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Nuclear Forces

- Lecture 3 -

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Prolog

In Lecture 2, we have seen how beautifully the meson theory of nuclear forces works. This may suggest that we are done with the theory of nuclear forces.

Well, as it turned out, the fundamental theory of the strong interaction is QCD. Thus, if we want to understand the nuclear force on the most fundamental level, then we have to base it upon QCD.

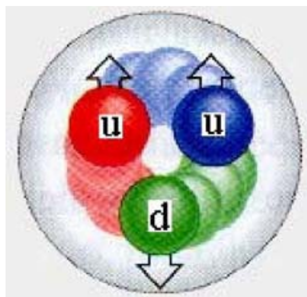
This is what we will try now.

Lecture 3: Low-energy QCD and the nuclear force

- The nuclear force in the light of QCD
- QCD-based models ("quark models")
- The symmetries of low-energy QCD

The nuclear force in the light of QCD

If nucleons do not overlap, ….



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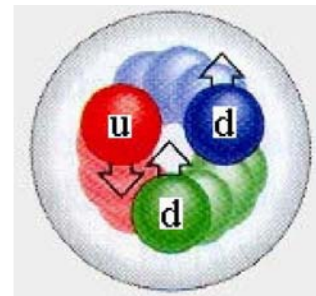
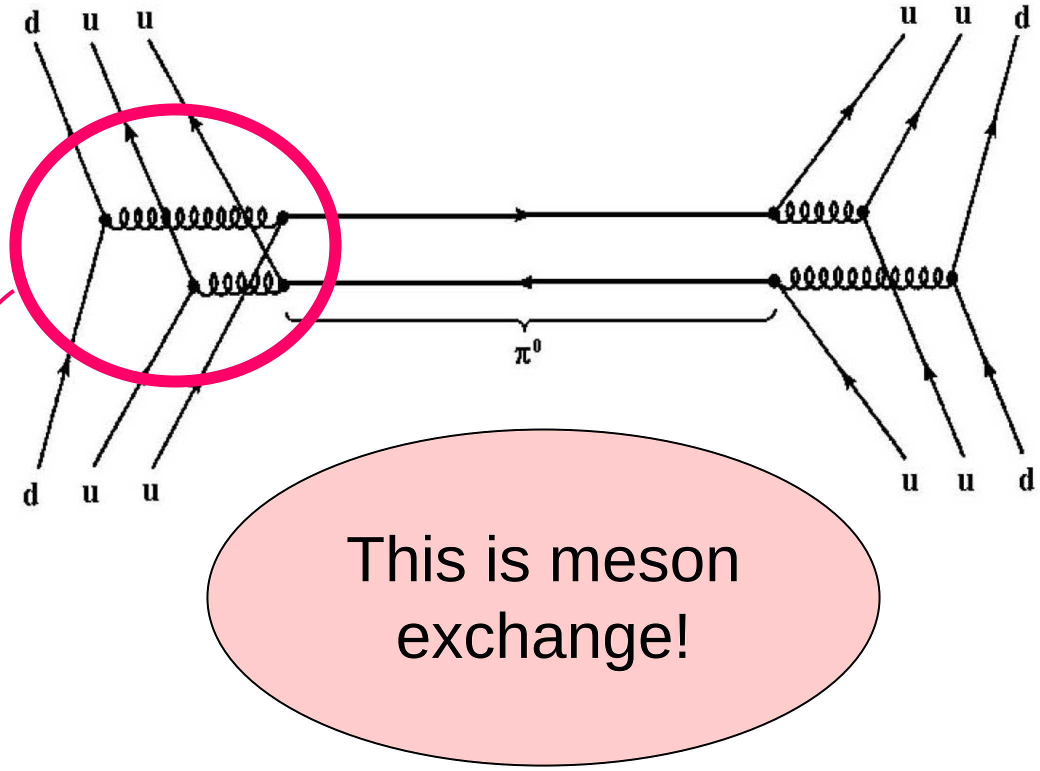


then we have two separated colorless objects. In lowest order, they do not interact. This is analogous to the interaction between two neutral atoms. Like the Van der Waals force, **the nuclear force is a residual interaction**. Such interactions are typically weak as compared to the "pure" version of the force (Coulomb force between electron and proton, strong force between two quarks, respectively).

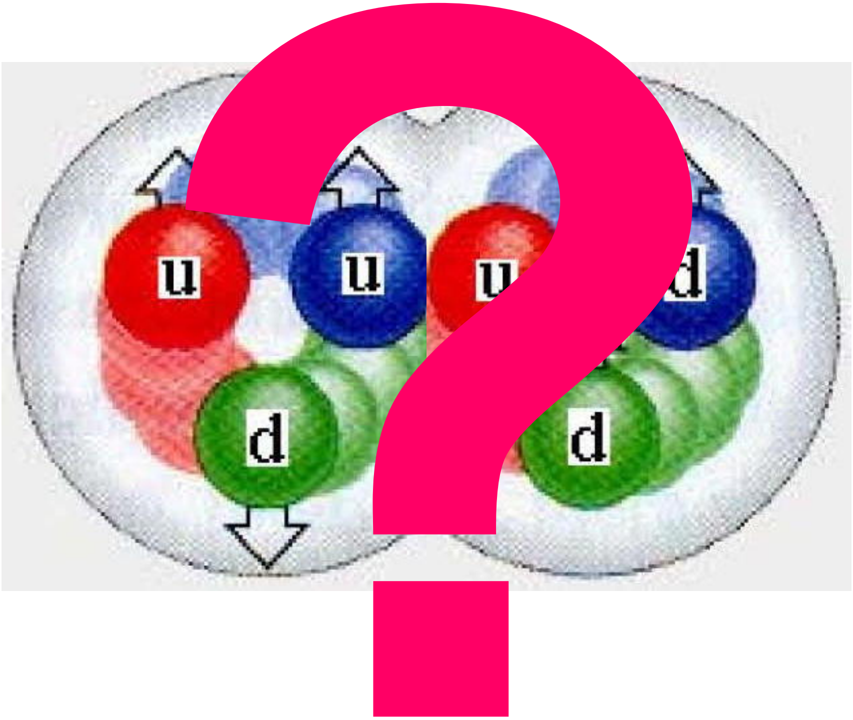
Non-overlapping nucleons, cont'd

One way they could interact is this:



But notice that if you want to claim to do QCD, then you have to calculate this vertex from quark and gluon exchanges. Good luck!

When nucleons DO overlap, we have a six-quark problem with non-perturbative interactions between the quarks (non-perturbative gluon-exchanges). A formidable problem!



So far, nobody has been able to calculate this.
Maybe one day it can be done by lattice QCD.

QCD-based models (“quark models”)

To make 2- and 3-quark (hadron spectrum) or 5- or 6-quark (hadron-hadron interaction) calculations feasible, simple assumptions on the quark-quark interactions are made. For example, the quark-quark interaction is assumed to be just one-gluon-exchange or even meson-exchange.

This is, of course, not real QCD. It is a model; a “QCD-inspired” model which, however, does not make it much better.

The only positive thing one can say about these models is that they make a connection between the hadron spectrum and the hadron-hadron interaction.

Conclusion

“QCD-inspired” models are not the solution.

If we want to do something that is truly QCD based, then we have to try something totally different.

To get ready for this new approach, let’s take a close look at the properties of low-energy QCD.

The symmetries of low-energy QCD

- The QCD Lagrangian
- Symmetries of the QCD Lagrangian
- Broken symmetries

The QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = \sum_{f=\substack{u,d,s,\\c,b,t}} \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4}\mathcal{G}_{\mu\nu,a}\mathcal{G}_a^{\mu\nu}$$

with $q_f = \begin{pmatrix} q_{f,r} \\ q_{f,g} \\ q_{f,b} \end{pmatrix}$, where r=red, g=green, and b=blue,

and the gauge-covariant derivative

$$D_\mu \begin{pmatrix} q_{f,r} \\ q_{f,g} \\ q_{f,b} \end{pmatrix} = \partial_\mu \begin{pmatrix} q_{f,r} \\ q_{f,g} \\ q_{f,b} \end{pmatrix} - ig \sum_{a=1}^{8} \frac{\lambda_a^C}{2} \mathcal{A}_{\mu,a} \begin{pmatrix} q_{f,r} \\ q_{f,g} \\ q_{f,b} \end{pmatrix}$$

and the gluon field tensor

$$\mathcal{G}_{\mu\nu,a} = \partial_\mu \mathcal{A}_{\nu,a} - \partial_\nu \mathcal{A}_{\mu,a} + g f_{abc} \mathcal{A}_{\mu,b} \mathcal{A}_{\nu,c}$$

Symmetries of the QCD Lagrangian

Quark masses

$$\begin{pmatrix} m_u = 0.005\,\mathrm{GeV} \\ m_d = 0.009\,\mathrm{GeV} \\ m_s = 0.175\,\mathrm{GeV} \end{pmatrix} \ll 1\,\mathrm{GeV} \leq \begin{pmatrix} m_c = (1.15 - 1.35)\,\mathrm{GeV} \\ m_b = (4.0 - 4.4)\,\mathrm{GeV} \\ m_t = 174\,\mathrm{GeV} \end{pmatrix}$$

Assuming $m_u,\ m_d \approx 0,$

the **QCD Lagrangian for just up and down quarks** reads

$$\mathcal{L}^0_{\mathrm{QCD}} = \sum_{l=u,d} \bar{q}_l i \not{D}\, q_l - \frac{1}{4} \mathcal{G}_{\mu\nu,a} \mathcal{G}^{\mu\nu}_a \,.$$

Symmetries of the QCD Lagrangian, cont'd

Introduce projection operators

"Right-handed" $$P_R = \frac{1}{2}(1+\gamma_5) = P_R^\dagger$$

"Left-handed" $$P_L = \frac{1}{2}(1-\gamma_5) = P_L^\dagger$$

Properties

Complete $$P_R + P_L = 1$$

Idempotent $$P_R^2 = P_R, \quad P_L^2 = P_L$$

Orthogonal $$P_R P_L = P_L P_R = 0$$

Symmetries of the QCD Lagrangian, cont'd

Consider Dirac spinor for a particle of large energy or zero mass:

$$u(\vec{p}, \pm) = \sqrt{E+m} \begin{pmatrix} \chi_\pm \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi_\pm \end{pmatrix} \quad \xrightarrow{E \gg m} \quad \sqrt{E} \begin{pmatrix} \chi_\pm \\ \pm\chi_\pm \end{pmatrix} \equiv u_\pm(\vec{p})$$

where we assume that the spin in the rest frame is either parallel or antiparallel to the direction of momentum

$$\vec{\sigma} \cdot \hat{p} \chi_\pm = \pm \chi_\pm .$$

Thus, for mass-less particles, helicity/chirality eigenstates exist

$$(\Sigma \cdot \hat{p})\, u_\pm = \gamma_5\, u_\pm = \pm u_\pm$$

and the projection operators project them out

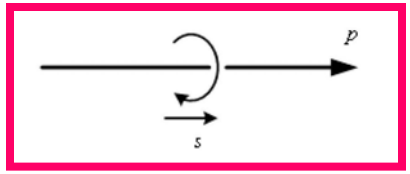
$$u_+ \equiv u_R \equiv P_R\, u$$

$$u_- \equiv u_L \equiv P_L\, u$$

Symmetries of the QCD Lagrangian, cont'd

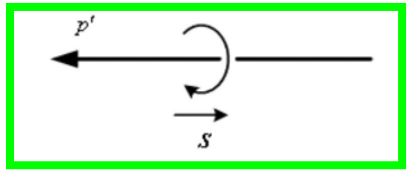
For zero-mass quarks,

quark field of **right-handed chirality**



$q_R = P_R q,$

quark field of **left-handed chirality**:



$q_L = P_L q.$

The QCD Lagrangian can be written

$$\mathcal{L}^0_{\mathrm{QCD}} = \sum_{l=u,d} \left(\bar{q}_{R,l} i \not{D}\, q_{R,l} + \bar{q}_{L,l} i \not{D}\, q_{L,l} \right) - \frac{1}{4} \mathcal{G}_{\mu\nu,a} \mathcal{G}^{\mu\nu}_a .$$

$\Rightarrow \quad SU(2)_L \times SU(2)_R$ symmetry **= "Chiral Symmetry"**

(Because the mass term is absent; mass term destroys chiral symmetry.)

Interim summary

$$\mathcal{L}_{\rm QCD} = \sum_{f=\substack{u,d,s,\\c,b,t}} \bar{q}_f (i \not{D} - m_f) q_f - \frac{1}{4} \mathcal{G}_{\mu\nu,a} \mathcal{G}^{\mu\nu}_a$$

⬇

$$\mathcal{L}^0_{\rm QCD} = \sum_{l=u,d,.} \bar{q}_l i \not{D}\, q_l - \frac{1}{4} \mathcal{G}_{\mu\nu,a} \mathcal{G}^{\mu\nu}_a .$$

⬇

$$\mathcal{L}^0_{\rm QCD} = \sum_{l=u,d} (\bar{q}_{R,l} i \not{D}\, q_{R,l} + \bar{q}_{L,l} i \not{D}\, q_{L,l}) - \frac{1}{4} \mathcal{G}_{\mu\nu,a} \mathcal{G}^{\mu\nu}_a .$$

(approximate) Chiral Symmetry

Chiral symmetry and Noether currents

Global $SU(2)_L \times SU(2)_R$ symmetry

$$q_L \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix} \longmapsto \exp\left(-i \sum_{a=1}^{3} \Theta_a^L \frac{\tau_a}{2}\right) \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$$q_R \equiv \begin{pmatrix} u_R \\ d_R \end{pmatrix} \longmapsto \exp\left(-i \sum_{a=1}^{3} \Theta_a^R \frac{\tau_a}{2}\right) \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

where τ_a $(a = 1, 2, 3)$ are the usual Pauli matrices, which—here— play the role of the generators of the $SU(2)$ flavor group. Obviously, right-handed and left-handed mass-less quarks never mix.

Noether's Theorem ⇒ Six conserved currents

3 left-handed currents $L_a^\mu = \bar{q}_L \gamma^\mu \frac{\tau_a}{2} q_L$ with $\partial_\mu L_a^\mu = 0$

3 right-handed currents $R_a^\mu = \bar{q}_R \gamma^\mu \frac{\tau_a}{2} q_R$ with $\partial_\mu R_a^\mu = 0$

Alternatively, ...

$SU(2)_V \times SU(2)_A$

$$q \equiv \begin{pmatrix} u \\ d \end{pmatrix} \longmapsto \exp\left(-i \sum_{a=1}^{3} \Theta_a^V \frac{\tau_a}{2}\right) \begin{pmatrix} u \\ d \end{pmatrix}$$

$$q \equiv \begin{pmatrix} u \\ d \end{pmatrix} \longmapsto \exp\left(-i \sum_{a=1}^{3} \gamma_5 \Theta_a^A \frac{\tau_a}{2}\right) \begin{pmatrix} u \\ d \end{pmatrix}$$

3 vector currents $$V_a^\mu = R_a^\mu + L_a^\mu = \bar{q}\gamma^\mu \frac{\tau_a}{2} q \quad \text{with} \quad \partial_\mu V_a^\mu = 0$$

3 axial-vector currents $$A_a^\mu = R_a^\mu - L_a^\mu = \bar{q}\gamma^\mu \gamma_5 \frac{\tau_a}{2} q \quad \text{with} \quad \partial_\mu A_a^\mu = 0$$

6 conserved "charges" which are likewise generators of $SU(2)_V \times SU(2)_A$

$$Q_a^V = \int d^3x \, V_a^0 = \int d^3x \, q^\dagger(t,\vec{x}) \frac{\tau_a}{2} q(t,\vec{x}) \quad \text{with} \quad \frac{dQ_a^V}{dt} = 0$$

$$Q_a^A = \int d^3x \, A_a^0 = \int d^3x \, q^\dagger(t,\vec{x}) \gamma_5 \frac{\tau_a}{2} q(t,\vec{x}) \quad \text{with} \quad \frac{dQ_a^A}{dt} = 0$$

Note: "vector" is Isospin!

Explicit symmetry breaking due to non-zero quark masses

The mass term that we neglected breaks chiral symmetry explicitly:

$$-\sum_{l=u,d} \bar{q}_l m_l q_l = -\bar{q} M q = -(\bar{q}_R M q_L + \bar{q}_L M q_L)$$

with

$$\begin{aligned} M &= \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} \\ &= \frac{1}{2}(m_u + m_d) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{2}(m_u - m_d) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ &= \frac{1}{2}(m_u + m_d)\, I + \frac{1}{2}(m_u - m_d)\, \tau_3 \end{aligned}$$

Both terms break $SU(2)_A$, but the 1st term is invariant under $SU(2)_V$.

⇒ if $m_u = m_d$, then there is $SU(2)_V$ symmetry

(=Isospin symmetry).

Spontaneous symmetry breaking

If the ground state of QCD (=hadron spectrum) has the same symmetry as the Lagrangian (namely, chiral symmetry), then “parity doublets” exist; because:

Let $|i,+\rangle$ denote an eigenstate of $H^0_{\rm QCD}$ with eigenvalue E_i,

$$H^0_{\rm QCD}|i,+\rangle = E_i|i,+\rangle,$$

having positive parity,

$$P|i,+\rangle = +|i,+\rangle,$$

such as, e.g., a member of the ground state baryon octet (in the chiral limit). Defining $|\phi\rangle = Q^a_A|i,+\rangle$, because of $[H^0_{\rm QCD}, Q^a_A] = 0$, we have

$$H^0_{\rm QCD}|\phi\rangle = H^0_{\rm QCD}Q^a_A|i,+\rangle = Q^a_A H^0_{\rm QCD}|i,+\rangle = E_i Q^a_A|i,+\rangle = E_i|\phi\rangle,$$

i.e, the new state $|\phi\rangle$ is also an eigenstate of $H^0_{\rm QCD}$ with the same eigenvalue E_i but of opposite parity:

$$P|\phi\rangle = PQ^a_A P^{-1}P|i,+\rangle = -Q^a_A(+|i,+\rangle) = -|\phi\rangle.$$

Spontaneous symmetry breaking, cont'd

What would parity doublets look like?

Nucleons of positive parity: p(1/2+,938.3), n(1/2+,939.6), I=1/2;

nucleons of negative parity: N(1/2-,1535), I=1/2.

But, the masses are very different: NOT a parity doublet!

A meson of negative parity: $\rho(1-,770),\ I=1.$

the "same" with positive parity: $a_1(1+,1260),\ I=1.$

But again, the masses are very different: NOT a parity doublet!

Conclusion: Parity doublets are not observed in the low-energy hadron spectrum.

→ **Chiral symmetry is spontaneously broken.**

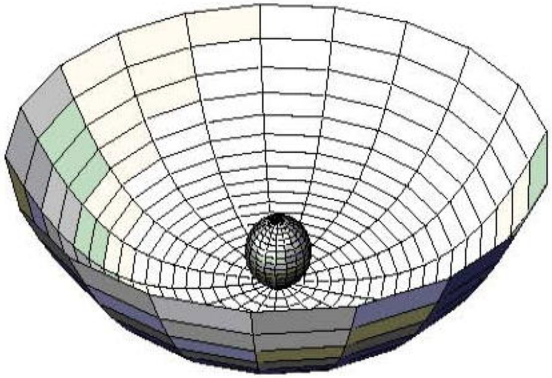
Definition:

When the ground state does not have the same symmetries as the Langrangian, then one speaks of "spontaneous symmetry breaking".

Classical examples

Lagrangian and ground state have both rotational symmetry.

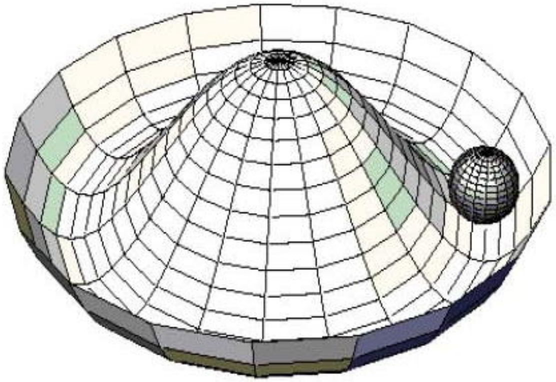
No symmetry breaking.



Lagrangian has rot. symmetry;

groundstate has not.

Symmetry is spontaneously broken.



Goldstone's Theorem

When a continuous symmetry is broken, then there exists a (massless) boson with the quantum numbers of the broken generator.

Here: The 3 axial generators $Q_a^A \ (a = 1,2,3)$ are broken, therefore, 3 pseudoscalar bosons exist: the 3 pions π^+, π^0, π^-

This explains the small mass of the pion. The pion mass is not exactly zero, because the u and d quark masses are no exactly zero either.

More about Goldstone Bosons

The breaking of the axial symmetry in the QCD ground state (QCD vacuum) implies that the ground state is not invariant under axial transformations, i.e.

$$Q_a^A \mid 0\rangle \neq 0 \ .$$

Thus, a physical state must be generated by the axial charge,

$$\mid \phi_a \rangle \equiv Q_a^A \mid 0\rangle \ .$$

Since H^0_{QCD} commutes with Q_a^A, we have

$$H^0_{QCD} \mid \phi_a \rangle = Q_a^A \ H^0_{QCD} \mid 0\rangle = 0 \ .$$

The energy of the state is zero. The state is energetically degenerate with the vacuum.

It is a massless pseudoscalar boson with vanishing interaction energy.

→ **Goldstone bosons interact weakly.**

Summary

QCD in the u/d sector has approximate chiral symmetry; but this symmetry is broken in two ways:

- **explicitly broken,**

  because the u and d quark masses are not exactly zero;

- **spontaneously broken:**

  $$SU(2)_L \times SU(2)_R \cong SU(2)_V \times SU(2)_A \longrightarrow SU(2)_V$$

  i.e., in the QCD ground state,

  axial symmetry is broken,

  while isospin symmetry is intact.

- **There exist 3 Goldstone bosons: the pions**