# 核衝突ダイナミクスと状態方程式
# Nuclear collision dynamics and the equation of state

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We want to measure EOS.

Measure $T$, $P$ and $\rho$ of matter ...

Prepare matter in the state we want to measure $\cdots\cdots$ HI collisions

What are taking place in collisions?

- High density
- Low density

# EOS and Collision Dynamics

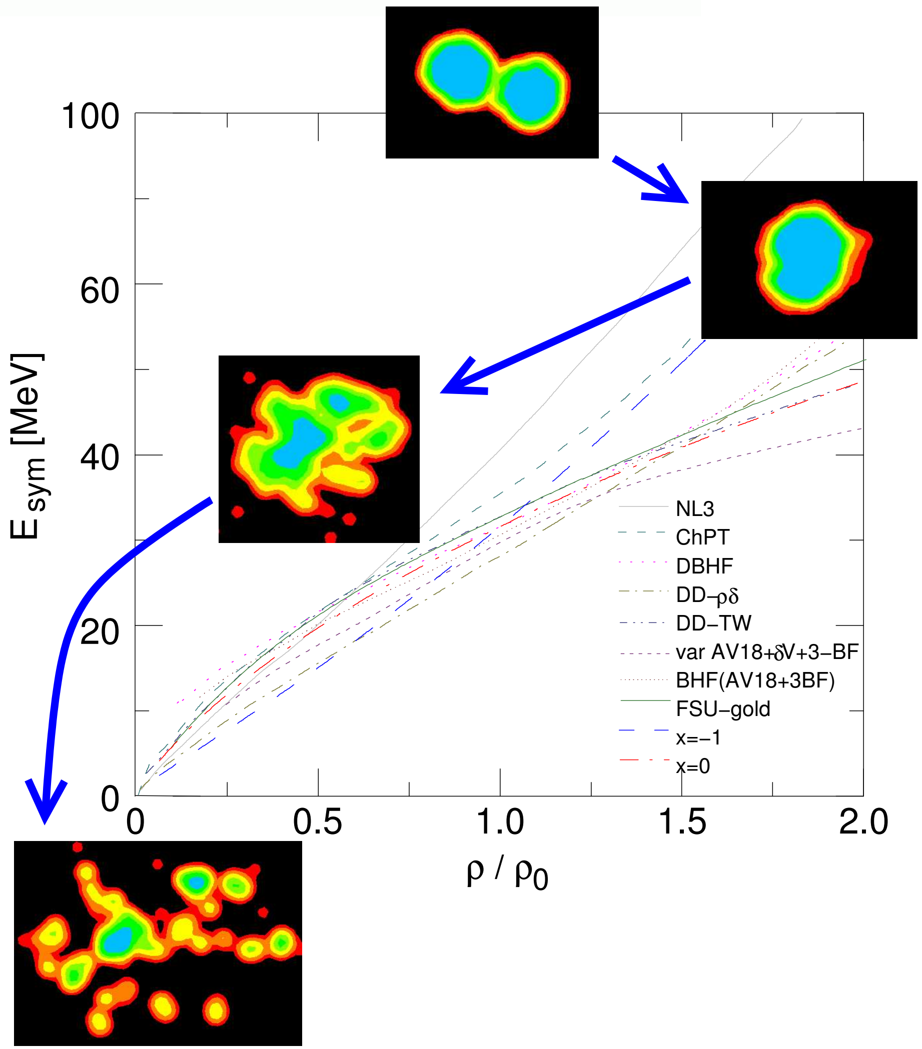
Energy of nuclear matter

$$E(\rho,\delta)/A = E(\rho,0)/A + E_{\mathrm{sym}}(\rho)\delta^2$$

$$\delta = (\rho_n - \rho_p)/\rho$$

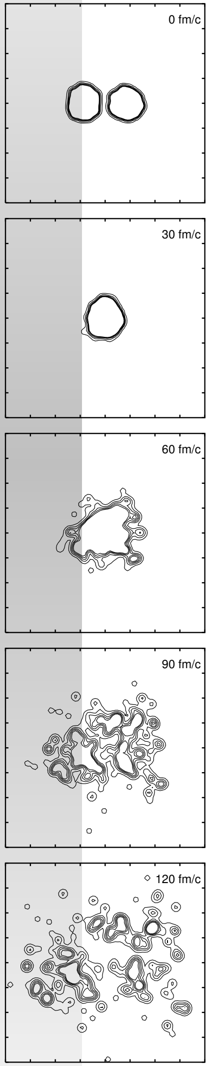
- $E(\rho,0)$ (Symmetric matter $\rho_n = \rho_p$)
- $E_{\mathrm{sym}}(\rho)$: Symmetry energy
- Depends on temperature $T$

  free energy rather than energy
- LG phase transition (two components)
- Effective masses $m_n^*(\rho,\delta)$, $m_p^*(\rho,\delta)$
- NN cross sections $\sigma_{NN}(\rho,\delta)$

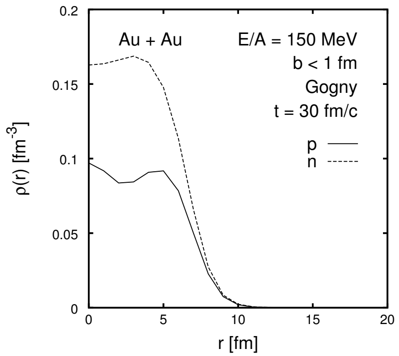


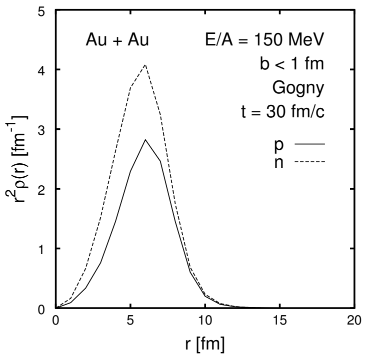
# Isospin Effects in High Density Region

$^{197}$Au + $^{197}$Au at 150 MeV/u, $b < 1$ fm, $t = 30$ fm/$c$

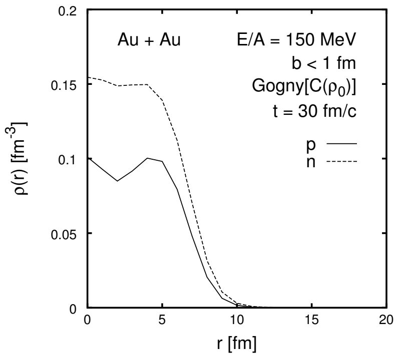


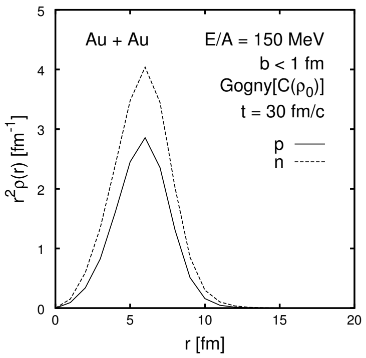

Gogny

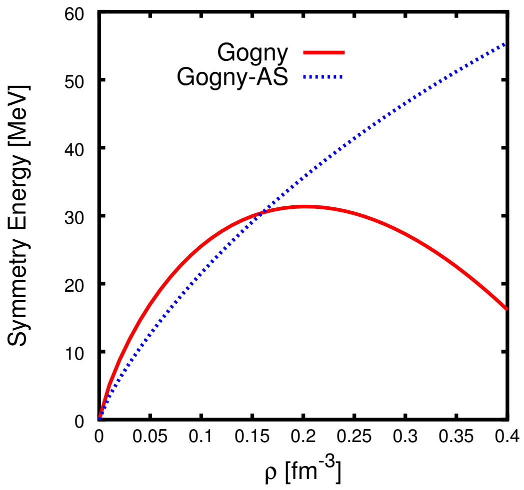


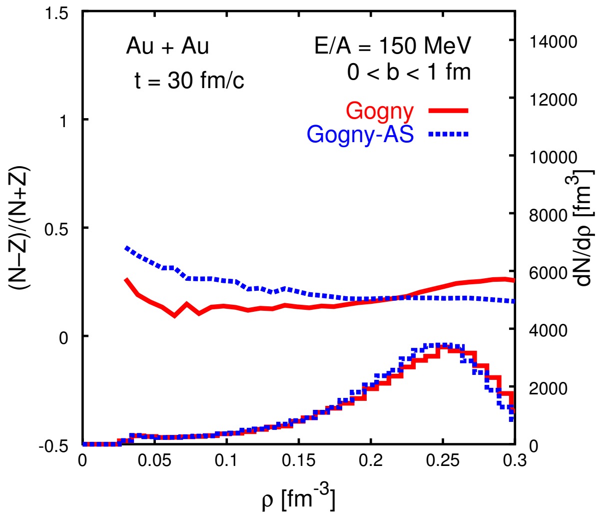




Gogny-AS



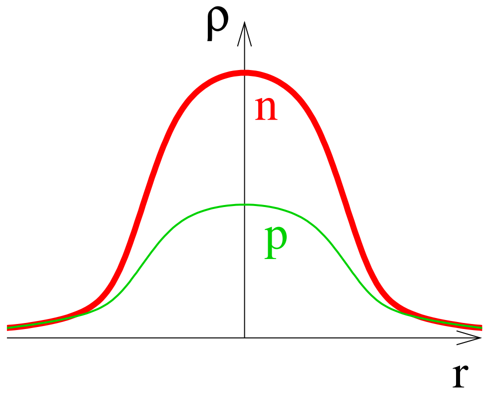



# Probes of High Density Matter

Compressed state ⇔ Observables

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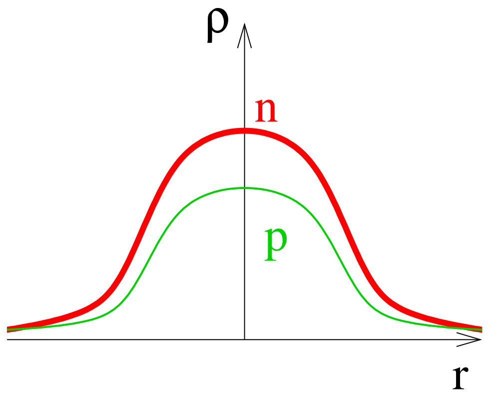
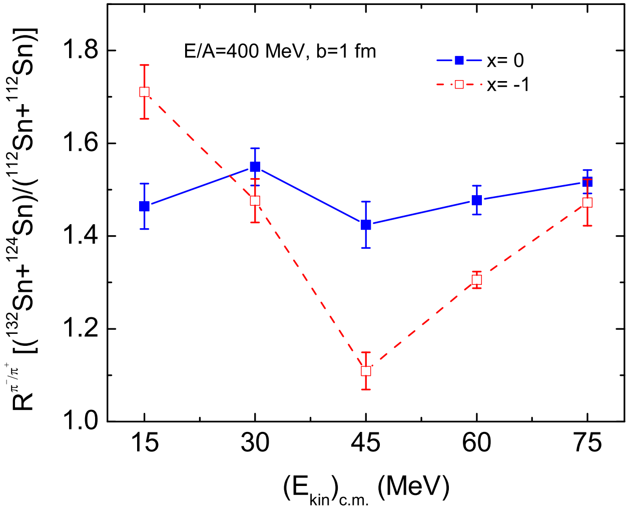



- $\pi^-/\pi^+$ ratio

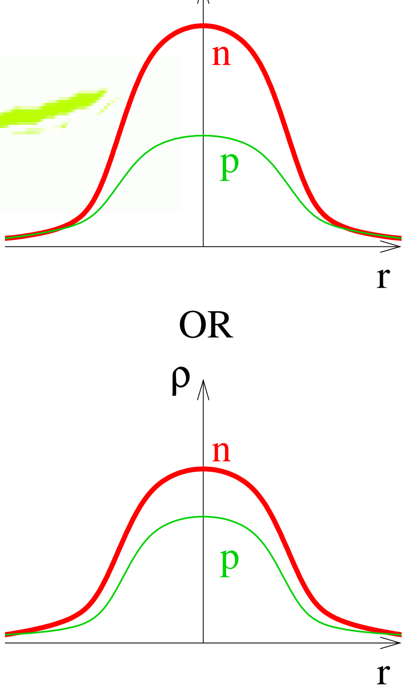
$$\pi^-/\pi^+ \approx \left(\frac{\rho_n}{\rho_p}\right)^2, \qquad R = \frac{(\pi^-/\pi^+)(^{124}\mathrm{Sn} + {}^{124}\mathrm{Sn})}{(\pi^-/\pi^+)(^{112}\mathrm{Sn} + {}^{112}\mathrm{Sn})}$$




Yong et al., PRC73(2006)034603.

- Difference of neutron flow and proton flow

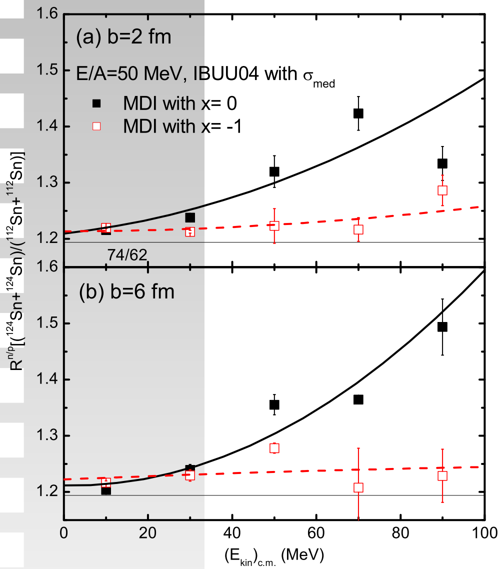
# Neutron and Proton Flows




Double ratio of neutron-proton spectra

$$R = \frac{(Y_n/Y_p)(^{124,132}\mathrm{Sn} + {}^{124}\mathrm{Sn})}{(Y_n/Y_p)(^{112}\mathrm{Sn} + {}^{112}\mathrm{Sn})}$$

$E/A = 50$ MeV

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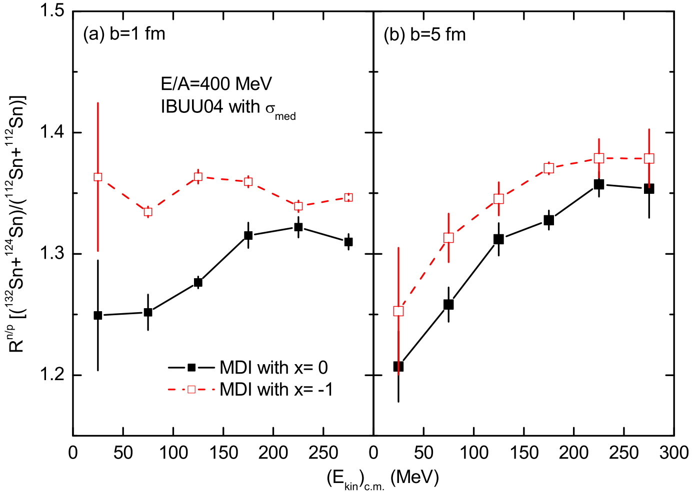
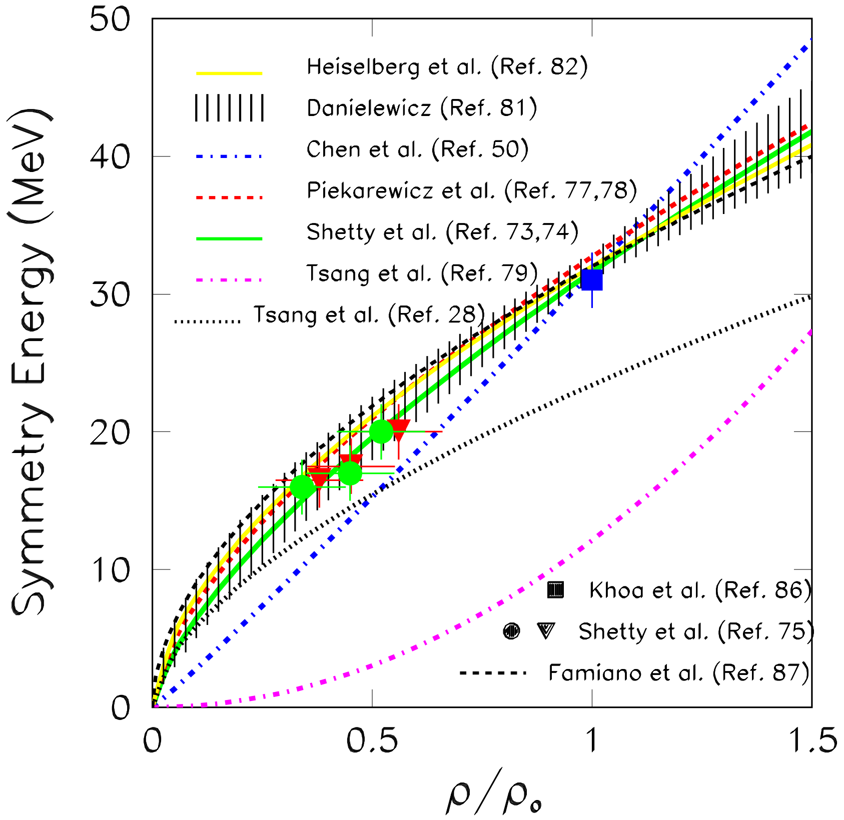
$E/A = 400$ MeV




Li et al., PLB(2006)378.

# Experiments ⇒ Lowdensity EOS

Shetty et al., PRC 76 (2007) 024606



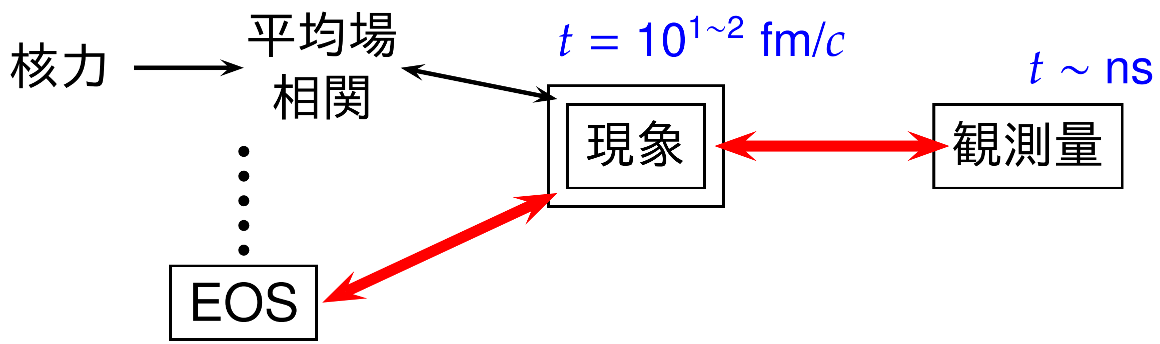

**Observables:** Isoscaling, Isospin diffusion,

Neutron/proton emission ratio, Giant resonances,

Binding energy and neutron skin,

Neutron star calc., ...

# Approach to measure EOS




- 反応の中間段階における特徴的現象の研究・探索

  例： $\rho_n/\rho_p$ の異常な振舞（高密度，多重破砕，ネック形成...）

- 平均場等と現象の関連（動力学計算）

  例： $\rho_n/\rho_p$ が有効相互作用パラメータを反映するか

- EOS（≡ 熱化学平衡の物質の性質）と現象の関連

  熱化学平衡が実現しているか，動的効果が重要か

- 現象（中間段階の物理量）と観測量との関連

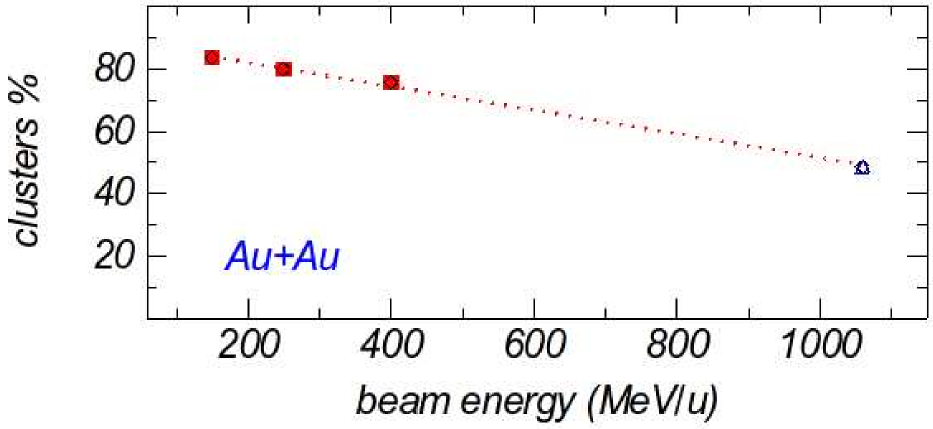
  例： $\pi^-/\pi^+$ を測れば，$\rho_n/\rho_p$ を測ったことになるか

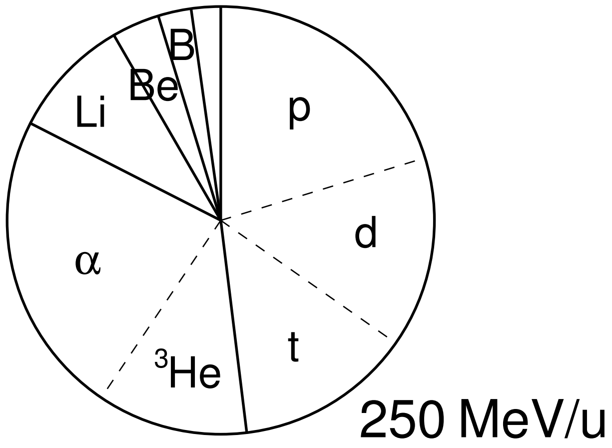
<u>このあとの話題</u>

- クラスターの重要性
- 反応系と熱平衡系の統一的記述

# Clusters are important

- Many experimental observables (to probe high and low densities) are related to clusters and fragments. ($t/^3$He, isoscaling etc)
- Clusters and fragments are the main part of the total system.




For example, four nucleons in the gas at $T = 10$ MeV.

- Uncorrelated: $\langle E \rangle = \frac{3}{2} T \times 4 = 60$ MeV
- $\alpha$ cluster: $\langle E \rangle = -28.3\,\text{MeV} + \frac{3}{2} T = -13.3$ MeV

Can we satisfy with “coalescence” ?

# VUU Equation

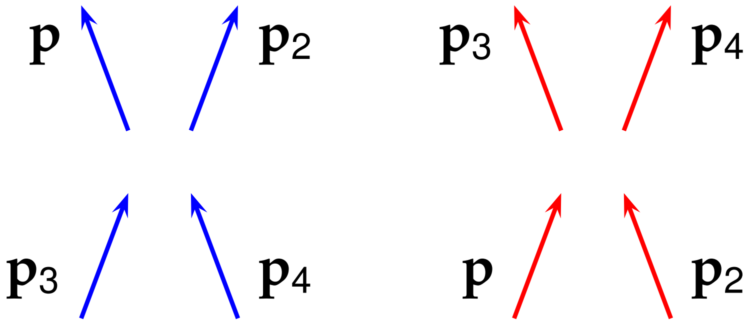
VUU Equation（BUU Equation, BNV Equation)

$$\frac{\partial f}{\partial t} = \frac{\partial h}{\partial \mathbf{r}} \cdot \frac{\partial f}{\partial \mathbf{p}} - \frac{\partial h}{\partial \mathbf{p}} \cdot \frac{\partial f}{\partial \mathbf{r}} + I_{\rm coll}$$

Collision term

$$I_{\rm coll} = \int \frac{d\mathbf{p}_2}{(2\pi\hbar)^3} \int d\Omega\, |v| \left(\frac{d\sigma}{d\Omega}\right)_v \Big\{ f(\mathbf{r},\mathbf{p}_3,t) f(\mathbf{r},\mathbf{p}_4,t) \big[1 - f(\mathbf{r},\mathbf{p},t)\big]\big[1 - f(\mathbf{r},\mathbf{p}_2,t)\big]$$
$$- f(\mathbf{r},\mathbf{p},t) f(\mathbf{r},\mathbf{p}_2,t) \big[1 - f(\mathbf{r},\mathbf{p}_3,t)\big]\big[1 - f(\mathbf{r},\mathbf{p}_4,t)\big] \Big\}$$

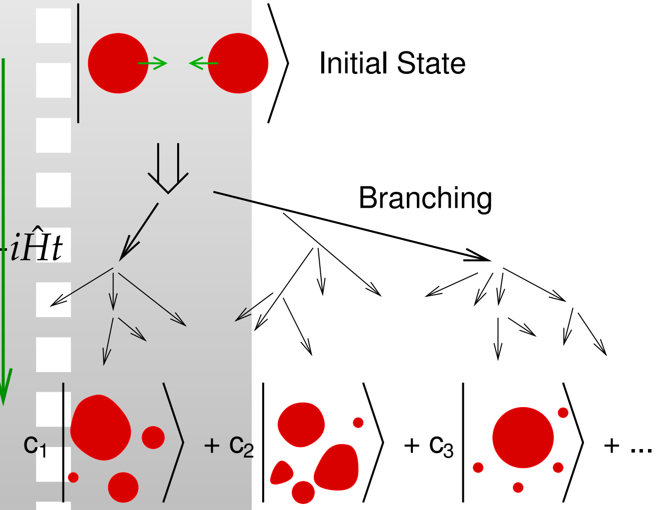
Gain term　　Loss term




$\mathbf{p}, \mathbf{p}_2, \Omega \Rightarrow \mathbf{p}_3, \mathbf{p}_4$(Energy and momentum conservation)

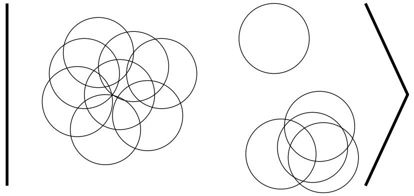
$$v = |\mathbf{p} - \mathbf{p}_2| / M^*$$

# Antisymmetrized Molecular Dynamics (AMD)




AMD wave function

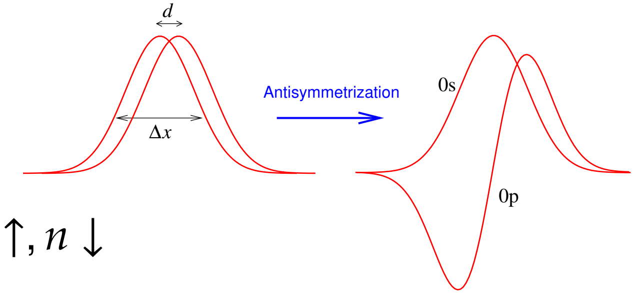
$$|\Phi(Z)\rangle = \det_{ij}\Big[\exp\Big\{-\nu\Big(\mathbf{r}_j - \frac{\mathbf{Z}_i}{\sqrt{\nu}}\Big)^2\Big\}\chi_{\alpha_i}(j)\Big]$$



$$\mathbf{Z}_i = \sqrt{\nu}\mathbf{D}_i + \frac{i}{2\hbar\sqrt{\nu}}\mathbf{K}_i$$

$\nu$ : Width parameter = (2.5 fm)$^{-2}$

$\chi_{\alpha_i}$ : Spin-isospin states = $p\uparrow, p\downarrow, n\uparrow, n\downarrow$




Stochastic equation of motion for the wave packet centroids $Z$:

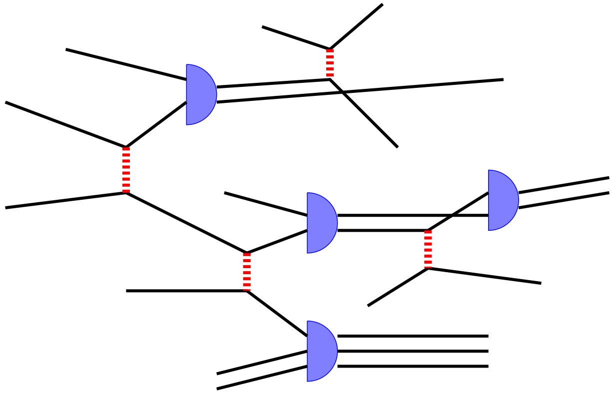
$$\frac{d}{dt}\mathbf{Z}_i = \{\mathbf{Z}_i, \mathcal{H}\}_{\text{PB}} + \text{(NN collisions)} + \Delta\mathbf{Z}_i(t)$$

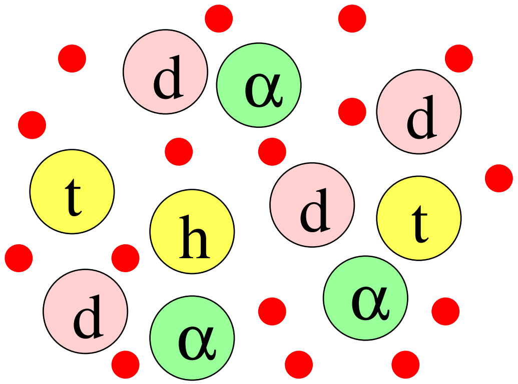
- One-body motion in the mean field
- Two-nucleon collisions

# Clusters in Collision Dynamics

Extension of AMD to respect cluster correlations

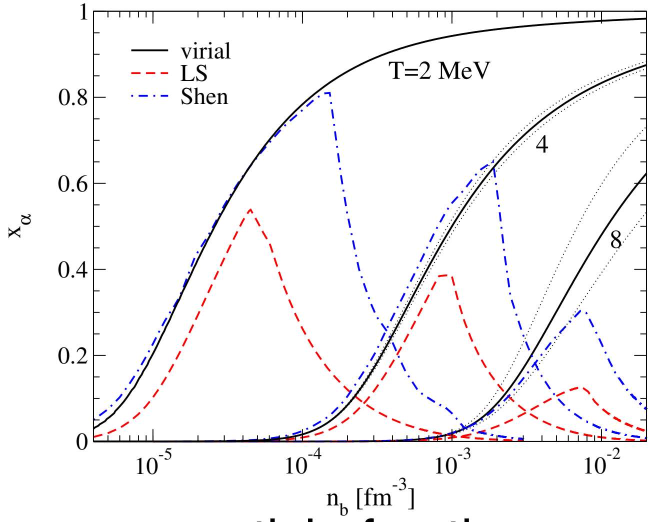
- Cluster formation
- Propagation
- Breakup



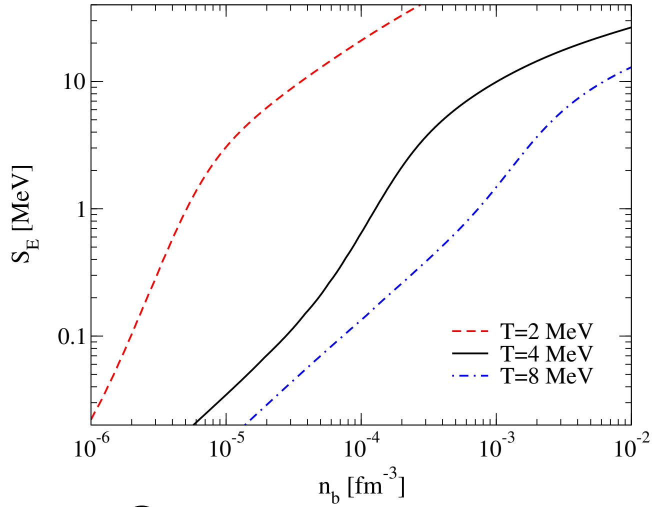



Low density EOS

Horowitz and Schwenk, NPA776 (2006) 55.



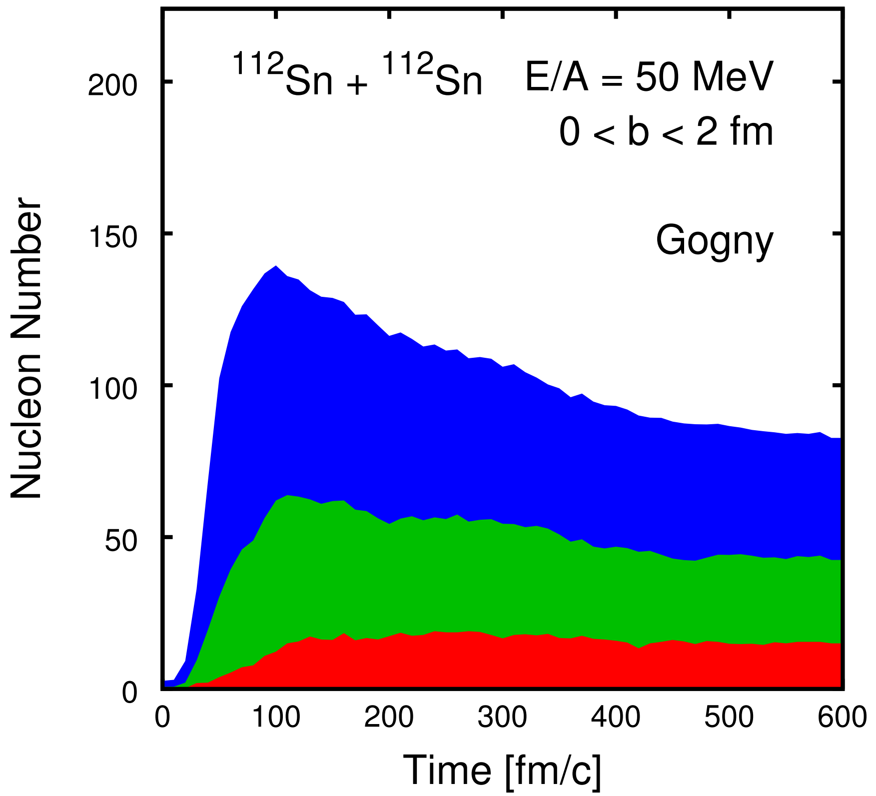

$\alpha$-particle fraction




Symmetry energy

# Time evolution of number of clusters

Number of nucleons in correlated clusters




Non-clustered

(2N)

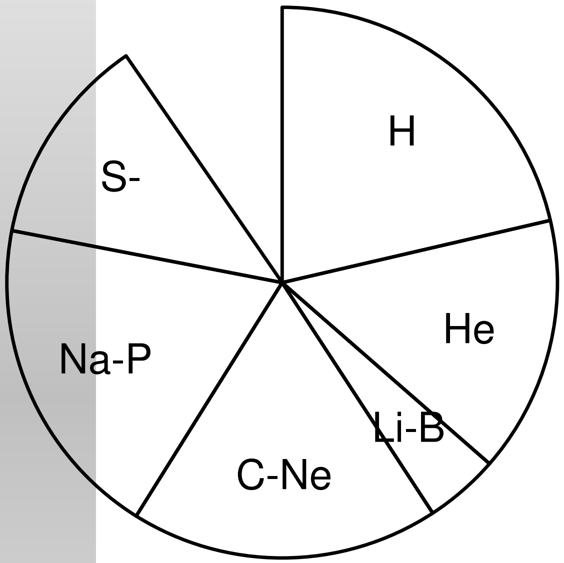
(3N)

(4N)

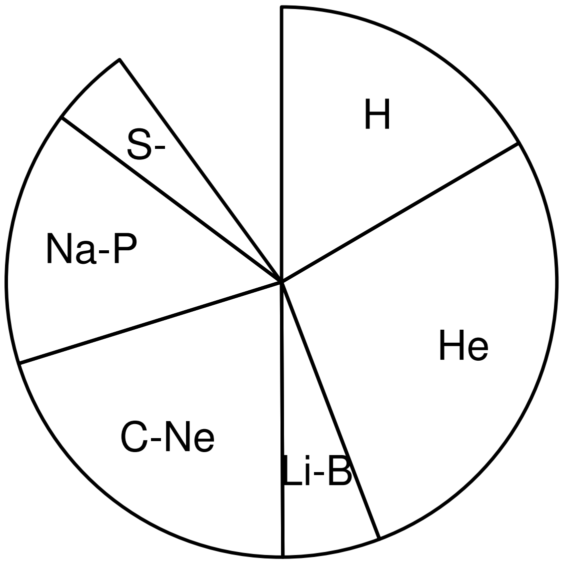
# Effects of cluster correlations

$^{40}$Ca + $^{40}$Ca, $E/A = 35$ MeV, filtered violent collisions

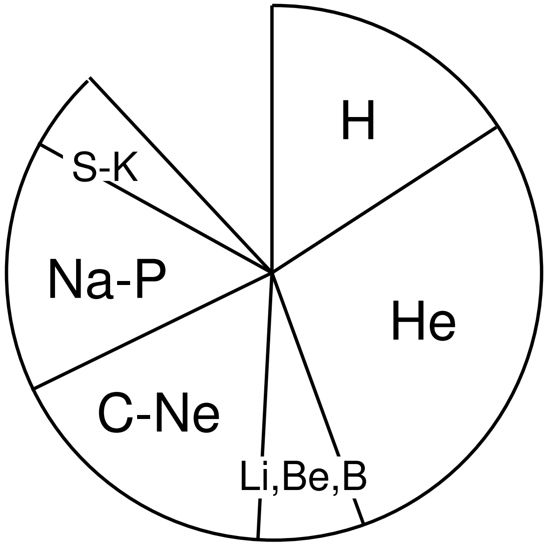
w/o cluster correlations




with cluster correlations




experiment



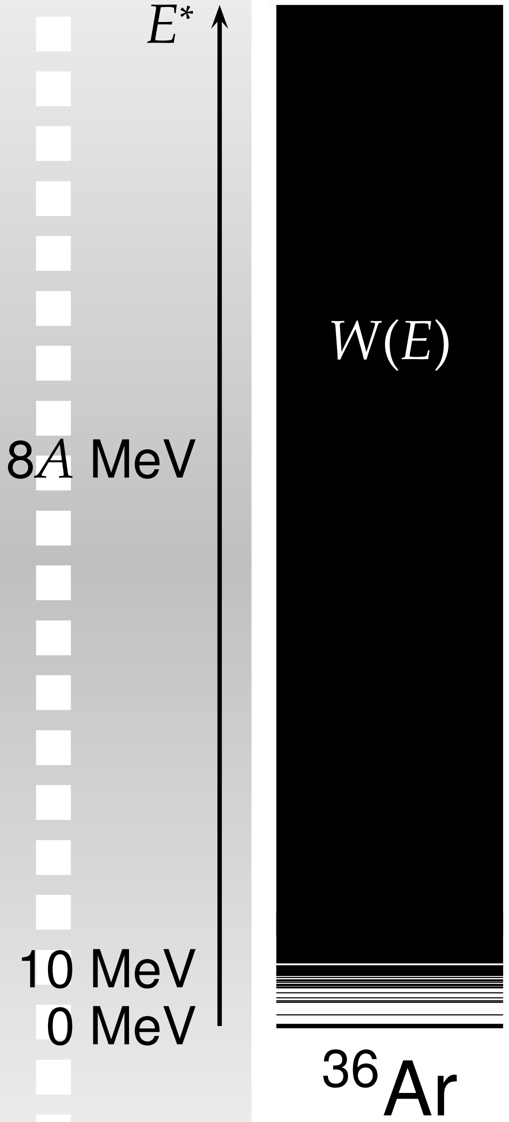

w/o cluster correlations:

| | |
|---|---|
| p | 6.7 |
| d | 1.5 |
| t | 0.3 |
| $^3$He | 0.3 |
| $\alpha$ | 2.7 |

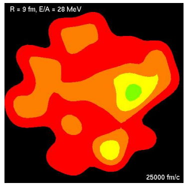
with cluster correlations:

| | |
|---|---|
| p | 4.4 |
| d | 1.8 |
| t | 0.5 |
| $^3$He | 0.6 |
| $\alpha$ | 5.0 |

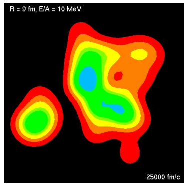
# Low density matter (Liquid-gas phase transition)



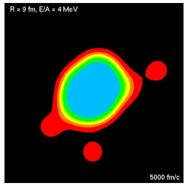

核子・クラスター気体



液相気相相転移



$W(E) \approx e^{2\sqrt{aE^*}}$



$V = \frac{4}{3}\pi(9\ \mathrm{fm})^3$

低密度非一様核物質が実現
（平均場の並進対称性の破れ）

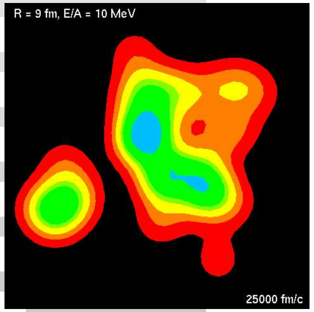
- 多体相関（クラスター）
- 相転移（二成分系）
- 一様物質の不安定性

天体現象との直接的関連

- 状態方程式
- パスタ相，$\nu$ との相互作用
- 元素合成 (?)

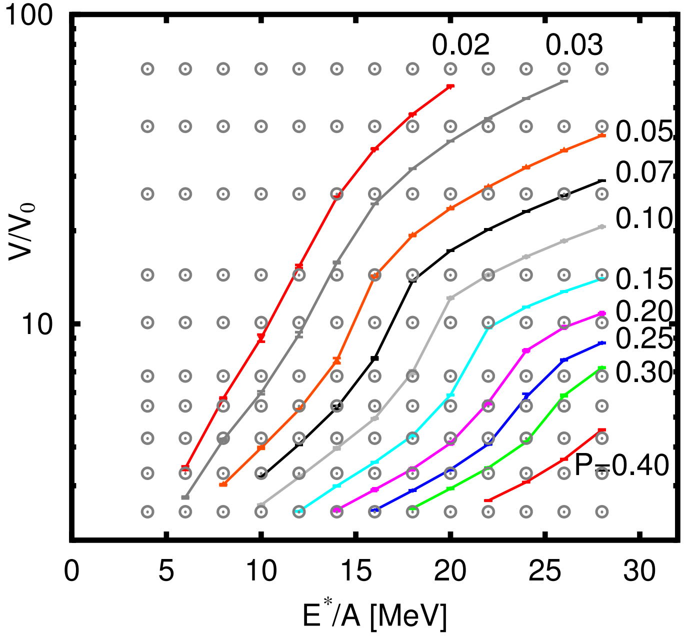
# Equilibrium ensembles and caloric curves

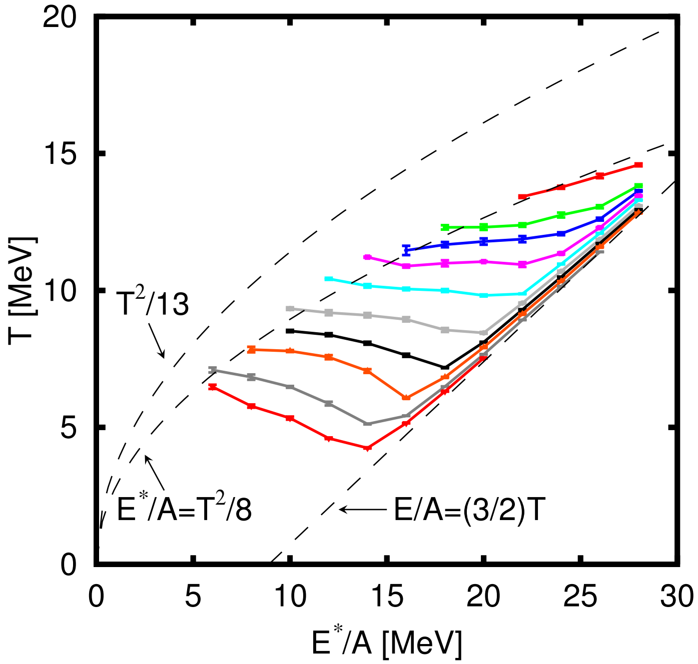
Microcanonical ensemble ⇐ Simply solve the time evolution for a long time




- Total energy: $E$
- Volume: $V = \frac{4}{3}\pi R^3$ (reflections at the wall of container)
- Neutron and proton numbers: $N = 18$, $Z = 18$

⇒ Temperature $T(E, V)$ and Pressure $P(E, V)$



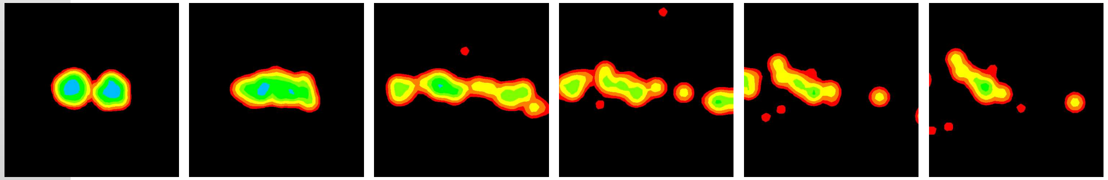



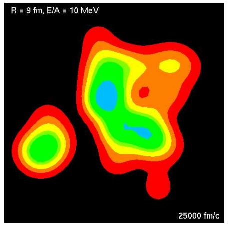

25,000 fm/$c$ × 130 combinations of $(E, V)$ ⇒ 300 CPU · hours

# Comparison of reaction and equilibrium

T. Furuta, Doctor Thesis, Tohoku University, 2007.

$^{40}$Ca + $^{40}$Ca, $E/A$ = 35 MeV, $b$ = 0



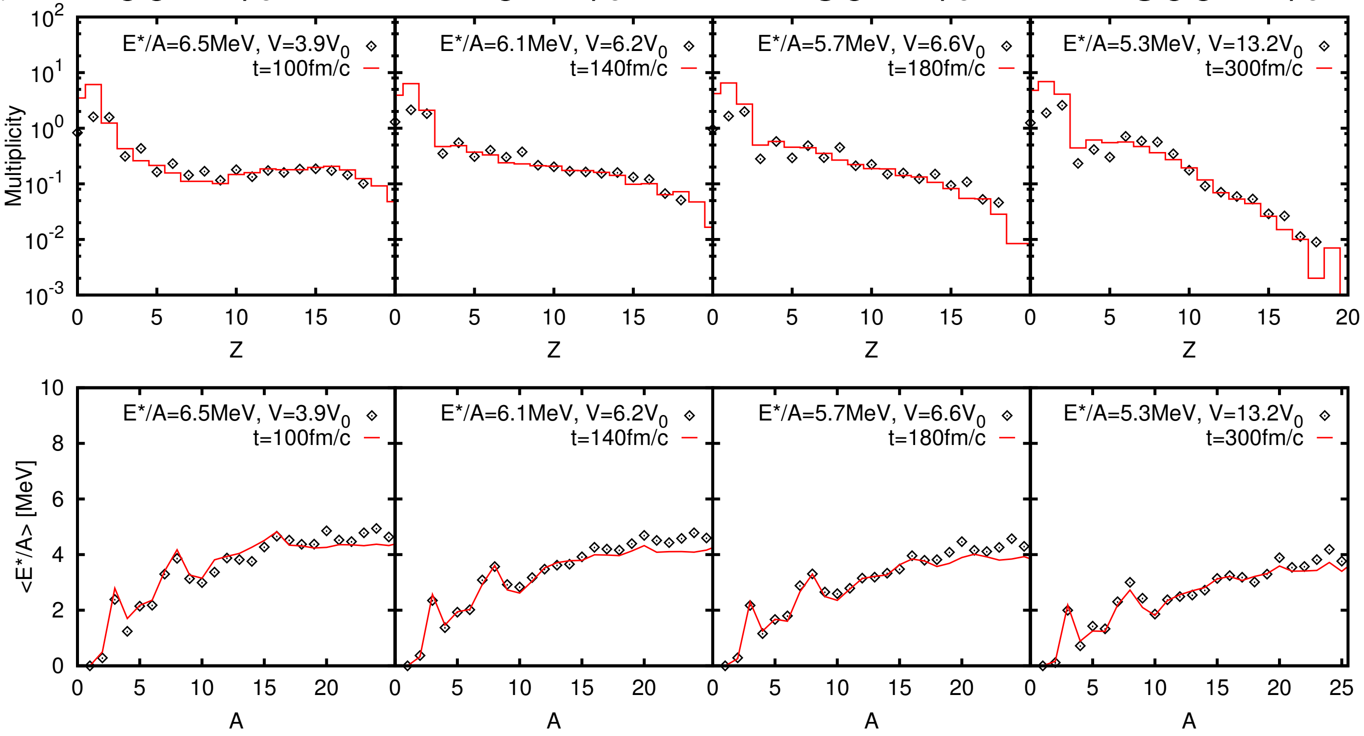



$\{$States at the reaction time $t\}$ $\;=\overset{?}{=}=\;$ Equilibrium ensemble$(E, V, A)$

$t$ = 100 fm/$c$ 140 fm/$c$ 180 fm/$c$ 300 fm/$c$

$M(Z)$

$\langle E^*/A \rangle$



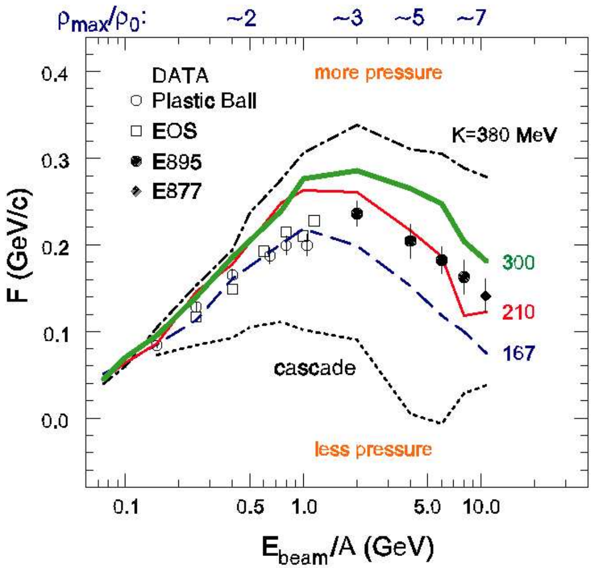
# Summary

状態方程式を測るために必要なこと

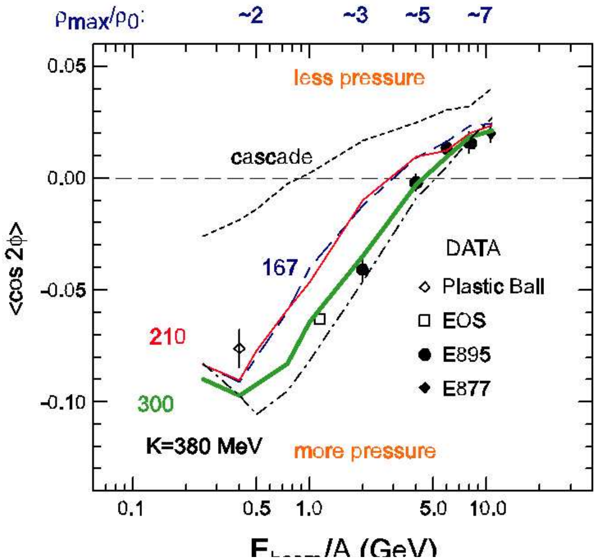
- 測りたい状況の物質を作ること
  — 核衝突では（ある程度）実現している.
  - 反応と熱平衡系を同一の枠組みで記述する.
    - 例： 液相気相相転移と多重破砕を AMD で比較.
- 実現した状況（密度，温度，圧力，組成）を実験の観測量と関係づけること
  — 理論研究が必要.
  - 十分正確に反応を記述する.
    - 反応初期はあまり問題ないと思う.
    - 反応後期（低密度）・フラグメント形成は，重要課題.
    - 例： AMD におけるクラスター相関
  - 既存の研究の多くは，計算の入力（有効核力・平均場など）と終状態との関係を調べていると思われる.

# High Density EOS and Flow

## Transverse Flow




## Eliptic Flow




Danielewicz et al.,

Science 298(2002)1592.

$$\tan\phi = p_x/p_y$$

$\langle\cos 2\phi\rangle$ ：膨張速度の指標

⇓

状態方程式