New Era of Nuclear Physics in the Cosmos -the r-process nucleosynthesis, RIKEN, Sep. 25, 25, 2008
核物理から見た宇宙 -rプロセス研究の新時代に向かって 理研2008年9月25-26日

原子核質量公式とr-過程元素合成

Nuclear mass formulae and the r-process nucleosynthesis

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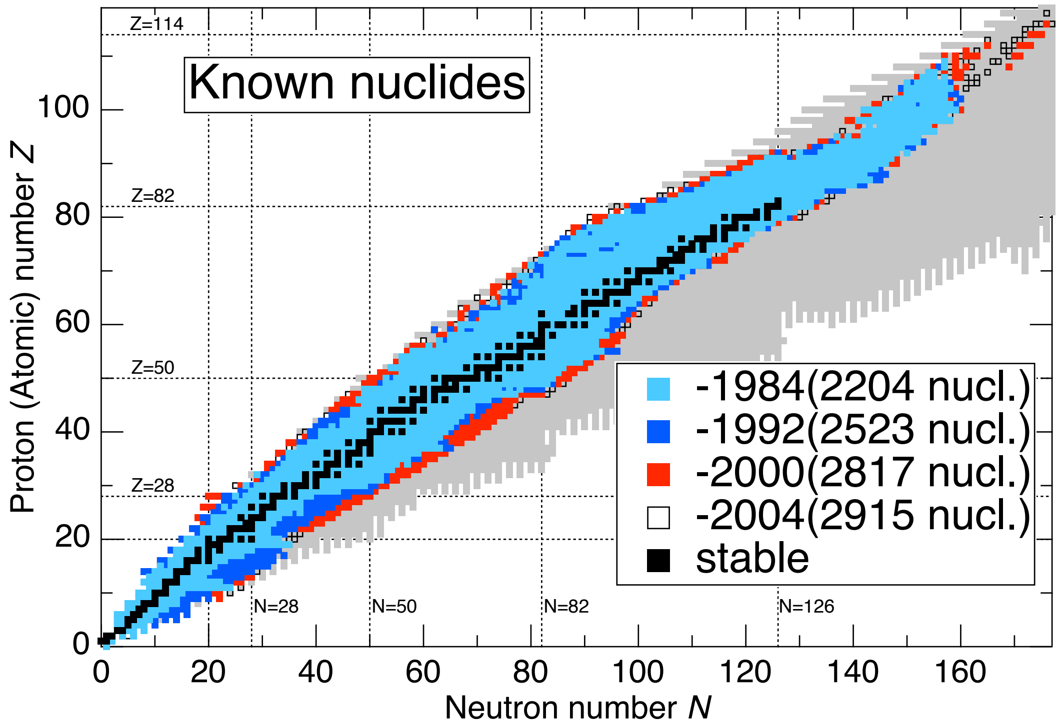
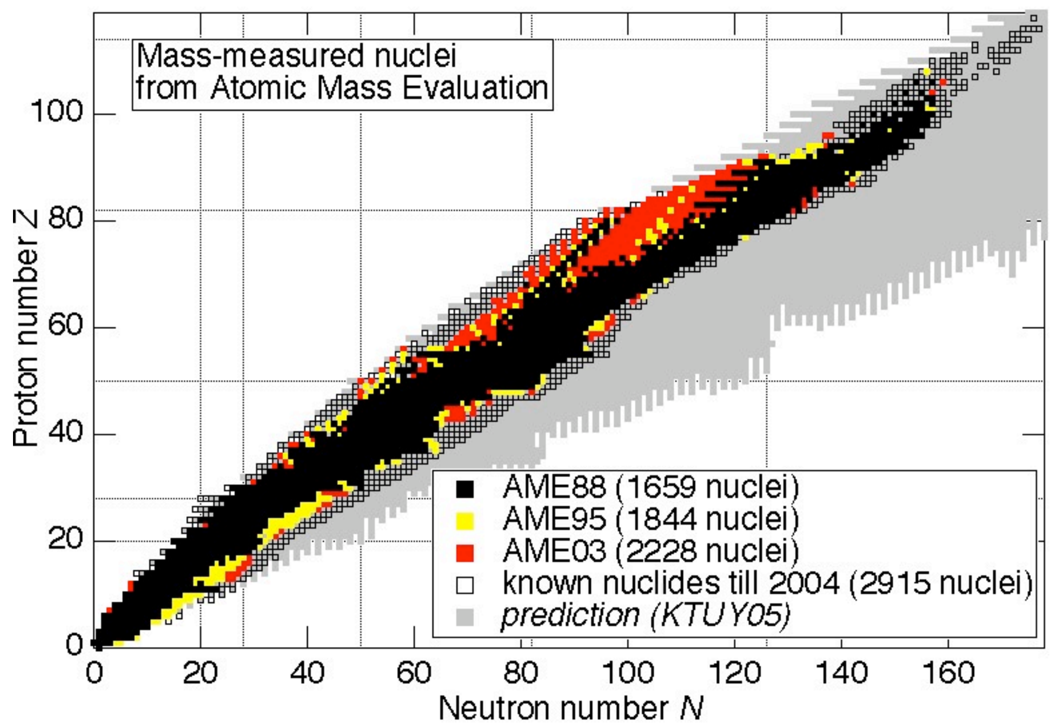
1. Bulk properties of nuclear mass formulae
2. r-process nucleosynthesis

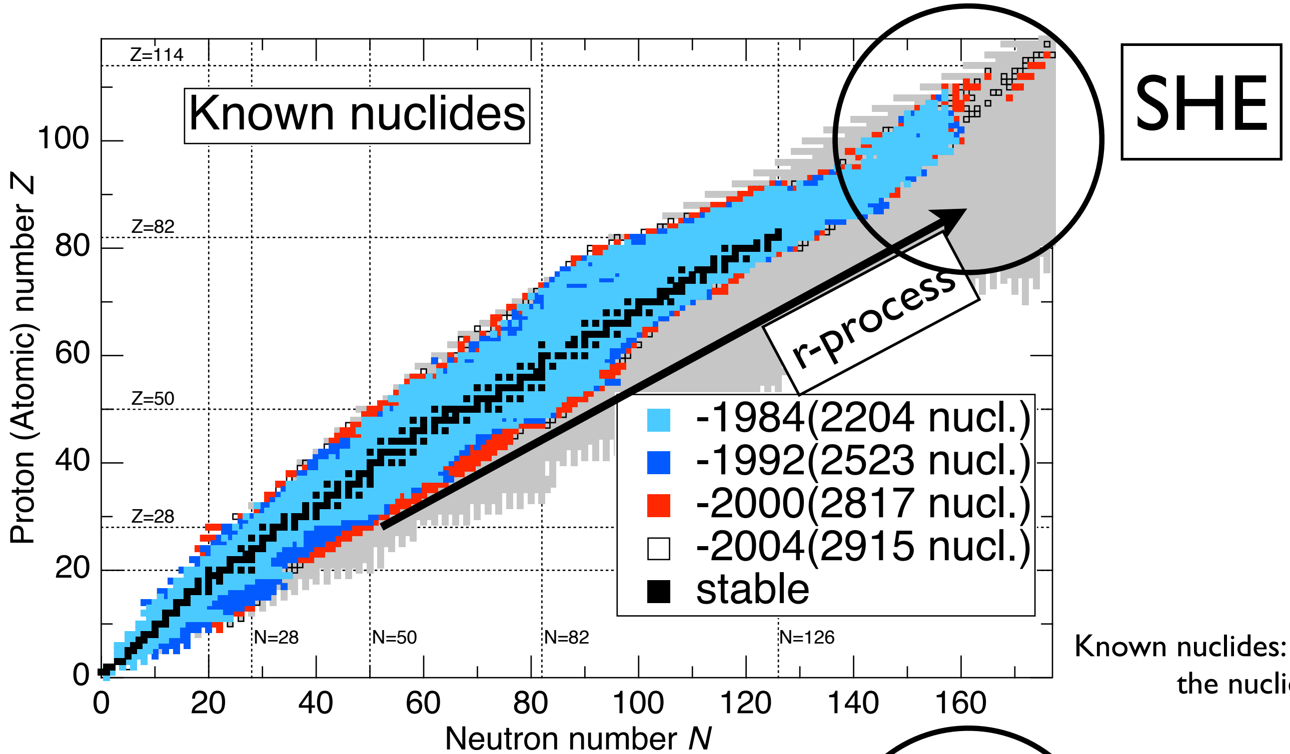
(3. Effect of β-delayed fission to r-process abundance)



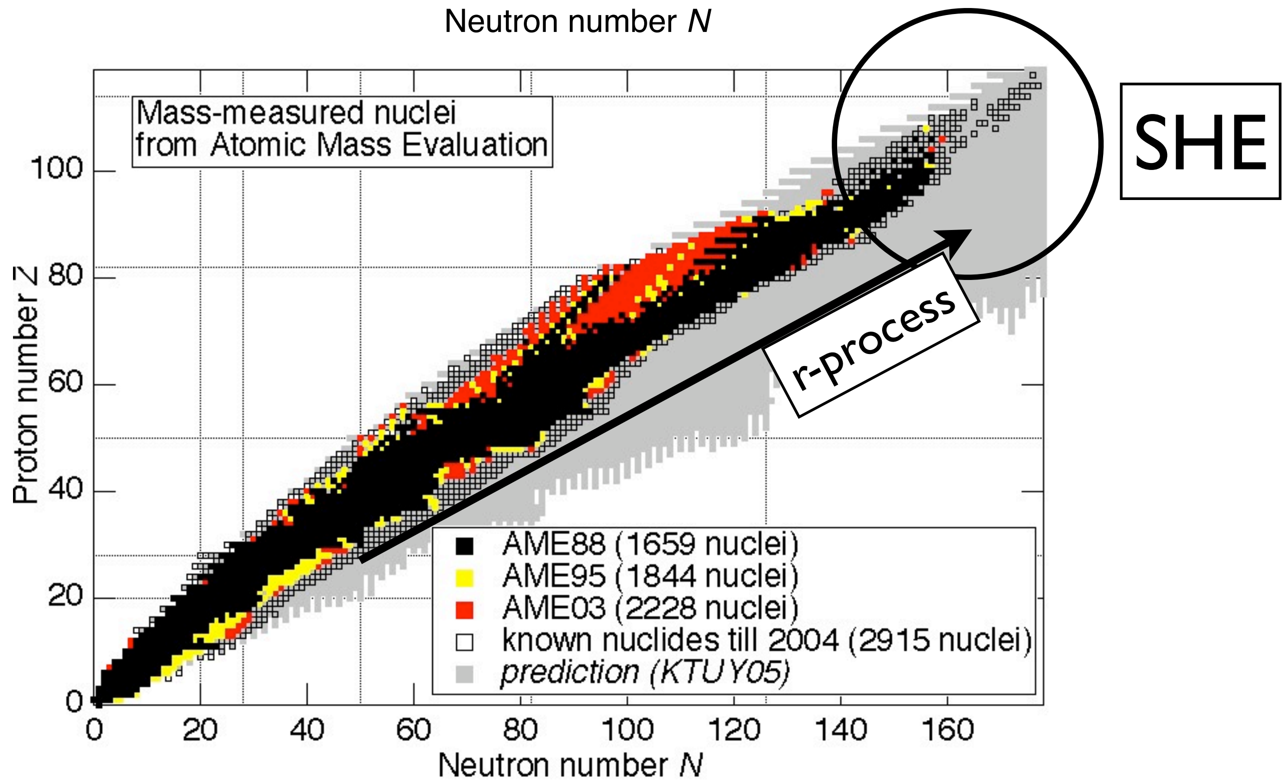
Known nuclides
Z=114
Z=82
Z=50
Z=28
N=28
N=50
N=82
N=126
-1984(2204 nucl.)
-1992(2523 nucl.)
-2000(2817 nucl.)
-2004(2915 nucl.)
stable
Proton (Atomic) number Z
Neutron number N



Mass-measured nuclei
from Atomic Mass Evaluation
AME88 (1659 nuclei)
AME95 (1844 nuclei)
AME03 (2228 nuclei)
known nuclides till 2004 (2915 nuclei)
prediction (KTUY05)
Proton number Z
Neutron number N



Known nuclides: taken from Chart of the nuclides by JAERI



Importance of nuclear masses for the r-process nucleosynthesis

Neutron-rich nuclei($A \gtrsim 80$)=>quite difficult to measure so far.

- Q_β, S_n, ...=>require to estimate λ_β, λn, ...
- Estimations of decay modes of α, β and SF =>important in the SHE region
- Nuclear structure (magicity, isomer)=>gives abundance pattern
- mass curvature=> reach 3rd peak (+U,Th) or not
- . . .

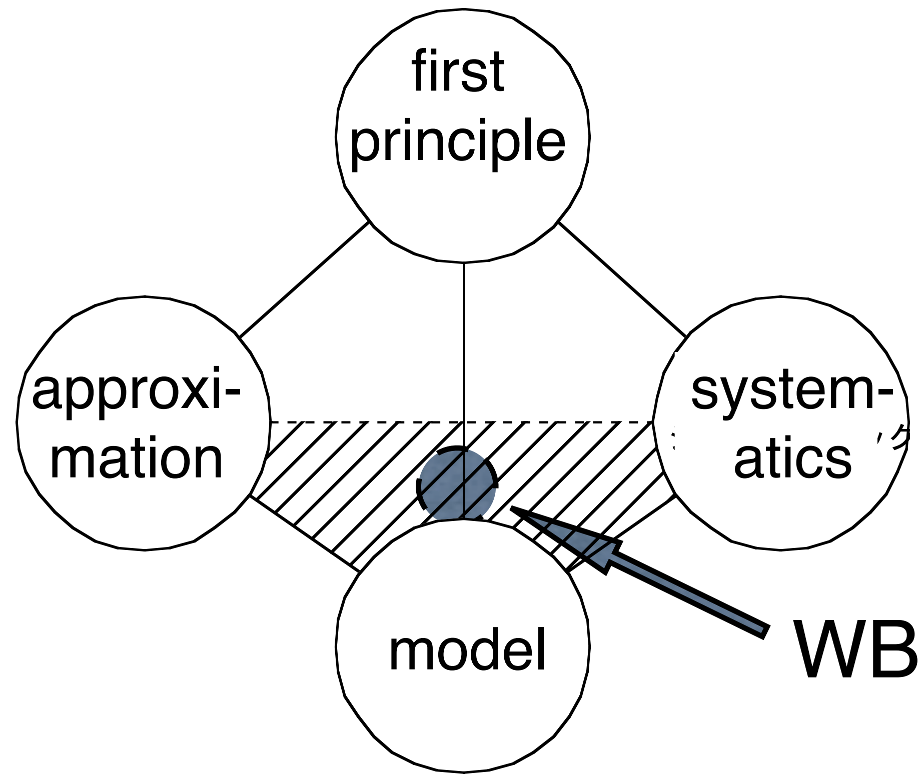
II. Phenomenological Mass Formulas

- First-principle calculation

(calculation from the realistic nuclear force)

To treat nuclear masses with the first principle is not appropriate way from consideration on properties of nuclear force, many-body force, computational time, and accuracy for the present and in the near future.

Theoretical approaches on nuclear masses



⇒ Nuclear masses are estimated from various viewpoints for various purposes.

Systematics・Phenomenology

- **Garvey-Kelson-like mass systematics**

  focusing on relation between mass values and Z, N

  Comay-Kelson-Zidon, Jänecke-Masson (1988)

- **Empirical shell term**

  focusing on Bulk part (**WB-like**)+deviation（**Shell term**）

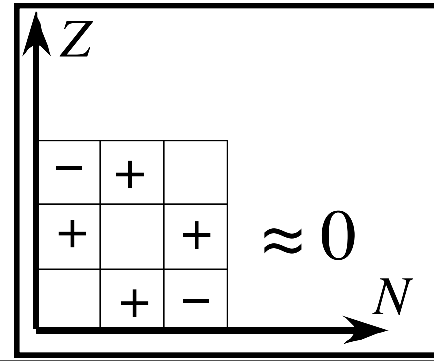
  <u>Tachibana-Uno-Yamada-Yamada</u> (1988)

- **Phenomenological shell model calculation**

  Polynomials of particle and hole numbers,
  obliged to assume magic numbers in advance.

  Liran-Zeldes (1976)

- **...**



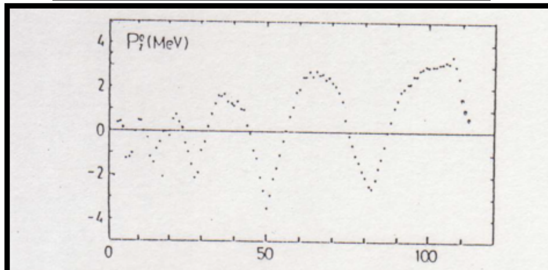
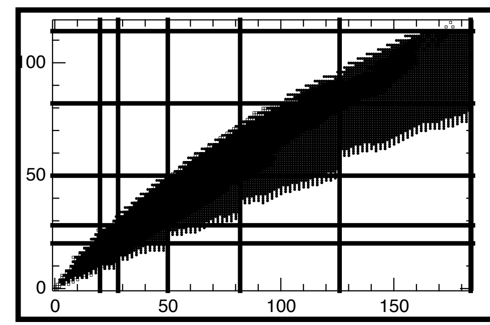


Figure 1. The proton shell parameters P^0_Z. The open circles are extrap
olated values.



Properties

・Good reproduction of masses for known nuclei + good prediction for unknown nuclei (quite) near mass-measured nuclides. (300-600 keV)

・<u>No predictable power for superheavy nuclei (next magic number, etc.)</u>

・<u>No deformation is obtained.</u>

Hilf-Groote-Takahashi(HGT,1976)

Macroscopic part: deformed liquid-drop (Myers-Swiatecki-type)

Microscopic part

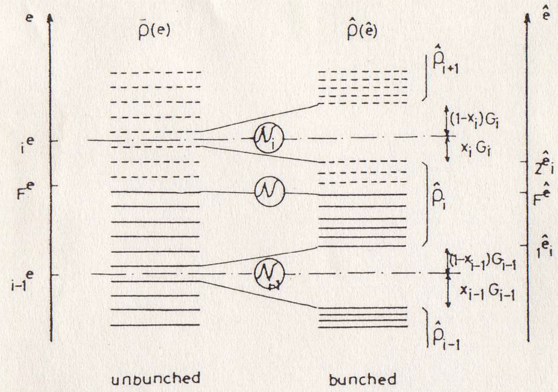


Fig.1 Schematic illustration of bunching procedure. $\mathcal{N}_i$ and $\mathcal{N}_{i-1}$ are the magic numbers sandwiching the total neutron or proton number, $\mathcal{N}$. The Fermi surface is denoted by $_F e$.

k	N_k	η_k	ξ_{k-1}
1	2	–	–
2(j_0)	8	0.0222	-0.0000
3	14	0.0772	0.0577
4	20	-0.0289	-0.0119
5	28	0.1177	0.0439
6	40	-0.0346	0.0322
7	50	0.1665	0.0510
8	82	0.0516	0.0540
9	126	0.0648	0.0452
10	184	0.0603	0.0478

k	Z_k	η_k	ξ_{k-1}
1	2	–	–
2(j_0)	8	-0.0050	0.0019
3	14	0.0588	0.0722
4	20	-0.0388	0.0467
5	28	0.0840	0.0293
6	40	0.0436	0.0573
7	50	0.1369	-0.0236
8	82	0.0405	0.0326
9	114	0.0438	0.0896

For the nucleus with $N_{i-1} < N \leq N_i$ with N_i and N_{i-1} being the neutron magic numbers, we have

$$\widetilde{M}_n(N,I) = C_n A^{-2/3}[(1-3\bar{\epsilon})(1+\bar{\delta})/(1+I)]^{2/3} \times$$
$$\times \left[\widehat{S}_i(N) + \sum_{j=j_0}^{i-1} \widehat{S}_j(N_j) + c_n\right]$$

where

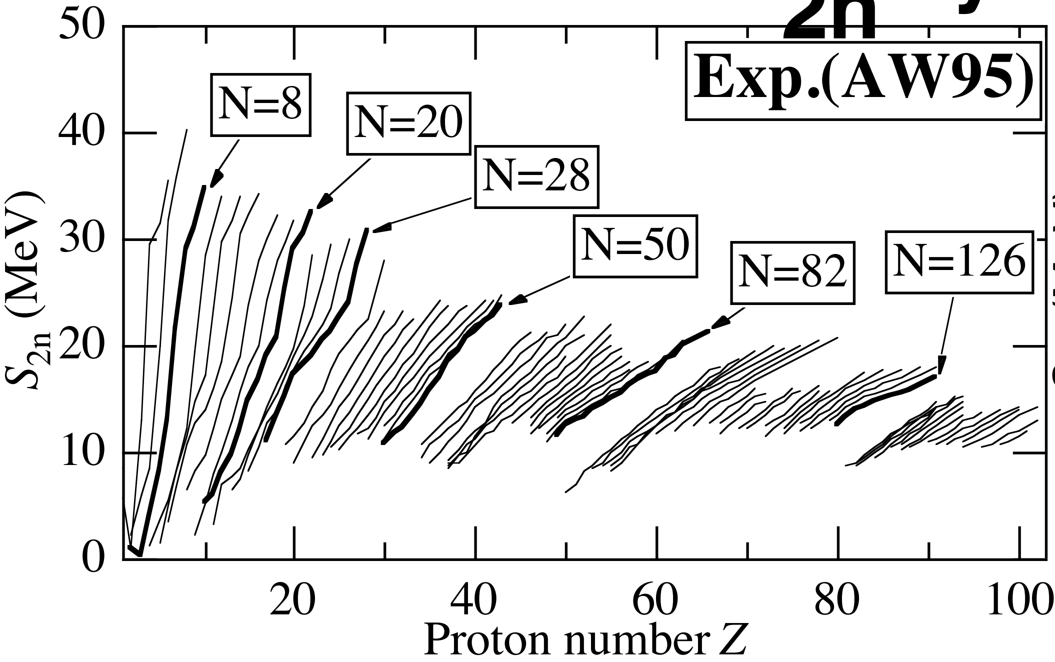
$$\widehat{S}_k(N) = \left[S_{k-1}^{(-)}(N)\cdot\xi_{k-1} - S_k^{(+)}(N)\cdot\eta_k\right]/(1-\xi_{k-1}-\eta_k)$$

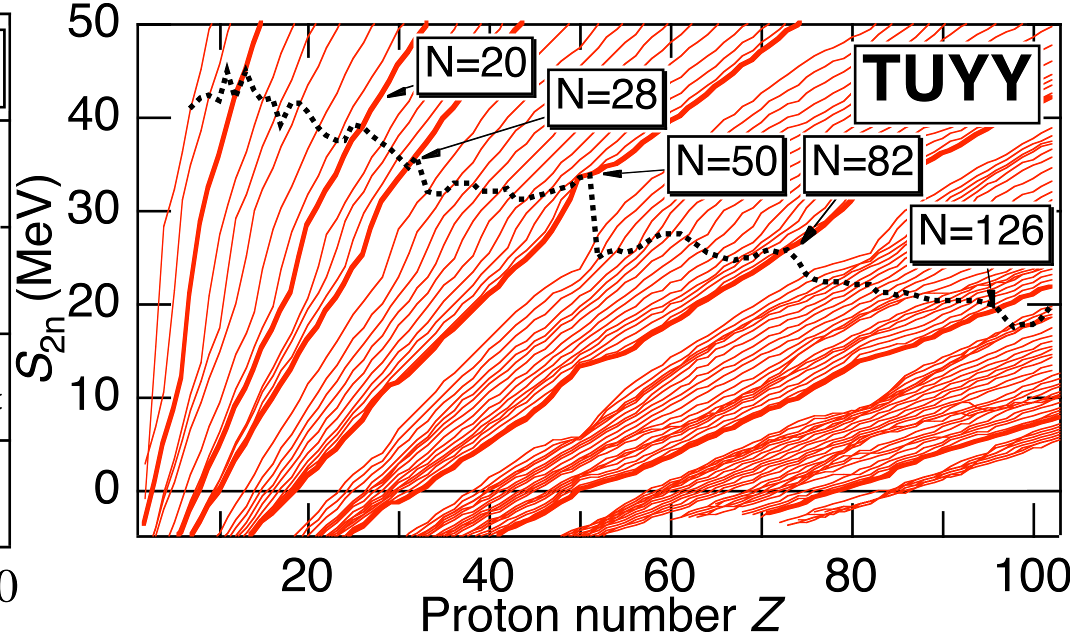
with

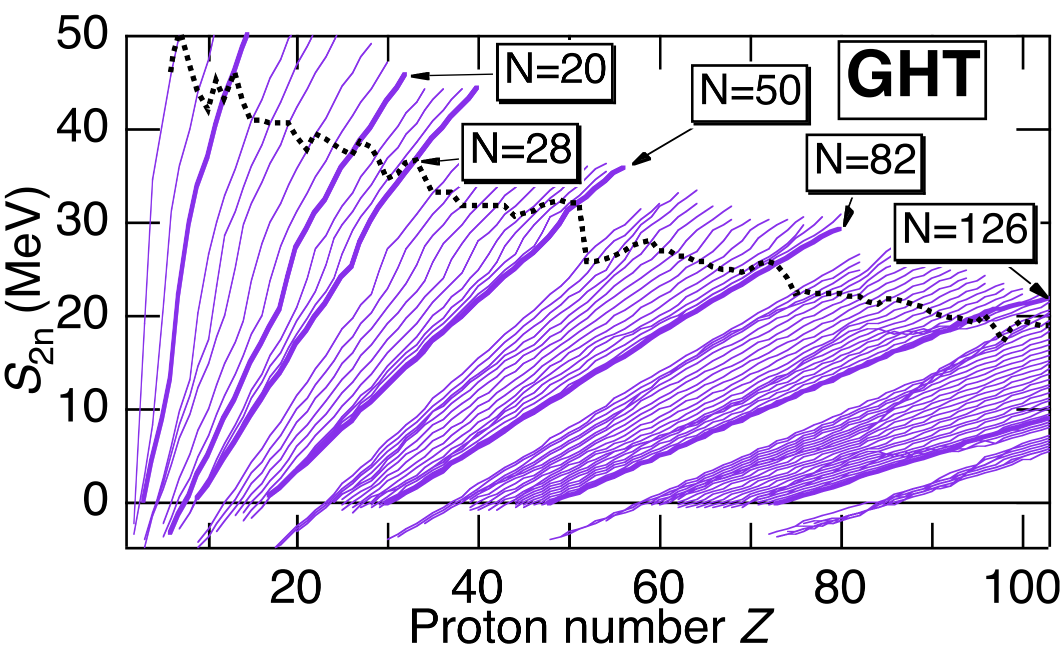
$$S_{k-1}^{(+)}(N) = \frac{2}{5}N^{2/3}(N-N_{k-1}) - \frac{3}{5}N_{k-1}(N^{2/3}-N_{k-1}^{2/3}),$$

$$S_k^{(-)}(N) = \frac{2}{5}N^{2/3}(N_k-N\quad) + \frac{3}{5}N_k(N^{2/3}-N_{k-1}^{2/3})$$
$$-\frac{2}{5}N_{k-1}^{2/3}(N_k-N_{k-1}).$$

S_{2n} systematics







Mass Model, Approximation

Recent mass formulas:

- are designed for nuclei **with Z, N=8 to $^{310}[126]_{184}$ or more**
- have the RMS dev. from exp. masses. **of 600-800 keV**
- give **deformation parameters** $\beta 2$, $\beta 4$... and **fission barriers**

- **Density functional theory** <- recent project

by M.Stoitsov, etc.

- **Hartree-Fock method with Skyrme force**

Strong short-range force => δ-function => HF calc.
ETFSI (1995), HFBCS (2001), HFB (2002-)

by S.Goriely

- **Liquid-drop model**

Deformed liquid-drop part+Micro. (folded Yukawa)
FRDM (1995), FRLDM (2002),

by P.Möller

- **Mass formula with spherical-basis shell term**

Phenom. gross (WB-like)+spherical-basis shell part
KUTY (2000), KTUY (2005) Koura, Uno, Tachibana, Yamada

micro (-like)

macro+ micro

phenom.

III. KTUY (KUTY) mass formula

(Koura, Uno, Tachibana, Yamada, NPA674, 2000)
(Koura, Tachibana, Uno, Yamada, PTP113, 2005)

$$M(Z, N) = M_{\mathrm{gross}}(Z, N) + M_{\mathrm{eo}}(Z, N) + M_{\mathrm{shell}}(Z, N)$$

$M_{\mathrm{gross}}(Z, N)$: gross term

$$M_{\mathrm{gross}}(Z,N) = M_{\mathrm{H}} Z + M_{\mathrm{n}} N + a(A) A + b(A) \left| N - Z \right| + c(A)(N-Z)^2 / A + E_{\mathrm{C}}(Z,N) - k_{\mathrm{el}} Z^{2.39}$$

$$\begin{cases} a(A) = a_1 + a_2 A^{-1/3} + a_3 A^{-2/3} + a_4 (A + \alpha_{\mathrm{a}})^{-1} & \longleftarrow \text{Volume, Surface, etc...} \\ b(A) = \quad + b_4 (A + \alpha_{\mathrm{b}})^{-1} & \longleftarrow \text{Wigner term} \\ c(A) = c_1 + c_2 A^{-1/3} + c_3 A^{-2/3} + c_4 (A + \alpha_{\mathrm{c}})^{-1} & \longleftarrow \text{Asymmetry term} \end{cases}$$

$$E_{\mathrm{C}} = a_{\mathrm{C}}(Z,N) \frac{Z^2}{A^{1/3}} \left[1 - 0.76 C_x \frac{1}{Z^{2/3}} \right] \quad \longleftarrow \text{Coulomb with an exchange term}$$

$M_{\mathrm{shell}}(Z, N)$: shell term

Spherical nuclei

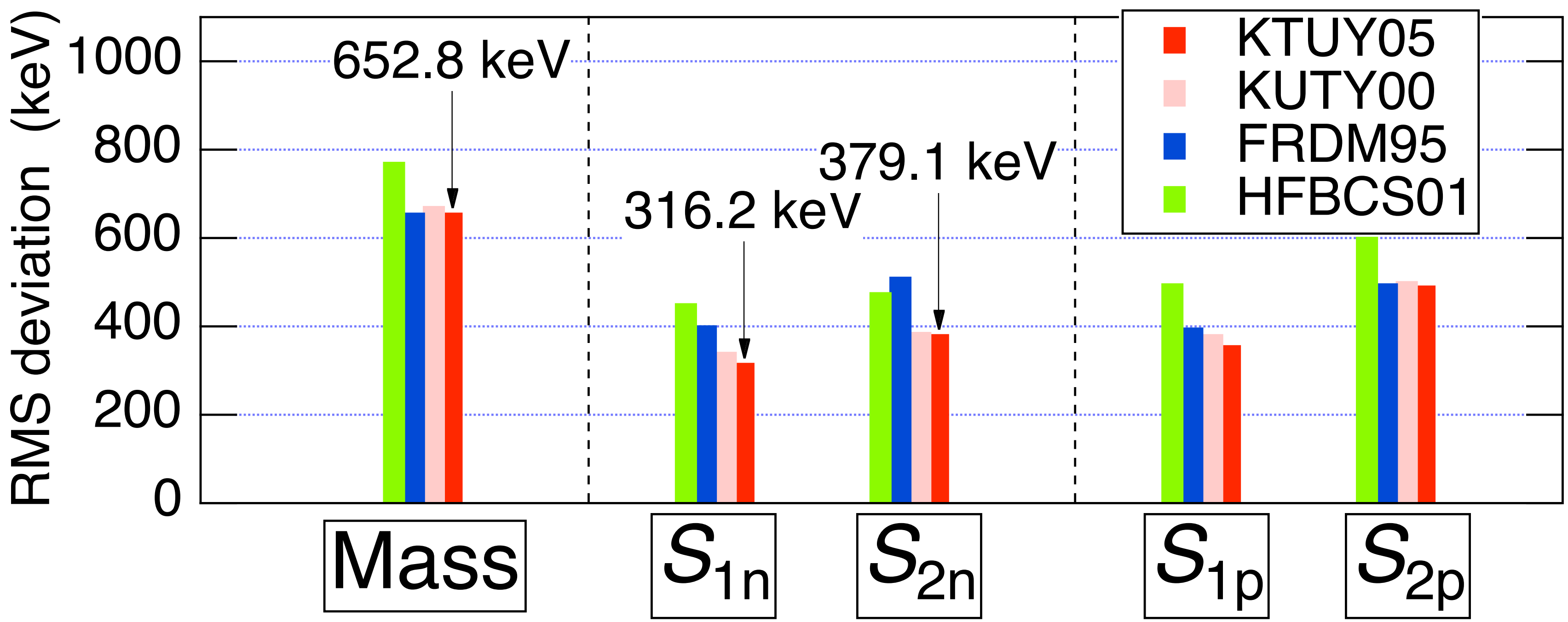
Calculated from **Spherical single-particle potential** for any nuclei (includes the BCS paring, reduction)

Deformed nuclei (Spherical-basis condieration)

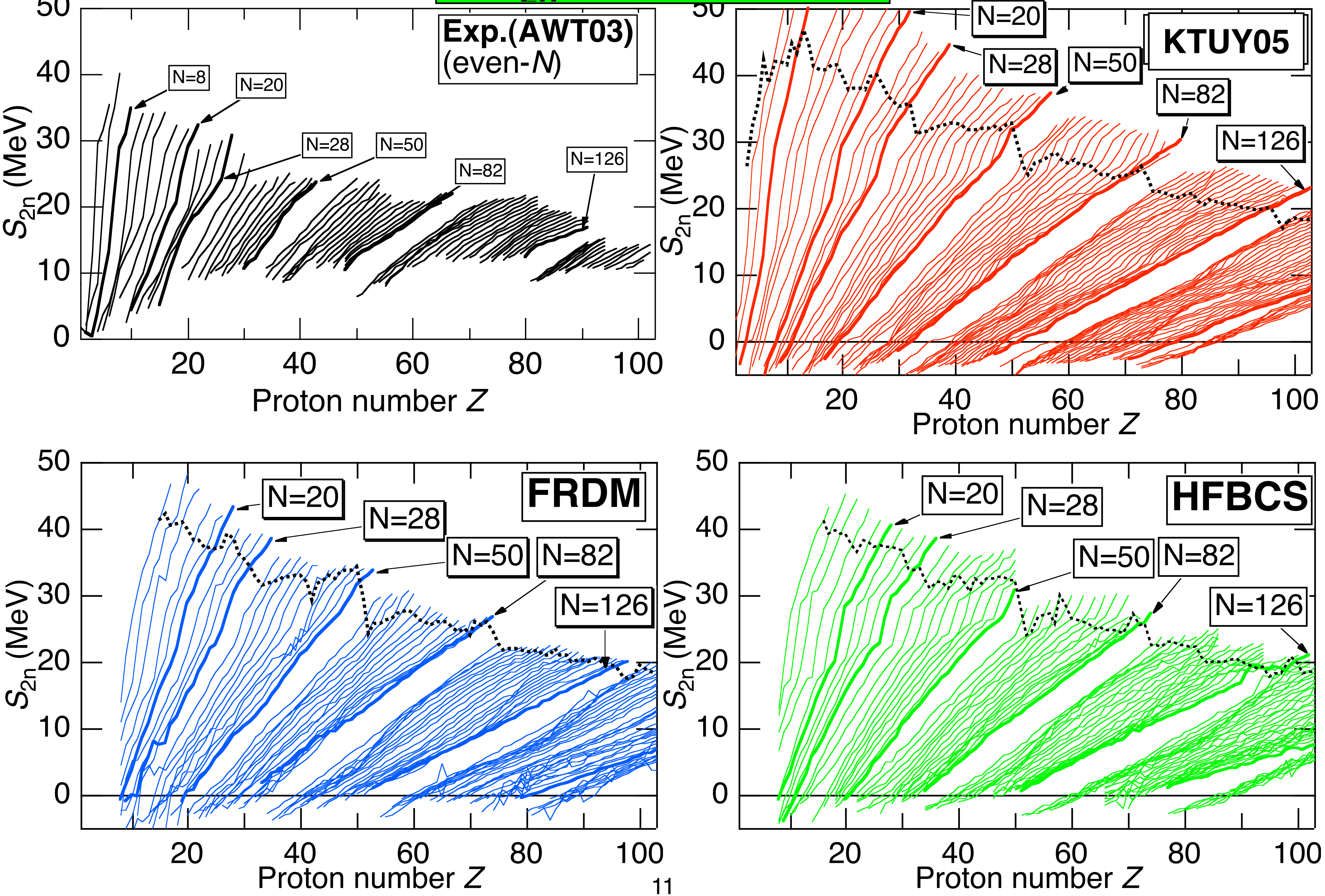
Obtained by **an appropriate mixture** of the above spherical shell energies + liquid-drop deform. energies

RMS dev. of masses and separation energies from Exp.

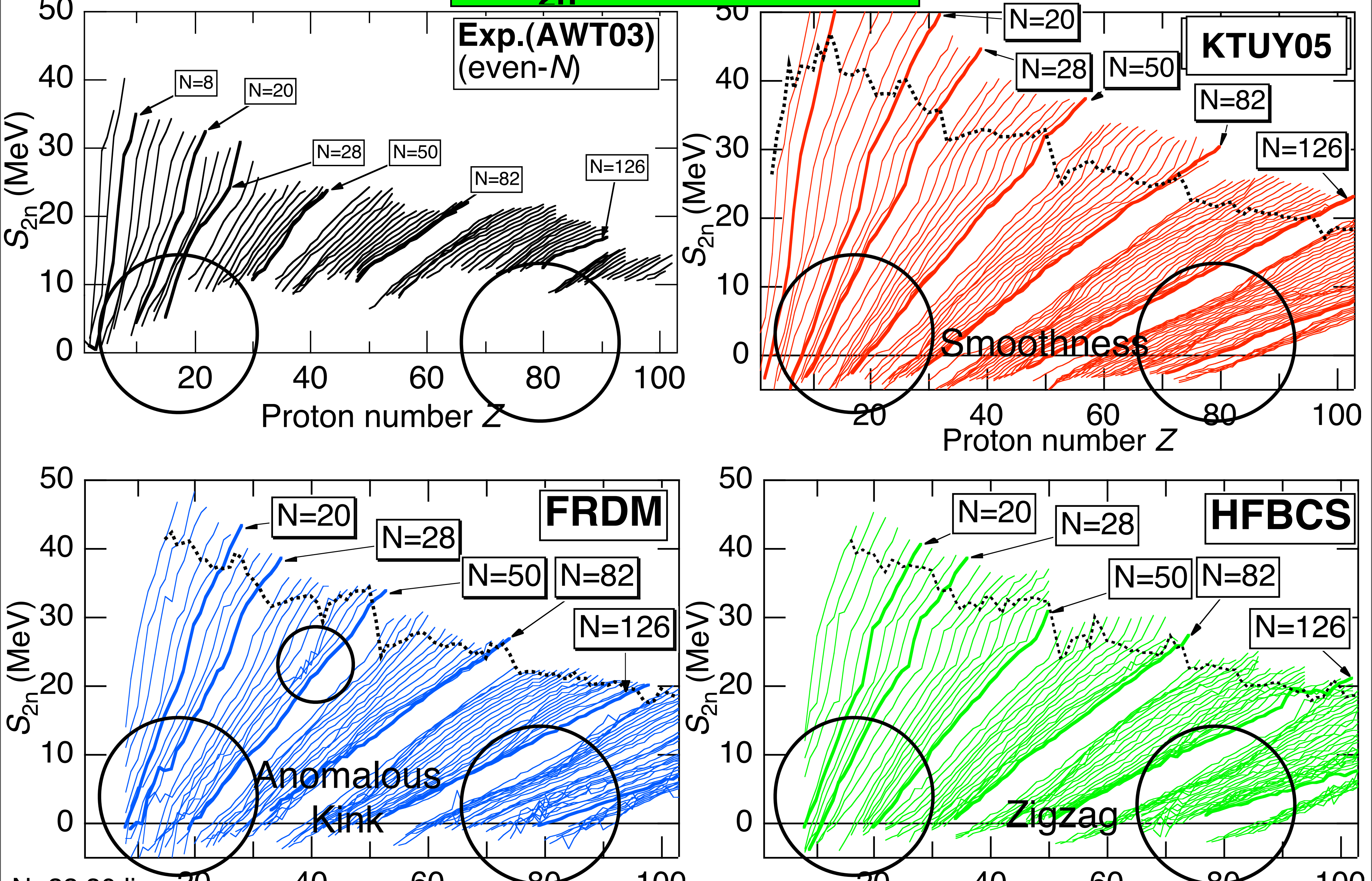
Mass formula	Masses	Neutron sep.		Proton sep.	
		S_n	S_{2n}	S_p	S_{2p}
Z, *N*≥2	(2219 nuclei)	(2054 nuclei)	(1997 nuclei)	(2016 nuclei)	(1897 nuclei)
KTUY03(05)	**666.7 keV**	**352.9 keV**	**442.0 keV**	**389.9 keV**	**532.2 keV**
KUTY(00)	**689.8 keV**	**389.1 keV**	**462.4 keV**	**473.6 keV**	**558.2 keV**
Z, *N*≥8	(2149 nuclei)	(1988 nuclei)	(1937 nuclei)	(1948 nuclei)	(1835 nuclei)
KTUY03(05)	**652.8 keV**	**316.2 keV**	**379.1 keV**	**353.0 keV**	**490.1 keV**
KUTY(00)	**670.7 keV**	**339.5 keV**	**386.5 keV**	**379.2 keV**	**499.7 keV**
FRDM(95)	655.5 keV	399.3 keV	511.7 keV	395.2 keV	493.6 keV
HFBCS-1(01)	770.7 keV	451.8 keV	477.3 keV	496.5 keV	603.8 keV



S_{2n} systematics



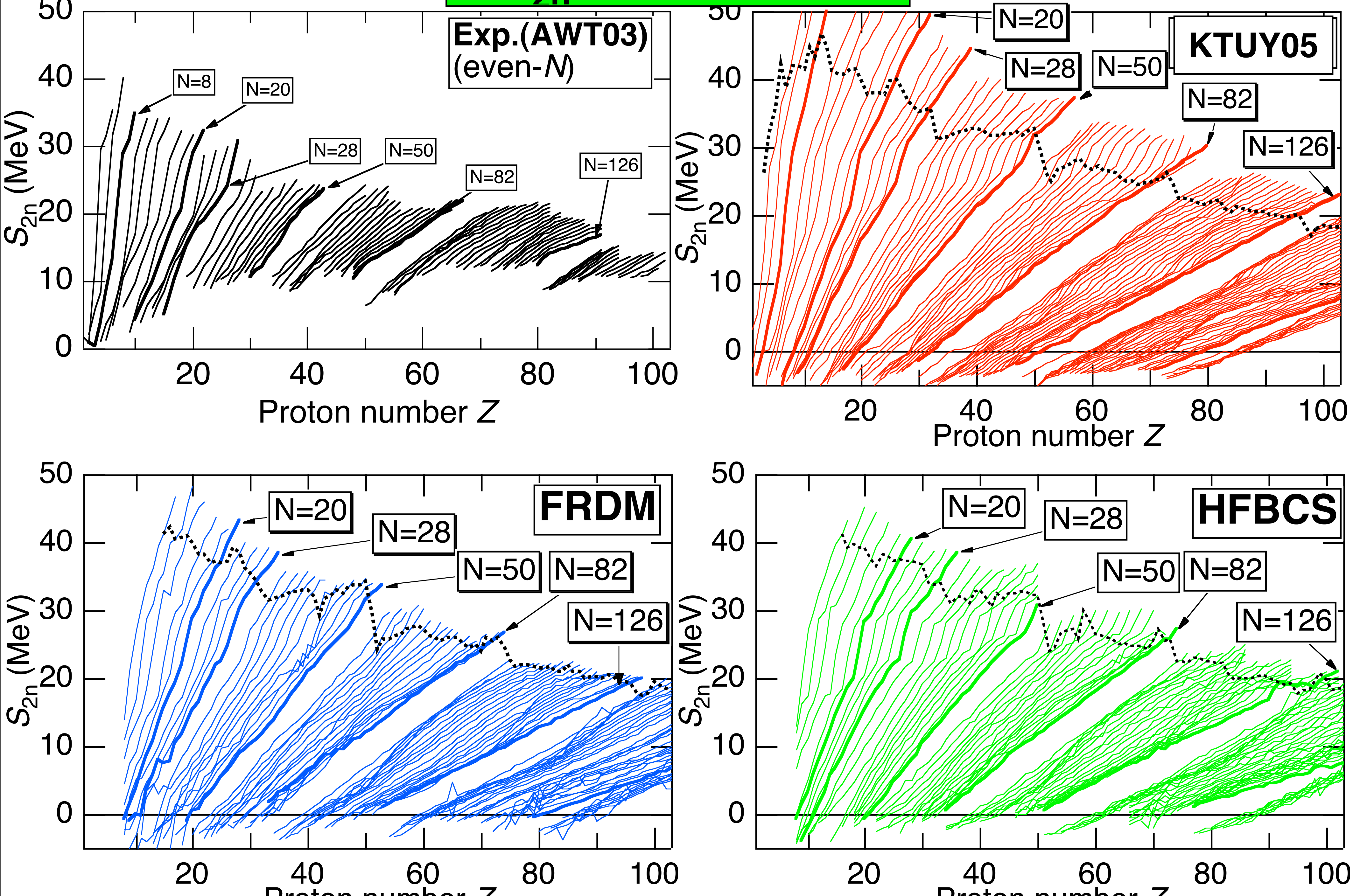
S_{2n} systematics



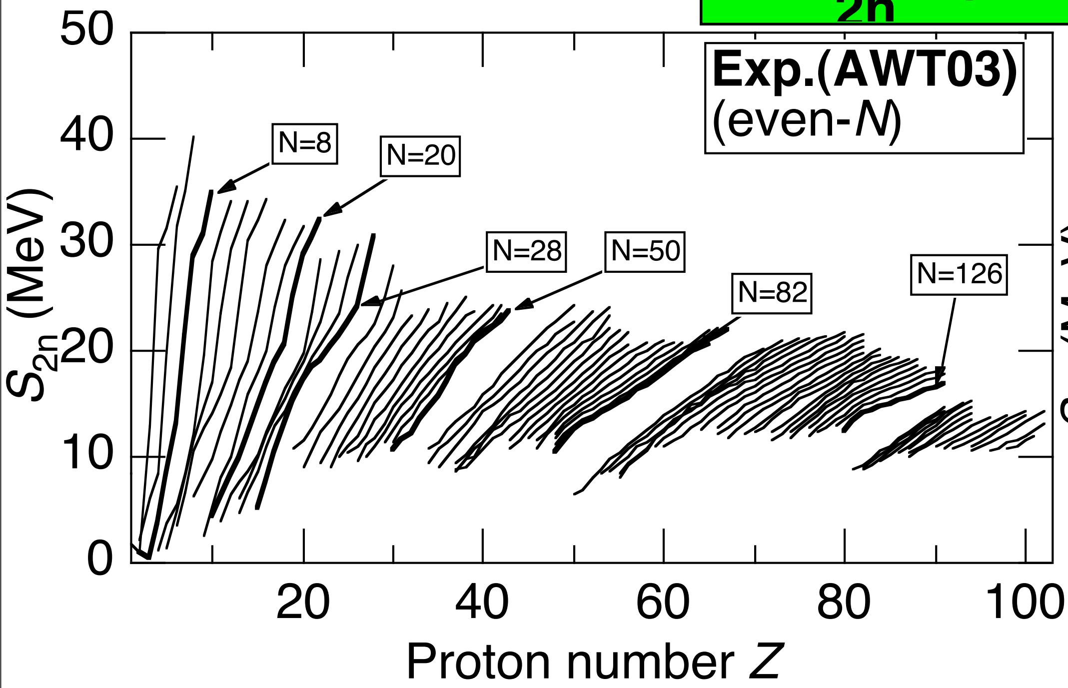
N=22,30 line
->crossing

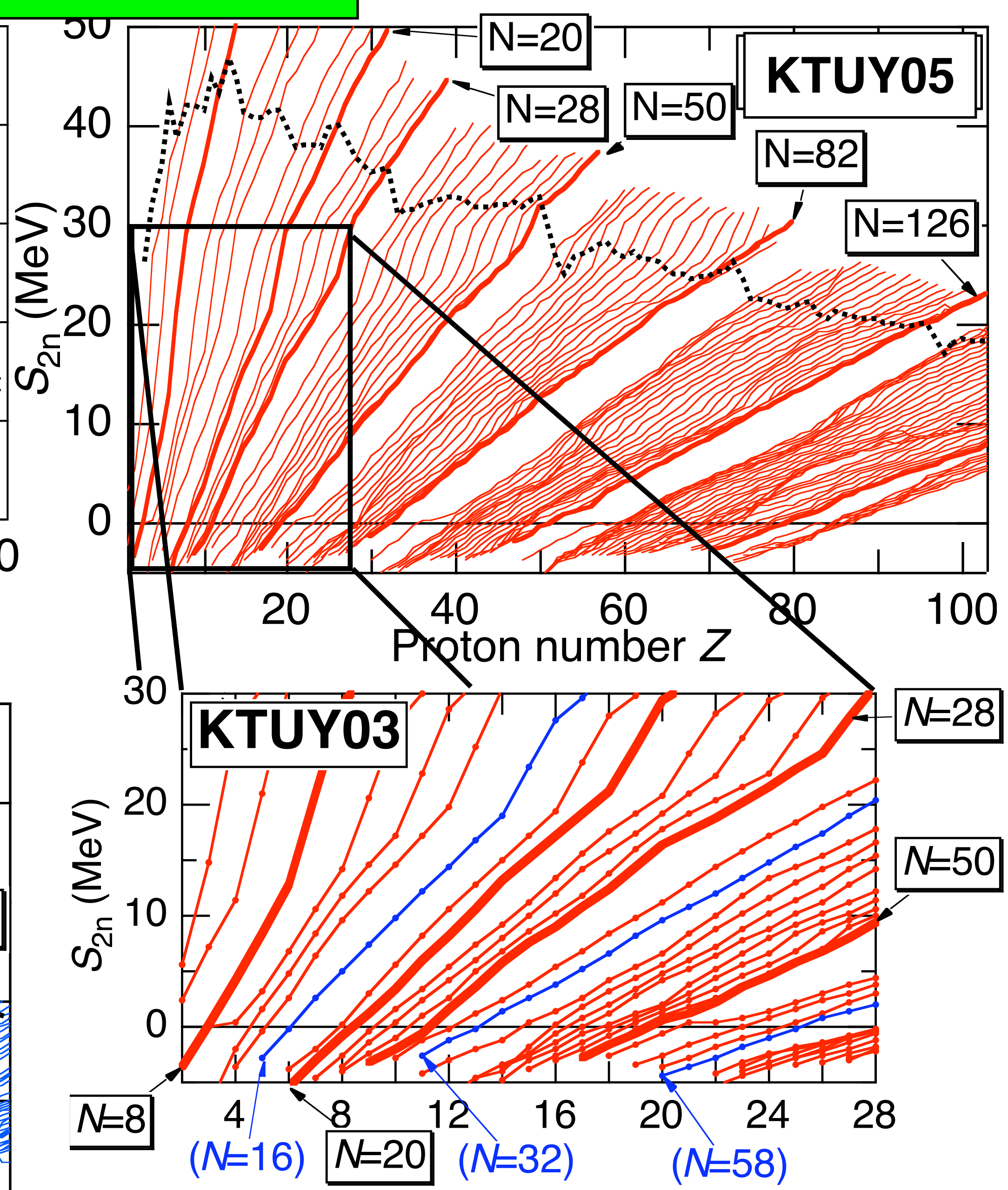
N=20,28 gap
->already weak

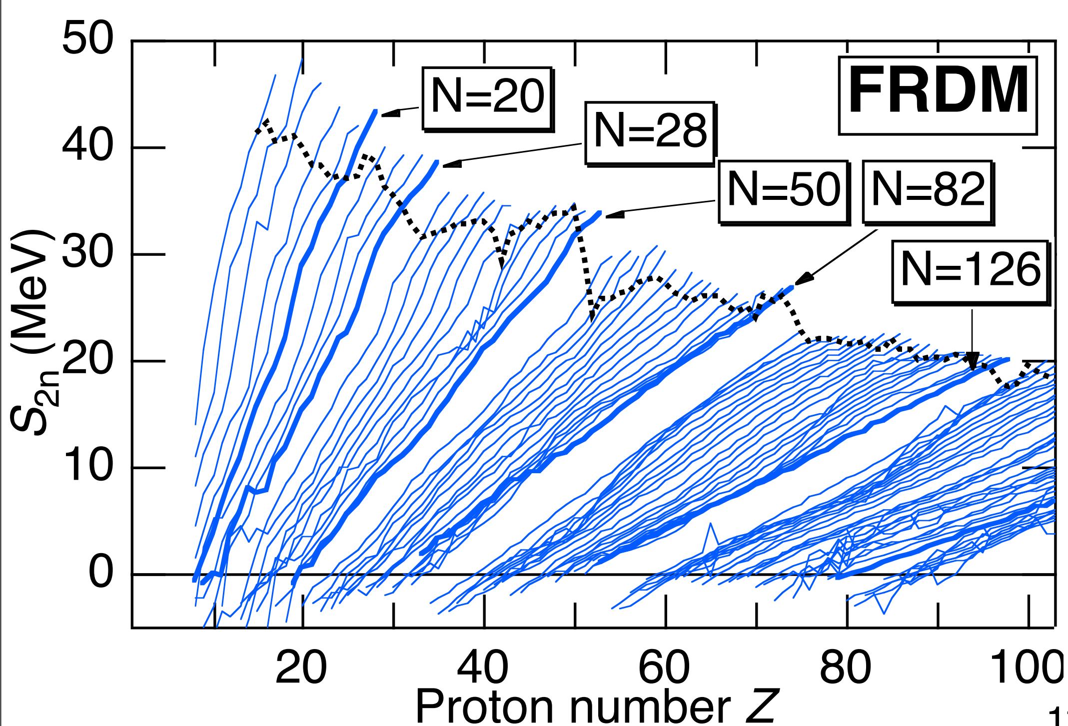
S_{2n} systematics



S_{2n} systematics

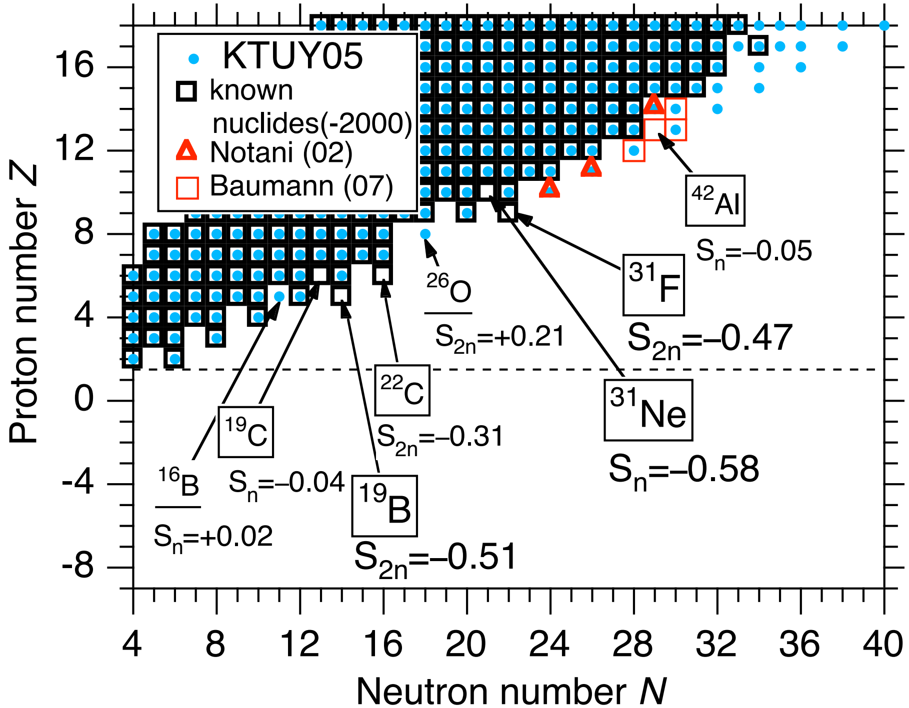






N=14,20,28,50 gap
->decreasing

Light neutron-drip line

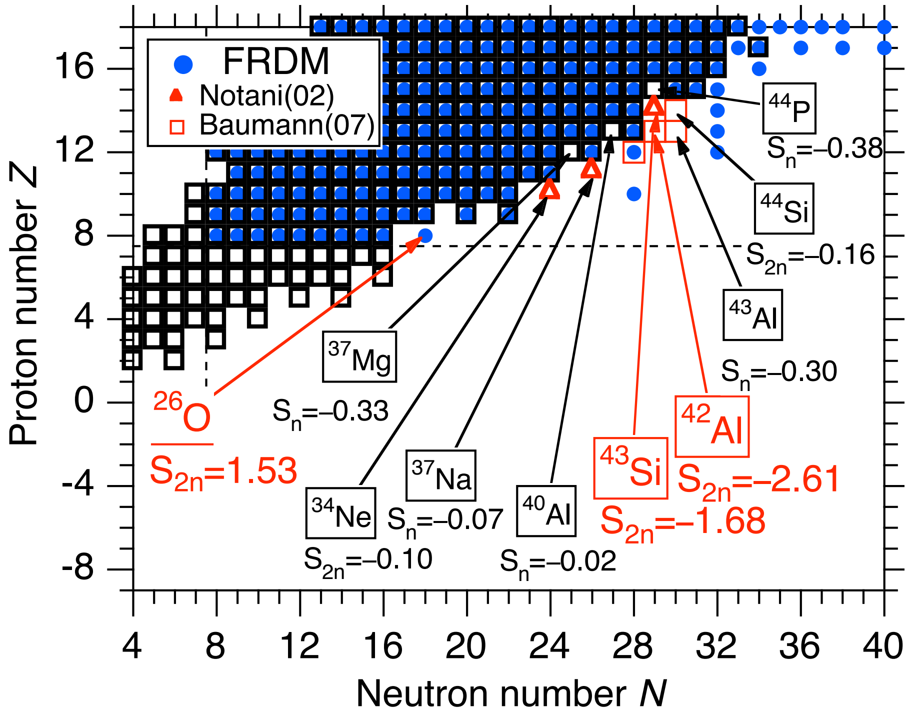


RMS dev. : 0.35MeV for S_n

0.44MeV for S_{2n}

Fail to predict 8 nuclei whether bound/unbound

All the n-sep. energy of these nuclei are within 1.5 times of RMS dev.



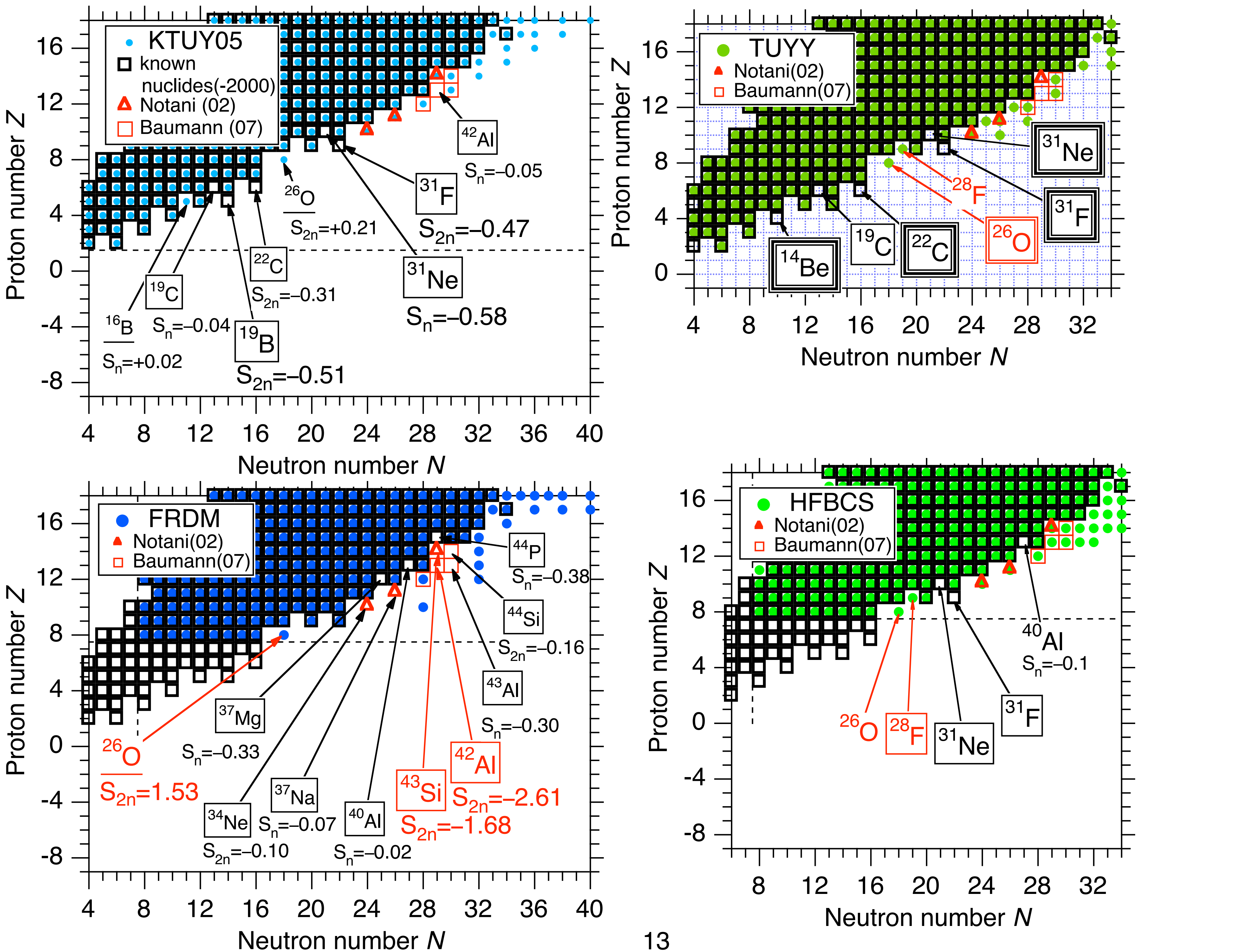
RMS dev. : 0.40MeV for S_n

0.51MeV for S_{2n}

Fail to predict 10 nuclei whether bound/unbound

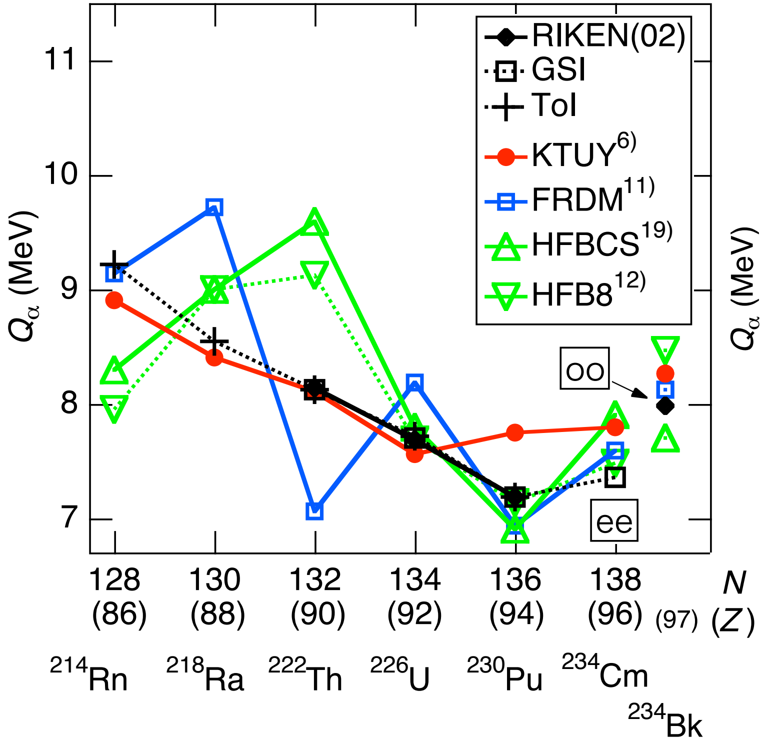
The n-sep. energy of the most nuclei considered are within 1.5 times of RMS dev, while those of three nuclei, ^{26}O, ^{43}Si, ^{42}Al, exceed 3.0 times of its RMS dev.

Light neutron-drip line



α-decay Q-values of heavy and superheavy nuclei

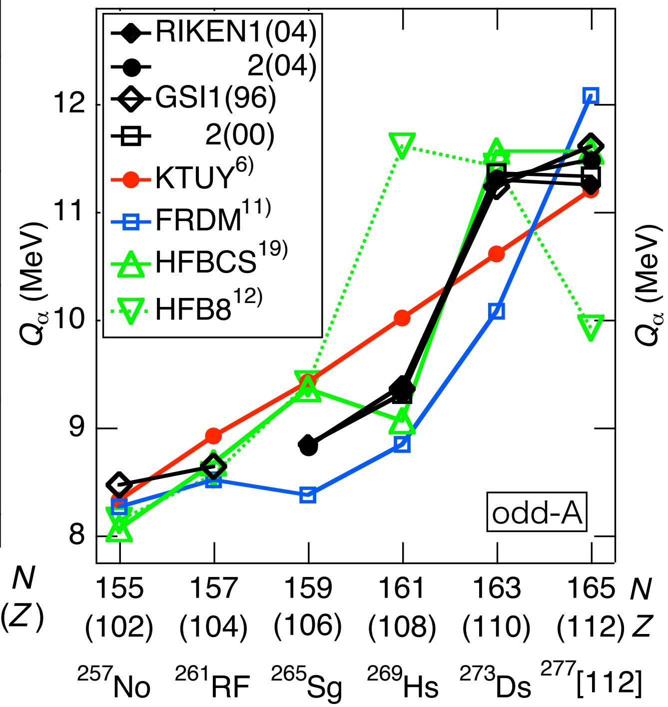
related to ^{234}Bk



Exp.: gradually decreasing to N=136

KTUY	O or △
FRDM	X
HFBCS	X
HFB8	X

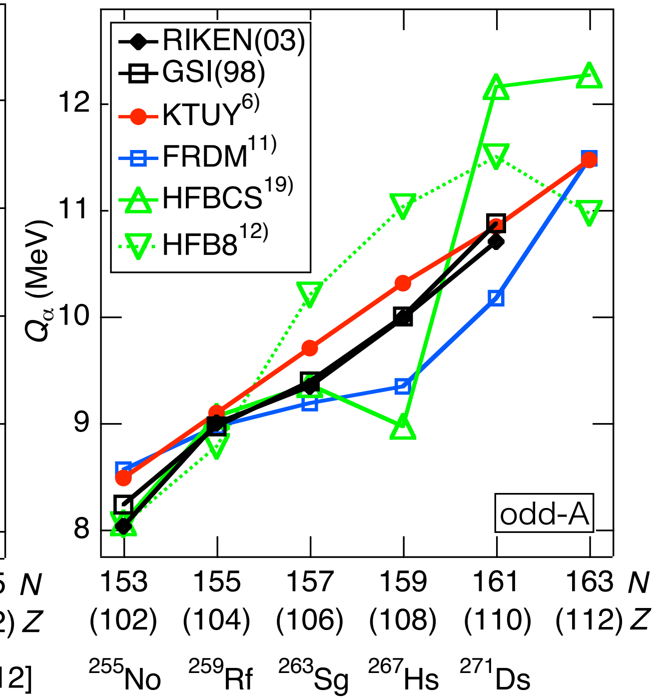
^{277}[112]



Exp.: steep increasing at N=162
suppressed at N=164

KTUY	X
FRDM	△
HFBCS	O
HFB8	X

^{271}[110] (^{275}[112])



Exp.: gradually increasing

KTUY	O
FRDM	△
HFBCS	X
HFB8	X or △

Black mark: exp. data

*Measured Q_α of odd-*A* nuclei is not surely represented the g.s.-to g.s. decay.

R-process nucleosynthesis

-Check the mass formulas as astrophysical data-

- **Canonical model**

Steady flow +Waiting point Approximation

Neutron-number density (N_n) and temperature (T_9) are constants

(n,γ)-(γ,n) equilibrium is established over an irradiation time τ

- **Saha equation** *(gives the r-process path)*

$$\log \frac{Y(Z,A+1)}{Y(Z,A)} = \log \frac{G(Z,A+1)}{G(Z,A)} - 34.075 + \log N_{\rm n} + 1.5 \log \left(\frac{A}{A+1} T_9 \right) + \frac{5.04}{T_9} S_{\rm n}(Z,A+1)$$

⇩

$$S_{2\rm n}(Z,A+2)/2 \leq S_{\rm a}^0 \equiv (34.075 - \log N_{\rm n} + 1.5 \log T_9) T_9/5.04 \leq S_{2\rm n}(Z,A)/2$$

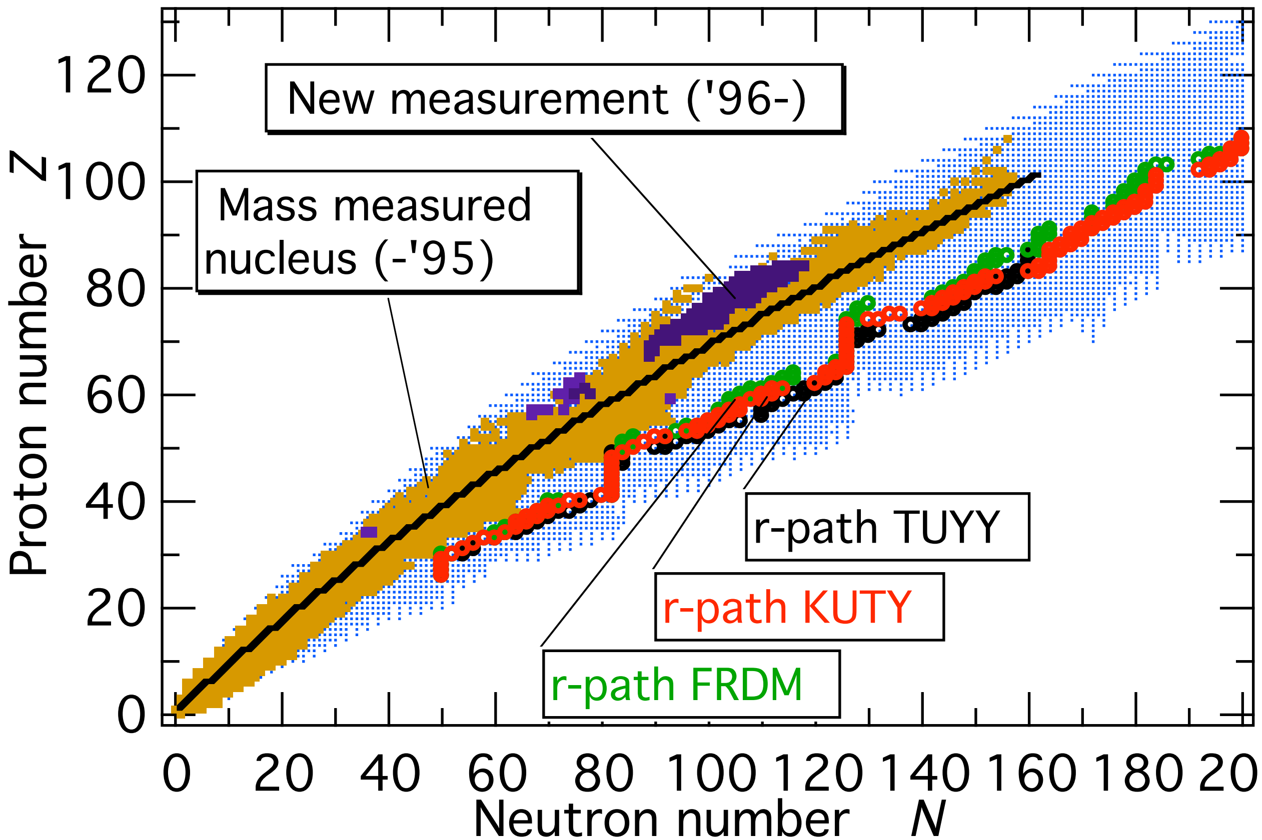
S_{2n}: 2-neutron separation energy

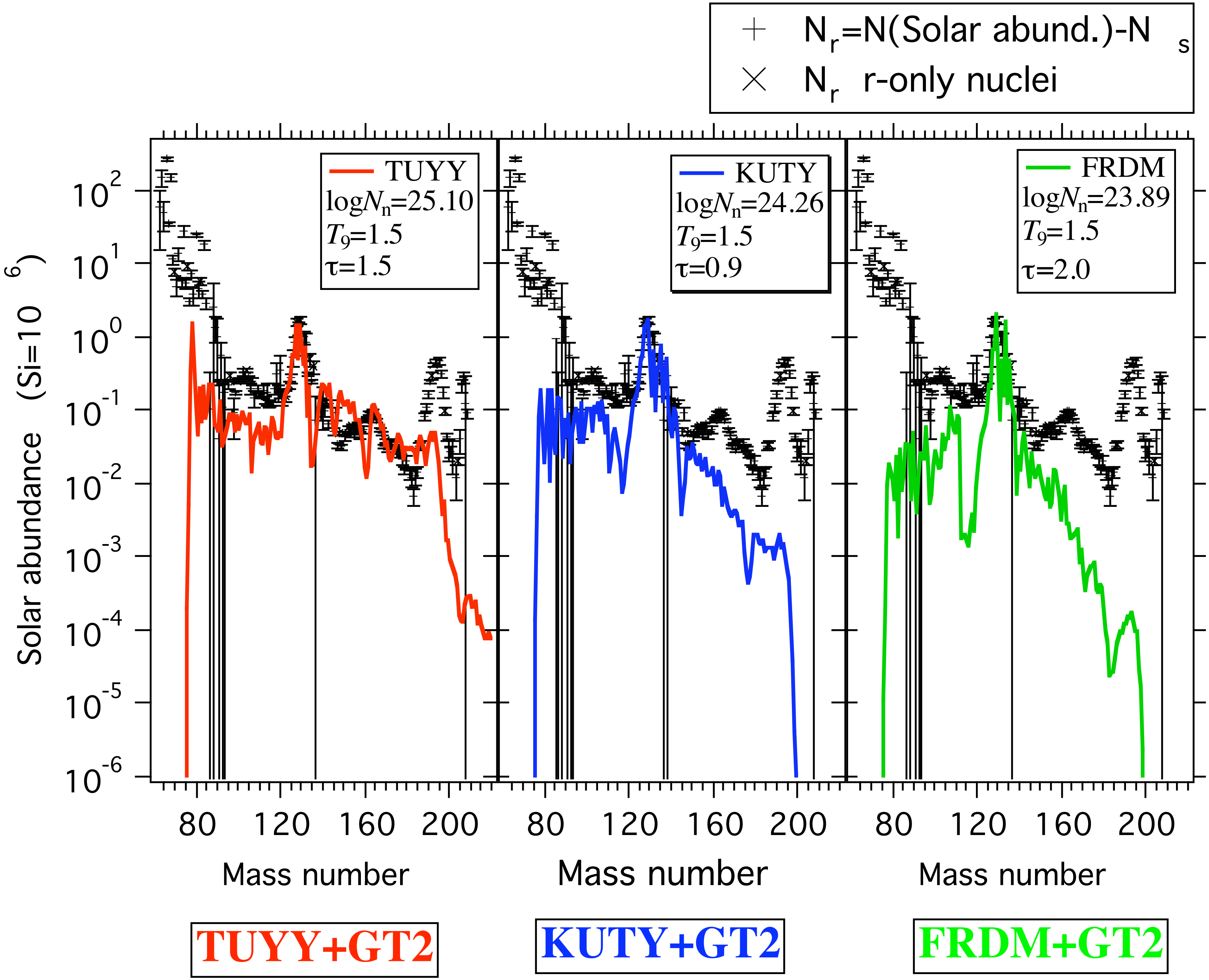
- **Time evolution** $dY/dt = -\lambda_Z Y_Z(t) + \lambda_{Z-1} Y_{Z-1}(t)$

Y_Z: abundance, λ_Z: β-deacy constant (gross theory 2nd)

N_n,T_9,τ: chosen to reproduce the abundance peak at *A*=130 (obs.)

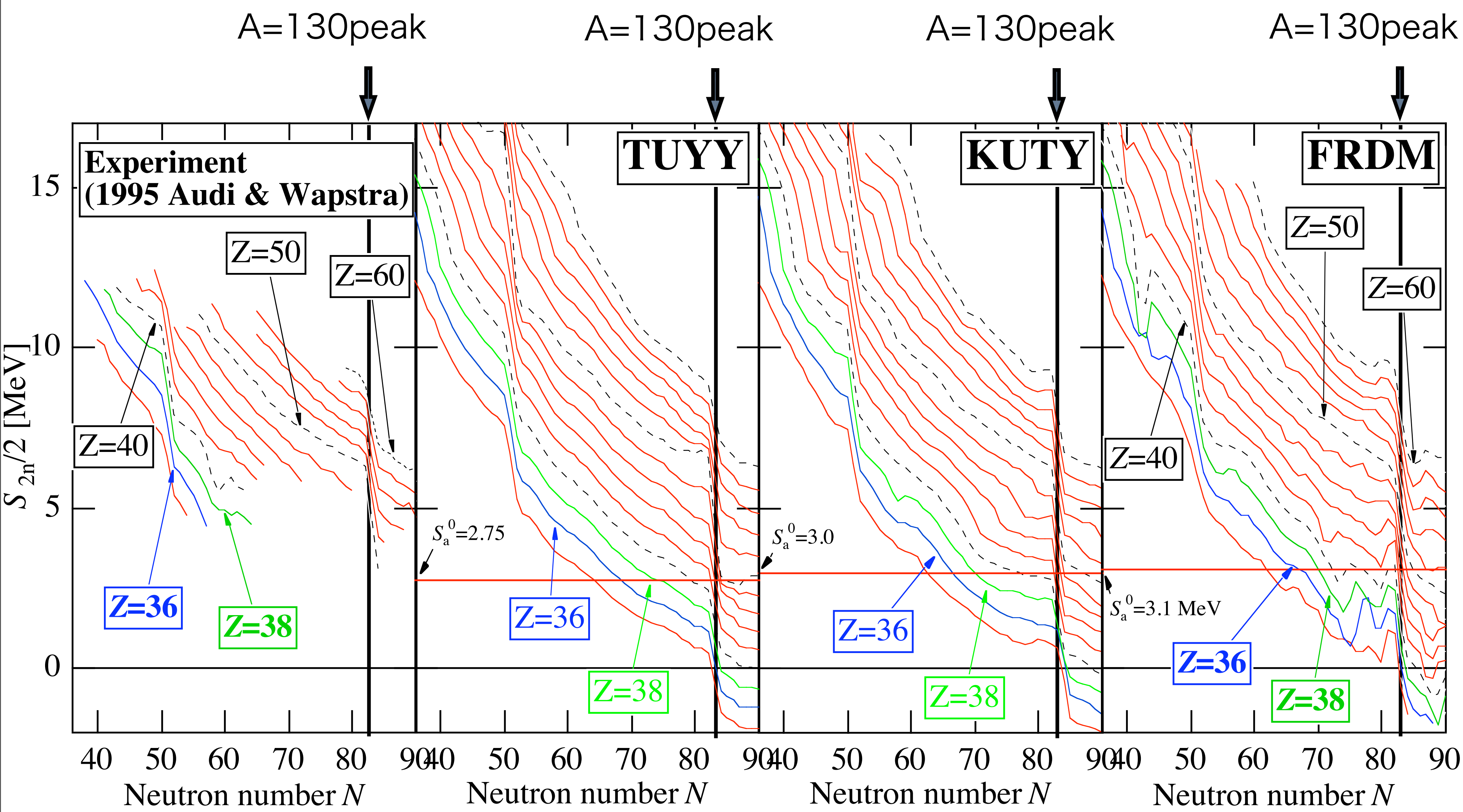
S_{2n},Q_β: estimated from mass formulas (TUYY, KUTY, FRDM)


New measurement ('96-)
Mass measured nucleus (-'95)
r-path TUYY
r-path KUTY
r-path FRDM
Proton number Z
Neutron number N
0
20
40
60
80
100
120
140
160
180
200



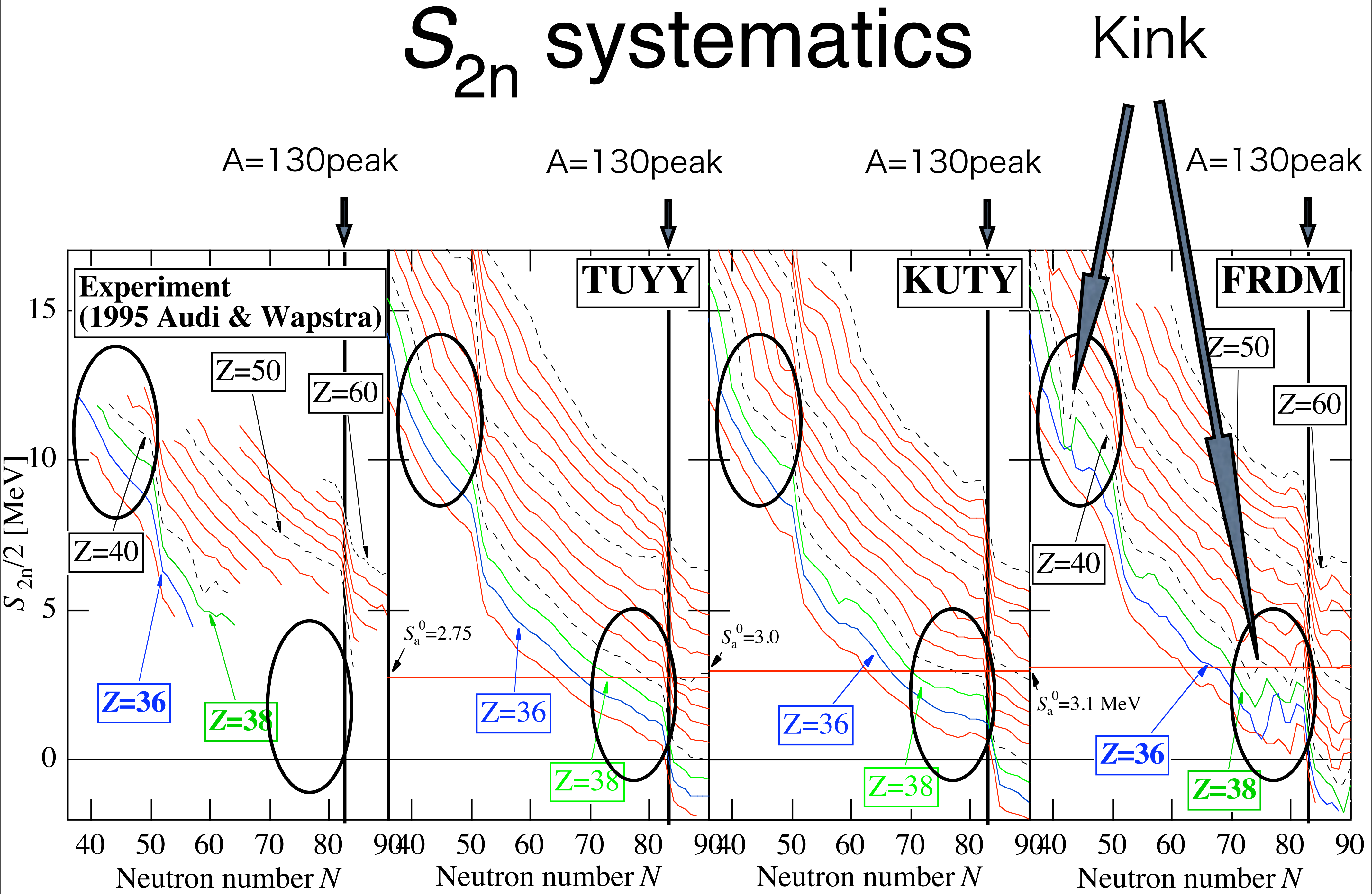
Nr=N(Solar abund.)-Ns
Nr r-only nuclei
TUYY
logNn=25.10
T9=1.5
τ=1.5
KUTY
logNn=24.26
T9=1.5
τ=0.9
FRDM
logNn=23.89
T9=1.5
τ=2.0
Solar abundance (Si=10^6)
Mass number
80 120 160 200
TUYY+GT2
KUTY+GT2
FRDM+GT2

S_{2n} systematics



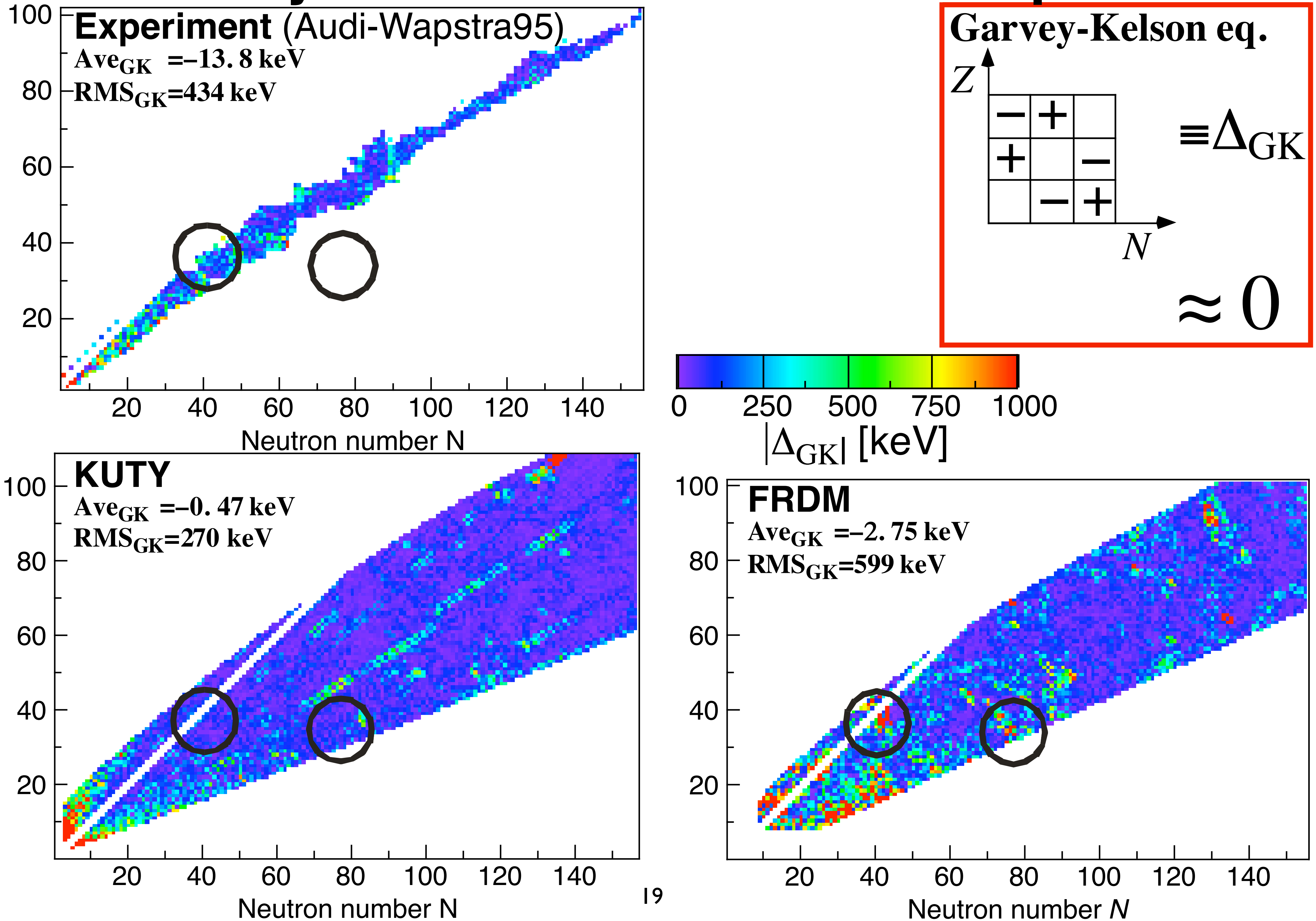
Experiment **TUYY** **KUTY** **FRDM**

S_{2n} systematics

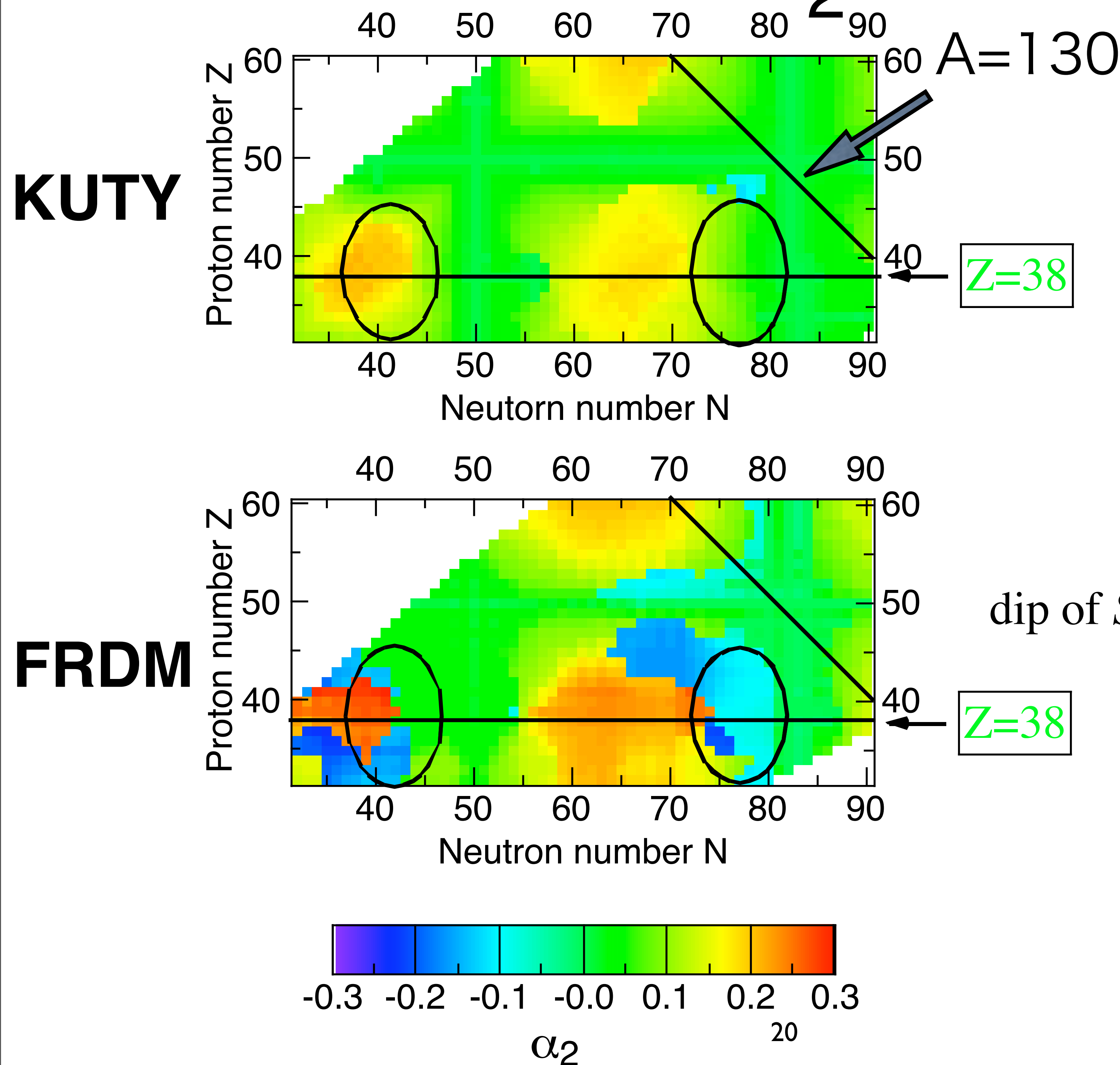


Experiment **TUYY** **KUTY** **FRDM**

Garvey-Kelson mass relationship



Deformation parameter α_2



dip of S_{2n} <=> change of α_2
(FRDM)

Influence of shell-quenching far from stability

B. Chen et al. / Physics Letters B 355 (1995) 37–44

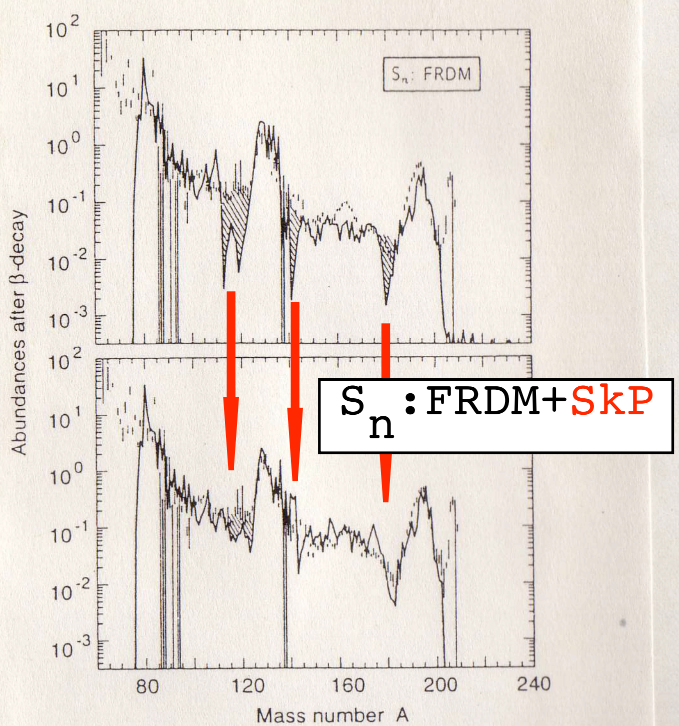


Fig. 2. r-process abundance fits obtained with ten equidistant neutron-density components from 10^{20} cm^{-3} to 3×10^{24} cm^{-3} according to Fig. 1. In the upper part, the result is presented for FRDM [10] masses with the $T_{1/2}$ and P_n values from the QRPA calculations according to Ref. [11]. In the lower part, masses of spherical nuclei around $N = 82$ have been replaced by masses from HFB calculations with the Skyrme force SkP. The quenching of the $N = 82$ shell gap (see Fig. 4) leads to a filling of the abundance troughs around $A \simeq 120$ and 140, and to a better overall reproduction of the heavy-mass region.

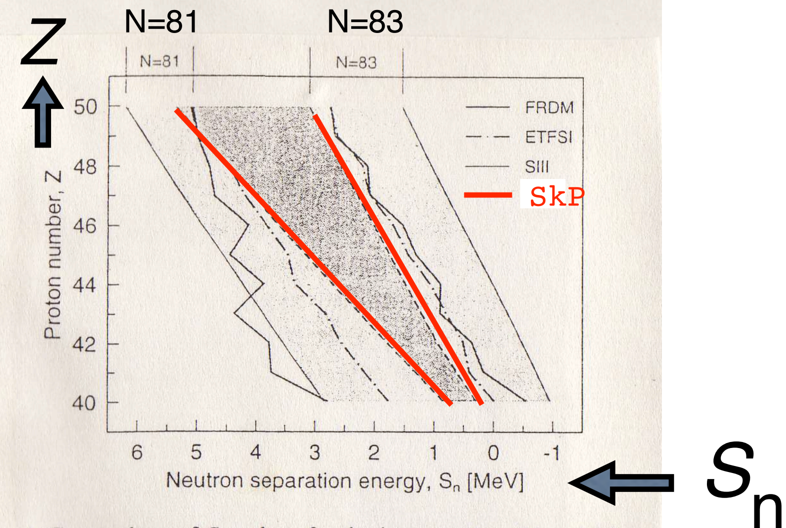
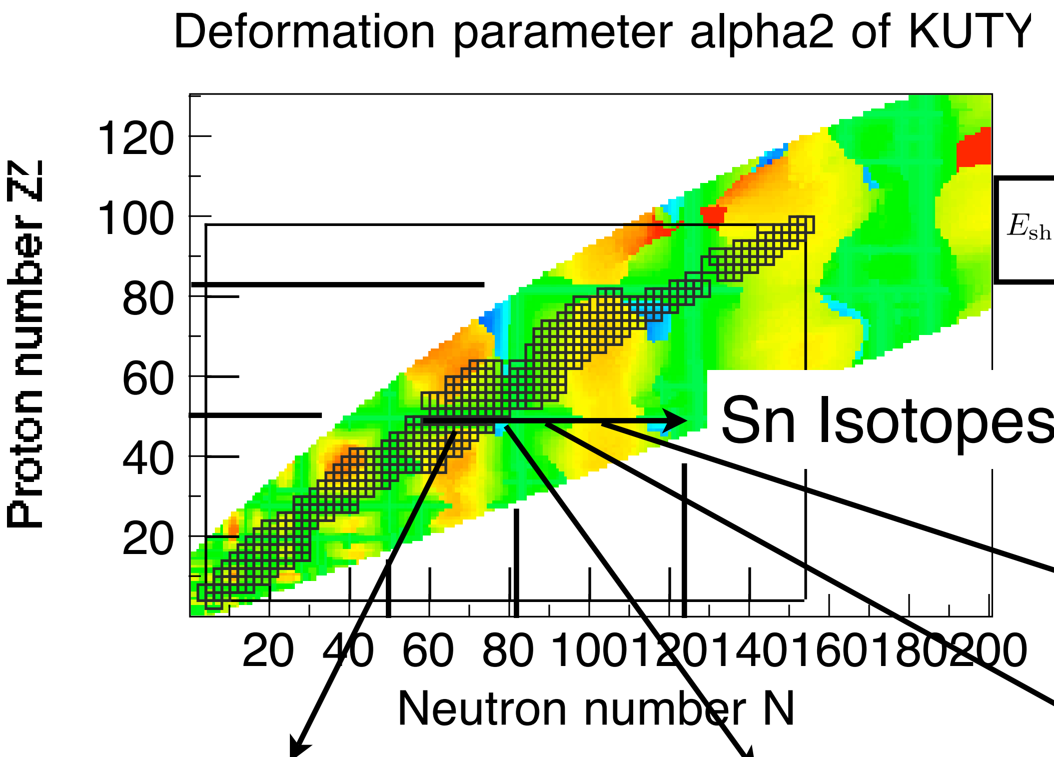


Fig. 4. Comparison of S_n values for the isotones $N = 81$ and 83 as predicted by different mass models. The difference $\delta S_n = [S_n(N = 81) - S_n(N = 83)]$ is a measure of the $N = 82$ shell strength and is shaded for SIII (light) and SkP (dark). The shell quenching with distance from stability for SkP, in contrast to SIII, can be recognized. Masses of odd-odd nuclei have not been calculated in our SIII study.

shell-quenching
=> decreasing of dips
around the peaks
(Chen et al. 1995)

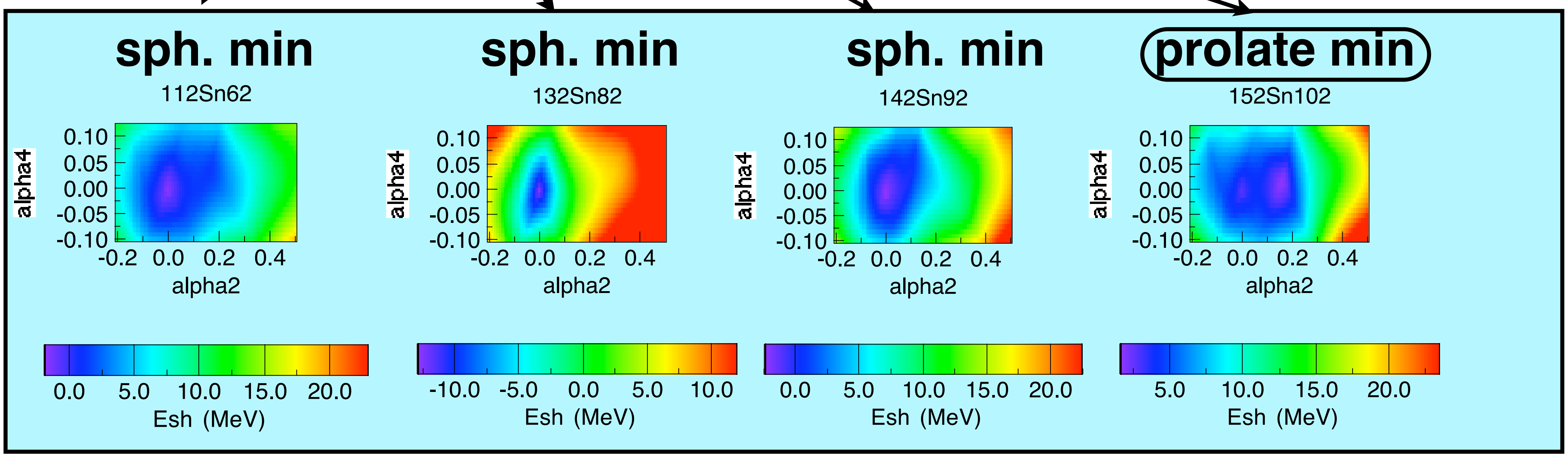
Kink of S_{2n}, or shell quenching?

Potential energy surface



$$E_{\rm sh}(Z,N) = \sum_{\rm def.}(< E^{\rm sph}_{\rm sh}(Z,N) >_{\rm def.} + \Delta E_{\rm S}(Z,N) - \Delta E_{\rm C}(Z,N) - \Delta E_{\rm pro}(Z,N))$$

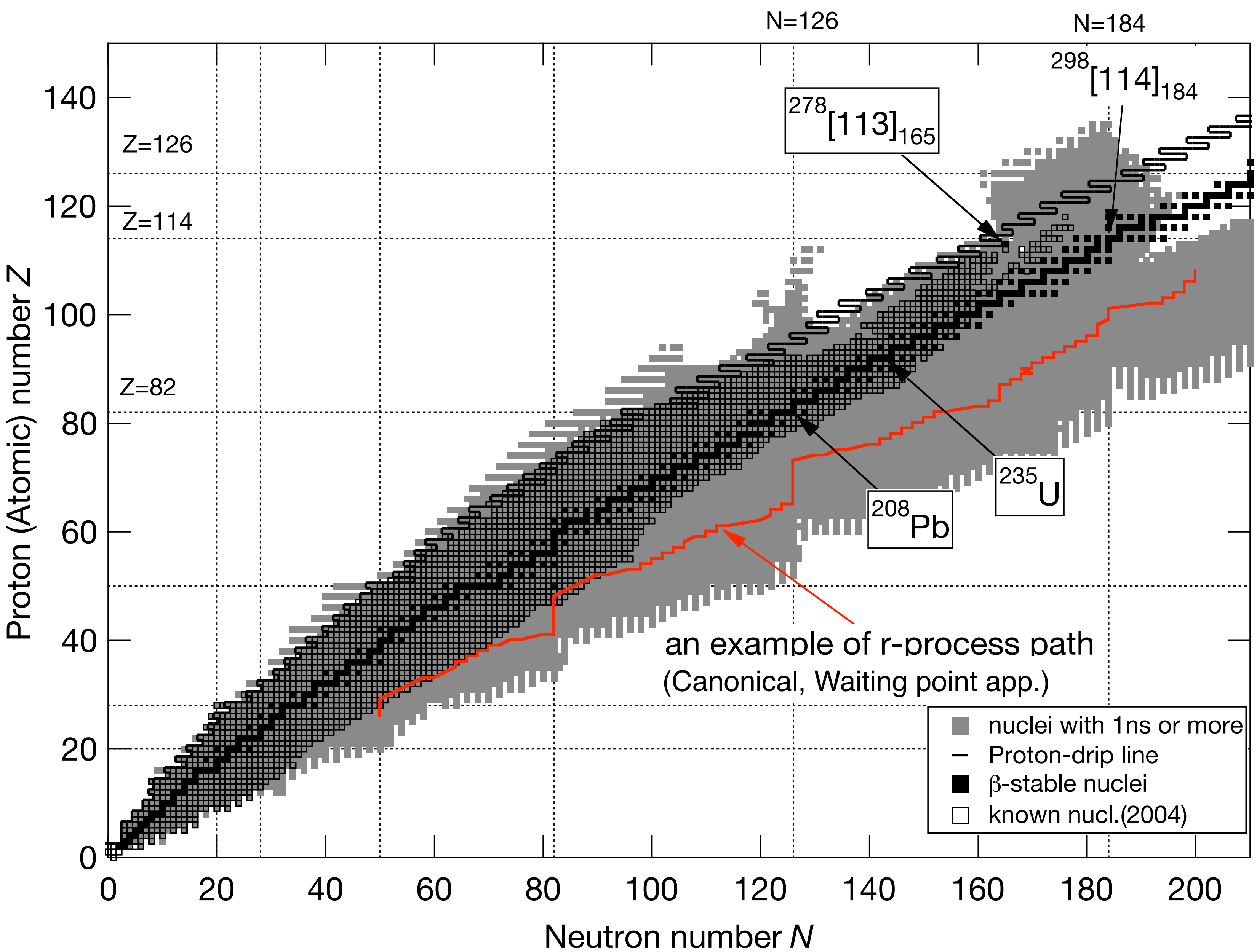
Possibility of shape isomer



$^{112}Sn_{62}$ $^{132}Sn_{82}$ $^{142}Sn_{92}$ $^{152}Sn_{102}$

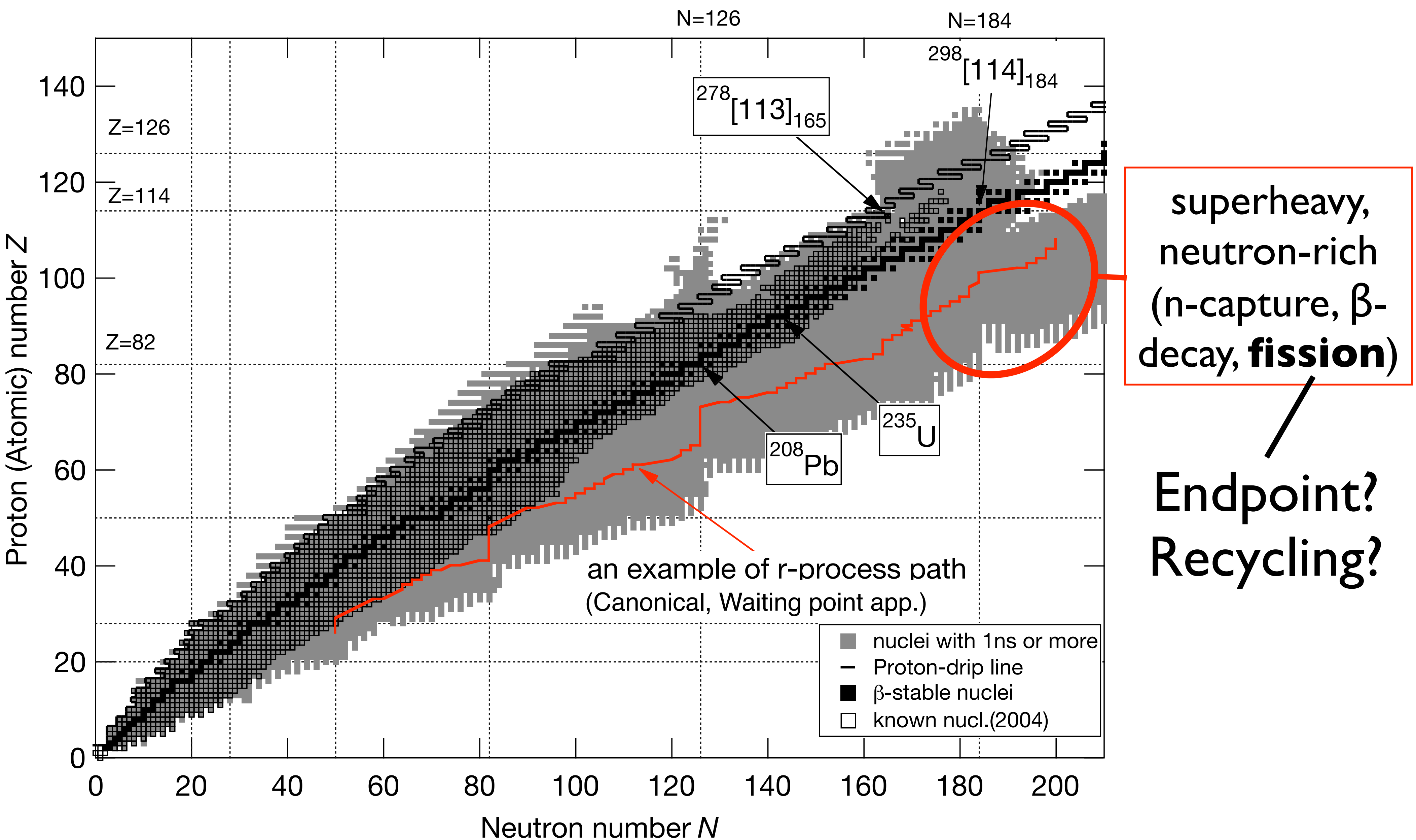
Systematical Study of Tin deformation (Shape coexistence)

r-process mass region



Calculated from the KTUY mass formula

r-process mass region

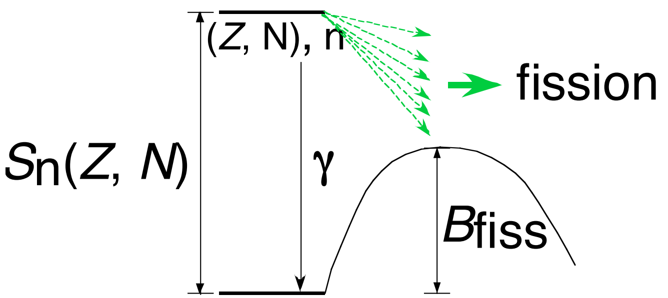


Calculated from the KTUY mass formula

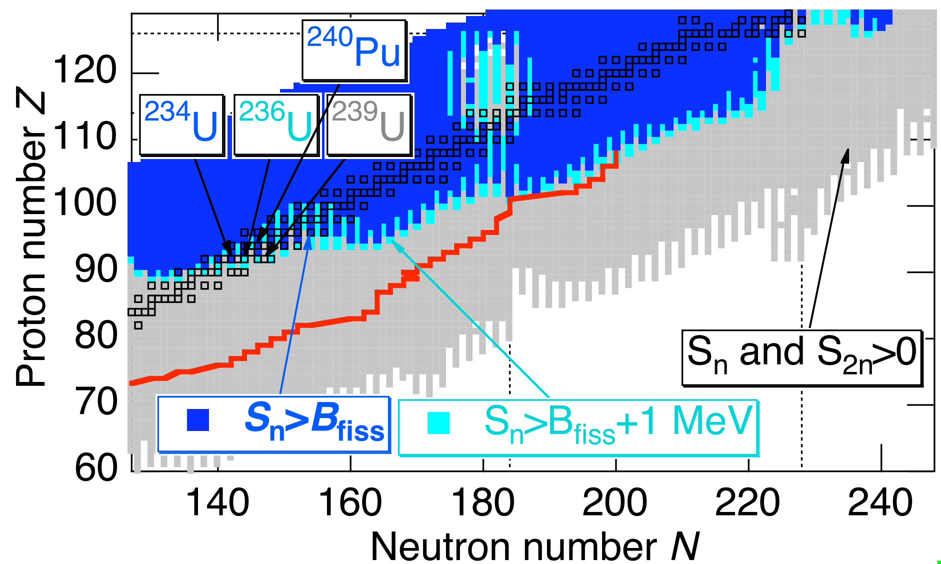
Region of β-delayed and n-induced fission

Nuclear masses and fission barrier:
KTUY (Koura-Tachibana-Uno-Yamada) mass formula

- n-induced fission

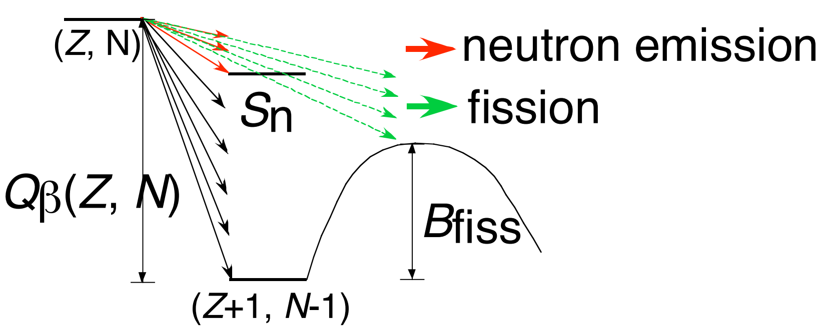


neutron-induced fission

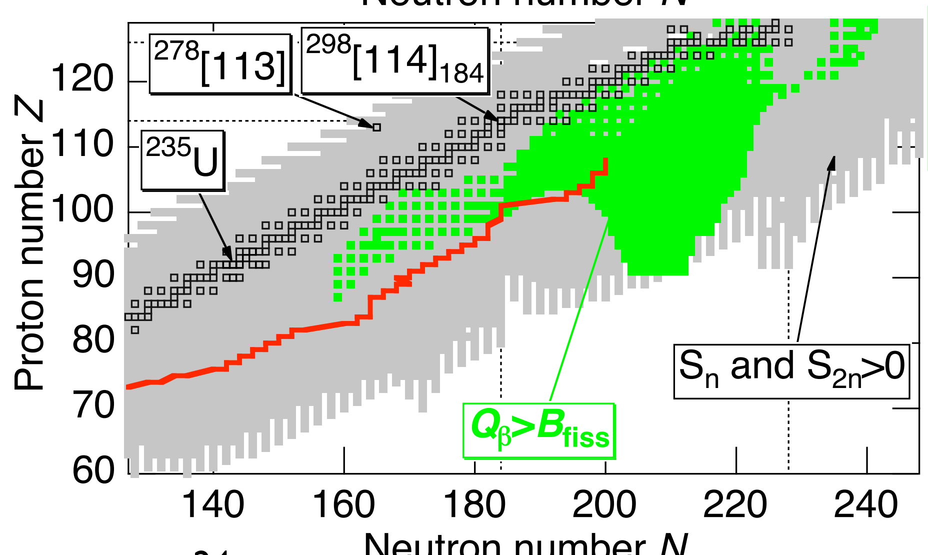


$S_n > B_{fiss}$

- β-delayed fission



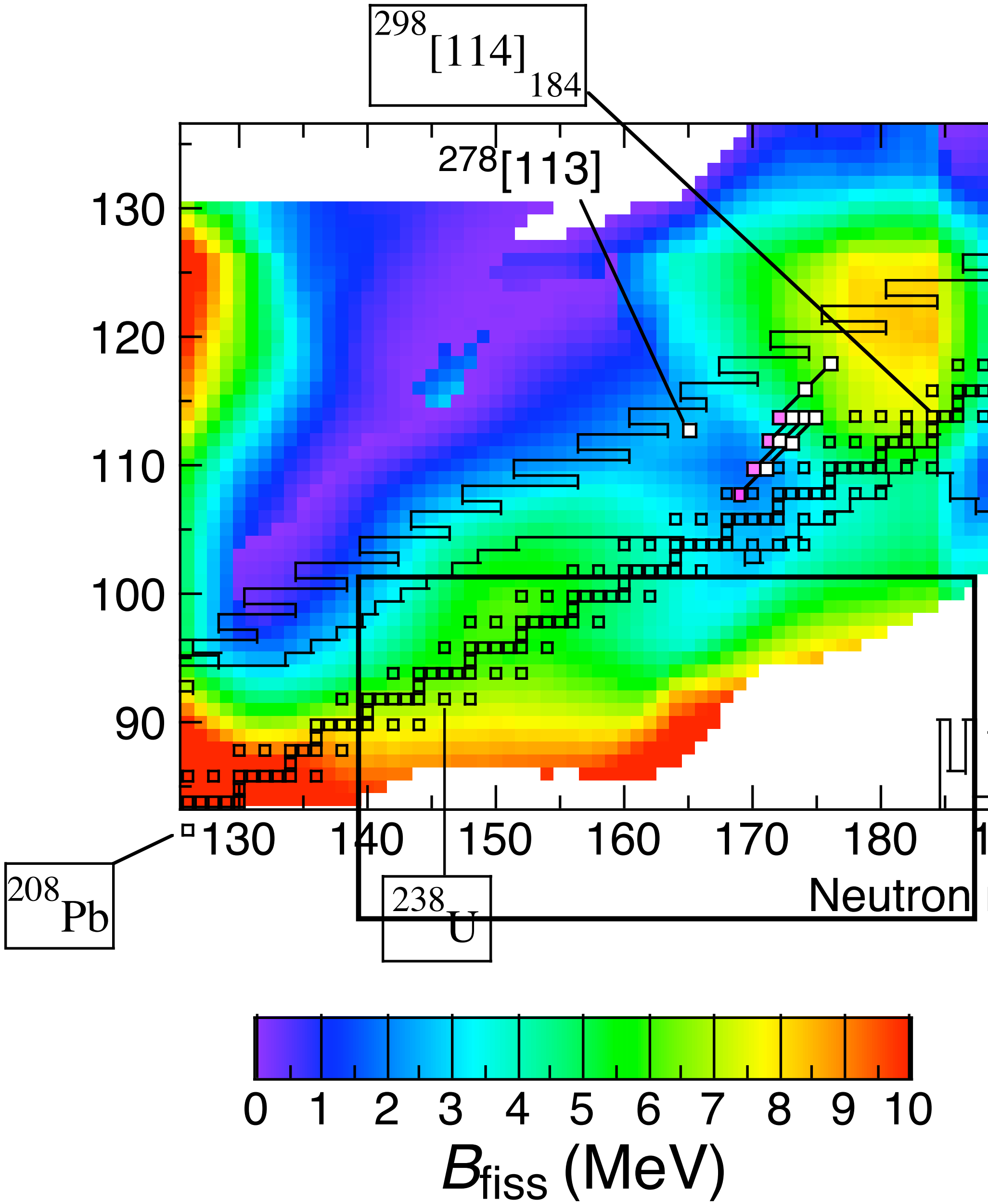
β-delayed neutron emission and β-delayed fission



$Q_\beta > B_{fiss}$

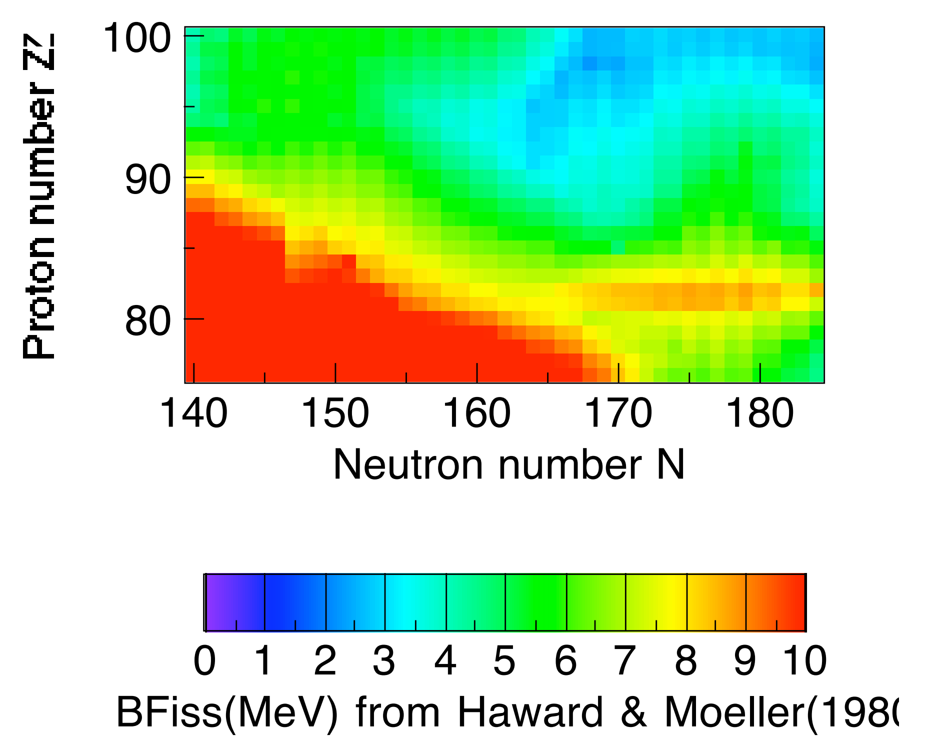
Koura

Fission barrier height



Fission Barrier height of Howard &Möller (1980):

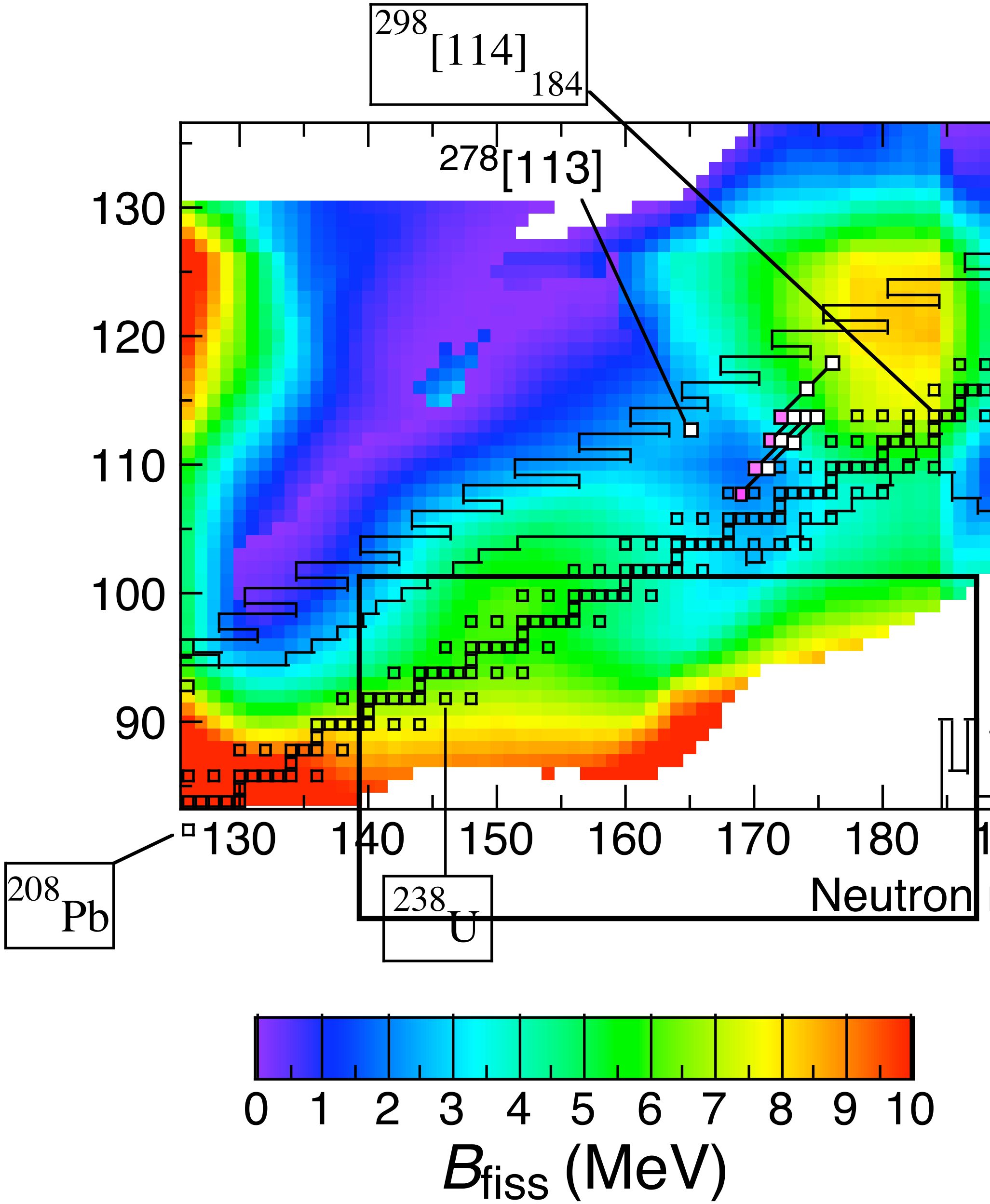
Used for calc. of β-delayed fission prob. from QRPA (Klapdor, 88)



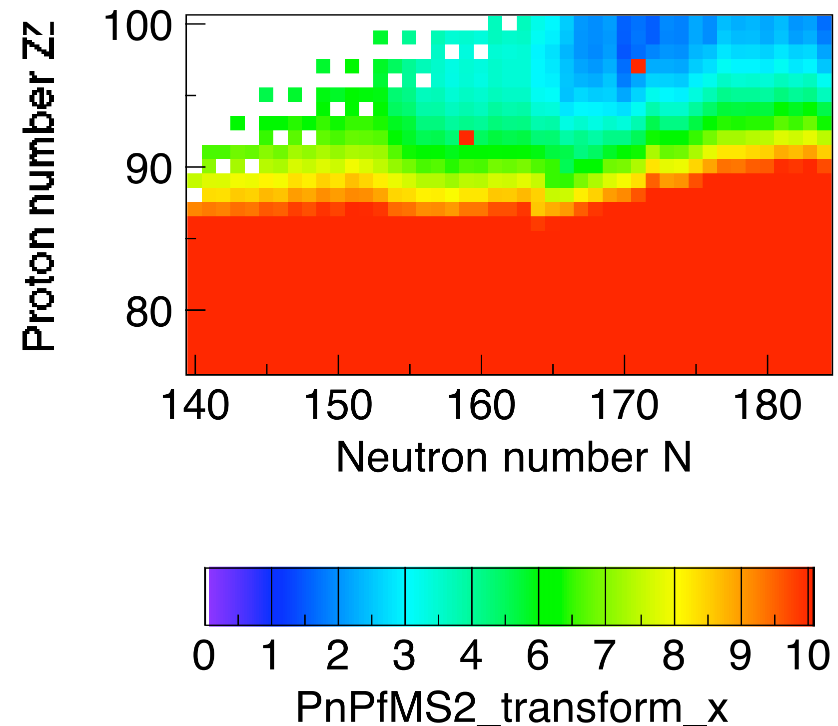
RMS dev. from exp. :~1MeV

Koura

Fission barrier height



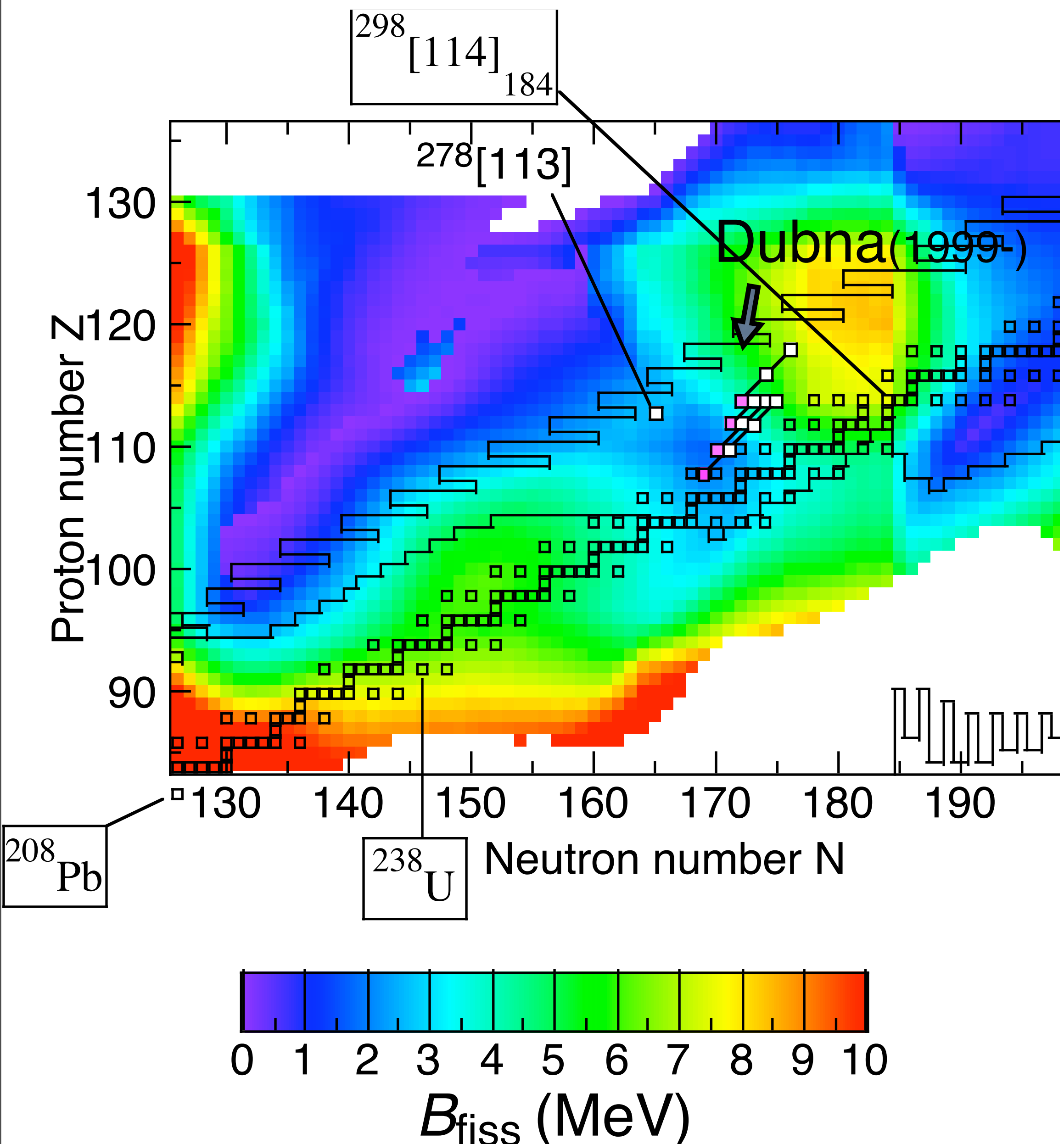
Fission Barrier height of Myers &Swiatecki (1996):



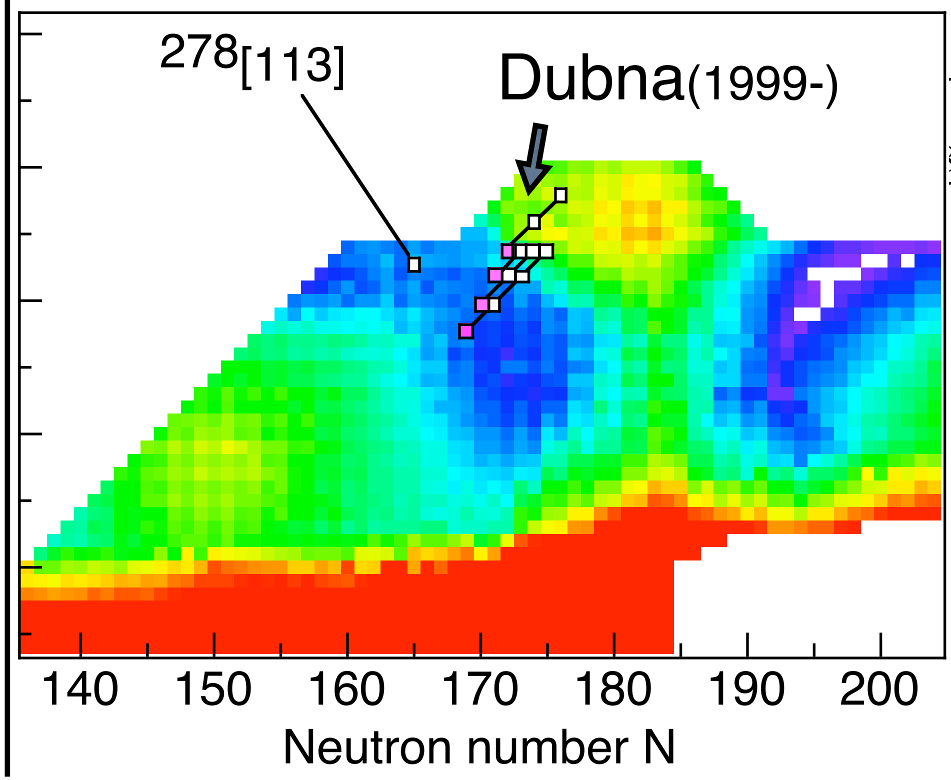
RMS dev. from exp. :~1MeV

Koura

Fission barrier height of KUTY



ETFSI*) (β_2, β_4, β_6 deformation)



*) Mamdouh,et al NPA679 (2001)

Koura

Summary

Bulk properties of nuclear masses:

- **Notable differences in the n-rich region among mass formulae**
  - -Kink of the S2n systematics , Garvey-Kelson discrepancy, etc.

Relation between the r-process nucleosynthesis cal. and mass formulae:

- **R-process nucleosynthesis calculation with a canonical model**
  - -Abundance deficient both sides of A=130 <= kink of S2n sys.

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