CNS Summer School, Univ. of Tokyo,
at Wako campus of RIKEN,
Aug. 18-23, 2005

Nuclear Forces

- Lecture 4 -

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From last Lecture

- **The nuclear force in the light of QCD**
- **QCD-based models (“quark models”)**

Conclusion

"QCD-inspired" models are not the solution.

If we want to do something that is truly QCD based, then we have to try something totally different.

To get ready for this new approach, let's take a close look at the properties of low-energy QCD.

The symmetries of low-energy QCD

**QCD in the u/d sector has approximate chiral symmetry;
but this symmetry is broken in two ways:**

- **explicitly broken,**

  because the u and d quark masses are not exactly zero;

- **spontaneously broken:**

  $$SU(2)_L \times SU(2)_R \simeq SU(2)_V \times SU(2)_A \longrightarrow SU(2)_V$$

  i.e., in the QCD ground state,
  axial symmetry is broken,
  while isospin symmetry is intact.

- **There exist 3 Goldstone bosons: the pions**

<u>Lecture 4:</u> Effective Field Theory (EFT) and nuclear forces

- An EFT for low-energy QCD: Why and how?
- Power counting
- The hierarchy of nuclear forces
- The new generation of chiral NN potentials
- Many-body forces
- First applications in nuclear structure
- Conclusions

We want to describe the low-energy scenario of QCD (that we studied in Lecture 3) by an Effective Field Theory (EFT).
Why and how?

QCD at low energy is non-perturbative and, therefore, not solvable analytically in terms of the “fundamental” degrees of freedom (dof), quarks and gluons.

We need to use dof which interact such that a perturbative approach is possible. Goldstone bosons typically interact weakly. Therefore, let’s use pions and nucleons.

To ensure the connection with QCD, we must observe the same symmetries as low-energy QCD, particularly, spontaneously broken chiral symmetry.

Such a theory is called an Effective Field Theory (EFT).

Further justification for this approach is provided by

Weinberg's

"Folk Theorem"

If one writes down the most general possible Langrangian, including all terms consistent with assumed symmetry principles, and then calculates matrix elements with this Langrangian to any given order of perturbation theory, the result will simply be the most general possible S-matrix consistent with analyticity, perturbative unitarity, cluster decomposition, and the assumed symmetry principles.

Physica **96A**, *327 (1979)*

So, for doing EFT, we have to take the following steps:

- Write down the most general Lagrangian
  including **all** terms consistent with the assumed symmetries, particularly, spontaneously broken chiral symmetry. (Note: There will be infinitely many terms.)
- Calculate Feynman diagrams.
  (Note: There will be infinitely many diagrams.)
- Find a scheme for assessing the importance of the various diagrams
  (because we cannot calculate infinitely many diagrams).

The organizational scheme

"Power Counting"

Organize the contributions in terms of $\left(\frac{Q}{\Lambda}\right)^{\nu}$;

where Q denotes a momentum (derivative) or a pion mass (m_π);

Λ is the chiral symmetry breaking scale, $\Lambda \approx 1$ GeV;

and $\nu \geq 0$.

⟺ Chiral Perturbation Theory (ChPT)

The Lagrangian

$$\mathcal{L} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN}$$

with

$$\mathcal{L}_{\pi\pi} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi\pi}^{(4)} + \ldots$$

$$\mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \ldots$$

$$\mathcal{L}_{NN} = \mathcal{L}_{NN}^{(0)} + \mathcal{L}_{NN}^{(2)} + \mathcal{L}_{NN}^{(4)} + \ldots$$

where the superscript refers to the number of derivaties or pion mass insertions.

The pi-N Lagrangian with one derivative (=lowest order or "leading" order)

$$\mathcal{L}^{(I)}_{\pi N} = \bar{\Psi}_N \left(i\gamma^\mu D_\mu - M_N + \frac{g_A}{2}\gamma^\mu \gamma_5 u_\mu \right) \Psi_N$$

with

$$\begin{aligned}
D_\mu &= \partial_\mu + \Gamma_\mu \\
\Gamma_\mu &= (\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger)/2 \approx i\boldsymbol{\tau} \cdot (\boldsymbol{\pi} \times \partial_\mu \boldsymbol{\pi})/4f_\pi^2 + \ldots \\
u_\mu &= i(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger) \quad \approx -\boldsymbol{\tau} \cdot \partial_\mu \boldsymbol{\pi}/f_\pi + \ldots \\
U &= \xi\xi \\
\xi &= e^{i\boldsymbol{\tau}\cdot\boldsymbol{\pi}/2f_\pi} \qquad \approx 1 + \frac{i\boldsymbol{\tau}\cdot\boldsymbol{\pi}}{2f_\pi} - \frac{\boldsymbol{\pi}^2}{8f_\pi^2} + \ldots
\end{aligned}$$

and

$$\begin{aligned}
f_\pi &= 92.4 \text{ MeV} \\
g_A &= g_{\pi NN}\, f_\pi/M_N = 1.29
\end{aligned}$$

equivalent to $g^2_{\pi NN}/4\pi = 13.67$

Non-linear realization of chiral symmetry; Callan, Coleman, Wess, and Zumino, PR 177, 2247 (1968).

pi-N Lagrangian, cont'd

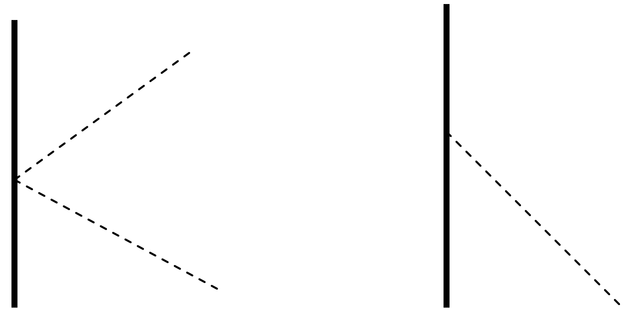
The M_N term is a problem.

$\Rightarrow$ Perform a $1/M_N$ expansion to remove it.

"Heavy Baryon Chiral Perturbation Theory"

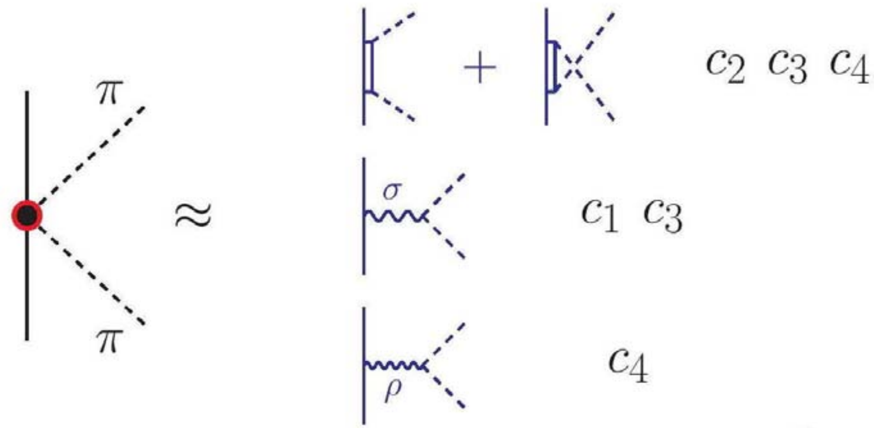
$$
\begin{aligned}
\mathcal{L}_{\pi N}^{(I)} &= \bar{\Psi}_N \left(i\gamma^\mu D_\mu - M_N + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu \right) \Psi_N \\
&\approx \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N, M_N}^{(2)} + \mathcal{L}_{\pi N, M_N}^{(3)} + \ldots
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{\pi N}^{(1)} &= \bar{N} \left(iD_0 - \frac{g_A}{2} \vec{\sigma} \cdot \vec{u} \right) N \\
&\approx \bar{N} \left[i\partial_0 - \frac{1}{4f_\pi^2} \boldsymbol{\tau} \cdot (\boldsymbol{\pi} \times \partial_0 \boldsymbol{\pi}) - \frac{g_A}{2f_\pi} \boldsymbol{\tau} \cdot (\vec{\sigma} \cdot \vec{\nabla}) \boldsymbol{\pi} \right] N + \ldots
\end{aligned}
$$



pi-N Lagrangian with two derivatives
("next-to-leading" order)

$$\begin{aligned}\mathcal{L}^{(2)}_{\pi N, c_i} \quad = \quad & \bar{N}\Big[2\,c_1\,m_\pi^2\,(U+U^\dagger) \\ & + \left(c_2 - \frac{g_A^2}{8M_N}\right)u_0^2 \\ & + c_3\,u_\mu\,u^\mu \\ & + \frac{i}{2}\left(c_4 + \frac{1}{4M_N}\right)\vec{\sigma}\cdot(\vec{u}\times\vec{u})\Big]\,N\end{aligned}$$



The Langrangian, cont'd

- The pi-pi Lagrangian at order two (leading order):

$$\mathcal{L}^{(2)}_{\pi\pi} = \frac{f_\pi^2}{4} \ \mathrm{Tr} \left[\partial_\mu U \partial^\mu U^\dagger + m_\pi^2 \left(U + U^\dagger \right) \right]$$

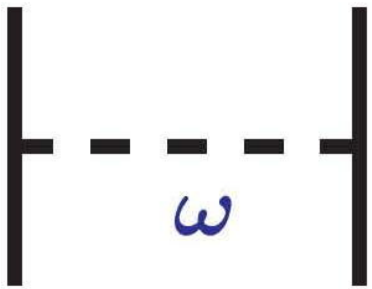
- The N-N Lagrangian at order zero (leading order):

$$\mathcal{L}^{(0)}_{NN} = -\frac{1}{2} C_S \bar{N} N \bar{N} N - \frac{1}{2} C_T \bar{N} \vec{\sigma} N \bar{N} \vec{\sigma} N$$

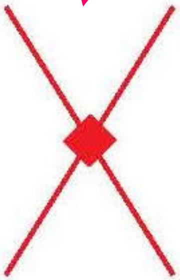
"Contact terms"

What are contact terms?

Consider the contribution from the exchange of a heavy meson



$$\frac{1}{m_\omega^2+Q^2} \approx \frac{1}{m_\omega^2}\left[1 - \frac{Q^2}{m_\omega^2} + \frac{Q^4}{m_\omega^2} - \cdots\right]$$

≈  +  +  + ...

Contact terms take care of the short range contributions.

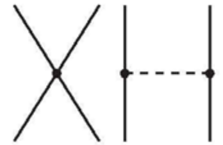
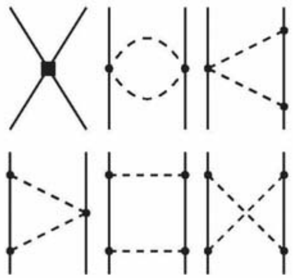
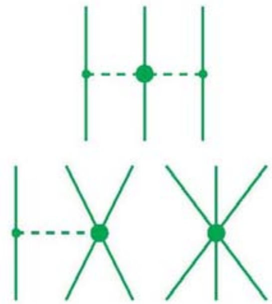
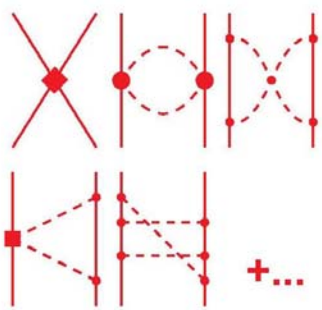
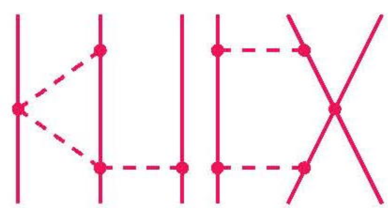
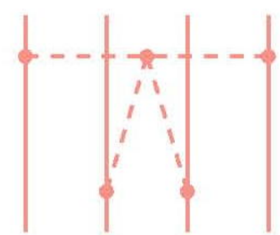
The Diagrams

Power counting for two-nucleon diagrams:

$$\nu = 2 \times Loops \; + \sum_{all\ vertices\ j} \Delta_j$$

with

$$\Delta_j = d_j + \frac{n_j}{2} - 2 \geq 0$$

		2N forces	3N forces	4N forces
Leading Order	Q^0 LO			
Next-to Leading Order	Q^2 NLO			
Next-to-Next-to Leading Order	Q^3 N^{2}LO	 +...		
Next-to-Next-to-Next-to Leading Order	Q^4 N^{3}LO	 +...	 +...	 +...

Explicit mathematical expressions for some of the diagrams just shown

Operators that occur in the expressions, and notation:

$$
\begin{aligned}
V(\vec{p}\,', \vec{p}) = \quad & V_C \; + \tau_1 \cdot \tau_2 \, W_C \\
& + [\, V_S \; + \tau_1 \cdot \tau_2 \, W_S \;]\, \vec{\sigma}_1 \cdot \vec{\sigma}_2 \\
& + [\, V_T \; + \tau_1 \cdot \tau_2 \, W_T \;]\, \vec{\sigma}_1 \cdot \vec{q}\; \vec{\sigma}_2 \cdot \vec{q} \\
& + [\, V_{LS} + \tau_1 \cdot \tau_2 \, W_{LS}] \left(-i\vec{S} \cdot (\vec{q} \times \vec{k}) \right) \\
& + [\, V_{\sigma L} + \tau_1 \cdot \tau_2 \, W_{\sigma L}]\, \vec{\sigma}_1 \cdot (\vec{q} \times \vec{k}\,)\; \vec{\sigma}_2 \cdot (\vec{q} \times \vec{k}\,)
\end{aligned}
$$

with $\vec{p}\,'$ and $\vec{p}$ the final and initial nucleon momenta in the center-of-mass (CM) frame, respectively, and

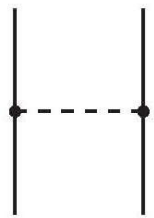
$\vec{q} \equiv \vec{p}\,' - \vec{p}$	the momentum transfer
$\vec{k} \equiv \frac{1}{2}(\vec{p}\,' + \vec{p})$	the average momentum
$\vec{S} \equiv \frac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2)$	the total spin

Explicit expressions, cont'd.
Leading order (LO)

Pion-exchange contribution

Contact contributions

Q^0



Static, non-relativistic

One-Pion-Exchange (OPE)

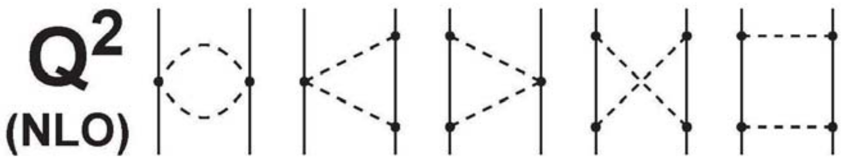
$$V_{\rm OPE} \equiv -\frac{g_A^2}{4f_\pi^2}\frac{\vec\sigma_1\cdot\vec q\,\vec\sigma_2\cdot\vec q}{q^2+m_\pi^2}\,\tau_1\cdot\tau_2$$

$$V^{(0)}(\vec{p}\,',\vec p) = C_S + C_T\,\vec\sigma_1\cdot\vec\sigma_2$$

2 parameters

Explicit expressions, cont'd.
Next-to-Leading order (NLO)

Pion-exchange contributions



"leading order 2π exchange"

$$W_C = -\frac{L(q)}{384\pi^2 f_\pi^4}\left[4m_\pi^2(5g_A^4 - 4g_A^2 - 1) + q^2(23g_A^4 - 10g_A^2 - 1) + \frac{48g_A^4 m_\pi^4}{w^2}\right]$$

$$V_T = -\frac{1}{q^2}V_S = -\frac{3g_A^4 L(q)}{64\pi^2 f_\pi^4}$$

where

$$L(q) \equiv \frac{w}{q}\ln\frac{w+q}{2m_\pi}$$

and

$$w \equiv \sqrt{4m_\pi^2 + q^2}$$

Contact contributions

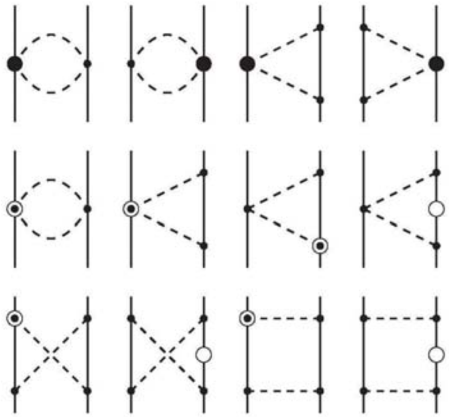
$$\begin{aligned} V^{(2)}(\vec{p}', \vec{p}) &= C_1 q^2 + C_2 k^2 \\ &+ \left(C_3 q^2 + C_4 k^2\right)\vec{\sigma}_1 \cdot \vec{\sigma}_2 \\ &+ C_5\left(-i\vec{S}\cdot(\vec{q}\times\vec{k})\right) \\ &+ C_6(\vec{\sigma}_1\cdot\vec{q})\,(\vec{\sigma}_2\cdot\vec{q}) \\ &+ C_7(\vec{\sigma}_1\cdot\vec{k})\,(\vec{\sigma}_2\cdot\vec{k}) \end{aligned}$$

7 parameters

Explicit expressions, cont'd. Next-to-Next-to-Leading order (NNLO)

Pion-exchange contributions

Q^3
(N^2LO)



"sub-leading 2π exchange"

$$
\begin{aligned}
V_C &= \frac{3g_A^2}{16\pi f_\pi^4}\left\{\frac{g_A^2 m_\pi^5}{16M_N w^2} - \left[2m_\pi^2(2c_1 - c_3) - q^2\left(c_3 + \frac{3g_A^2}{16M_N}\right)\right]\widetilde{w}^2 A(q)\right\} \\
W_C &= \frac{g_A^2}{128\pi M_N f_\pi^4}\left\{3g_A^2 m_\pi^5 w^{-2} - \left[4m_\pi^2 + 2q^2 - g_A^2(4m_\pi^2 + 3q^2)\right]\widetilde{w}^2 A(q)\right\} \\
V_T &= -\frac{1}{q^2}V_S = \frac{9g_A^4\widetilde{w}^2 A(q)}{512\pi M_N f_\pi^4} \\
W_T &= -\frac{1}{q^2}W_S = -\frac{g_A^2 A(q)}{32\pi f_\pi^4}\left[\left(c_4 + \frac{1}{4M_N}\right)w^2 - \frac{g_A^2}{8M_N}(10m_\pi^2 + 3q^2)\right] \\
V_{LS} &= \frac{3g_A^4\widetilde{w}^2 A(q)}{32\pi M_N f_\pi^4} \\
W_{LS} &= \frac{g_A^2(1 - g_A^2)}{32\pi M_N f_\pi^4}w^2 A(q)
\end{aligned}
$$

with

$$A(q) \equiv \frac{1}{2q}\arctan\frac{q}{2m_\pi} \quad \text{and} \quad \widetilde{w} \equiv \sqrt{2m_\pi^2 + q^2}$$

Contact contributions

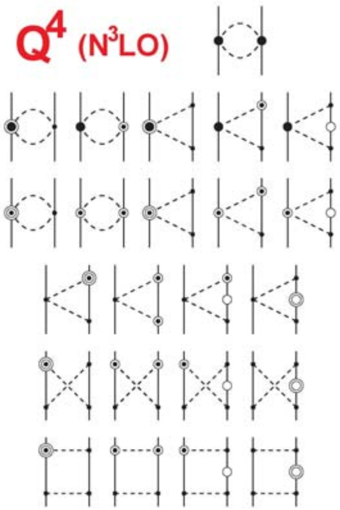
--- None ---

Explicit expressions, cont'd.
Next-to-Next-to-Next-to-Leading order (NNNLO)

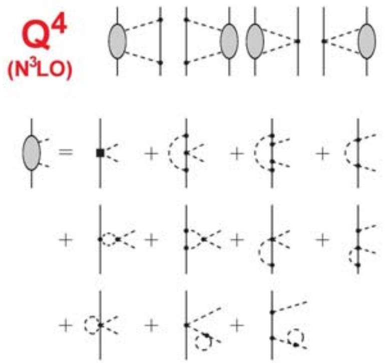
Pion-exchange contributions

--- Two-pion exchange ---

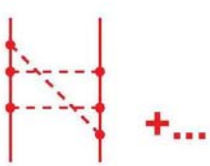
One-Loop



Two-Loop



--- Three-pion exchange ---



negligible
(Kaiser)

Contact contributions

$$
\begin{aligned}
V^{(4)}(\vec{p'},\vec{p}) = {} & D_1 q^4 + D_2 k^4 + D_3 q^2 k^2 + D_4 (\vec{q}\times\vec{k})^2 \\
& + \left(D_5 q^4 + D_6 k^4 + D_7 q^2 k^2 + D_8 (\vec{q}\times\vec{k})^2\right) \vec{\sigma}_1\cdot\vec{\sigma}_2 \\
& + \left(D_9 q^2 + D_{10} k^2\right) \left(-i\vec{S}\cdot(\vec{q}\times\vec{k})\right) \\
& + \left(D_{11} q^2 + D_{12} k^2\right) (\vec{\sigma}_1\cdot\vec{q})\,(\vec{\sigma}_2\cdot\vec{q}) \\
& + \left(D_{13} q^2 + D_{14} k^2\right) (\vec{\sigma}_1\cdot\vec{k})\,(\vec{\sigma}_2\cdot\vec{k}) \\
& + D_{15} \left(\vec{\sigma}_1\cdot(\vec{q}\times\vec{k})\ \vec{\sigma}_2\cdot(\vec{q}\times\vec{k})\right)
\end{aligned}
$$

15 parameters

Note

The Feynman diagrams shown define the NN-NN amplitude, calculated perturbatively. This is fine, e.g., for the calculation of NN scattering in peripheral partial waves.

However: The NN system has a bound state, the deuteron, and the NN scattering lengths in S-waves are large. This can be obtained only in a non-perturbative calculation.

What do we do about this?

Define a potential that is applied in the usual (non-perturbative) way in which we always use potentials; namely, insert the potential in a Schroedinger or Lippmann-Schwinger equation (to obtain a bound state, a scattering length, a scattering amplitude, …).

And how do we define this potential? As the sum of the perturbative diagrams.

Defining the potential

$$\bar{V}(\vec{p}\,', \vec{p}) \equiv \left\{ \begin{array}{c} \text{sum of irreducible} \\ \pi + 2\pi \text{ diagrams} \end{array} \right\} + \text{ contacts}$$

Define

$$V(\vec{p}\,', \vec{p}) \equiv \sqrt{\frac{M}{E_{p'}}}\, \bar{V}(\vec{p}\,', \vec{p})\, \sqrt{\frac{M}{E_p}}$$

and apply in Lippmann-Schwinger Equation

$$T(\vec{p}\,', \vec{p}) = V(\vec{p}\,', \vec{p}) + \int d^3p'' V(\vec{p}\,', \vec{p}\,'') \frac{M}{p^2 - p''^2 + i\epsilon} T(\vec{p}\,'', \vec{p})$$

Iteration of V in the LS Eq. (ladder-diagram loops) requires cutting V for high momenta, therefore

$$V(\vec{p}\,', \vec{p}) \longmapsto V(\vec{p}\,', \vec{p})\, e^{-(p'/\Lambda)^{2n}}\, e^{-(p/\Lambda)^{2n}}$$

$$\approx V(\vec{p}\,', \vec{p}) \left\{ 1 - \left[\left(\frac{p'}{\Lambda} \right)^{2n} + \left(\frac{p}{\Lambda} \right)^{2n} \right] + \ldots \right\}$$

$\Lambda = 0.5$ GeV

"Weinberg Power Counting"

Parameters

- pi-pi Lagrangian: $\mathcal{L}^{(2)}_{\pi\pi} = \frac{f_\pi^2}{4}\, \mathrm{Tr}\left[\partial_\mu U \partial^\mu U^\dagger + m_\pi^2 \left(U + U^\dagger\right)\right];$

  m_π and f_π are fixed ($f_\pi = 92.4$ MeV). No free parameters.

- pi-N Lagrangians: $\mathcal{L}^{(1)}_{\pi N} = \bar{N}\left[i\partial_0 - \frac{1}{4f_\pi^2}\boldsymbol{\tau}\cdot(\boldsymbol{\pi}\times\partial_0\boldsymbol{\pi}) - \frac{g_A}{2f_\pi}\boldsymbol{\tau}\cdot(\vec{\sigma}\cdot\vec{\nabla})\boldsymbol{\pi}\right]N$

  $g_A = 1.29$, no free parameters.

  $\mathcal{L}^{(2)}_{\pi N}$: 4 parameters.

  $\mathcal{L}^{(3)}_{\pi N}$: 4 parameters.

  In principal fixed by pi-N data; but cf. Table on next slide.

- N-N Lagrangian ("Contacts"): 2+7+15=24 essentially free parameters.

The free parameters are used to adjust the potential to the empirical NN phase shifts and data.

Parameters, cont'd
pi-N Lagrangian parameters

TABLE I. Low-energy constants applied in the N^{3}LO NN potential (column 'NN'). The c_i belong to the dimension-two πN Lagrangian and are in units of GeV^{-1}, while the $\bar{d}_i$ are associated with the dimension-three Lagrangian and are in units of GeV^{-2}. The column 'πN' shows values determined from πN data.

		NN	πN
$\mathcal{L}^{(2)}_{\pi N}$:	c_1	–0.81	-0.81 ± 0.15^a
	c_2	2.80	3.28 ± 0.23^b
	c_3	–3.20	-4.69 ± 1.34^a
	c_4	5.40	3.40 ± 0.04^a
$\mathcal{L}^{(3)}_{\pi N}$:	$\bar{d}_1 + \bar{d}_2$	3.06	3.06 ± 0.21^b
	$\bar{d}_3$	–3.27	-3.27 ± 0.73^b
	$\bar{d}_5$	0.45	0.45 ± 0.42^b
	$\bar{d}_{14} - \bar{d}_{15}$	–5.65	-5.65 ± 0.41^b

Parameters, cont'd.

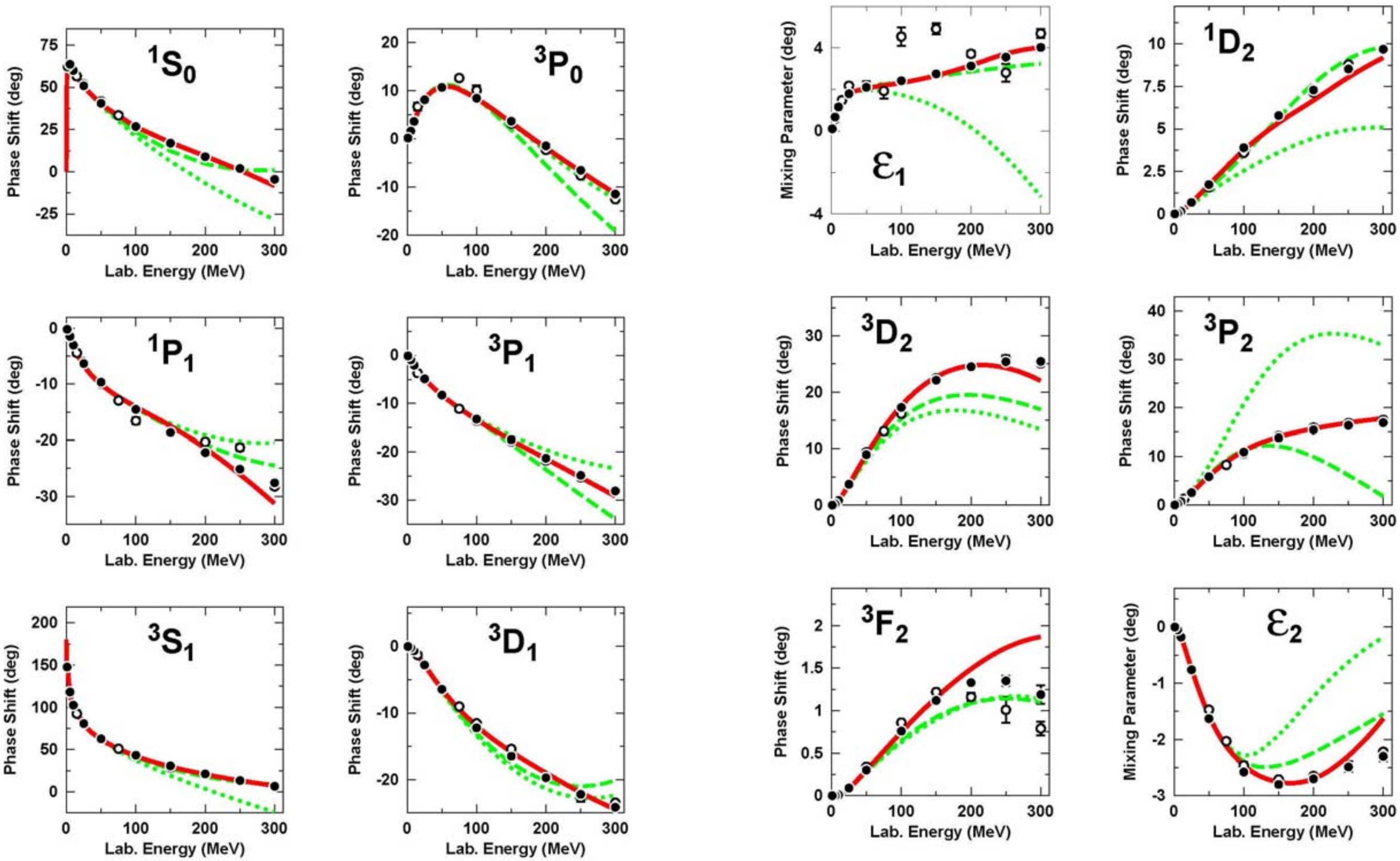
of contact parameters compared to phenomenological fits

NUMBER OF PARAMETERS
for the np potential

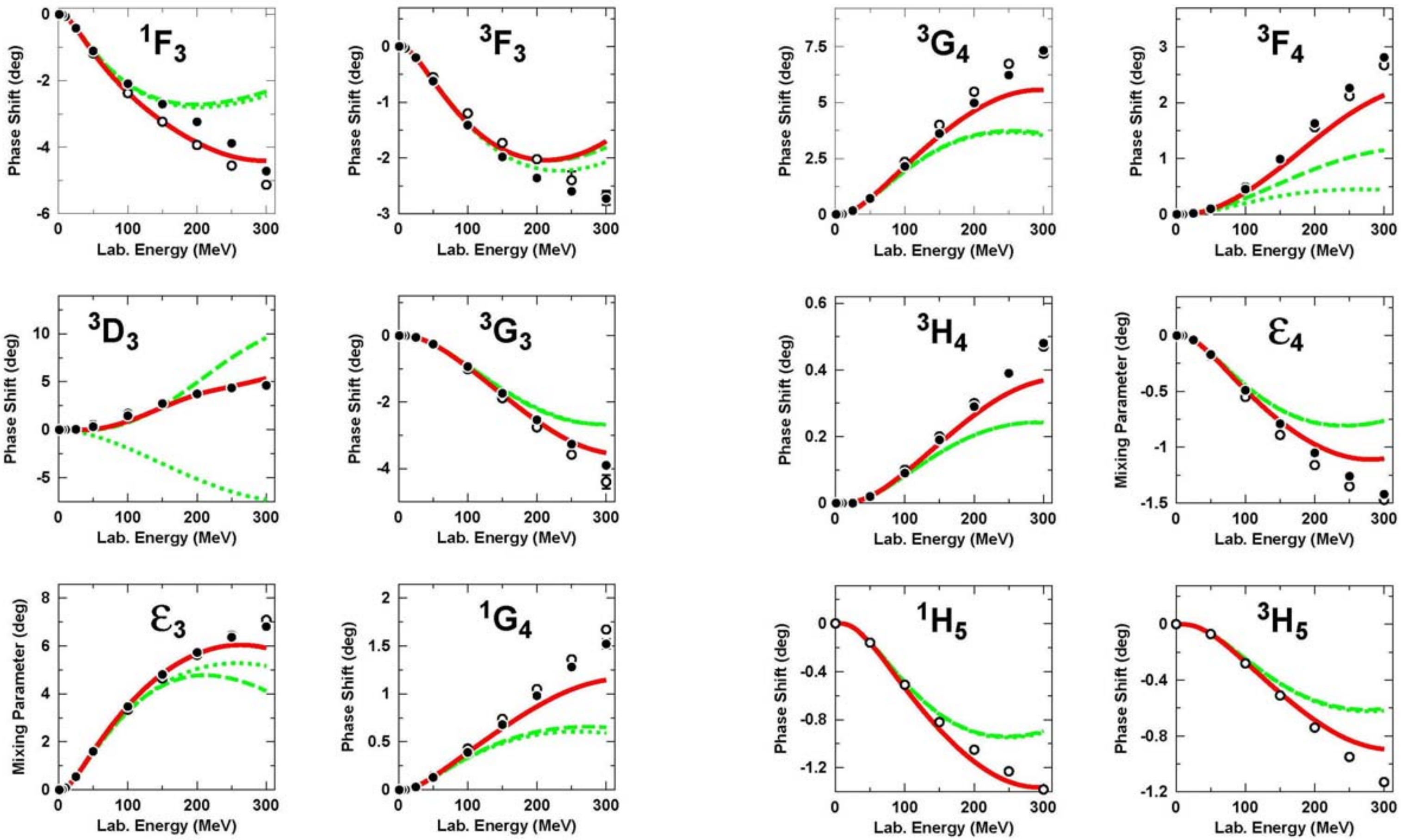
	Nijmegen PWA93	CD-Bonn "high precision"	NLO Q^2 (NNLO)	N^{3}LO Q^4
$^1S_0$	3	4	2	4
$^3S_1$	3	4	2	4
$^3S_1$-$^3D_1$	2	2	1	3
$^1P_1$	3	3	1	2
$^3P_0$	3	2	1	2
$^3P_1$	2	2	1	2
$^3P_2$	3	3	1	2
$^3P_2$-$^3F_2$	2	1	0	1
$^1D_2$	2	3	0	1
$^3D_1$	2	1	0	1
$^3D_2$	2	2	0	1
$^3D_3$	1	2	0	1
$^3D_3$-$^3G_3$	1	0	0	0
$^1F_3$	1	1	0	0
$^3F_2$	1	2	0	0
$^3F_3$	1	2	0	0
$^3F_4$	2	1	0	0
$^3F_4$-$^3H_4$	0	0	0	0
$^1G_4$	1	0	0	0
$^3G_3$	0	1	0	0
$^3G_4$	0	1	0	0
$^3G_5$	0	1	0	0
Total	35	38	9	24

Phase shifts up to 300 MeV

Red Line: N3LO Potential by Entem & Machleidt, PRC 68, 041001 (2003).
Green dashed line: N2LO Potential, and
green dotted line: NLO Potential
by Epelbaum et al., Eur. Phys. J. A15, 543 (2002).



Phase shifts, cont’d



χ^2/datum for the reproduction of the 1999 *np* database

Bin (MeV)	# of data	N^3LO	NNLO	NLO	AV18
0–100	1058	1.06	1.71	5.20	0.95
100–190	501	1.08	12.9	49.3	1.10
190–290	843	1.15	19.2	68.3	1.11
0–290	2402	1.10	10.1	36.2	1.04

N3LO Potential by Entem & Machleidt, PRC 68, 041001 (2003).
N2LO and NLO Potential by Epelbaum et al., Eur. Phys. J. A15, 543 (2002).

Deuteron Properties

	N3LO	CD-Bonn	AV18	Empirical
Binding energy B_d (MeV)	2.224575	2.224575	2.224575	2.224575(9)
Asymptotic S state A_S (fm$^{-1/2}$)	0.8843	0.8846	0.8850	0.8846(9)
Asymptotic D/S state η	0.0256	0.0256	0.0250	0.0256(4)
Matter radius r_d (fm)	1.978[a]	1.970[a]	1.971[a]	1.9754(9)
Quadrupole moment Q_d (fm^2)	0.285[b]	0.280[b]	0.280[b]	0.286(1)
D-state probability P_D (%)	4.51	4.85	5.76	

[a] With MEC and rel. corrections (Friar, Martorell & Sprung).

[b] Including MEC and rel. corrections in the amount of 0.010 fm^2 (Henning).

Charge-dependence:
There is a pp, np, and nn potential.

CHARGE DEPENDENCE

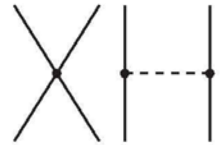
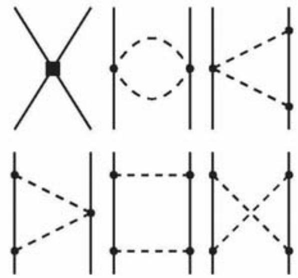
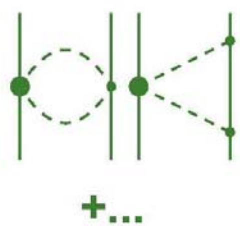
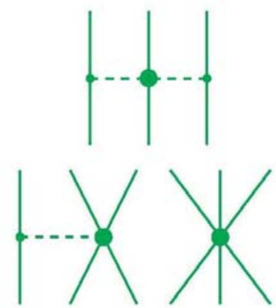
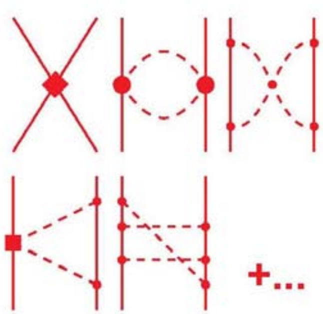
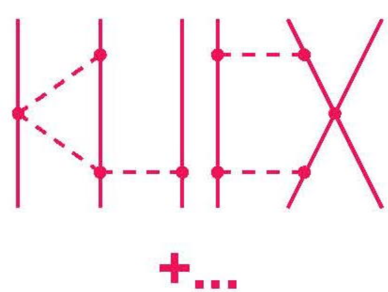
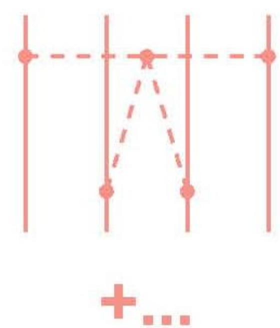
Classification[a]	Order	Contribution
LØ	αQ^{-2}	Δm_π in OPE
		Coulomb (pp)
NLØ	αQ^0	Δm_π in LO TPE
		LO $\pi\gamma$ exchange
		Q^0 CIB contact
		Q^0 CSB contact

[a] Using the notation of Walzl *et al.*.

TABLE IV. Scattering lengths (a) and effective ranges (r) in units of fm. (a^C_{pp} and r^C_{pp} refer to the pp parameters in the presence of the Coulomb force. a^N and r^N denote parameters determined from the nuclear force only and with all electromagnetic effects omitted.)

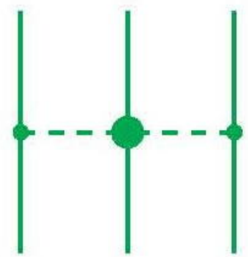
	N^{3}LO[a]	Experiment[b]
	$^1S_0$	
a^C_{pp}	–7.8188	–7.8196 ± 0.0026
r^C_{pp}	2.795	2.790 ± 0.014
a^N_{pp}	–17.083	
r^N_{pp}	2.876	
a^N_{nn}	–18.900	–18.9 ± 0.4
r^N_{nn}	2.838	2.75 ± 0.11
a_{np}	–23.732	–23.740 ± 0.020
r_{np}	2.725	2.77 ± 0.05
	$^3S_1$	
a_t	5.417	5.419 ± 0.007
r_t	1.752	1.753 ± 0.008

N3LO potential by Entem & Machleidt, PRC 68, 041001 (2003).

		2N forces	3N forces	4N forces
Leading Order	Q^0 LO			
Next-to Leading Order	Q^2 NLO			
Next-to-Next-to Leading Order	Q^3 N^2LO			
Next-to-Next-to-Next-to Leading Order	Q^4 N^3LO			

Three-nucleon forces at N2LO

TPE-3NF



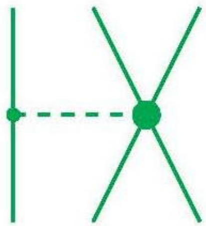
$$V^{3\mathrm{NF}}_{\mathrm{TPE}} = \left(\frac{g_A}{2f_\pi}\right)^2 \frac{1}{2} \sum_{i\neq j\neq k} \frac{(\vec{\sigma}_i \cdot \vec{q}_i)(\vec{\sigma}_j \cdot \vec{q}_j)}{(q_i^2+m_\pi^2)(q_j^2+m_\pi^2)} \, F^{\alpha\beta}_{ijk} \, \tau_i^\alpha \tau_j^\beta$$

with $\vec{q}_i \equiv \vec{p}_i{}' - \vec{p}_i$, where $\vec{p}_i$ and $\vec{p}_i{}'$ are the initial and final momenta of nucleon i, respectively, and

$$F^{\alpha\beta}_{ijk} = \delta^{\alpha\beta}\left[-\frac{4c_1 m_\pi^2}{f_\pi^2} + \frac{2c_3}{f_\pi^2}\,\vec{q}_i\cdot\vec{q}_j\right] + \frac{c_4}{f_\pi^2}\sum_\gamma \epsilon^{\alpha\beta\gamma}\,\tau_k^\gamma\,\vec{\sigma}_k\cdot[\vec{q}_i\times\vec{q}_j]$$

No new parameters!

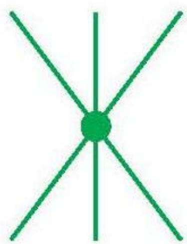
OPE-3NF



$$V^{3\mathrm{NF}}_{\mathrm{OPE}} = D\,\frac{g_A}{8f_\pi^2}\sum_{i\neq j\neq k}\frac{\vec{\sigma}_j\cdot\vec{q}_j}{q_j^2+m_\pi^2}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)(\vec{\sigma}_i\cdot\vec{q}_j)$$

One new parameter, D.

Contact-3NF



$$V^{3\mathrm{NF}}_{\mathrm{ct}} = E\,\frac{1}{2}\sum_{j\neq k}\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k\,.$$

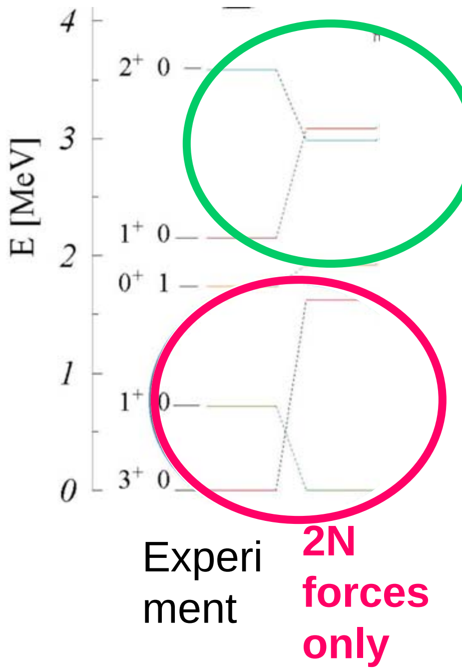
One new parameter, E.

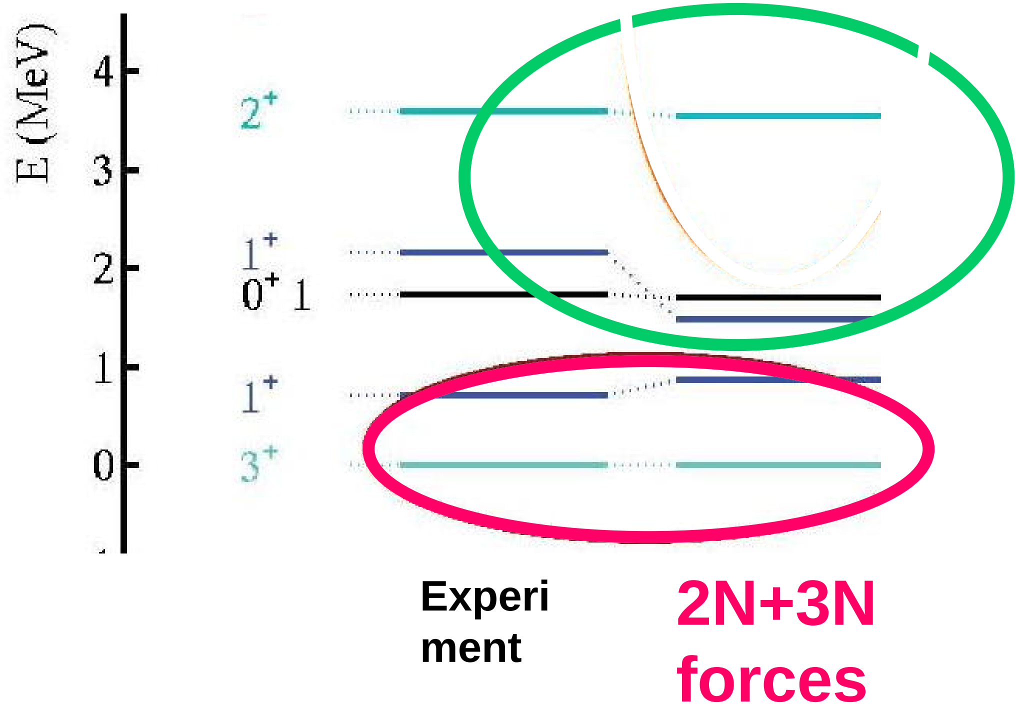
Strategy: Adjust D and E to two few-nucleon observables, e.g., the triton and alpha-particle binding energies. Then predict properties of other light nuclei.

Applications in microscopic nuclear structure: Calculating the properties of light nuclei using chiral 2N and 3N forces

$\boxed{^{10}B}$

"No-Core Shell Model " Calculations
by P. Navratil et al., LLNL

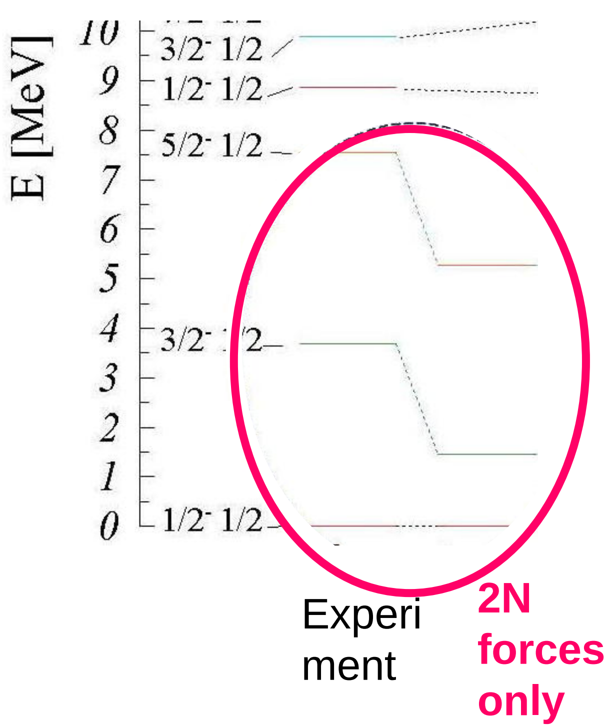




Microscopic nuclear structure, cont'd

^{13}C

"No-Core Shell Model " Calculations
by P. Navratil et al., LLNL



Conclusions

- After 70 years of struggle, we have now a proper theory for nuclear forces that is based upon the fundamental theory of strong interactions, QCD. It is chiral effective field theory in which the pion plays a fundamental role. Thus, the original Yukawa idea that there is ONE MESON that makes the nuclear force is re-instated, for low-energy nuclear physics.
- Based upon chiral perturbation theory (ChPT), quantitative NN potentials have been successfully developed and are available for nuclear structure applications.
- In ChPT, two- and many-body forces are generated on an equal footing. There is no arbitrariness anymore.
- **A new and exciting era has started for microscopic nuclear structure!**