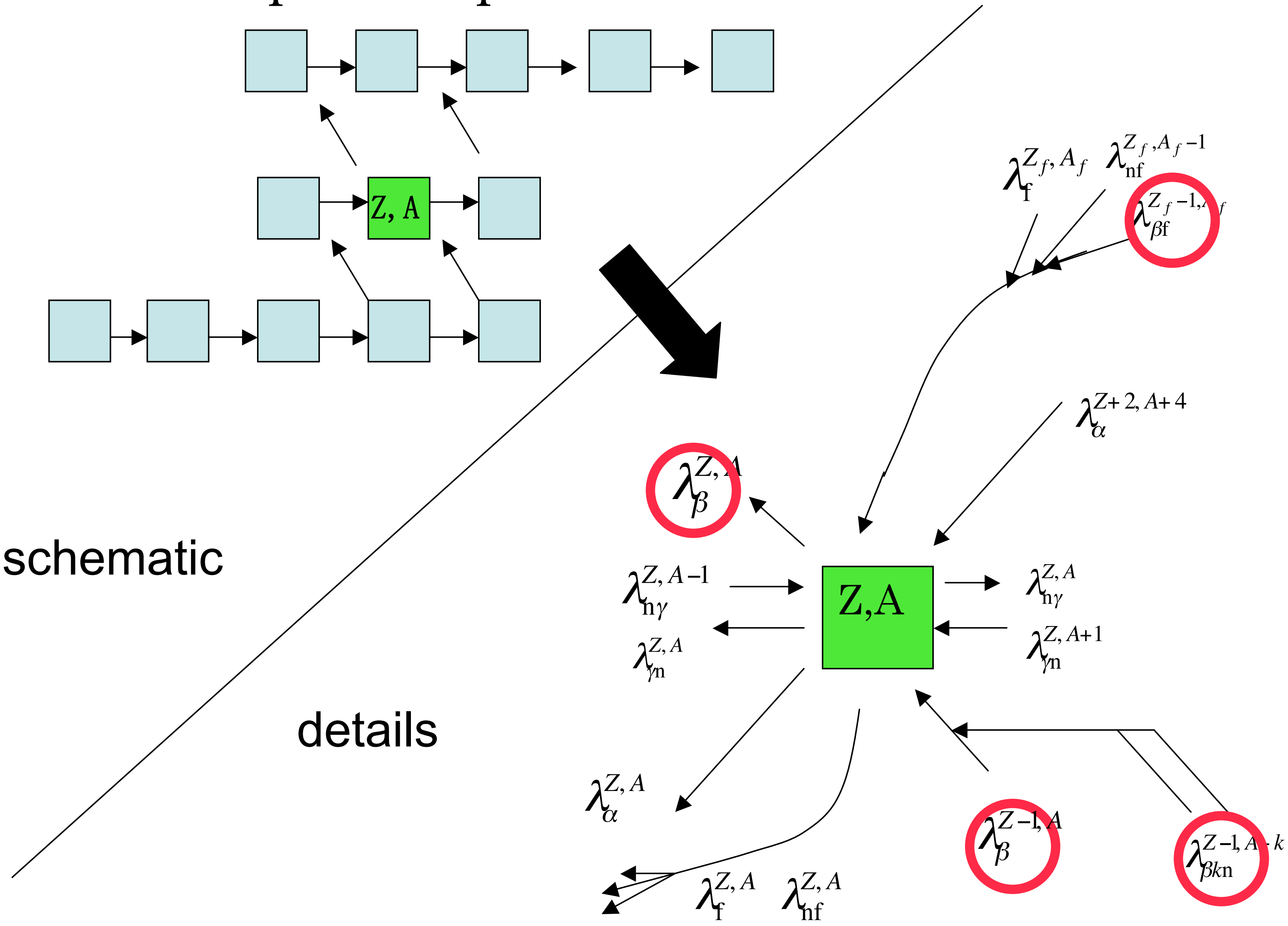
β-decay properties as input data for the calculation of r-process nucleosynthesis

T. Tachibana

r-process path



$$
\begin{aligned}
\frac{dN(Z,A)}{dt} = {} & N(Z,A-1)\lambda_{n\gamma}^{Z,A-1} + N(Z,A+1)\lambda_{\gamma n}^{Z,A+1} \\
& + N(Z-1,A)\lambda_{\beta}^{Z-1,A} + \sum_{k} N(Z-1,A+k)\lambda_{\beta k n}^{Z-1,A+k} \\
& + N(Z+2,A+4)\lambda_{\alpha}^{Z+2,A+4} \\
& - N(Z,A)[\lambda_{n\gamma}^{Z,A} + \lambda_{\gamma n}^{Z,A} + \lambda_{\beta}^{Z,A} + \lambda_{f}^{Z,A} + \lambda_{nf}^{Z,A} + \lambda_{\alpha}^{Z,A}] \\
& + \sum_{f} y_{Z,A}(Z_f,A_f)\lambda_{f}^{Z_f,A_f} N(Z_f,A_f) \\
& + \sum_{f} y_{Z,A}^{\beta}(Z_f,A_f)\lambda_{\beta f}^{Z_f-1,A_f} N(Z_f-1,A_f) \\
& + \sum_{f} y_{Z,A}^{n}(Z_f,A_f)\lambda_{nf}^{Z_f,A_f-1} N(Z_f,A_f-1)
\end{aligned}
$$

Necessary nuclear data for λs:

Mass formula (shell energy,
pairing energy,
Q-value、
fission barrier,
deformation parameter….)

β-decay model (half-life,
β-delayed neutron emission,
β-delayed fission ….)

Cross section

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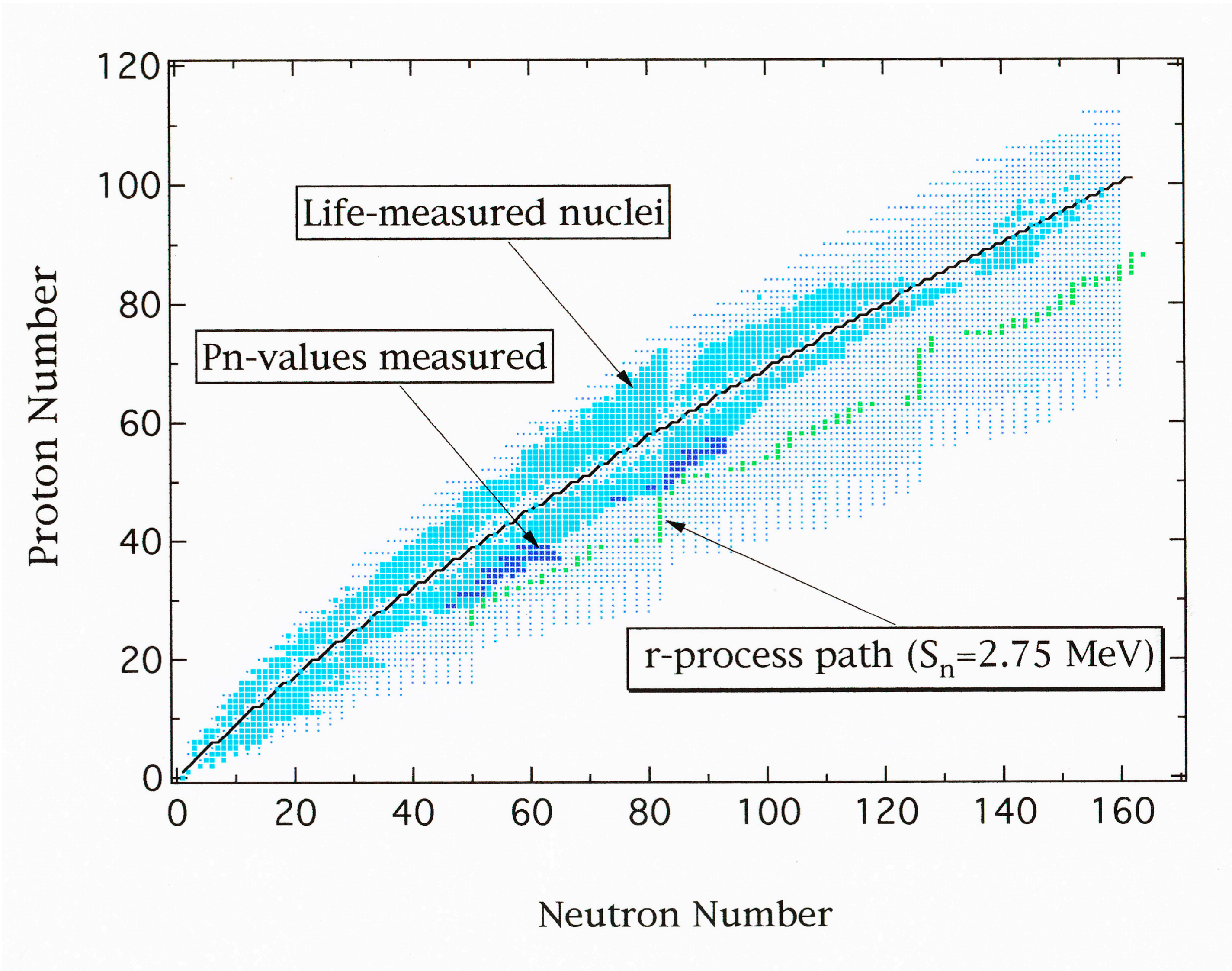
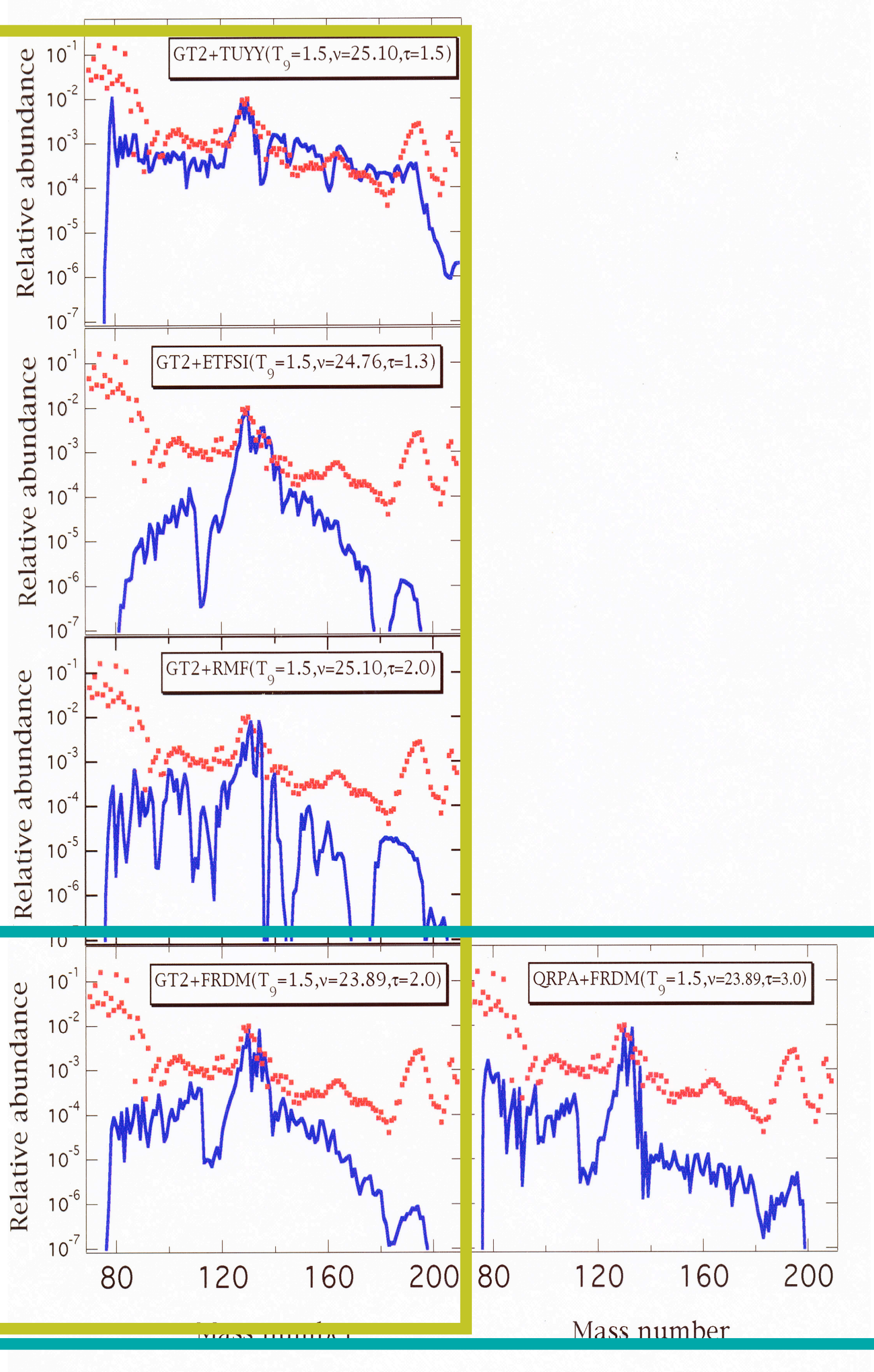
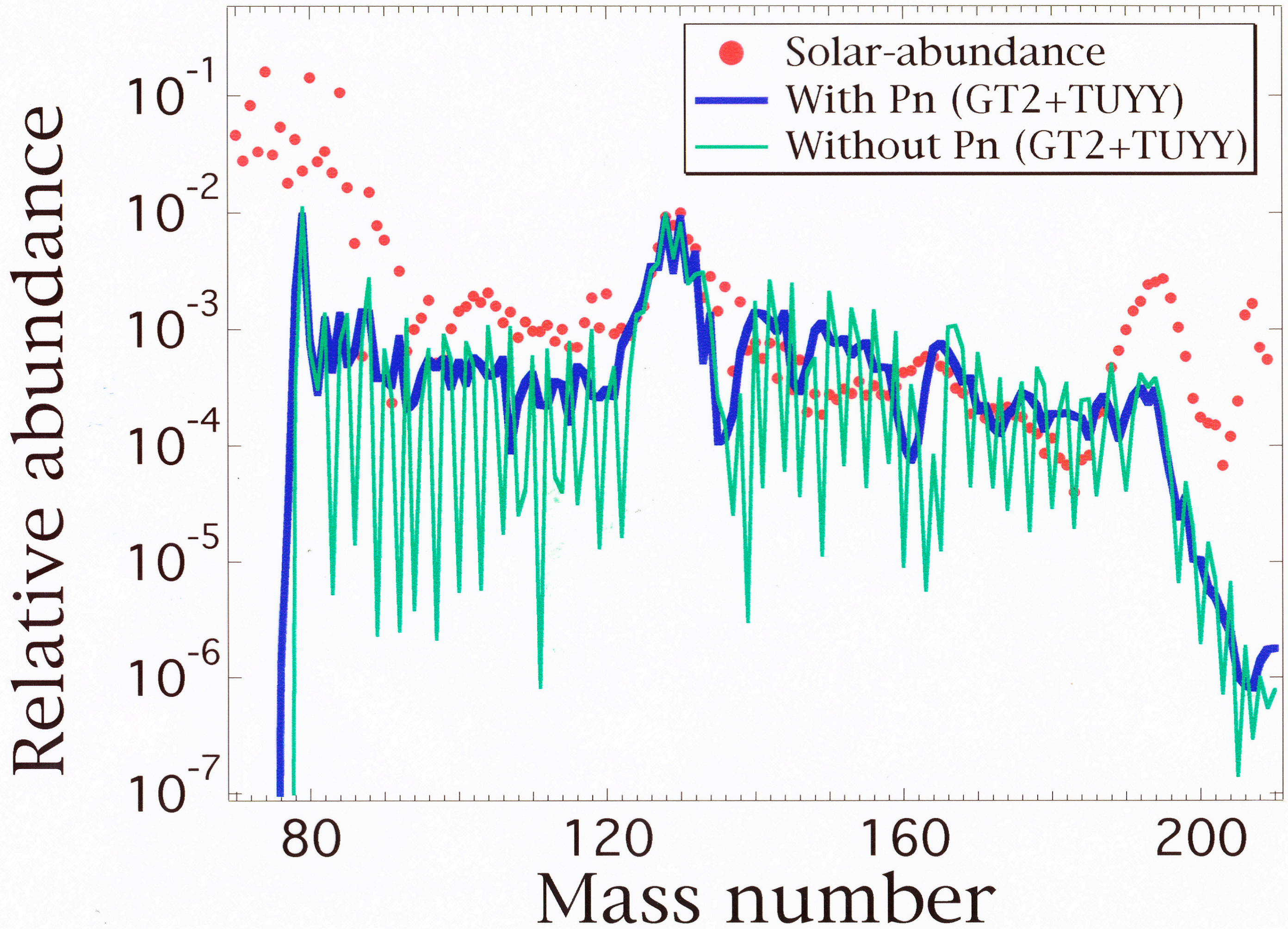
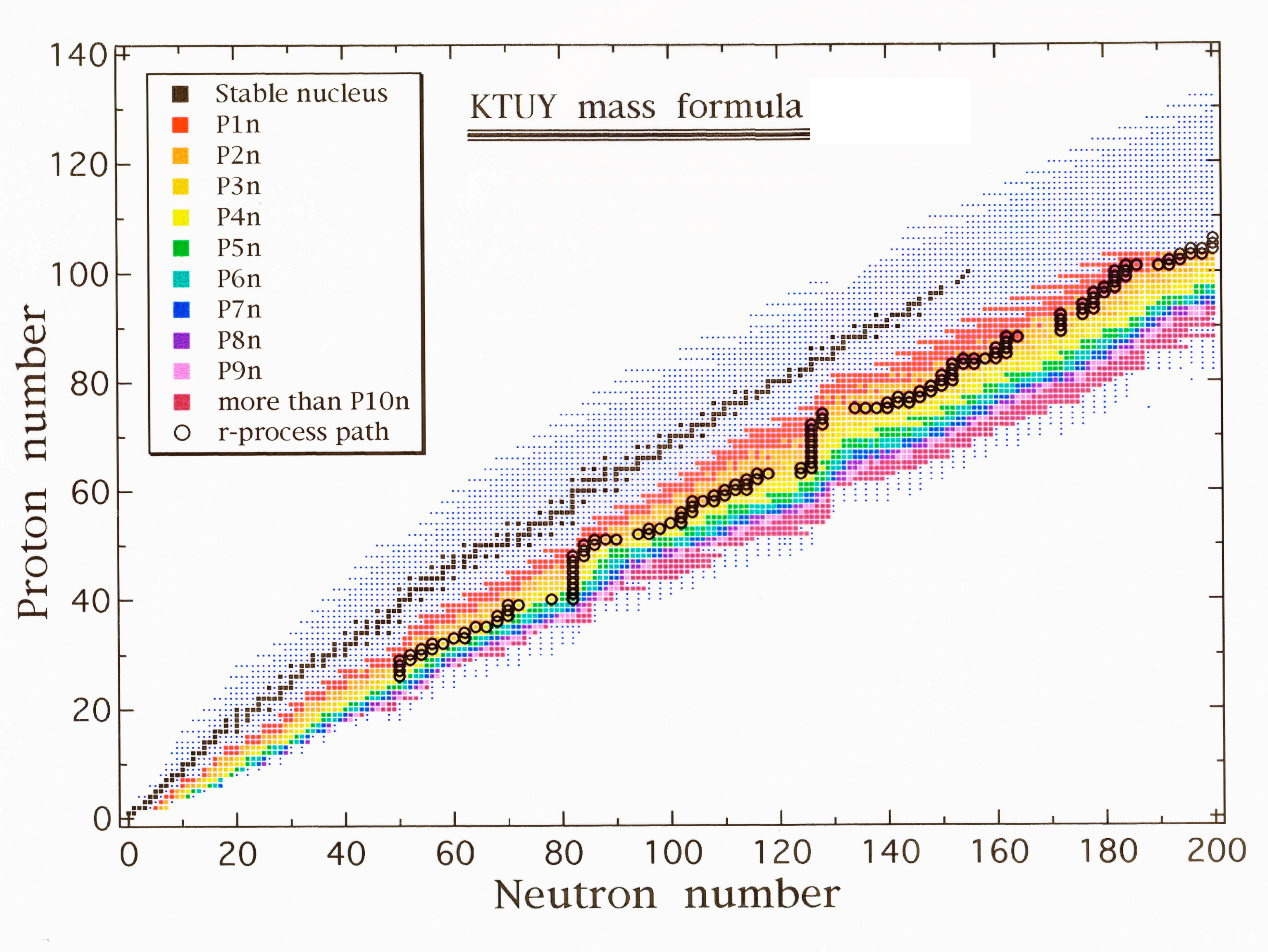


We need a beta-decay model



GT2+TUYY(T9=1.5,ν=25.10,τ=1.5)
GT2+ETFSI(T9=1.5,ν=24.76,τ=1.3)
GT2+RMF(T9=1.5,ν=25.10,τ=2.0)
GT2+FRDM(T9=1.5,ν=23.89,τ=2.0)
QRPA+FRDM(T9=1.5,ν=23.89,τ=3.0)
Relative abundance
Mass number


Solar-abundance
With Pn (GT2+TUYY)
Without Pn (GT2+TUYY)
Relative abundance
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
80
120
160
200
Mass number


KTUY mass formula
Stable nucleus
P1n
P2n
P3n
P4n
P5n
P6n
P7n
P8n
P9n
more than P10n
r-process path
Proton number
Neutron number
0
20
40
60
80
100
120
140
160
180
200

Estimation of β-strength function

(1) Gross theory [ver.1, ver.2, SGT]

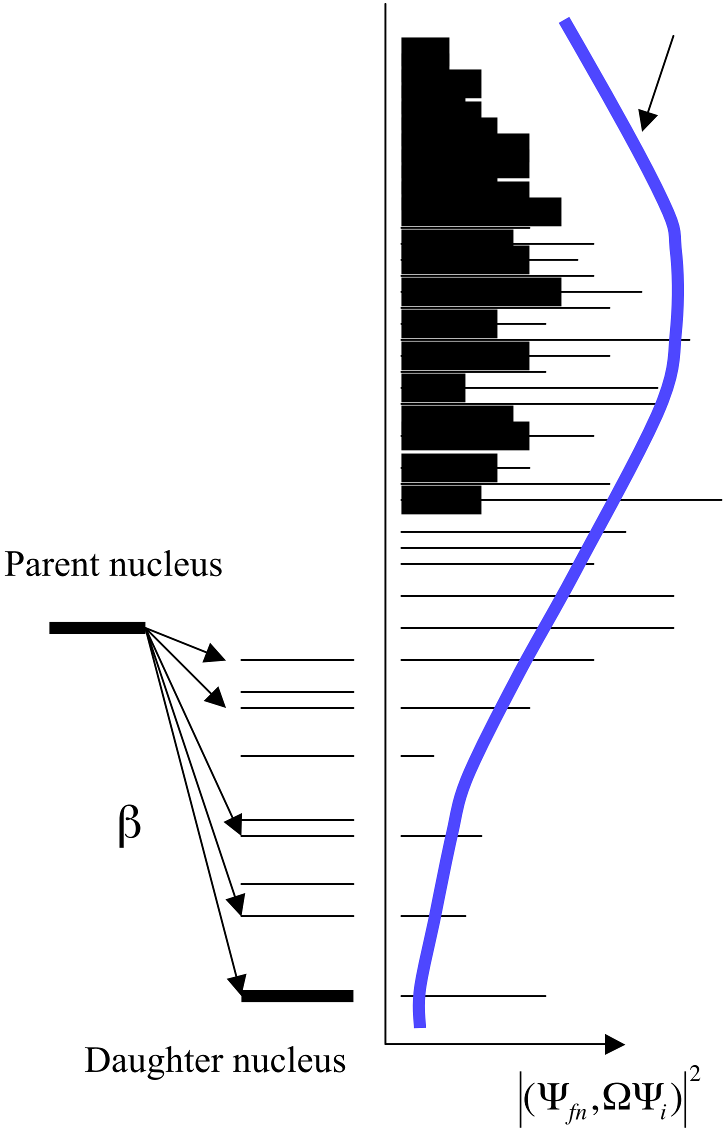
(2) QRPA

(3) Shell model

(4) Systematics

β – strength function

$$|\mathrm{M}_{\Omega}(E)|^2 = \overline{|(\Psi_f, \Omega\Psi_i)|}^2 \rho(E)$$



β – decay rate

The Fermi transition: $\lambda_{\mathrm{F}} \approx \dfrac{C}{2\pi^3}|g_V|^2 \int_{-Q}^{0} |M_{\mathrm{F}}(E)|^2 f(-E)dE$

The Gamow - Teller transition: $\lambda_{\mathrm{GT}} \approx \dfrac{C}{2\pi^3}|g_A|^2 3\int_{-Q}^{0} |M_{\mathrm{GT}}(E)|^2 f(-E)dE$

The first - forbidden transition (with rank = 0,1,2):

$$\lambda_1^{(0)} \approx \frac{C}{2\pi^3(\hbar/m_e c)^2}|g_A|^2 \int_{-Q}^{0} |M_{\sigma\mathrm{r}}(E)|^2 f_{1A}^{(0)}(-E)dE$$

$$\lambda_1^{(1)} \approx \frac{C}{2\pi^3(\hbar/m_e c)^2}[|g_V|^2 \int_{-Q}^{0} |M_{\mathrm{r}}(E)|^2 f_{1V}^{(1)}(-E)dE$$

$$+ |g_A|^2 \int_{-Q}^{0} |M_{\sigma\times\mathrm{r}}(E)|^2 f_{1A}^{(1)}(-E)dE$$

$$\lambda_1^{(2)} \approx \frac{C}{2\pi^3(\hbar/m_e c)^2}|g_A|^2 \int_{-Q}^{0} \sum_{ij} |M_{B_{ij}}(E)|^2 f_1(-E)dE$$

where $C = \dfrac{m_e c^4}{\hbar^7}$ (m_e is the electron mass), and $g_V \approx 1.4 \times 10^{-49}$ erg cm^3

and $g_A \approx -1.2 g_V$ are the coupling constants.

In the gross theory, the strength function $|M_\Omega(E)|^2$ is assumed to be given by

$$|M_\Omega(E)|^2 = \int_{\varepsilon_{min}}^{\varepsilon_{\max}} D_\Omega(E,\varepsilon)\; W(E,\varepsilon)\; \frac{dn_1}{d\varepsilon}\; d\varepsilon$$

One-particle strength function ($D_\Omega(E,\varepsilon)$)

Weight function to take into account the Pauli exclusion principle ($W(E,\varepsilon)$)

The neutron (proton) energy distribution ($\frac{dn_1}{d\varepsilon}$)

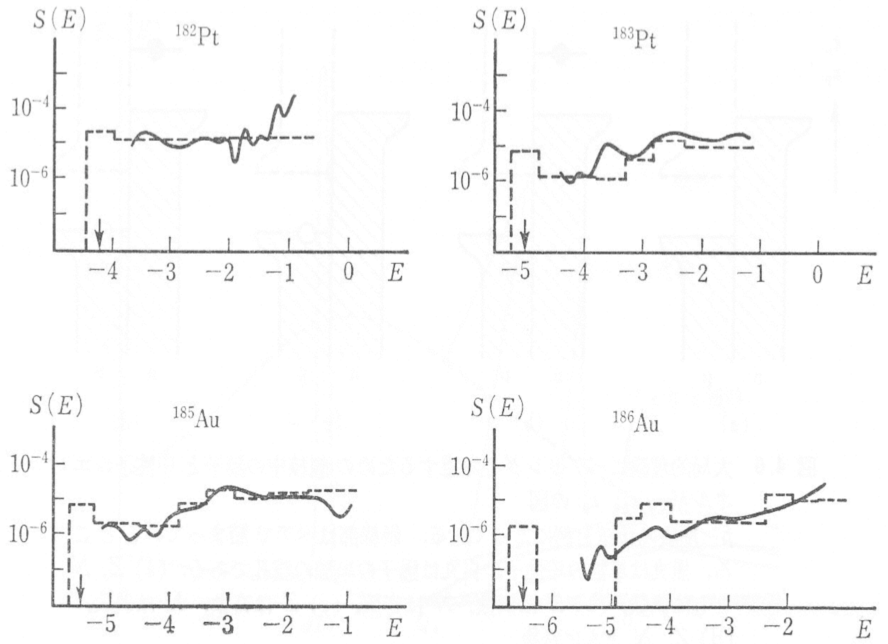
The distribution of the strength function could be obtained from the sum rules,

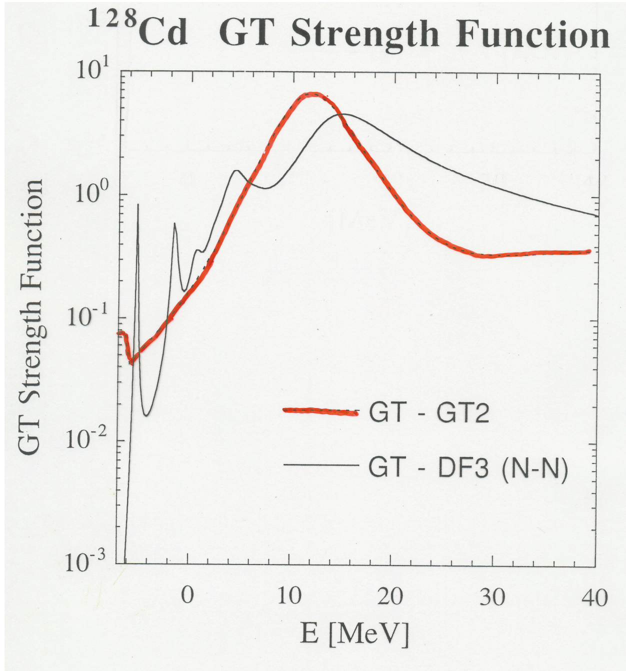
$$\int_{-\infty}^{\infty} |M_\Omega(E,\varepsilon)|^2 dE = (\Psi_0, \Omega^* \Omega \Psi_0), \qquad (1)$$

$$\int_{-\infty}^{\infty} |M_\Omega(E,\varepsilon)|^2 E dE = (\Psi_0, \Omega^* [H, \Omega] \Psi_0), \qquad (2)$$

$$\int_{-\infty}^{\infty} |M_\Omega(E,\varepsilon)|^2 E^2 dE = (\Psi_0, [\Omega^*, H][H, \Omega] \Psi_0) \qquad (3)$$

Examples of β-strength function





—— Experimental strength function

----- Calculated strength function by the gross theory

Gross theory

QRPA

β-decay half-life, delayed neutron emission probability and delayed fission probability are given by,

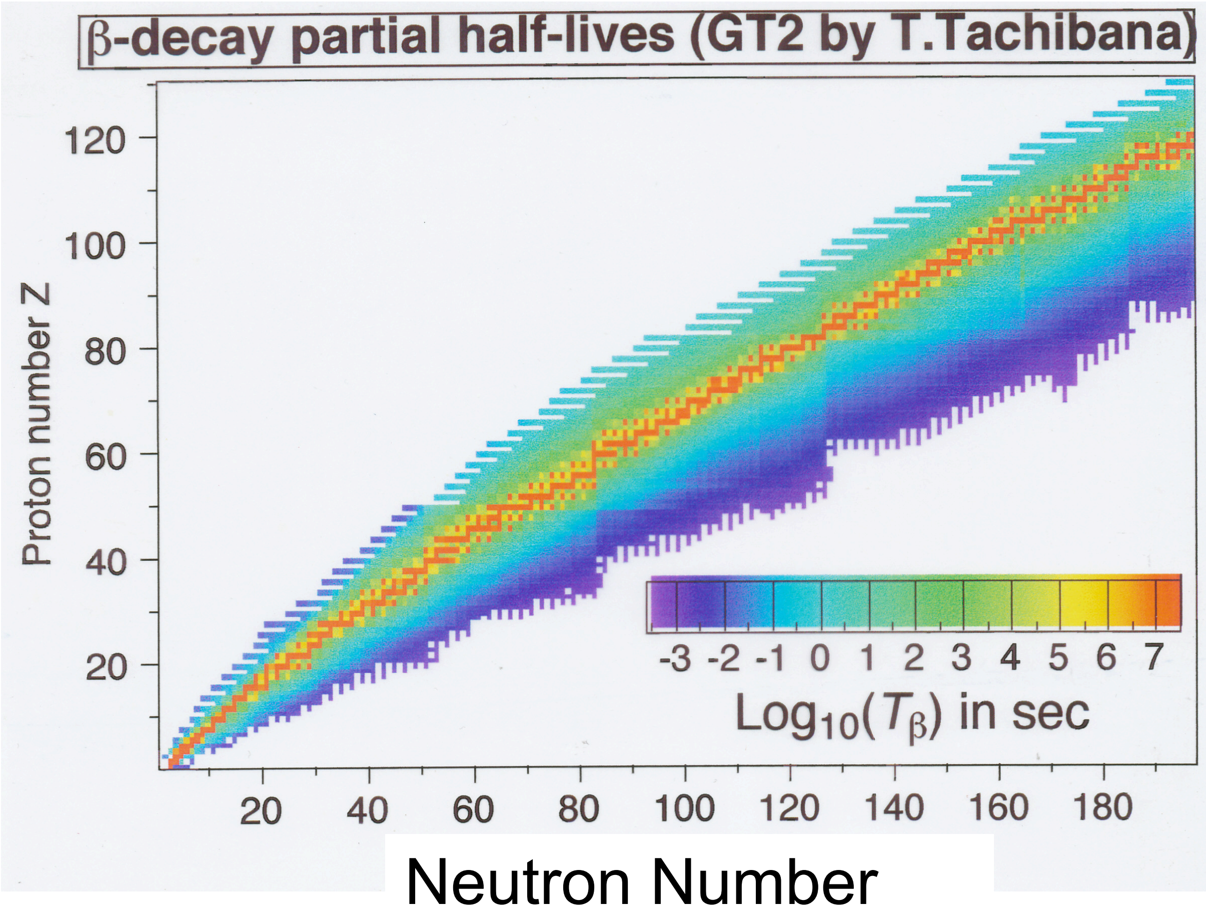
$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{C\int_{-Q}^{0} S_{\beta}(E) f(-E) dE},$$

$$P_{1\mathrm{n}} = \frac{C}{\lambda}\int_{-Q}^{0} S_{\beta}(E) f(-E) \frac{\Gamma_{\mathrm{n}}}{\Gamma_{\mathrm{n}} + \Gamma_{\mathrm{f}} + \Gamma_{\gamma}} dE,$$

$$P_{\mathrm{f}} = \frac{C}{\lambda}\int_{-Q}^{0} S_{\beta}(E) f(-E) \frac{\Gamma_{\mathrm{f}}}{\Gamma_{\mathrm{n}} + \Gamma_{\mathrm{f}} + \Gamma_{\gamma}} dE,$$

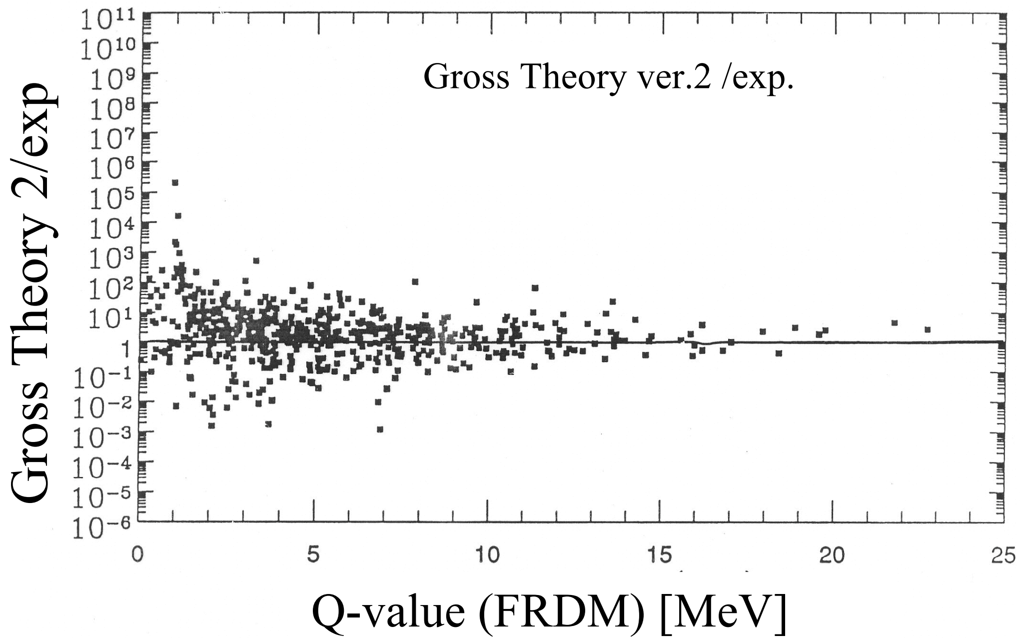
$S_{\beta}(E)$ is obtained from

$$\left|M_{\Omega}(E)\right|^{2} \quad (\Omega = \text{Fermi, Gamow – Teller, 1st forbidden})$$

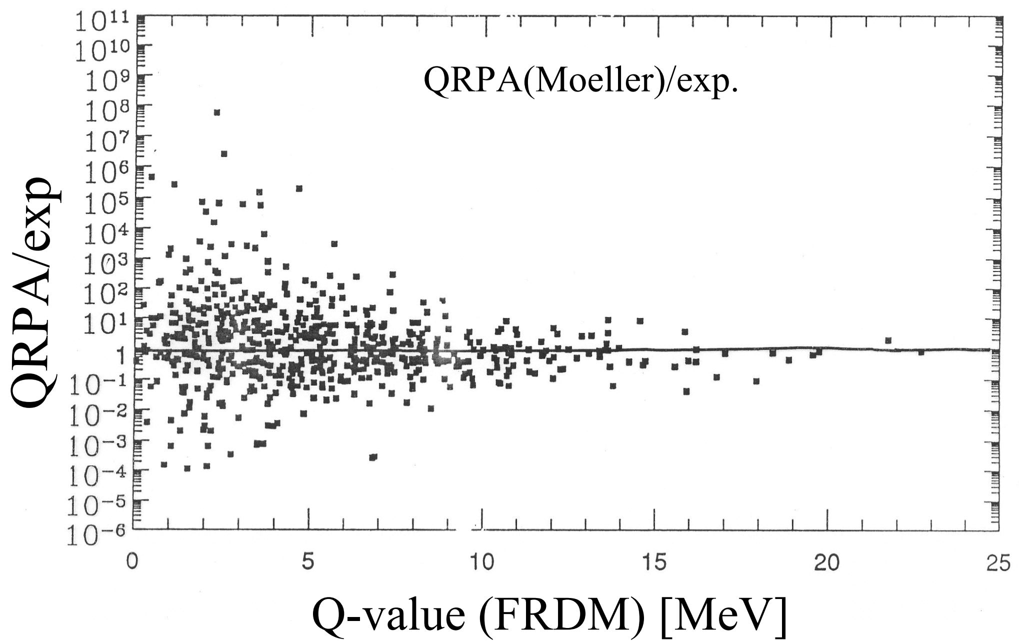

β-decay partial half-lives (GT2 by T.Tachibana)
Proton number Z
120
100
80
60
40
20
20
40
60
80
100
120
140
160
180
-3 -2 -1 0 1 2 3 4 5 6 7
Log10(Tβ) in sec
Neutron Number

Comparison between β-decay models

Transition	ΔJ^{π}	V	A	Gross theory	QRPA (Moeller)	QRPA (Klapdor)	QRPA (Goriely)
Fermi	0+	$\tau^{\pm}$		○	○	○	○
G-T	1+		$\sigma\tau^{\pm}$	○	○	○	○
1st forbidden	0-		$(\sigma \bullet r)\tau^{\pm},\ \gamma_5\tau^{\pm}$	○	△	×	○
	1-	$r\tau^{\pm},\ \alpha\tau^{\pm}$	$(\sigma \times r)\tau^{\pm}$	○	△	×	○
	2-		$B_{ij}\tau^{\pm}$	○	△	×	○



Gross Theory ver.2 /exp.
Gross Theory 2/exp
Q-value (FRDM) [MeV]



QRPA(Moeller)/exp.
QRPA/exp
Q-value (FRDM) [MeV]

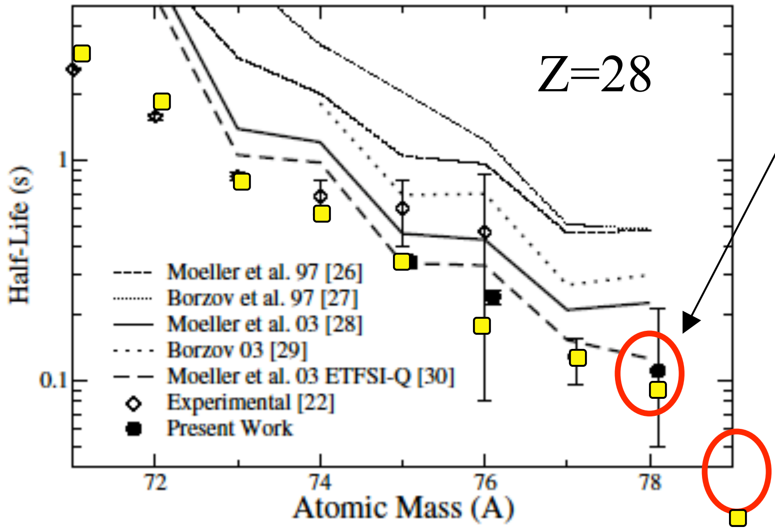
Half-lives of nuclei beyond ^{78}Ni at RIBF

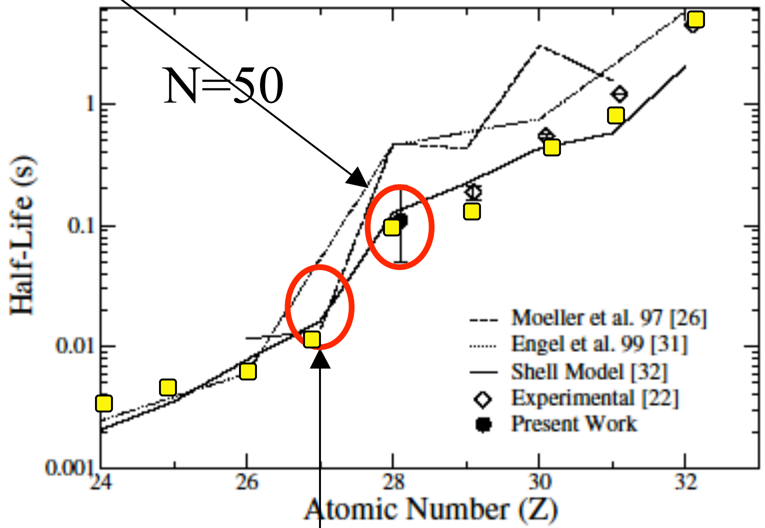
Magicity at Z=28 and N=50 ?

P.T.H, PRL 94, 112501(2005)
^{78}Ni ~10 particles/week at MSU

more accurate measurement for ^{78}Ni at RIBF

^{78}Ni ~20 particles/day at RIBF at present





^{79}Ni at RIBF

^{77}Co at RIBF

: Gross theory
with KTUY mass formula

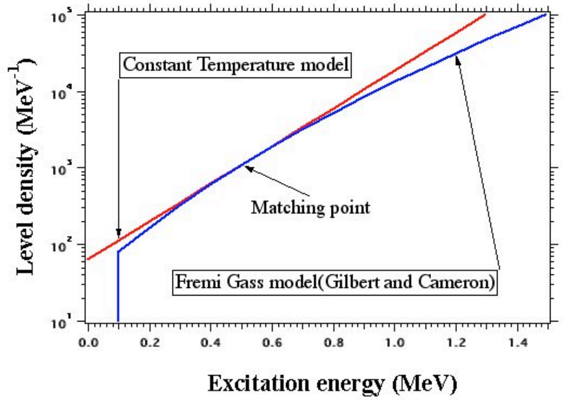
For the calculation of Γn and Γf

Nuclear level density

*Constant temperature model + collective effect
(for low energy region)

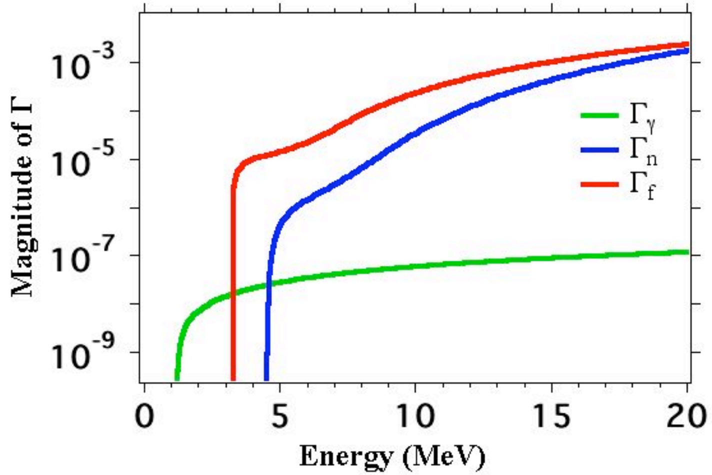
*Fermi gas model + shell effect + collective effect
(for high energy region)

Connection of two models

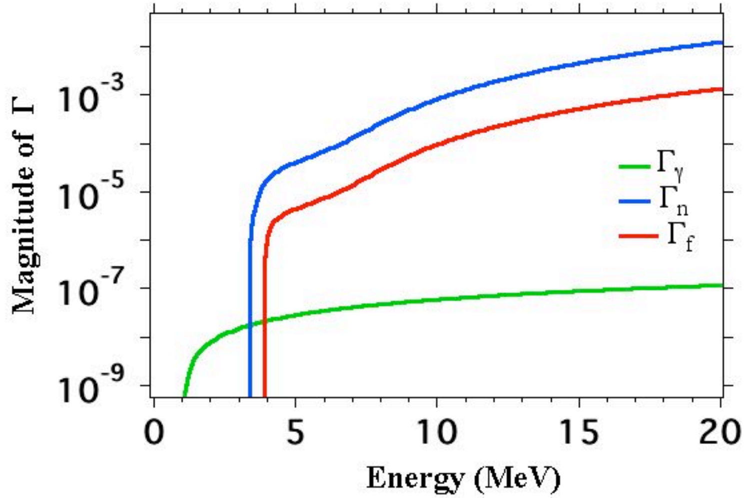


Examples of calculated Γn and Γf

$^{260}Pa_{169}$



$^{276}Np_{183}$



Evaluated P_f, P_n and P_γ values

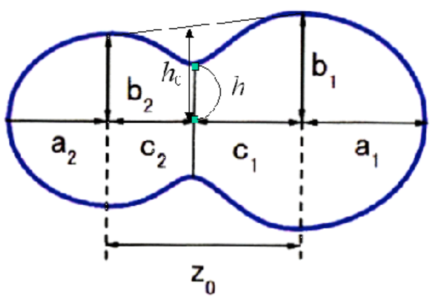
	260Pa			276Np		
	P_f	P_n	P_γ	P_f	P_n	P_γ
Thielemann et.al. (complete damping)	100%			25%		
Meyer et.al. (complete damping)	92%			83%		
Meyer et.al. (WKB barrier penetration)	97%	0%	3%	9%	84%	7%
Tachibana **[Preliminary]**	63%	2%	35%	0%	66%	34%

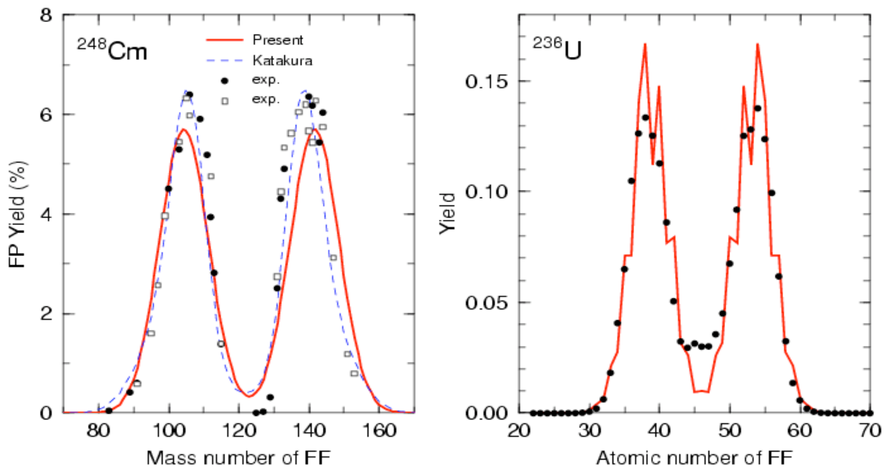
The fission data for the r-process calculation

We adopt the Fission Fragment Mass Distribution (FFMD) calculated by Ohta et. al.
They used two-center shell model and multi-dimensional Langevin calculation.

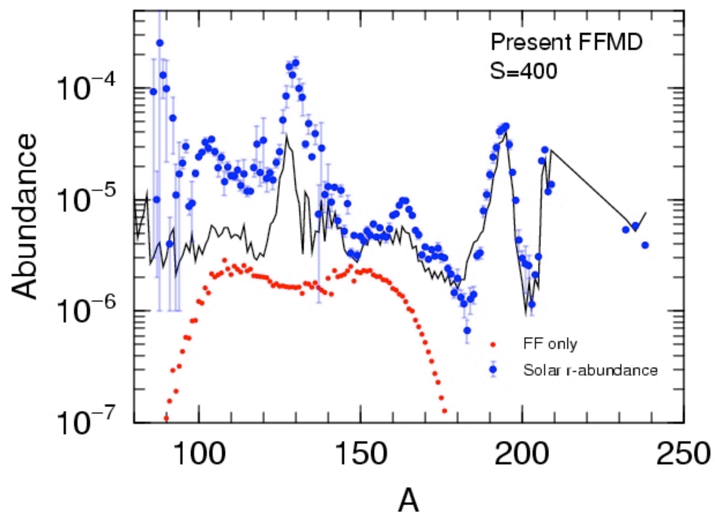
Liquid drop model + 2-center shell model

3-dimensional parameter space
- distance between fragments
- deformation of fragments
- mass asymmetry





The effect of the beta-delayed fission in the r-process calculation



Summary

- We showed how the β-decay model is important for the calculation of the r-process nucleosynthesis.
- We compared some β-decay values estimated by using different β-decay models.
- We calculated β-delayed fission probabilities with the use of the gross theory.