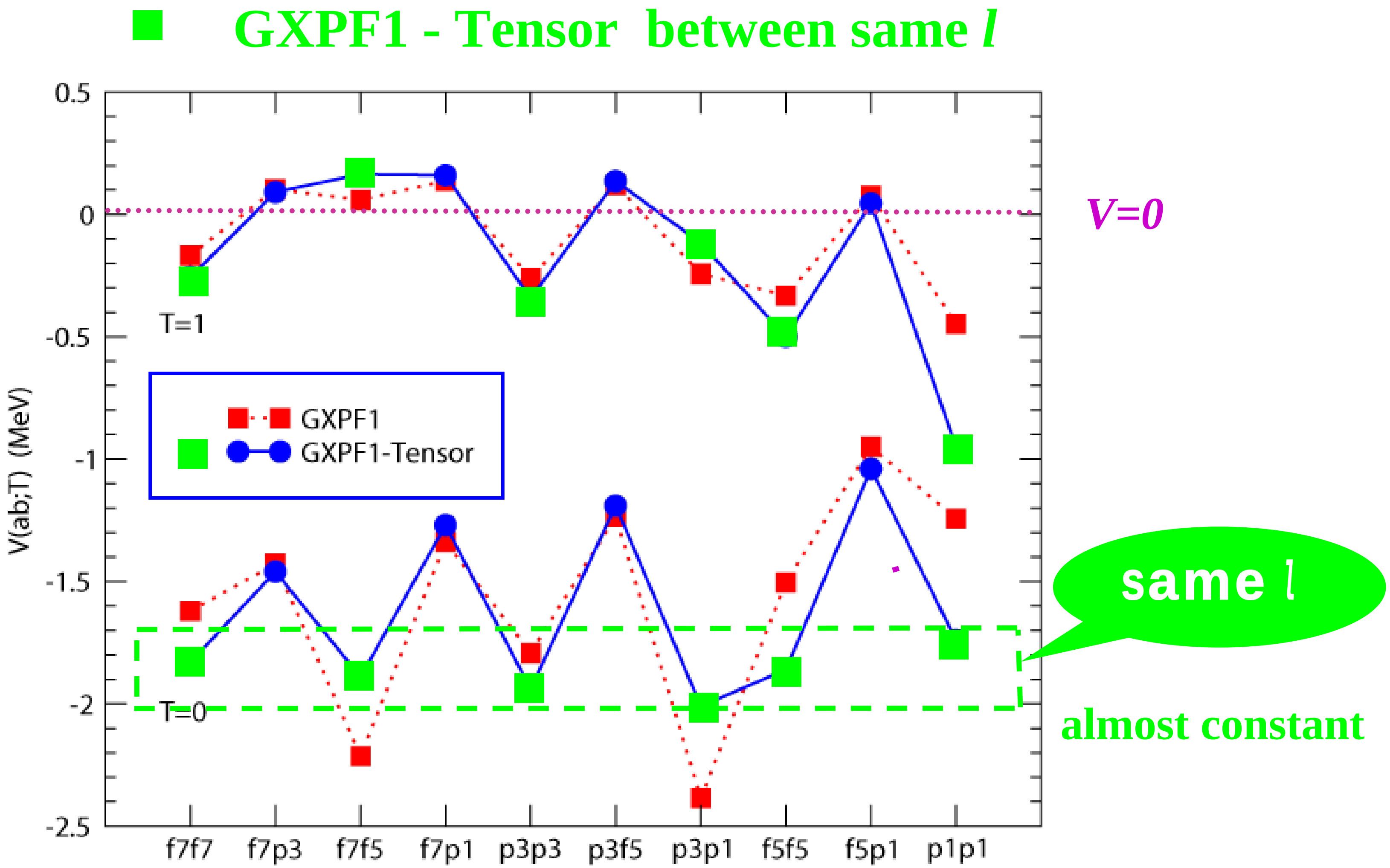
G

GXPF1

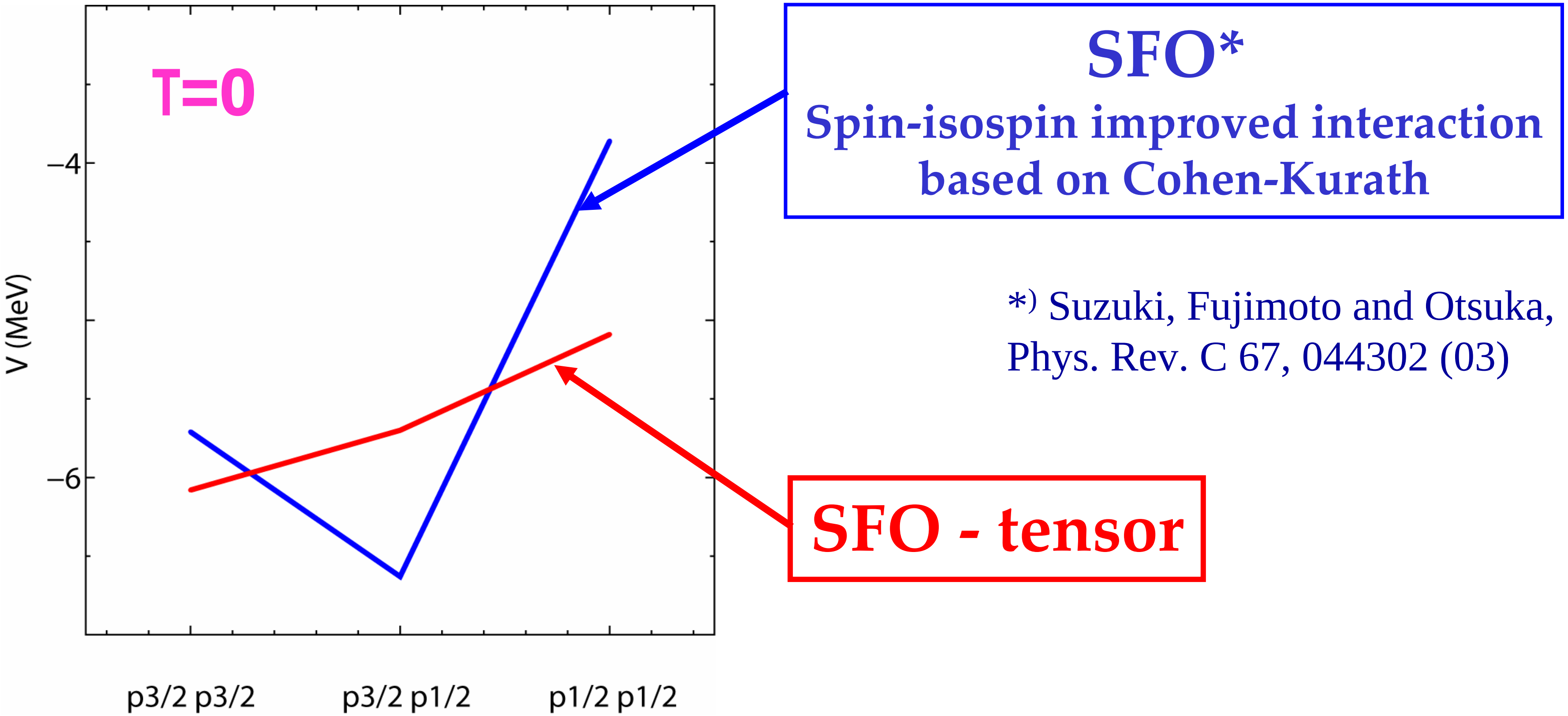


inversion between $f_{5/2}$ and $p_{1/2}$ is originally a tensor effect

Monopole interaction
after subtraction of tensor part

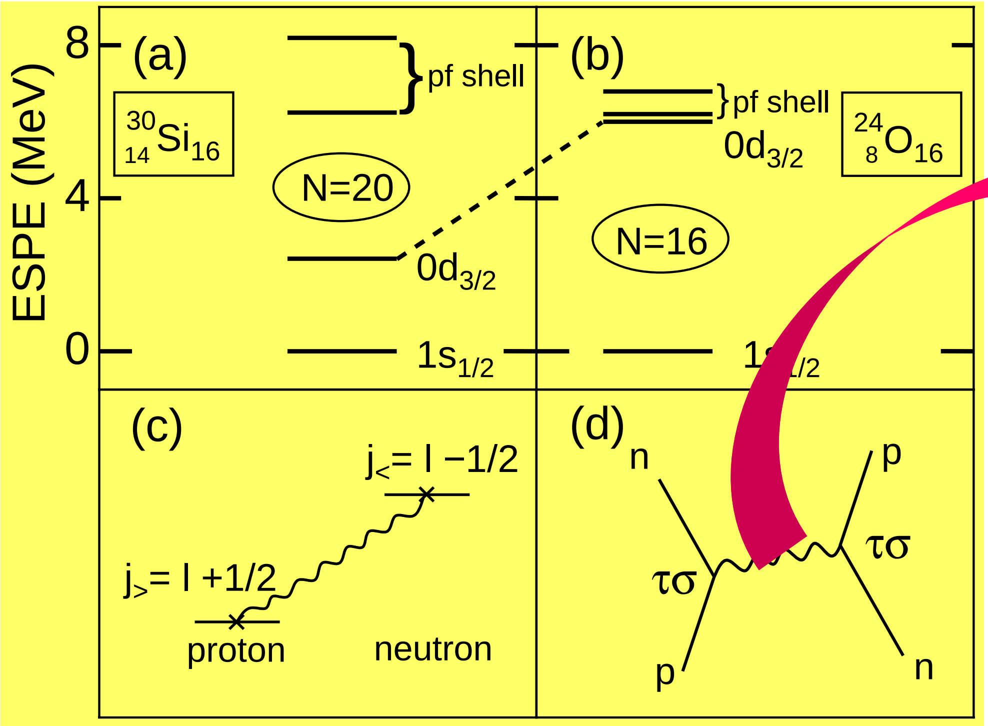


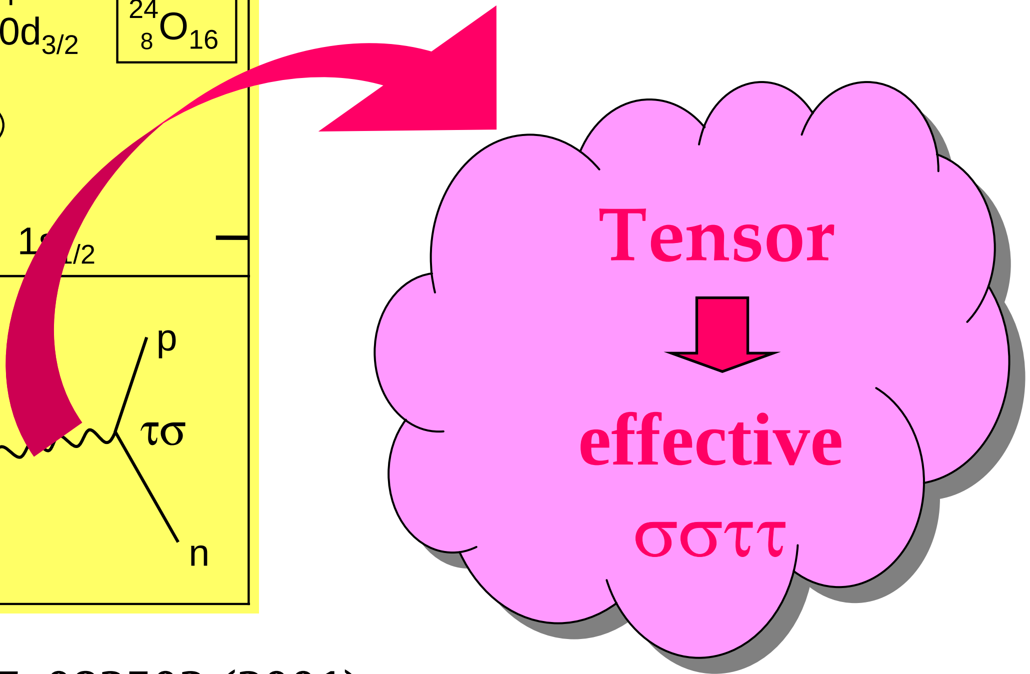
Monopole interaction in p-shell



*) Suzuki, Fujimoto and Otsuka, Phys. Rev. C 67, 044302 (03)

Tensor interaction is the primary origin of the p-n $j_>$-$j_<$ coupling also within a major shell (of a fixed parity).





Otsuka et al. Phys. Rev. Lett. 87, 082502 (2001)

Tensor interaction is the primary origin of the $\sigma\sigma\tau\tau$ effective interaction within a major shell (of a fixed parity).

This can be explained analytically at the long-range limit of of the tensor interaction.
(A good exercise of Racah algebra)

Tensor interaction is included in G-matrix-type calculations.

Implementation of tensor interaction into mean field calculations

Gogny interaction

(1+σσ+ττ+σσττ) (Gauss1 + Gauss2) + Density Dep.

finite range **zero range**

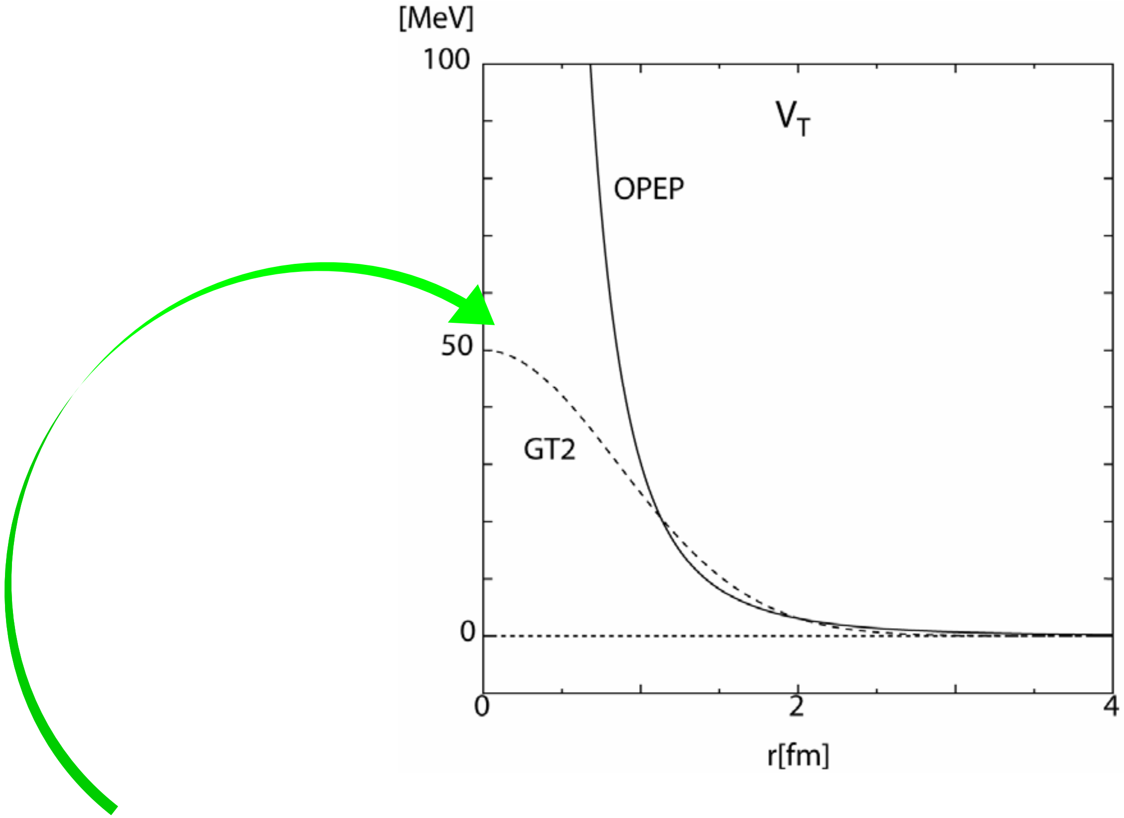
Tensor interaction is added

All parameters are readjusted

Nuclear matter properties reproduced with improvement of imcompressibility

Gogny-Tokyo interaction - 2 (GT2)

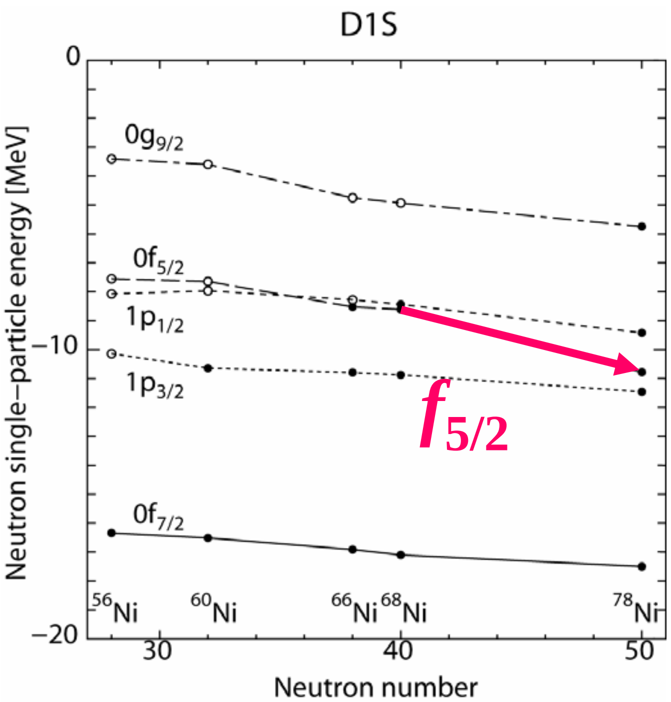
Tensor interaction actually used



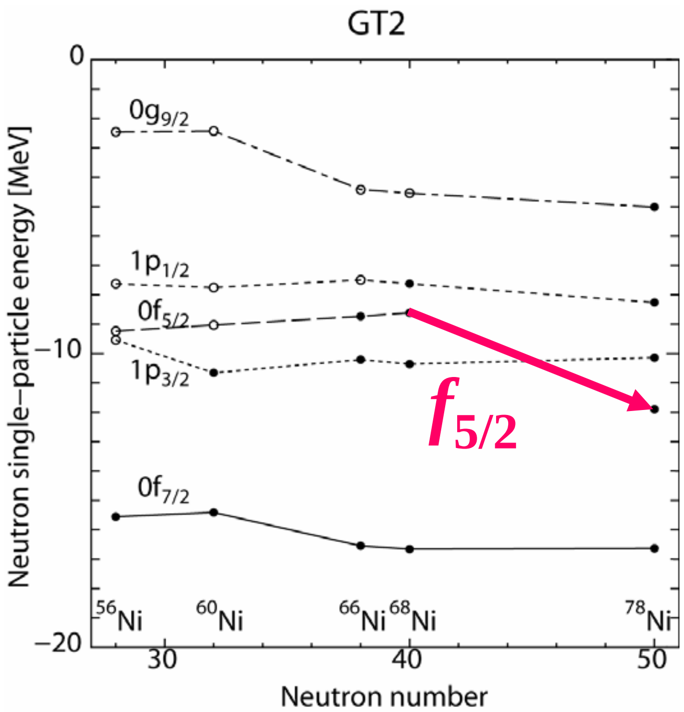
Regularized for short distance

Neutron effective single-particle energies of exotic Ni isotopes

Original (D1S)

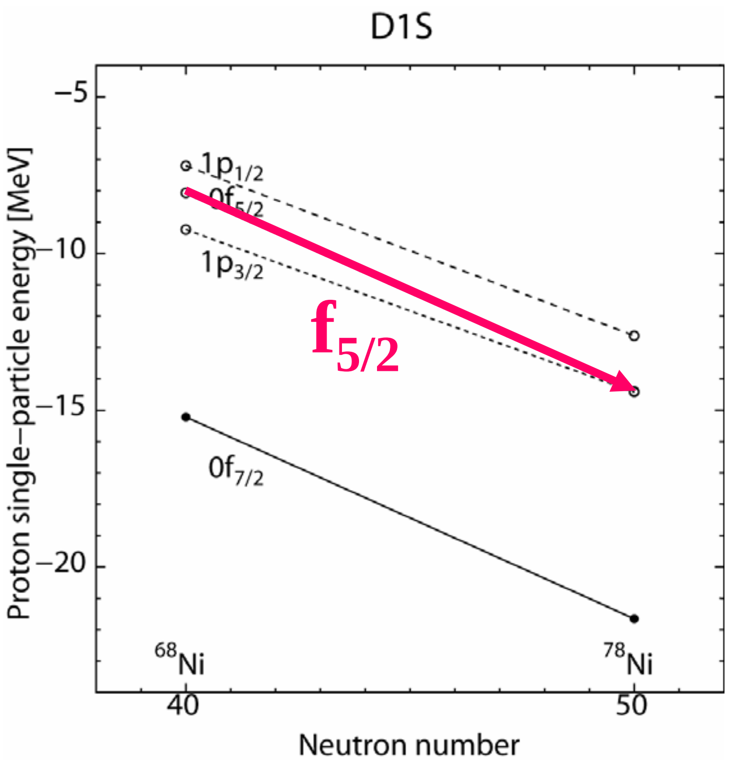


GT2 (incl. tensor)

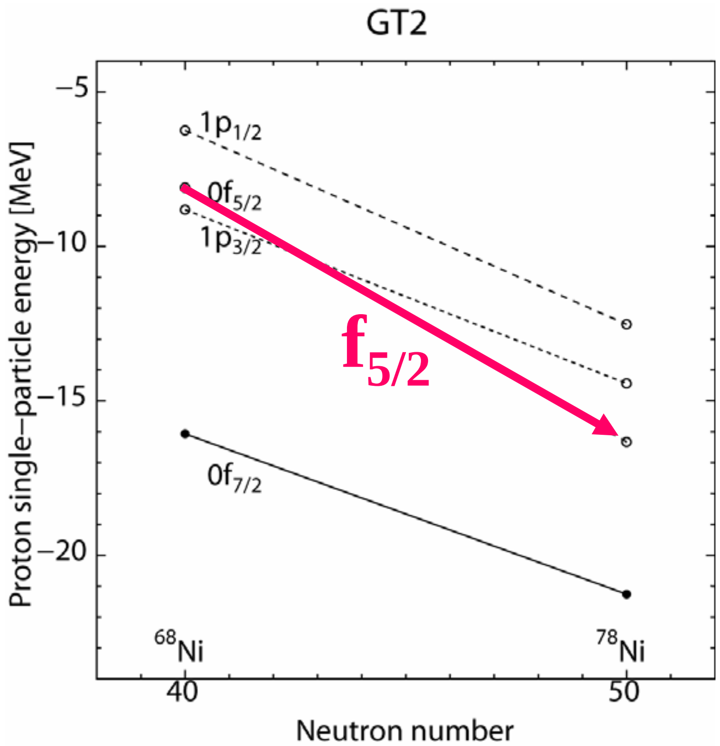


Proton effective single-particle energies of exotic Ni isotopes

Original (D1S)

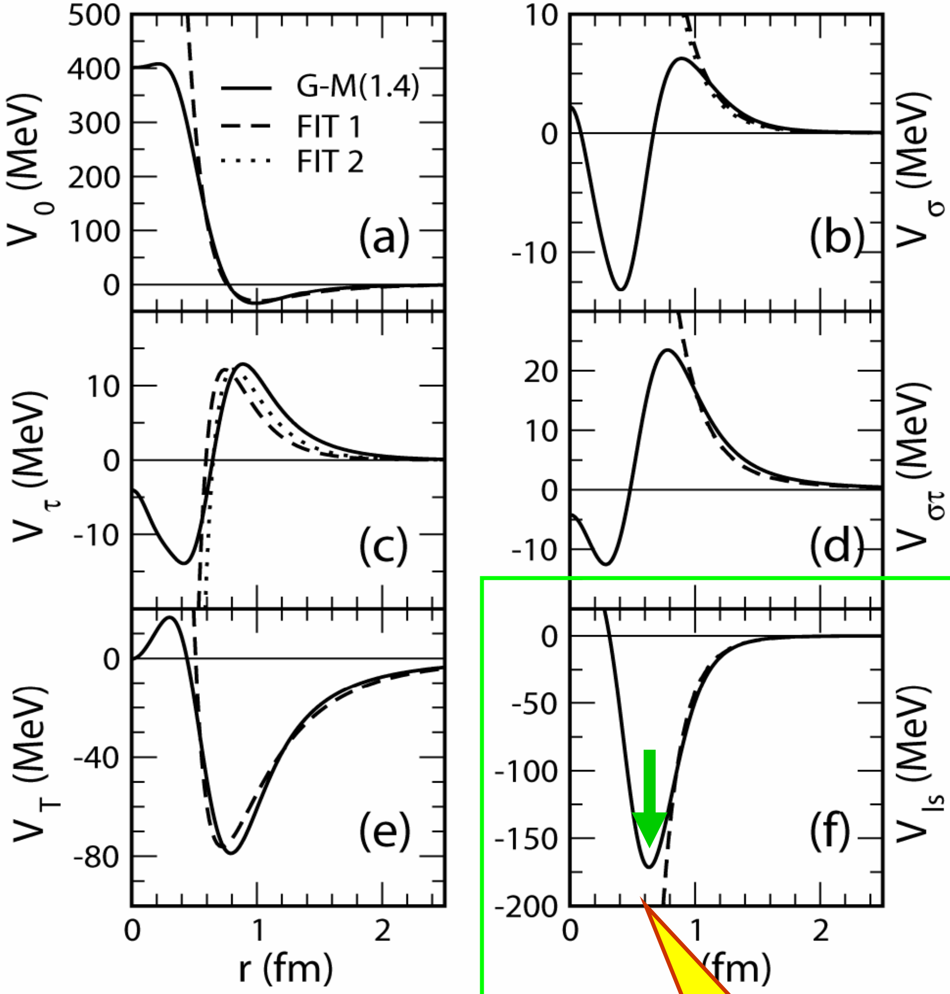


GT2 (incl. tensor)



Another origin of shell evolution:

2-body *LS* interaction



2-body *LS* interaction (odd potential)

***P*-wave**

Short range

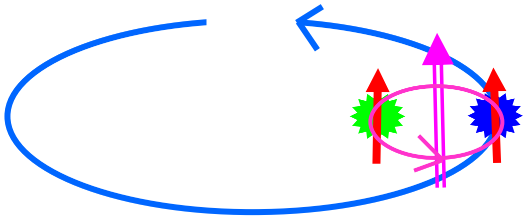
Intuitive Picture of 2-body *LS*

Orbital angular momentum of relative motion

spin of nucleon (parallel)

more likely

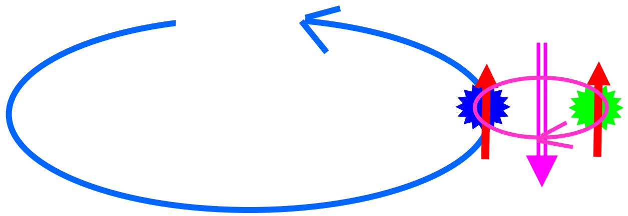
$j_>$



$LS>0$: attractive

less likely

$j_>$

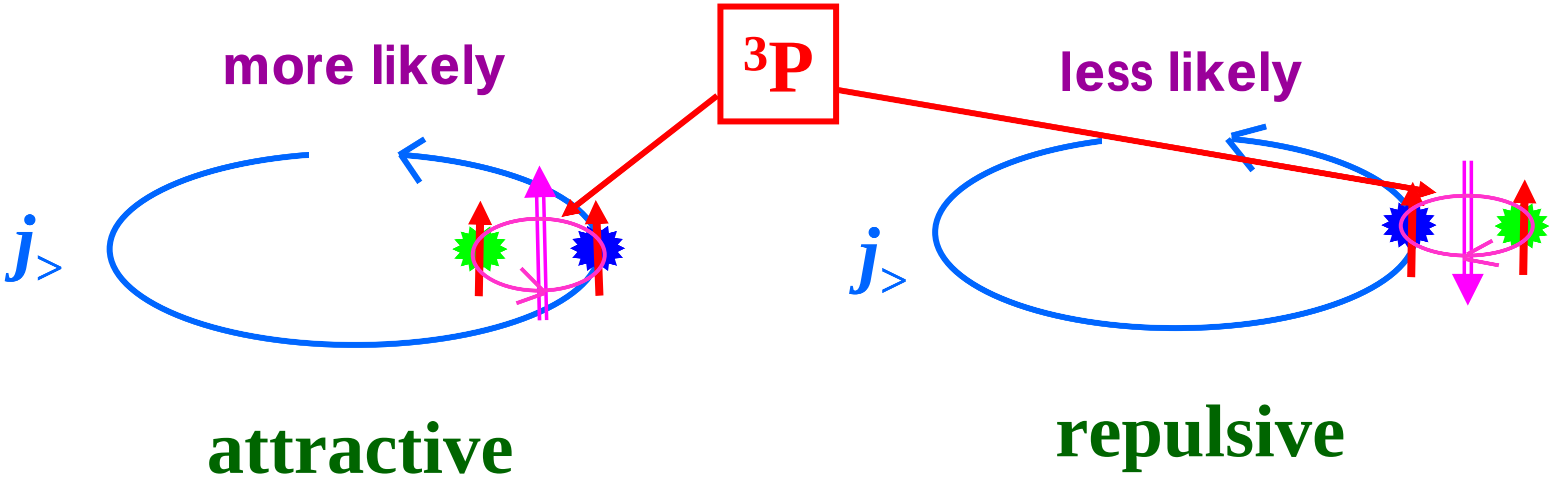


$LS<0$: repulsive

net effect : attractive

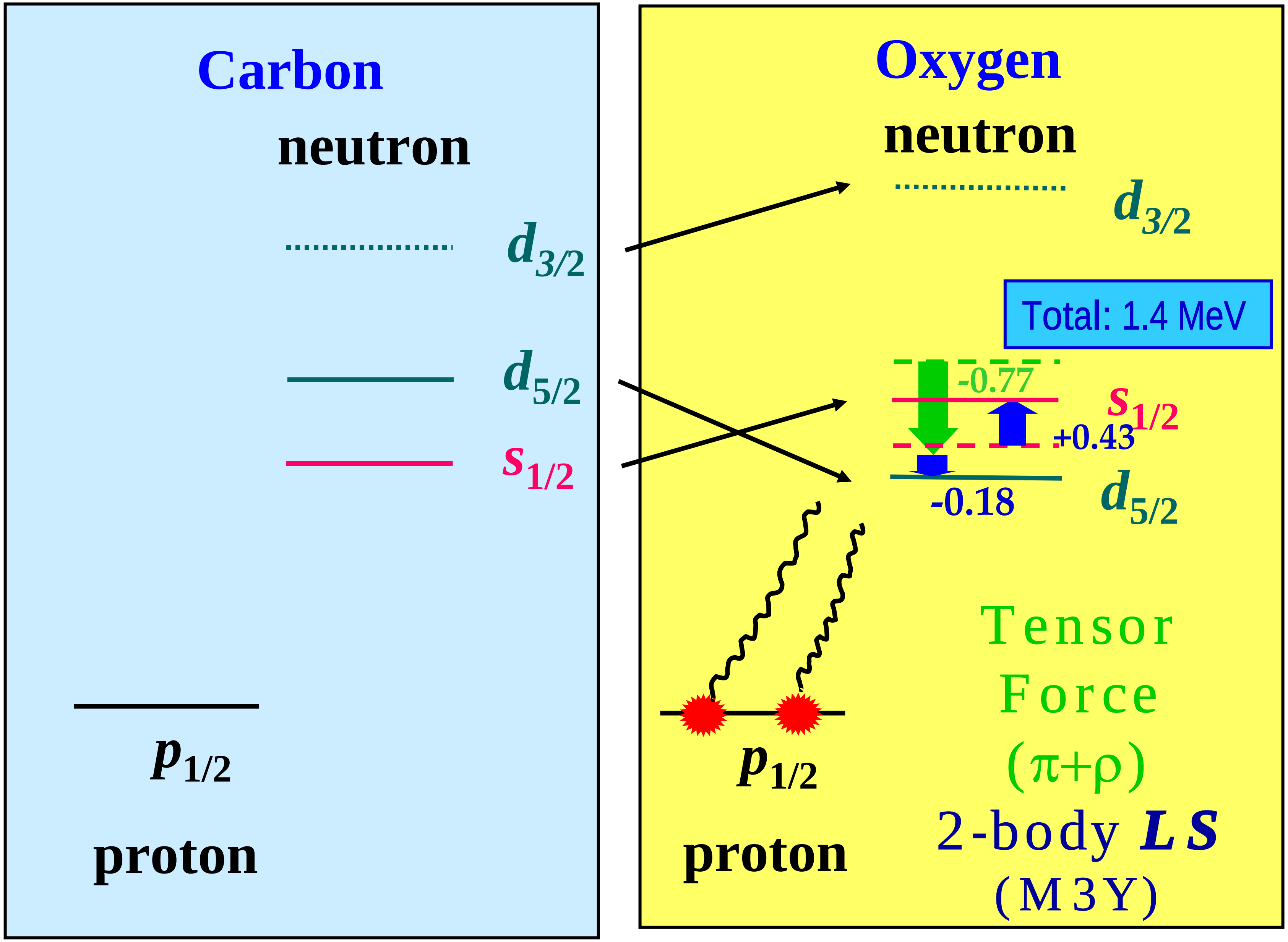
$j_<$: opposite

^3P channel dominance due to short range



large effect : s p

other combinations : higher partial waves observed change of $d_{5/2}$ - $s_{1/2}$: 1.6 MeV



2-body *LS* interaction is
one of the origins of spin-orbit splitting

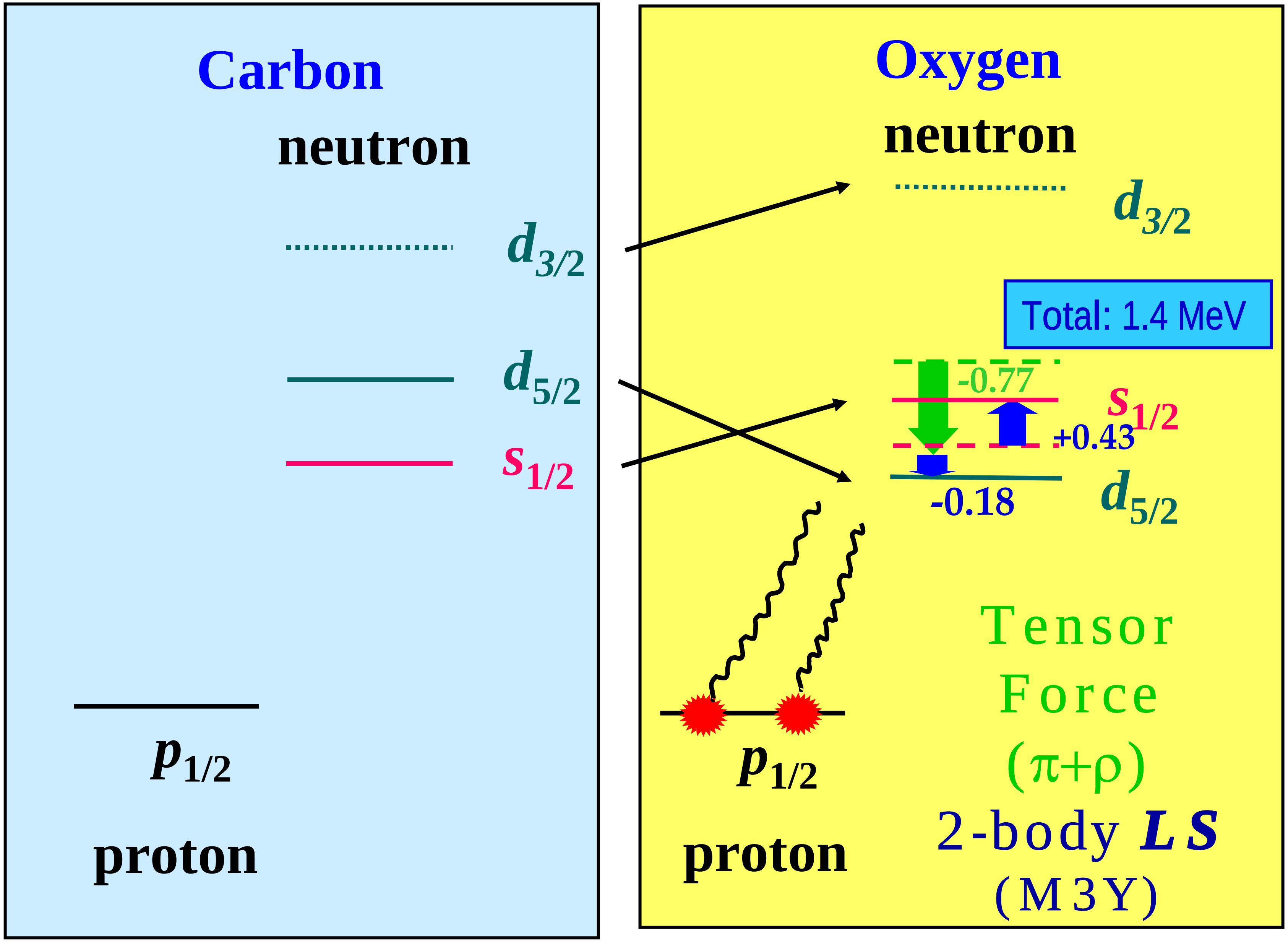
It shifts the position of *s* orbit
as the inverse mechanism.

In fact, the amount of the effect is largest
for *s* orbit due to ^{3}P dominance.

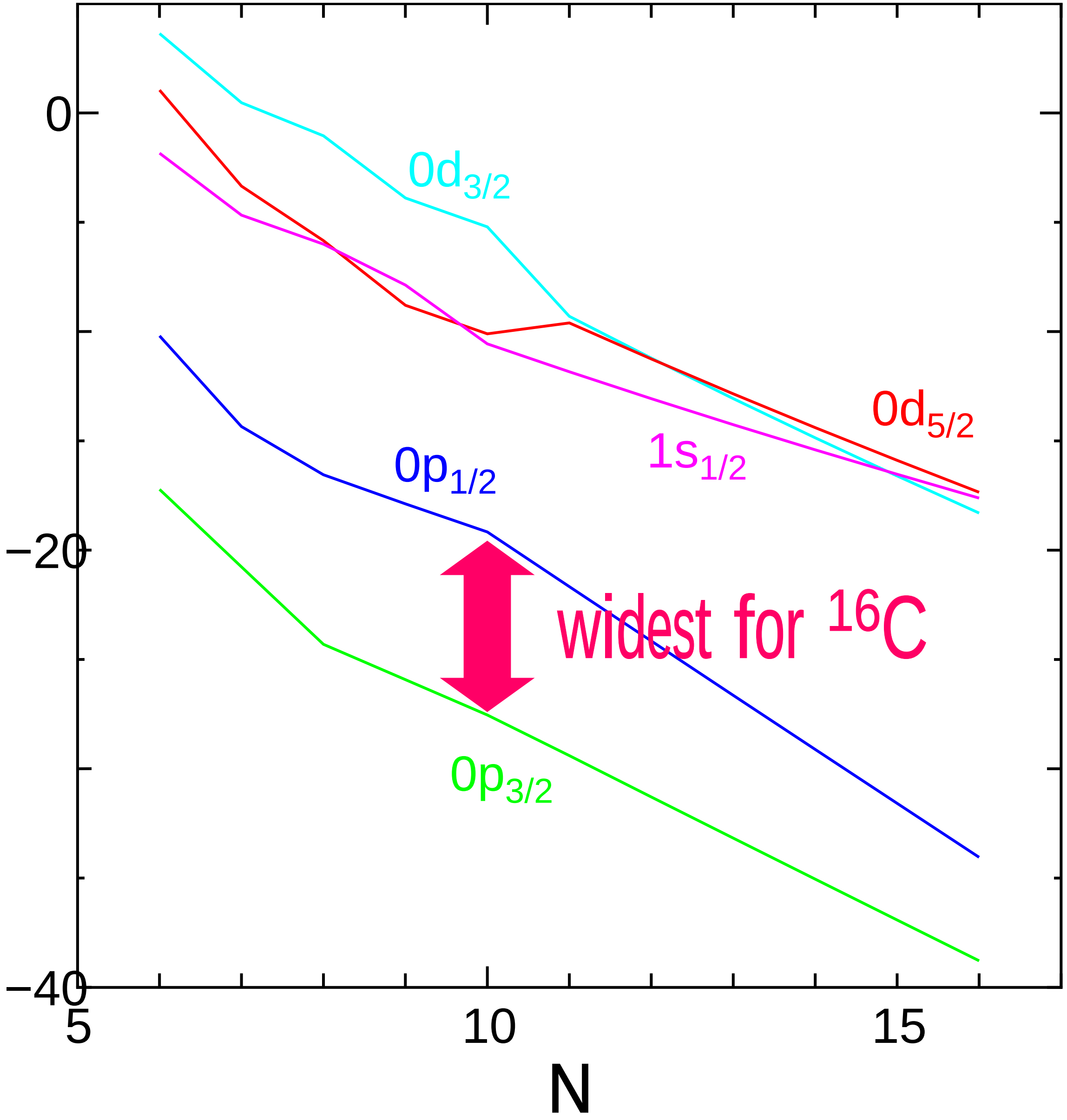
A systematic shell model calculation for Carbon isotopes

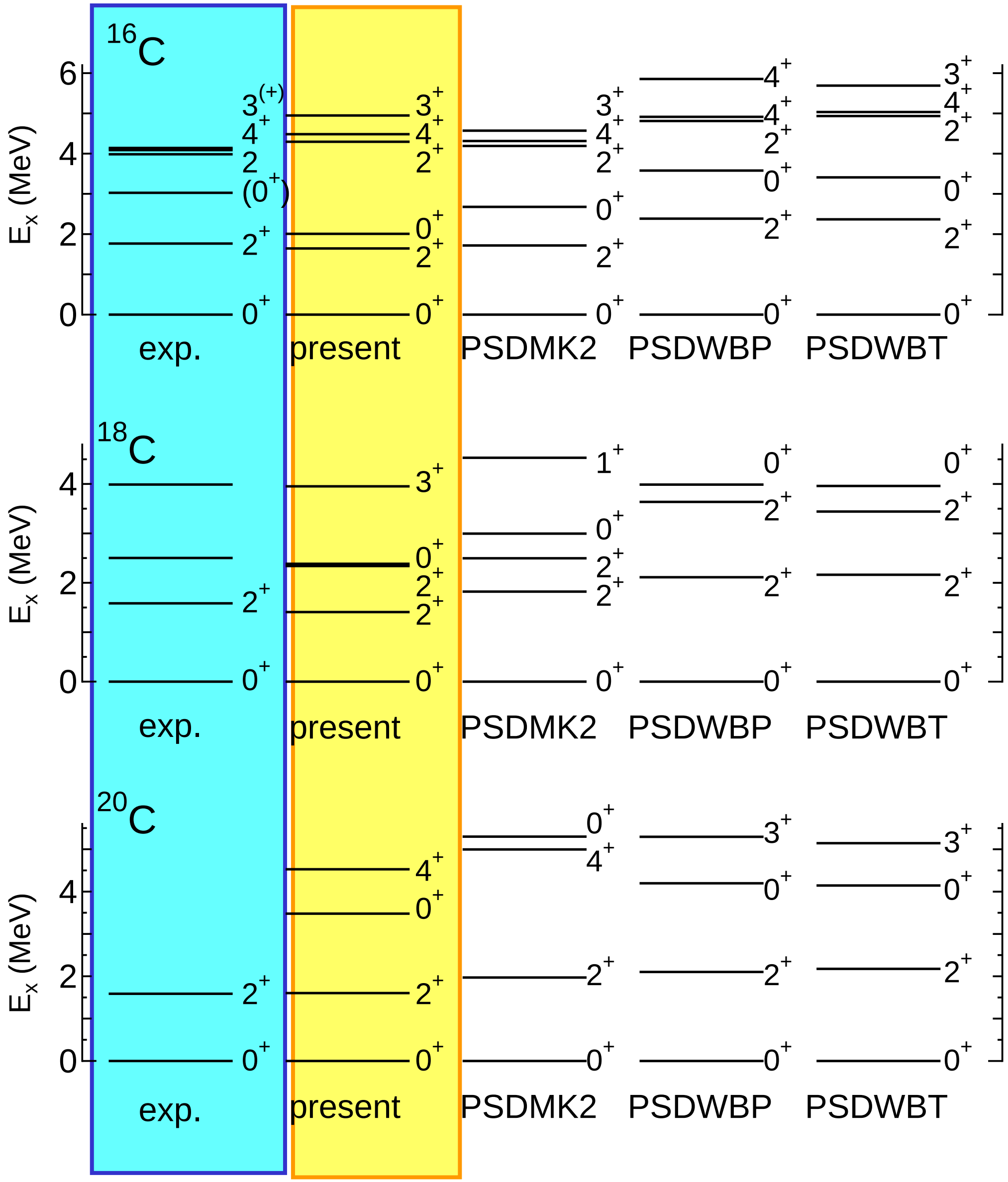
Millener-Kurath + modifications

observed change of $d_{5/2}$-$s_{1/2}$: 1.6 MeV



Effective single-particle energies for protons





^{16}C
^{18}C
^{20}C
E_x (MeV)
exp.
present
PSDMK2
PSDWBP
PSDWBT

E2 transition $0^+ \rightarrow 2^+$ in ^{16}C

Exp. : *B*(E2)=3.2 (6) e^2 fm^4

Imai et al., Phys. Rev. Lett. (03)

Shell model

e_p =1.3 e_n=0.5 *B*(E2)= 40 e^2 fm^4

e_p =1.2 e_n=0.1-0.2 *B*(E2)= 12 e^2 fm^4

Sagawa and Asahi, Phys. Rev. C63, 064310 (01)

Summary

Shell evolution due to spin-isospin interactions

Tensor interaction (long range)

drives $j_>$ or $j_<$ levels in a specific way

This is not necessarily a change of spin-orbit splitting.

is the dominant origin of shell evolution

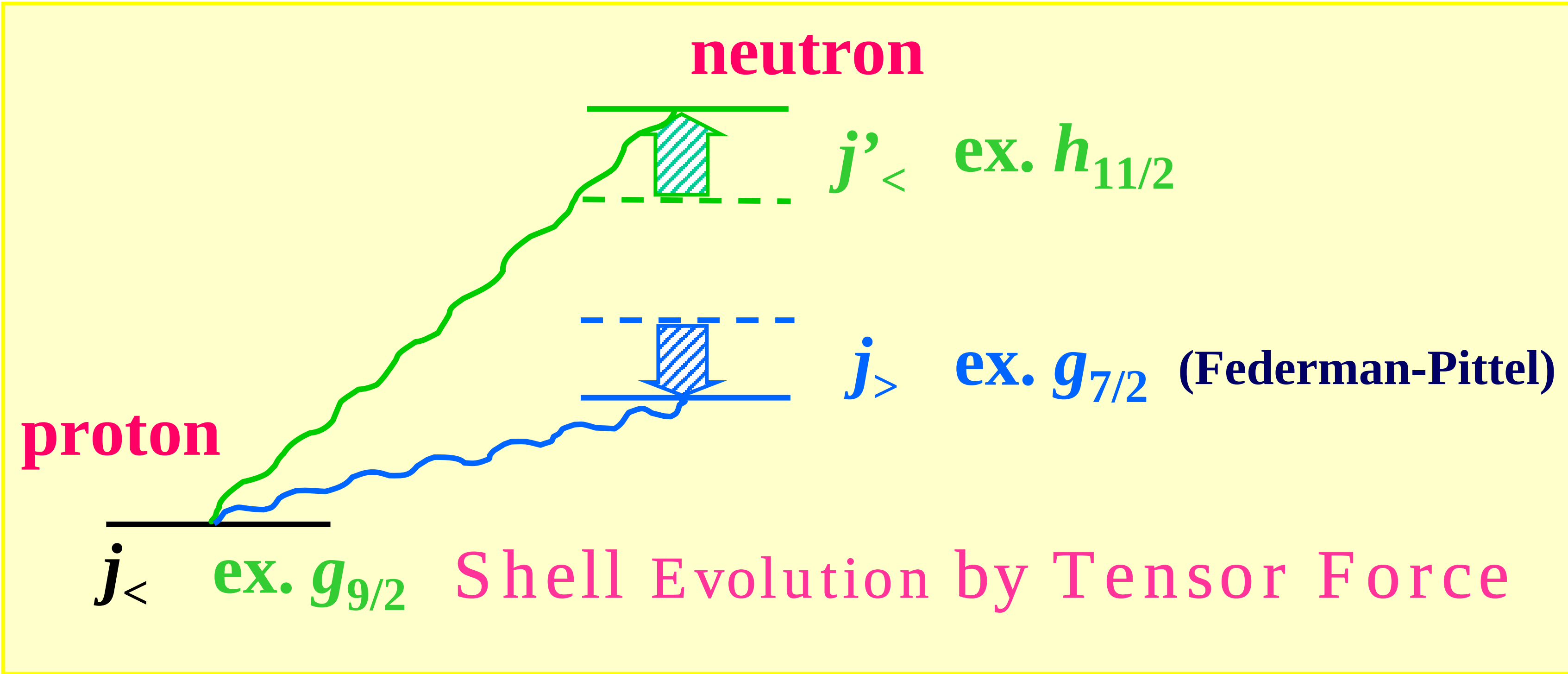
produces effects of similar magnitude to *neutron skin* (weakening of *ls* splitting)

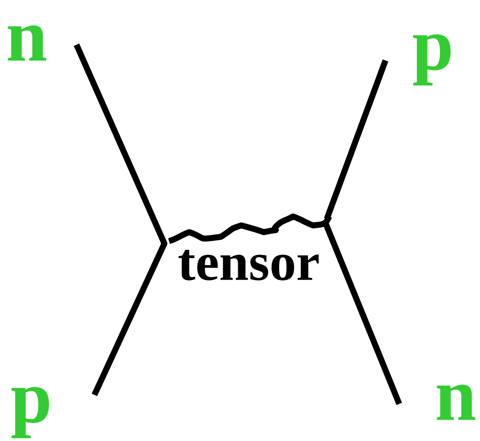
2-body *LS* interaction (short range)

special cases (*e.g.* between *s* and *p*)

carbon-oxygen $d_{5/2}$-$s_{1/2}$ inversion (same mechanism as ls splitting)

➡ Structure of exotic nuclei in many respects


neutron
j'< ex. h11/2
j> ex. g7/2 (Federman-Pittel)
proton
j< ex. g9/2
Shell Evolution by Tensor Force


n
p
tensor
p
n


exchange process

Exercise:

Find a combination of three orbits and a way to see the variation of their energies.

Collaborators

T. Suzuki	Nihon U.
R. Fujimoto	U. Tokyo
T. Matsuo	U. Tokyo
H. Grawe	GSI
Y. Akaishi	KEK

CNS

SUMMER SCHOOL 04

End