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Nuclear Forces

- Lecture 2 -

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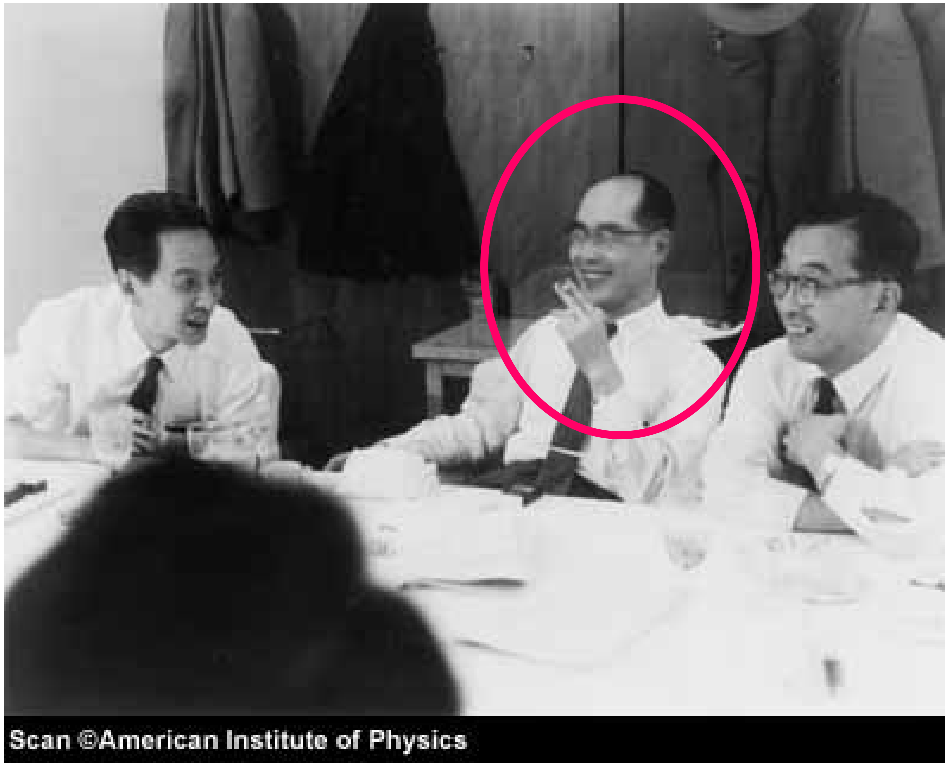
Lecture 2: The Meson Theory of Nuclear Forces

- Yukawa's historic idea
- The mesons
- How do those mesons couple to the nucleon?
- The One-Boson-Exchange Potential
- Closing remarks

Yukawa and his idea



S. Tomonaga, H. Yukawa, and S. Sakata in the 1950s.

From:

H. Yukawa, Proc. Phys. Math. Soc. Japan **17,** 48 (1935).

§ 2. Field describing the interaction

In analogy with the scalar potential of the electromagnetic field, a function $U(x, y, z, t)$ is introducd to describe the field between the neutron and the proton. This function will satisfy an equation similar to the wave equation for the electromagnetic potential.

Now the eqnation

$$\left\{\Delta-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right\}U=0 \qquad (1)$$

has only static solution with central symmetry $\frac{1}{r}$, except the additive and the multiplicative constants. The potential of force between the neutron and the proton should, however, not be of Coulomb type, but decrease more rapidly with distance. It can be expressed, for example, by

$$+ \text{ or } -g^2\frac{e^{-\lambda r}}{r}, \qquad (2)$$

where g is a constant with the dimension of electric charge, i. e., cm.$^{\frac{3}{2}}$ sec.$^{-1}$ gr.$^{\frac{1}{2}}$ and λ with the dimention cm.$^{-1}$

Since this function is a static solution with central symmetry of the wave equation

$$\left\{\Delta-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\lambda^2\right\}U=0, \qquad (3)$$

let this equation be assumed to be the correct equation for U in vacuum. In the presence of the heavy particles, the U-field interacts with them and causes the transition from neutron state to proton state.

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field. ~~According to the~~ ~~From the~~ The law of conservation of the electric charge demands that the quantum should have ~~+ or − e~~ the charge $+e$ or $-e$. The quantized field U corresponds to the operator which increases the number of the negatively charged quanta by one and decreases the number of the positively charged quanta by one. ~~The field~~ $\tilde{U}$, the complex conjugate of U, which does not commute with U, corresponds to the inverse operator.

Next, denoting

$$p_x = -i\hbar\frac{\partial}{\partial x}, \text{ etc} \qquad W = i\hbar\frac{\partial}{\partial t},$$

$$m_U c = \lambda\hbar$$

The wave equation for U in free space can be written in the form

$$\left\{p_x^2 + p_y^2 + p_z^2 - \frac{W^2}{c^2} + m_U^2 c^2\right\} U = 0, \qquad (12)$$

so that the quantum accompanying the field has the proper mass $m_U = \frac{\lambda\hbar}{c}$. Assuming, for example, $\lambda = 5 \times 10^{12}\ \text{cm}^{-1}$ we obtain for m_U a value 2×10^2 times as large as electron mass. Thus the result is rather surprising and ~~the existence of~~ such a quantum with large mass and positive or negative charge ~~was~~ has never

H. Yukawa: manuscript of the first paper showing the mass-range formula (November 1934). Reproduced by permission of Yukawa Hall Archival Library.

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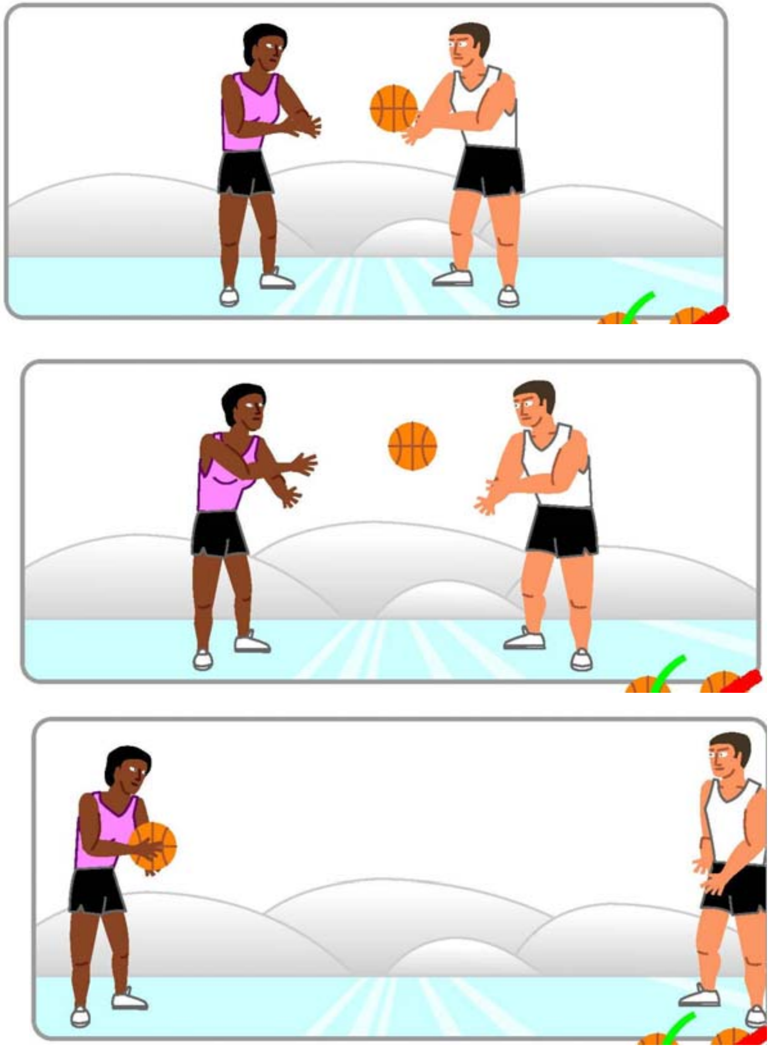
Repulsive force due to the exchange of a basket ball between two people standing on ice



LIGHT UNFLAVORED MESONS
$(S = C = B = 0)$

For $I = 1$ (π, b, ρ, a): $u\bar{d}$, $(u\bar{u} - d\bar{d})/\sqrt{2}$, $d\bar{u}$;
for $I = 0$ (η, η', h, h', ω, ϕ, f, f'): $c_1(u\bar{u} + d\bar{d}) + c_2(s\bar{s})$

$\pi^\pm$ $I^G(J^P) = 1^-(0^-)$

Mass $m = 139.57018 \pm 0.00035$ MeV
Mean life $\tau = (2.6033 \pm 0.0005) \times$
$c\tau = 7.8045$ m

π^0 $I^G(J^P$

Mass $m = 134.9766 \pm 0.0006$ Me
$m_{\pi^\pm} - m_{\pi^0} = 4.5936 \pm 0.0005$ M
Mean life $\tau = (8.4 \pm 0.6) \times 10^{-17}$
$c\tau = 25.1$ nm

pseudo scalar

η $I^G(J^{PC}) = 0^+(0^{-+})$

Mass $m = 547.75 \pm 0.12$ MeV [f] $(S = 2.6)$
Full width $\Gamma = 1.29 \pm 0.07$ keV [g]

$f_0(600)$ [i] or σ $I^G(J^{PC}) = 0^+(0^{++})$

Mass $m = (400–1200)$ MeV
Full width $\Gamma = (600–1000)$ MeV

$f_0(600)$ DECAY MODES	Fraction (
$\pi\pi$	dominant

scalar

$\rho(770)$ [j] $I^G(J^{PC}) = 1^+(1^{--})$

Mass $m = 775.8 \pm 0.5$ MeV
Full width $\Gamma = 150.3 \pm 1.6$ MeV
$\Gamma_{ee} = 7.02 \pm 0.11$ keV

$\rho(770)$ DECAY MODES	Fraction $(\Gamma_i/\Gamma$
$\pi\pi$	~ 100

vector

$\omega(782)$ $I^G(J^{PC}) = 0^-(1^{--})$

Mass $m = 782.$ … ± 0.1 … $= 1.7)$
Full width Γ …
Γ_{ee} …

ω … DECAY MODES	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
$\pi^+\pi^-$ …	(89.1 ± 0.7) %	S=1.1

Repulsive

STOP

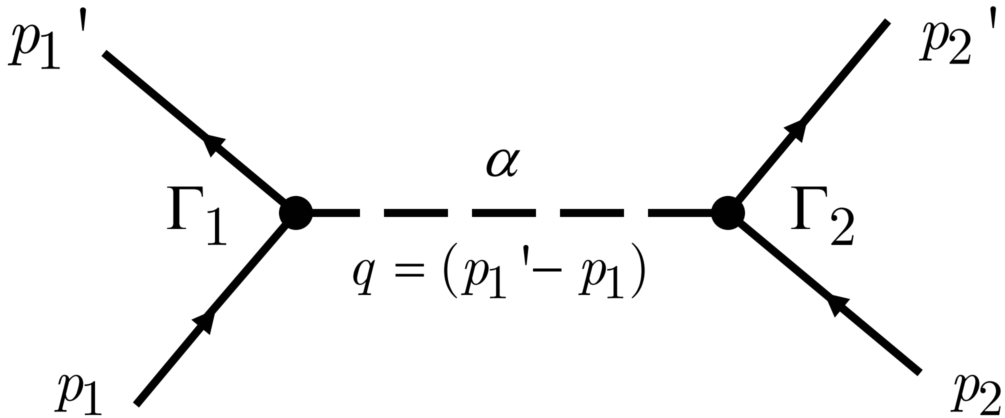
What do those mesons do to the NN interaction?

To find out, we have to do some calculations. Proper calculations are done in the framework of Quantum Field Theory. That means, we have to take the following steps:

- Write down appropriate Lagrangians for the interaction of the mesons with nucleons.

- Using those interaction Lagrangians, calculate Feynman diagrams that contribute to NN scattering.

Feynman diagram for NN scattering



$$\text{Amplitude: } F_\alpha(p', p) = \frac{\bar{u}_1' \Gamma_1 u_1 \; P_\alpha \; \bar{u}_2' \Gamma_2 u_2}{q^2 + m_\alpha^2}$$

with Dirac spinor $u(p,s) = \sqrt{\frac{E+M}{2M}} \begin{pmatrix} \chi_s \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+M}\chi_s \end{pmatrix} \approx \begin{pmatrix} \chi_s \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+M}\chi_s \end{pmatrix} \approx \begin{pmatrix} \chi_s \\ 0 \end{pmatrix}$

where $E = \sqrt{\vec{p}^2 + M^2}$ and χ_s is a two-component Pauli spinor.

Pseudo-vector coupling of a pseudo-scalar meson

Lagrangian: $$\mathcal{L}_{\pi NN} = -\frac{f_{\pi NN}}{m_\pi} \bar{\psi} \gamma^\mu \gamma_5 \vec{\tau} \psi \cdot \partial_\mu \vec{\varphi}^{(\pi)}$$

Vertex: i times the Lagrangian stripped off the fields

$$\Gamma_{\pi NN} = (i)^2 \frac{f_{\pi NN}}{m_\pi} \gamma^\mu \gamma_5 \vec{\tau} q_\mu \approx \frac{f_{\pi NN}}{m_\pi} (\vec{\sigma} \cdot \vec{q}) \vec{\tau}$$

Potential: i times the amplitude $(P_\pi = i, \; q^2 \approx -\vec{q}^{\,2})$

$$V_\pi = i F_\pi \approx -\frac{f_{\pi NN}^2}{m_\pi^2} \frac{(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q})}{\vec{q}^{\,2} + m_\pi^2} \vec{\tau}_1 \cdot \vec{\tau}_2$$

Pseudo-vector coupling of a pseudo-scalar meson, cont'd

Using the operator identity

$$(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q}) = \frac{\vec{q}^{\,2}}{3}\left[\vec{\sigma}_1 \cdot \vec{\sigma}_2 + S_{12}(\hat{q})\right]$$

with $S_{12}(\hat{q}) \equiv 3(\vec{\sigma}_1 \cdot \hat{q})(\vec{\sigma}_2 \cdot \hat{q}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2$ ("Tensor operator"),

the one-pion exchange potential (OPEP) can be written as

$$V_\pi = \frac{f_{\pi NN}^2}{3m_\pi^2} \frac{\vec{q}^{\,2}}{\vec{q}^{\,2} + m_\pi^2}\left[-\vec{\sigma}_1 \cdot \vec{\sigma}_2 - S_{12}(\hat{q})\right] \vec{\tau}_1 \cdot \vec{\tau}_2$$

Scalar coupling

Lagrangian: $$\mathcal{L}_{\sigma NN} = -g_\sigma \bar{\psi}\psi\, \varphi^{(\sigma)}$$

Vertex: $$\bar{u}(p')\Gamma_{\sigma NN}\, u(p) = -ig_\sigma\, \bar{u}(p')\; u(p) \approx -ig_\sigma \left(1 - \frac{(\vec{\sigma}\cdot\vec{p}\,')(\vec{\sigma}\cdot\vec{p})}{(E'+M)(E+M)}\right)$$

$$= -ig_\sigma \left(1 - \frac{\vec{p}\,'\cdot\vec{p} + i\vec{\sigma}\cdot(\vec{p}\,'\times\vec{p})}{(E'+M)(E+M)}\right) \approx -ig_\sigma \left(1 - \frac{\vec{k}^2 - \frac{1}{4}\vec{q}^2 - \vec{\sigma}\cdot\vec{L}}{4M^2}\right)$$

Potential: keeping all terms up to Q^2/M^2 $\left[P_\sigma = i,\ \vec{k} \equiv \frac{1}{2}(\vec{p}\,'+\vec{p}),\ \vec{L}\cdot\vec{S} = -\frac{i}{2}(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{q}\times\vec{k})\right]$

$$V_\sigma = iF_\sigma \approx \frac{g_\sigma^2}{\vec{q}^2 + m_\sigma^2}\left[-1 + \frac{\vec{k}^2}{2M^2} - \frac{\vec{q}^2}{8M^2} - \frac{\vec{L}\cdot\vec{S}}{2M^2}\right]$$

Vector coupling of a vector meson

Lagrangian: $$\mathcal{L}_{\omega NN} = -g_\omega \bar{\psi}\gamma^\mu \psi \, \varphi_\mu^{(\omega)}$$

Vertex:

$$\mu = 0: \quad \bar{u}(p')\Gamma^0_{\omega NN} u(p) = -ig_\omega \bar{u}(p')\gamma^0 u(p) \quad \approx -ig_\omega \left(1 + \frac{(\vec{\sigma}\cdot\vec{p}\,')(\vec{\sigma}\cdot\vec{p})}{(E'+M)(E+M)} \right)$$

$$\approx -ig_\omega \left(1 - \frac{\vec{\sigma}\cdot\vec{L}}{4M^2} \right), \text{ keeping only the } \vec{\sigma}\cdot\vec{L} \text{ term.}$$

Potential, including also the $\vec{\gamma}$ terms: $\left[P_\omega = -ig_{\mu\nu} + \ldots \right]$

$$V_\omega = iF_\omega \approx \frac{g_\omega^2}{\vec{q}^{\,2} + m_\omega^2} \left[1 - 3\frac{\vec{L}\cdot\vec{S}}{2M^2} \right]$$

Tensor coupling of a vector meson

Lagrangian: $$\mathcal{L}_{\rho NN}^{(\text{tensor})} = -\frac{f_\rho}{4M}\bar{\psi}\sigma^{\mu\nu}\vec{\tau}\psi\cdot(\partial_\mu\vec{\varphi}_\nu^{(\rho)} - \partial_\nu\vec{\varphi}_\mu^{(\rho)})$$

Vertex: $$\Gamma_{\rho NN}^{(\text{tensor})} = -\frac{f_\rho}{4M}\sigma^{\mu\nu}(q_\mu - q_\nu)\vec{\tau} = -\frac{f_\rho}{2M}\sigma^{\mu\nu}q_\mu\,\vec{\tau} \approx -\frac{f_\rho}{2M}(\vec{\sigma}\times\vec{q})\vec{\tau}$$

Potential: $$\left[P_\rho = -ig_{\mu\nu} + \ldots\right]$$

$$V_\rho^{(\text{tensor})} = iF_\rho^{(\text{tensor})} = -\frac{f_\rho^2}{4M^2}\frac{(\vec{\sigma}_1\times\vec{q})(\vec{\sigma}_2\times\vec{q})}{\vec{q}^{\,2}+m_\rho^2}\vec{\tau}_1\cdot\vec{\tau}_2$$

$$= -\frac{f_\rho^2}{4M^2}\frac{\vec{\sigma}_1\cdot\vec{\sigma}_2\,\vec{q}^{\,2} - (\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q})}{\vec{q}^{\,2}+m_\rho^2}\vec{\tau}_1\cdot\vec{\tau}_2$$

$$= \frac{f_\rho^2}{12M^2}\frac{\vec{q}^{\,2}}{\vec{q}^{\,2}+m_\rho^2}\left[-2\vec{\sigma}_1\cdot\vec{\sigma}_2 + S_{12}(\hat{q})\right]\vec{\tau}_1\cdot\vec{\tau}_2$$

Recall: We found the mesons below in PDG Table and asked:

What do they do?

Now, we have the answer. Let's summarize.

$\pi^{\pm}$ $I^G(J^P) = 1^-(0^-)$

Mass $m = 139.57018 \pm 0.00035$ MeV
Mean life $\tau = (2.6033 \pm 0.0005) \times$
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f_0(600) DECAY MODES	Fraction (Γ
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vector

$\omega(782)$ $I^G(J^{PC}) = 0^-(1^{--})$

Mass $m = 782.$ … ± 0.1 … 1.7)
Full width Γ … 8 MeV
Γ_{ee} … 0.02 ke…

ω … DECAY MODES	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
$\pi^+\pi^-$ …	(89.1 ±0.7) %	S=1.1

Repulsive

Summary

$\pi(138)$ $$V_\pi = \frac{f_{\pi NN}^2}{3m_\pi^2}\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2}\left[-\vec{\sigma}_1\cdot\vec{\sigma}_2 - S_{12}(\hat{q})\right]\vec{\tau}_1\cdot\vec{\tau}_2$$ **Long-ranged tensor force**

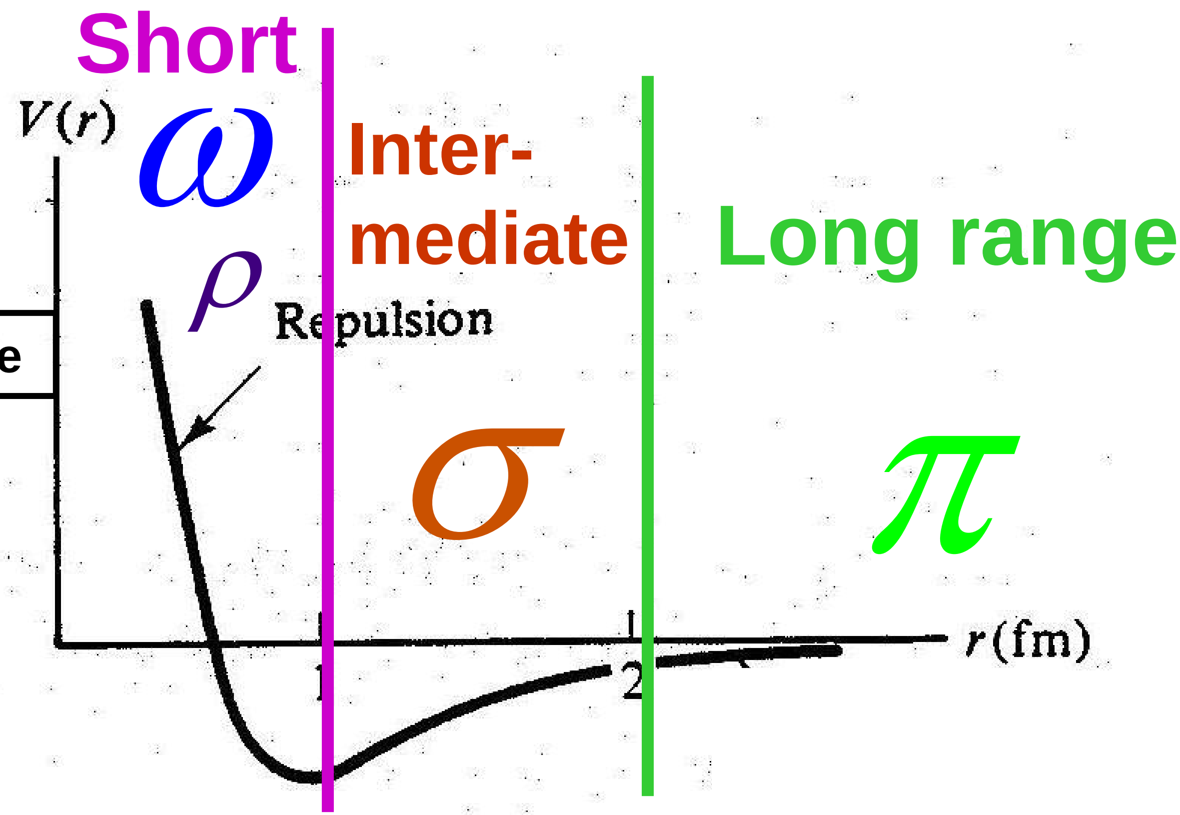
$\sigma(600)$ $$V_\sigma \approx \frac{g_\sigma^2}{\vec{q}^2+m_\sigma^2}\left[-1-\frac{\vec{L}\cdot\vec{S}}{2M^2}\right]$$ **intermediate-ranged, attractive central force plus LS force**

$\omega(782)$ $$V_\omega \approx \frac{g_\omega^2}{\vec{q}^2+m_\omega^2}\left[+1-3\frac{\vec{L}\cdot\vec{S}}{2M^2}\right]$$ **short-ranged, repulsive central force plus strong LS force**

$\rho(770)$ $$V\rho = \frac{f_\rho^2}{12M^2}\frac{\vec{q}^2}{\vec{q}^2+m_\rho^2}\left[-2\vec{\sigma}_1\cdot\vec{\sigma}_2 + S_{12}(\hat{q})\right]\vec{\tau}_1\cdot\vec{\tau}_2$$ **short-ranged tensor force, opposite to pion**

It's EVERYTHING we need to describe the nuclear force!

Summary:
Most important parts of the nuclear force



Central force

Tensor force π ρ

Spin-orbit force ω σ

The One-Boson Exchange Potential (OBEP)

$$V_{\text{OBEP}} = \sum_{\alpha=\pi,\sigma,\rho,\omega,\eta,a_0,\dots} V_\alpha$$

$\eta(548)$ is a pseudo-scalar meson with $I = 0$, therefore, V_η is given by the same expression as V_π, except that V_η carries no $(\vec{\tau}_1 \cdot \vec{\tau}_2)$ factor.

$a_0(980)$ is a scalar meson with $I = 1$, therefore, V_{a_0} is given by the same expression as V_σ, except that V_{a_0} carries a $(\vec{\tau}_1 \cdot \vec{\tau}_2)$ factor.

Some comments

- Note that the mathematical expressions for the various V_α given on previous slides are simplified (many approximations) --- for pedagogical reasons.
- For a serious OBEP, one should make few approximations. In fact, it is quite possible to apply essentially no approximations. This is known as the relativistic (momentum-space) OBEP. Examples are the OBEPs constructed by the "Bonn Group", the latest one being the "CD-Bonn potential" (R. M., PRC 63, 024001 (2001)).
- If one wants to represent the OBE potential in r-space, then the momentum-space OBE amplitudes must be Fourier transformed into r-space. The complete, relativistic momentum-space expressions do not yield analytic expressions in r-space after Fourier transform, i.e., it can be done only numerically. However, it is desirable to have analytic expressions. For this, the momentum-space expressions have to be approximated first, e.g., expanded up to Q^2/M^2, after which an analytic Fourier transform is possible. The expressions one gets by such a procedure are shown on the next slide. Traditionally, the Nijmegen group has taken this approach; their latest r-space OBEPs are published in: V. G. J. Stoks, PRC 49, 2950 (1994).

OBEP expressions in r-space

(All terms up to Q^2/M^2 are included.)

$$V_{ps}(m_{ps},\mathbf{r}) = \frac{1}{12}\frac{g_{ps}^2}{4\pi} m_{ps}\left\{\left(\frac{m_{ps}}{M}\right)^2\left[Y(m_{ps}r) - \frac{4\pi}{m_{ps}^3}\delta^{(3)}(\mathbf{r})\right]\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + Z(m_{ps}r)S_{12}\right\}$$

$$V_s(m_s,\mathbf{r}) = -\frac{g_s^2}{4\pi} m_s\left\{\left[1-\tfrac{1}{4}\left(\frac{m_s}{M}\right)^2\right]Y(m_sr) + \frac{1}{4M^2}\left[\nabla^2 Y(m_sr) + Y(m_sr)\nabla^2\right] + \tfrac{1}{2}Z_1(m_sr)\mathbf{L}\cdot\mathbf{S}\right\}.$$

$$\begin{aligned}V_v(m_v,\mathbf{r}) = \frac{g_v^2}{4\pi} m_v\Big\{&\left[1+\tfrac{1}{2}\left(\frac{m_v}{M}\right)^2\right]Y(m_vr) - \frac{3}{4M^2}\left[\nabla^2 Y(m_vr) + Y(m_vr)\nabla^2\right]\\ &+ \tfrac{1}{6}\left(\frac{m_v}{M}\right)^2 Y(m_vr)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 - \tfrac{3}{2}Z_1(m_vr)\mathbf{L}\cdot\mathbf{S} - \tfrac{1}{12}Z(m_vr)S_{12}\Big\}\\ &+ \tfrac{1}{2}\frac{g_vf_v}{4\pi} m_v\left[\left(\frac{m_v}{M}\right)^2 Y(m_vr) + \tfrac{2}{3}\left(\frac{m_v}{M}\right)^2 Y(m_vr)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 - 4Z_1(m_vr)\mathbf{L}\cdot\mathbf{S} - \tfrac{1}{3}Z(m_vr)S_{12}\right]\\ &+ \frac{f_v^2}{4\pi} m_v\left[\tfrac{1}{6}\left(\frac{m_v}{M}\right)^2 Y(m_vr)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 - \tfrac{1}{12}Z(m_vr)S_{12}\right]\end{aligned}$$

with

$$Y(x) = e^{-x}/x$$

$$Z(x) = \left(\frac{m_\alpha}{M}\right)^2\left(1+\frac{3}{x}+\frac{3}{x^2}\right)Y(x)$$

$$Z_1(x) = -\left(\frac{m_\alpha}{M}\right)^2\frac{1}{x}\frac{d}{dx}Y(x) = \left(\frac{m_\alpha}{M}\right)^2\left(\frac{1}{x}+\frac{1}{x^2}\right)Y(x)$$

and

$$S_{12} = 3\frac{(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{r})}{r^2} - \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$$

$$\nabla^2 = \frac{1}{r}\frac{\partial^2}{\partial r^2}r - \frac{\mathbf{L}^2}{r^2}$$

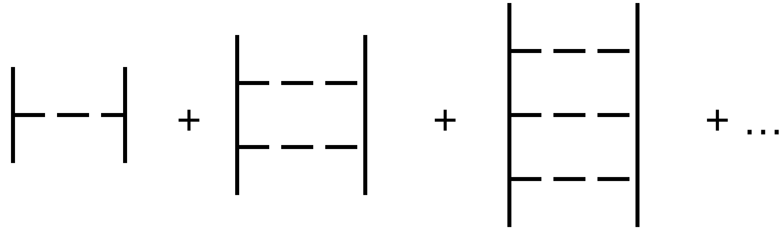
Does the OBE model contain “everything”?

- NO! It contains only the so-called iterative diagrams.

Lippmann-Schwinger eqn: $T = V + V \frac{1}{e} T$

$$T = V + V \frac{1}{e} V + V \frac{1}{e} V \frac{1}{e} V + \ldots$$

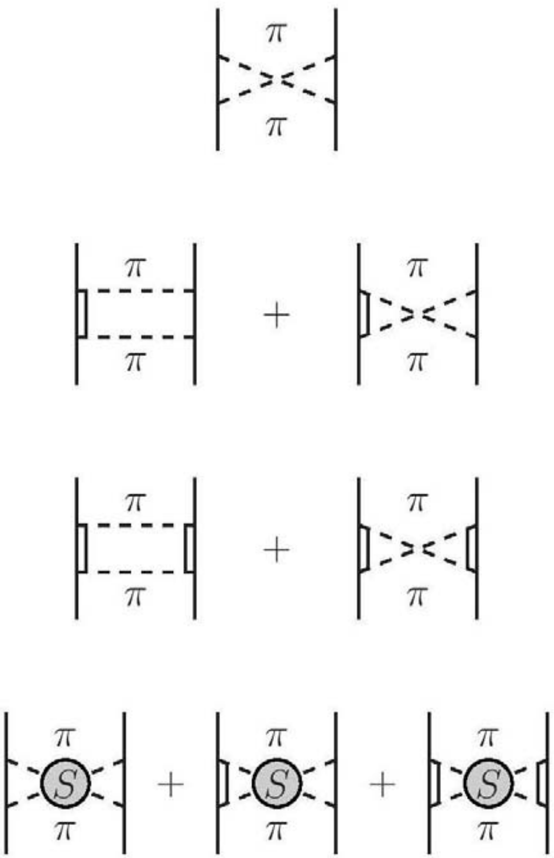
In diagrams:



“i-t-e-r-a-t-i-v-e”

- However: There are also non-iterative diagrams which contribute to the nuclear force (see next slide).

Some examples for non-iterative meson-exchange contributions not included in the OBE model (or OBEP).



The "Bonn Full Model" (or "Bonn Potential") contains these and other non-iterative contributions. It is the most comprehensive meson-model ever developed (R. M., Phys. Reports 149, 1 (1987)).

The "Paris Potential" is based upon dispersion theory and not on field theory. However, one may claim that, implicitly, the Paris Potential also includes these diagrams; M. Lacombe *et al.,* Phys. Rev. C 21, 861 (1980).

Reviews on Meson Theory

- Pedagogical introduction which also includes a lot of history: R. M., *Advances in Nuclear Physics* **19**, 189-376 (1989).
- The derivation of the meson-exchange potentials in all mathematical details is contained in: R. M., "The Meson Theory of Nuclear Forces and Nuclear Matter", in: *Relativistic Dynamics and Quark-Nuclear Physics,* M. B. Johnson and A. Picklesimer, eds. (Wiley, New York, 1986) pp. 71-173.
- Computer codes for relativistic OBEPs and phase-shift calculations in momentum-space are published in: R. M., "One-Boson Exchange Potentials and Nucleon-Nucleon Scattering", in: *Computational Nuclear Physics 2 – Nuclear Reactions,* K. Langanke, J.A. Maruhn, and S.E. Koonin, eds. (Springer, New York, 1993) pp. 1-29.